

A simple moisture transfer model for drying of sliced foods

Bülent Yesilata*, Mehmet Azmi Aktacir

Harran University, Mechanical Engineering Department, Osmanbey Campus, 63300 Sanliurfa, Turkey

ARTICLE INFO

Article history:

Received 1 January 2008

Accepted 30 March 2008

Available online 6 April 2008

Keywords:

Drying kinetics

Mathematical model

Moisture transfer

Slab

Effective diffusion coefficient

ABSTRACT

The mathematical formulation of mass transfer in drying processes is often based on the nonlinear unsteady diffusion equation. In general, numerical simulations are required to solve these equations. Very often, however, indirect and simplified methods neglecting fundamentals of the processes are used. In this work, a new mathematical model approach for the mass transfer occurring during drying of sliced foods is proposed. The model considers fundamentals of the drying process and takes internal resistance to moisture transfer into account. The parameters in the formulation have physical meaning and permit giving clear view of the moisture depletion process occurring during drying. The proposed model has an analytical solution and allows finding effective diffusion coefficient accurately. The verification of the model is made with basic drying experiments performed for chili red peppers sliced in slab form. The results reveal that there is nearly perfect match between the drying curves obtained by the model and the experiments.

© 2008 Elsevier Ltd. All rights reserved.

1. Introduction

Drying of industrial solid matters is of practical importance in engineering. New drying technologies along with better control and operational strategies have contributed to better quality dried products. Industrial drying processes have been fulfilled under the condition of controlling drying air in order to reduce crop losses, improve hygiene and quality of the dried products. This has caused a strong transition from traditional sun or shade drying to industrial driers despite of the fact that investment and operational costs of the latter are significantly higher. The other major issue to be solved for the drying industry is that there is yet no universally or even widely applicable drying theory. Most mathematical models of drying still remain specific to product-equipment for a variety of reasons [1]. There is therefore a great need for drying models to describe the drying process, help in its optimization, and assist in the effective design of dryers or improve existing drying systems [2,3].

In the literature, over 200 drying models (see [4,5], for excellent discussions on the subject) have been offered for various foods, which are formally characterized by two different, physical and empirical, approaches. Food science is experiencing a transition from empirically based approach for quantifying the dehydration of dry food particulates to physically based models that are mainly derived by assuming the food as a porous media [6–9]. For the time being it is not yet possible to judge the practical usefulness and reliability of these models that were mostly developed via com-

puter simulations. In general, there has been a need for more experimental validation and for establishing the usefulness of the approximate methods for fully modeling industrial drying processes [5,10,11]. The complete description and accurate prediction of the drying kinetics have not still been possible and different aspects of existing methods still need certain level of tuning due to the complex transport phenomena involved in food drying [12,13].

In this study, a new mathematical model for the mass transfer occurring during drying of sliced foods is proposed. The proposed model is validated with drying experiments performed for chili red pepper sliced in slab form. The drying kinetics of red peppers has been studied in the literature up to some extent. Most of previous works have examined mass transfer rate of red pepper slices using various empirical equations that are based on Fick's second diffusion law. These empirical equations were compared with their correlations obtained from basic curve-fitting approaches of experimental data. Either the Page model [14] or the modified Page model [2,15] produced the best fit for basic drying curves of various red peppers. These empirical equations on the other hand have not provided any physical information about internal moisture transfer mechanism.

2. Description of the proposed drying model

The schematic of the problem is given in Fig. 1. The solid body of the food that is sliced in strip-form is hypothetically divided into three different regions to explain logic of the model developed here. The initial ($t = 0$) moisture content of the solid matter is shown with w_{A0} . The stagnant hot air contacts with the surface at $z(t = 0) = 0$ when drying starts in the oven. A certain amount of

* Corresponding author. Tel.: +90 414 344 00 20x1094; fax: +90 414 344 00 31.
E-mail addresses: byesilata@harran.edu.tr, byesilata@yahoo.com (B. Yesilata).

