

## MATLAB USER GUIDE

1- Start the MATLAB program.  
You will get a window like



2- You can make simple calculations as follows

```
>>a=5; b=6; c=a*b; d=a/b; e=sin(a);
```

```
>>a=5, b=6, c=a*b,
```

3- Complicated calculations

```
>> a=5; b=6; c=7;  
>> y=2*a+3*b+log(c)
```

$$y=2a+3b+\ln(c)$$

```
>>z=a^2+b^3+exp(c)
```

$$z=a^2+b^3+e^c$$

## COMPLEX NUMBERS

Complex numbers are treated similar to real numbers.

```
>>a=1+2j,  
>>e=abs(a), d=angle(a), f=phase(a)  
e=2.236067, d=1.107, f=1.107  
abs : magnitude  
angle: angle  
phase: angle (pahse and angle are same)
```

In the above example

```
>>a=1+2j, d=abs(a)
```

$$\sqrt{1^2 + 2^2} = \sqrt{5} = 2.236067$$

```
>>e=angle(a)
```

$$\text{angle}(a)=\tan^{-1}(2/1)=1.107$$

Of course

$$d*\cos(e)=2.236067*\cos(1.10714)=1$$

and

$$d*\sin(e)=2.236067*\sin(1.10714)=2$$

Matlab uses **radian** in angle calculations.

If you want to convert to **degree**

$$\begin{aligned} >>g=e*180/\pi = 1.107 *180 /3.14 = 63.5 \\ g=63.5^\circ \end{aligned}$$

**Complex Number Addition Sustraction , Multiplication, Division**

```
>> a=1+3*j, b=3+4*j,
```

```
>> c= a+b,  
4+7j
```

```
>> d = a*b,  
-9+13j,
```

```
>> e=a/b  
0.6+0.2j
```

**Real and Imaginary parts**

```
>> g= -12 + 15j  
>>real(g)  
-12  
imag(g)  
15
```

**Example Problem:** Calculate the magnitude and phase of the following numbers.

a)3+4j, b)-3+4j, c)3-4j d)-3-4j

**Answer :**

```
>>abs(3+4j), angle(3+4j)*180/pi
```

```
>>abs(-3+4j), angle(-3+4j)*180/pi
```

```
>>abs(3-4j), angle(3-4j)*180/pi
```

```
>>abs(-3-4j), angle(-3-4j)*180/pi
```

## VECTORS

```
>>a=[ 7 2 5], b=[ 9 0 3]
```

```
>>c=1:5
```

```
c= 1 2 3 4 5
```

```
>>d=5:8
```

```
d= 5 6 7 8
```

```
>>e=0:2:10
```

```
0 2 4 6 8 10
```

```
>>f=0:0.1:0.6
```

```
0 0.1 0.2 0.3 0.4 0.5 0.6
```

```
>>g=zeros(1,6)
```

```
0 0 0 0 0 0
```

```
>>h=zeros(1,4)
```

```
0 0 0 0
```

```
>>k=ones(1,7)
```

```
1 1 1 1 1 1 1
```

```
>>m=ones(1,3)
```

```
1 1 1
```

## ADDITION AND SUBTRACTION

```
>>a=[ 2 8 10], b=[ 1 4 3]
```

```
>>c=a+b
```

```
  2  8 10
+  1  4  3
-----
  3 12 13
```

```
c=[ 3 12 13]
```

```
>>d=10*a
```

```
d=[ 20 80 100]
```

```
>>e=5*b
```

```
5 20 15
```

```
>>f=10*a+5*b
```

```
25 100 115
```

```
>>g=a-b
```

```
2 4 7
```

## NESTED VECTORS

```
>>h=[ 1:5]
```

```
1 2 3 4 5
```

```
>>k=[ 1:5 1:3]
```

```
1 2 3 4 5 1 2 3
```

```
>>m=[ 0:2:10 10:3:22]
```

```
0 2 4 6 8 10 13 16 19 22
```

```
>>a=[ 8 10 3], b=[ 4 7 8]
```

```
>>c=[ a b]
```

```
8 10 3 4 7 8
```

```
>>d=[ a a a]
```

```
8 10 3 8 10