Nomenclature

<p><i>A</i> the mass transfer area perpendicular to diffusion flux (cm²)</p> <p><i>D</i> diffusion coefficient (cm² min⁻¹)</p> <p><i>J_A</i> the rate of moisture depletion from the solid matter (g min⁻¹)</p> <p><i>k_m</i> the parabolic-rate constant in mass (g² cm⁻⁴ s⁻¹)</p> <p><i>k_z</i> the parabolic-rate constant in thickness (cm² s⁻¹)</p> <p><i>m</i> the mass (g)</p> <p><i>t</i> time (min)</p> <p><i>T</i> drying temperature (°C)</p> <p><i>v</i> the velocity of interface line (cm min⁻¹)</p> <p><i>w</i> the dimensionless moisture content</p> <p><i>z</i> the distance in 'z' direction (cm)</p> <p><i>Z</i> dimensionless similarity parameter in Eq. (9)</p>	<p><i>Greek</i></p> <p><i>φ_{lim}</i> dimensionless limitation factor in Eq. (6)</p> <p><i>ρ</i> density of sample (g cm⁻³)</p> <p><i>ψ</i> dimensionless parameter in Eq. (10)</p> <p><i>Δm</i> mass loss (g)</p> <p><i>Subscripts</i></p> <p><i>A</i> species A (moisture)</p> <p><i>B</i> species B (sliced solid food)</p> <p><i>D</i> dry</p> <p><i>eff</i> effective</p> <p><i>T</i> total</p> <p><i>0</i> initial</p> <p><i>S</i> equilibrium</p>
--	---

water is depleted from the region very next to the hot air (region I, the outer depletion region) instantaneously leading to much lower equilibrium concentration of *w_{AS}*. The original solid–gas interface, *z(t = 0) = 0*, is stationary because depletion of water content in the solid food should not cause complete layer movement. The interface between regions I and II moves with a velocity of *v(t)*, and its location is of interest to obtain a solution. Appropriate modeling of the problem thus requires a reference frame that moves with the hypothetical interface (i.e. *z(t > 0) = 0* always represents that interface). The amount of moisture transferred to the hot air can then be calculated since these parameters are directly related with the rate of moisture transfer from the solid matter.

The depleted moisture is assumed to be replaced with some type of vacancy (i.e. voids, pores) allowing water pass from region II. Further stage of drying continues with moisture transfer from the region II (inner depletion region) due to concentration gradient of (*w_{A0} - w_{AS}*) between its two interfaces. The initial moisture content in the very thin region III of the solid matter (*z → ∞*) thus remains always the same. The moisture is the only component that diffuses through the interface, the system can then be reduced from multi-diffusion to a binary diffusion problem (*A*: moisture and *B*: sliced food). The resulting diffusion equation for species 'A' is given as

$$\frac{\partial \rho_A}{\partial t} + (\nabla \rho_A v) = (\nabla \rho_{AB} \cdot \nabla w_A). \tag{1}$$

In the case of one dimensional mass transfer and constant material properties ($\rho = \rho_A + \rho_B$) along with the assumption of no-dependence of diffusion coefficient on position and composition ($D_{AB} = D_{BA} = D_{eff}$), the mass flux in a binary system can be described as

$$\frac{\partial \rho_A}{\partial t} + \frac{\partial}{\partial z}(\rho_A v) = \rho D_{AB} \frac{\partial^2 w_A}{\partial z^2}. \tag{2}$$

In Eq. (2), $D_{AB} = D_{eff}$ is the effective diffusion coefficient of species 'A' in 'B' (i.e. moisture in the sliced solid food). The second term repre-

sents diffusion of moisture due to the moving coordinate. Eq. (2) can be rewritten in simpler form by considering the relation of $v = v(t)$,

$$\frac{\partial w_A}{\partial t} + v(t) \frac{\partial w_A}{\partial z} = D_{AB} \frac{\partial^2 w_A}{\partial z^2}, \tag{3}$$

where *v(t)* denotes the velocity of the active coordinate system, which is directly related to the moisture transfer from the solid matter. A parabolic-rate constant in mass is defined below to find appropriate relationship between the mass loss and the velocity as

$$k_m = \frac{1}{t} \left(\frac{\Delta m(t)}{A} \right)^2, \tag{4}$$

where *t*, $\Delta m(t)$, and *A* are, respectively, drying time, mass loss, and area perpendicular to diffusion flux. Eq. (4), at a specified constant temperature, refers to a constant value of *k_m* since Δm is considered to be proportional to $t^{1/2}$, and the value of *k_m* in practice only varies with temperature [16]. The instantaneous mass variation of the solid matter, which is always measured in basic drying experiments, is described as,

$$\Delta m(t) = \rho_A A \phi_{lim} z(t) (w_{A0} - w_{AS}). \tag{5}$$

It is important to notice that there is a proportional relationship between *z(t)* and *v(t)*; such as $z(t) \propto \int v(t) dt$. The introduction of a dimensionless limitation factor (ϕ_{lim}) is necessary when one of the species is depleted from a solid body since mass transfer path has complex obstructions. The idea behind this definition is similar to Pilling–Bedworth ratio used in metal depletion from alloys [17] but the prediction of ϕ_{lim} is somewhat easier here. Considering maximum allowable location of *z(t)* at the end of the drying process is one of the best way to satisfy with physical mechanism since *z(t)* must never exceed the sample thickness. The instantaneous location of the interface can then accordingly be defined as a function of parabolic-rate constant in thickness (*k_z*),

$$\phi_{lim} z(t) = \sqrt{2k_z t}. \tag{6}$$

Such a definition is very beneficial to find appropriate relationship between parabolic-rate constants in mass and in thickness as given below,

$$k_z = \frac{k_m}{2[\phi_{lim} \rho_A (w_{A0} - w_S)]^2}. \tag{7}$$

The interface velocity *v(t)* becomes then equal to,

$$v(t) = \frac{dz(t)}{dt} = \sqrt{\frac{k_z}{\phi_{lim} 2t}}. \tag{8}$$

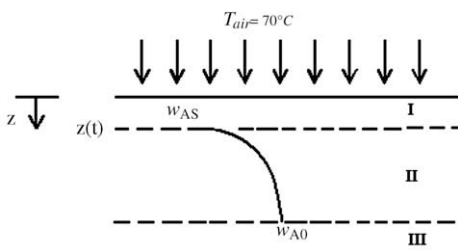


Fig. 1. The schematic of the moisture depletion model in the sliced food of slab form.

The following dimensionless parameters can be defined to solve Eq. (3),

$$w(z) = \frac{w_{A0} - w_A(z)}{w_{A0} - w_{AS}}; \quad Z(z, t) = \frac{z}{\sqrt{4D_{AB}t}} \quad (9)$$

with the boundary conditions of $w(Z=0) = 1$, $w(Z \rightarrow \infty) = 0$. The solution given below is well known for long years, after Arnold [18],

$$w(z) = \frac{1 - \operatorname{erf}(Z - \Psi)}{1 + \operatorname{erfc}(\Psi)} \quad (10)$$

where a dimensionless parameter of Ψ ,

$$\Psi = \left(\frac{k_z}{2D_{AB}} \right)^{1/2} \quad (11)$$

is used. The mass concentration profile determined in Eq. (9) can be used to calculate the rate of water depletion from the solid matter as follows:

$$J_A|_{z=0} = -\rho_A D_{AB} \left. \frac{\partial w_A}{\partial z} \right|_{z=0} \quad (12)$$

$$J_A = \rho_A A \exp(-\Psi^2) \frac{(w_{A0} - w_S)}{1 + \operatorname{erfc}(\Psi)} \sqrt{\frac{D_{AB}}{\pi t}} \quad (13)$$

The derivative is taken at $z=0$ because the reference frame is attached to the moving interface. In order to determine the location of the reference frame at a particular time, the velocity given by Eq. (8) is used. The rate of moisture depletion can now be used to predict mass loss of the specimen due to drying of the food. Due to the nature of basic drying experiments, the only unknown parameter in the Eq. (13) is the effective diffusion coefficient of D_{AB} . The main object of the approach presented here is to determine D_{AB} by using experimental mass transfer curve. For this purpose, the equation can be arranged to give the instantaneous mass loss,

$$\Delta m(t) = m_A(t) = J_A t. \quad (14)$$

The last equation is now in more suitable form to compare with the experimental curve since the mass variation of the food is usually measured in experiments. The model can allow estimation of D_{AB} by seeking the best match between the experimental curve and the curves calculated by the following equation:

$$m_T(t) = m_{T0} - m_A(t). \quad (15)$$

3. Experimental verification of the model

3.1. Experimentation

The experimental set-up used here consists of a drying oven with thermostat and digital indicator, and a highly accurate digital scale with sensitivity of 0.01 g. The mass transfer area and thickness of red pepper samples used in experiments are respectively $A=10 \text{ cm}^2$ ($10 \pm 0.01 \text{ cm} \times 1 \pm 0.01 \text{ cm}$) and $\delta = 0.21 \pm 0.01 \text{ cm}$. The samples were sliced in strip-form so that one dimension mass transfer model can be used. The three samples with identical dimensions were tested at the same conditions and the maximum deviations in time dependent mass variations remained within 2%. The arithmetic mean mass value of three samples at each specified time is taken into consideration for applying the mathematical model. The drying temperature of $70 \pm 1 \text{ }^\circ\text{C}$ was applied. The initial moisture contents of the samples were simply determined by considering the difference in fresh (wet) weight and dry weight that was determined after applying relatively high drying temperature of $T=110 \text{ }^\circ\text{C}$. During measurements of dry weights, the samples were kept in the oven until the difference between two successive mass measurements is less than 0.02 g.

3.2. Results

The variation of instantaneous masses for the three samples and the corresponding variations in mean moisture content with the drying time are shown in Fig. 2. The drying curve by using arithmetic mean values is indicated with solid line in Fig. 2a. The drying behaviors of all three samples are similar and the maximum deviation from the mean never exceeds 2%. The dimensionless moisture content of the red pepper (w_A) given in Fig. 2b is based on its dry weight and calculated with the following equation:

$$w_A(t) = \frac{m_A(t)}{m_T(t=0)} = \frac{m_T(t) - m_D}{m_{T0}}, \quad (16)$$

where m_T and m_D are respectively total (or wet) and dry weights of the sample. The initial weight of the sample is denoted by m_{T0} .

The application of the model for predicting effective mass diffusion coefficient of D_{AB} becomes now relatively easier since the instantaneous values of moisture (m_A) transferred to the hot air are known. The experimental values of m_A as a function of drying time are shown in Fig. 3a with filled symbols. The solid lines are obtained by using Eq. (14) along with Eq. (13). Eq. (14) enables us to determine D_{AB} by seeking the best match between the experimental and theoretical curves. The best match between these two curves is obtained for $D_{AB} = 0.0087 \text{ cm}^2/\text{min}$ ($1.45 \times 10^{-8} \text{ m}^2/\text{s}$). Eq. (14) can be considered to be very useful since the experimental determination of D_{AB} otherwise is quite difficult. The other two curves with mismatching values of D_{AB} are also included in the figure to demonstrate the usefulness of the method and the significant effect of D_{AB} on the drying rate.

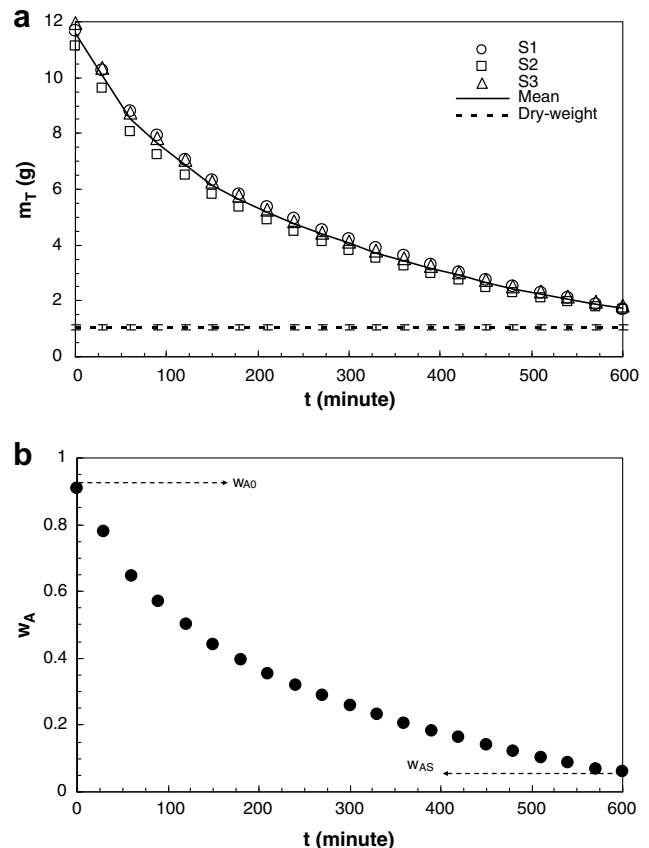


Fig. 2. (a) The variation of the sample mass with drying time (the symbols denote experimental data for samples 1, 2, and 3, and the solid line represents their mean values). The dashed line is for the mean dry mass, (b) the variation of the mean relative humidity with drying time.

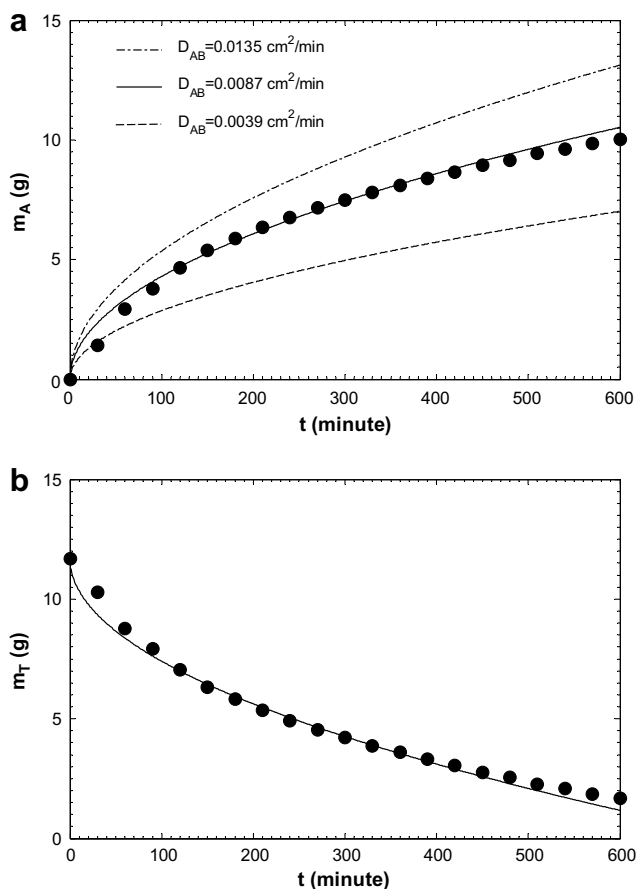


Fig. 3. (a) Variation of depleted moisture mass with drying time for various values of D_{AB} (the lines show calculations with the model and symbols show experimental data). The bold solid line represents the best matching value of D_{AB} , (b) mass variation of red pepper with drying time (the solid line show calculations with the best matching value of D_{AB} and the symbols show experimental data).

The comparison between the mass variations of the red pepper obtained from experimental measurements and the model with the predicted D_{AB} value of $0.0087 \text{ cm}^2/\text{min}$ is shown in Fig. 3b. The agreement of the two curves is nearly perfect with the exception of initial and final stages of drying. The deviations occurred in these regions can be expected due to the using constant mass diffusion coefficient during the whole process. It is well known that the initial mass transfer rate is higher due to the perfect contact of the outer surface with the hot air. The drying rate is conversely lower when approaching to the final stage due to shrinking of the mass transfer area. However, as can be seen from the figure, both initial and final stages last relatively for very short time and the deviations from experimental curve are also at acceptable level. The analytical model presented here can thus be considered as quiet beneficial for industrial drying of the red pepper.

The variation of instantaneous interface velocity calculated with the model, $v(t)$, is shown in Fig. 4a to demonstrate physical nature of the problem. The initial velocities are significantly higher and decrease then sharply. The model appears to overpredict the initial velocities, which may be one of the causes for that higher mass transfer rates than the real case are obtained in Fig. 3b. The decreasing rate of velocity becomes then much smoother until the end of drying process. These intermediate times confirm the parabolic-rate approach that result in nearly perfect agreement with experimental data during major part of drying.

The instantaneous locations of the hypothetical interface line are illustrated in Fig. 4b. The initial location is started from the ori-

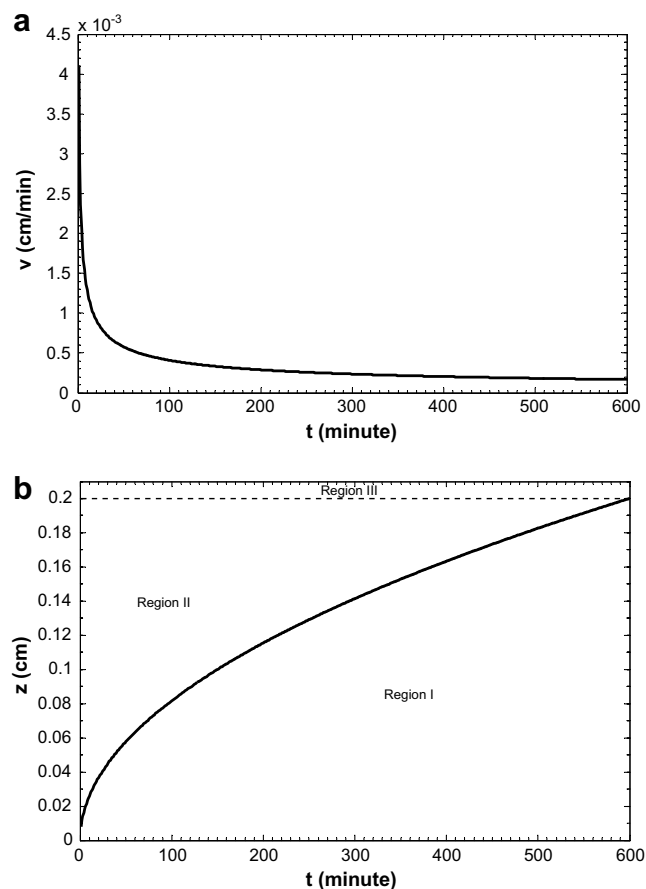


Fig. 4. (a) Variation of the interface velocity with drying time, and (b) variation of interface location with drying time.

ginal food surface ($z = 0$) and it moves with parabolic-rate. The final location is adjusted by ϕ_{lim} such that it does not pass through very thin outer region of 0.1 mm in thickness (region III or the bright shell of the red pepper).

Although the primary objective in this paper is to introduce major logic and utilization procedure of the proposed model, rather than a parametrical investigation, the model validation was verified at another air temperature ($T_{air} = 60 \text{ }^\circ\text{C}$) as well. The physical behaviors of the drying curves given in Fig. 3 remained nearly the same at this temperature, as expected. Only the values were shifted due to that predicted value of the effective diffusion coefficient was decreased to $0.0056 \text{ cm}^2/\text{min}$ ($0.93 \times 10^{-8} \text{ m}^2/\text{s}$). The effect of air temperature on the drying kinetics is indeed a well established matter, regardless of the nature and complexity of any reasonable model considered. It is well known that the air temperature proportionally affects the diffusion coefficient [2,5]. The characteristic drying curves of the same material at various drying temperatures collapse into a single curve by using an appropriate temperature shift factor. This factor is usually described by the Arrhenius law, where the logarithm of the diffusivity perfectly exhibits a linear behavior against the reciprocal of the absolute temperature [4]. The decrease in the effective diffusion coefficient with decreasing air temperature (from $70 \text{ }^\circ\text{C}$ to $60 \text{ }^\circ\text{C}$) obtained here is thus in accord with the Arrhenius law.

4. Conclusion

A simple unsteady and one dimensional diffusion model based on moisture depletion from solid body in slab form is introduced.

The model is validated with isothermal drying experiments performed for sliced chili red pepper. It is shown that by using this model, the effective diffusion coefficient of the depleted moisture in the solid food can accurately be predicted. This accomplishment allows calculating the instantaneous mass variation of the solid food during the drying process.

In contrast to most semi-theoretical models offered in the literature, which simplify general series solution of the Fick's second law, the proposed model considers the fundamentals of drying process for slab-shaped products. The parameters in the formulation have physical meaning and allow giving clear view of the moisture depletion process occurring during drying. The observations made so far indicate that the model could be applicable for wide ranges of products and external air conditions providing that significant shrinking do not occur.

References

- [1] A. Mujumdar, An overview of innovation in industrial drying: current status and R&D needs, in: *Drying of Porous Materials* by Stefan Kowalski, Springer, Netherlands, 2007, pp. 3–18.
- [2] A. Vega, P. Fito, A. Andres, R. Lemus, Mathematical modeling of hot-air drying kinetics of red bell pepper (var. Lamuyo), *Journal of Food Engineering* 79 (2007) 1460–1466.
- [3] S. Kooli, A. Fadhel, A. Farhat, A. Belghith, Drying of red pepper in open sun and greenhouse conditions: mathematical modeling and experimental validation, *Journal of Food Engineering* 79 (2007) 1094–1103.
- [4] J. Crank, *The Mathematics of Diffusion*, second ed., Oxford University Press, New York, NY, 1975.
- [5] W.J. Coumans, Models for drying kinetics based on drying curves of slabs, *Chemical Engineering and Processing* 39 (2000) 53–68.
- [6] J.R. Philip, D.A. de Vries, Moisture transport in porous materials under temperature gradients, *Transactions-American Geophysical Union* 38 (1957) 222–232.
- [7] K.M. Waananen, J.B. Litchfield, M.R. Okos, Classification of drying models for porous solids, *Drying Technology* 11 (1993) 1–40.
- [8] K.A. Landman, L. Pel, E.F. Kaasschieter, Analytic modelling of drying of porous materials, *Mathematical Engineering in Industry* 8 (2001) 89–122.
- [9] I.S. Saguy, A. Marabi, R. Wallach, New approach to model rehydration of dry food particulates utilizing principles of liquid transport in porous media, *Trends in Food Science and Technology* 16 (2005) 495–506.
- [10] I. Ceylan, M. Aktaş, H. Doğan, Mathematical modeling of drying characteristics of tropical fruits, *Applied Thermal Engineering* 25 (2007) 1931–1936.
- [11] A. Can, An analytical method for determining the temperature dependent moisture diffusivities of pumpkin seeds during drying process, *Applied Thermal Engineering* 27 (2007) 682–687.
- [12] M. Aversa, S. Curcio, V. Calabro, G. Iorio, An analysis of the transport phenomena occurring during food drying process, *Journal of Food Engineering* 78 (2007) 922–932.
- [13] M. Batista Lucia, A. da Rosa Cezar, A.A. Pinto Luiz, Diffusive model with variable effective diffusivity considering shrinkage in thin layer drying of chitosan, *Journal of Food Engineering* 81 (2007) 127–132.
- [14] I. Doymaz, M. Pala, Hot-air drying characteristics of red pepper, *Journal of Food Engineering* 55 (2002) 331–335.
- [15] E.K. Akpınar, Y. Bicer, C. Yildiz, Thin layer drying of red pepper, *Journal of Food Engineering* 59 (2003) 99–104.
- [16] B. Yesilata, Analytical modeling of unsteady aluminum depletion in thermal barrier coatings, *Turkish Journal of Engineering and Environmental Science* 25 (2001) 675–680.
- [17] N. Birks, G.H. Meier, *Introduction to High Temperature Oxidation of Metals*, Edward Arnold (Publishers) Ltd., London, 1983.
- [18] J.H. Arnold, Studies in diffusion: III unsteady-state vaporization and absorption, *Transactions A.I.Ch.E.* 40 (1944) 361–378.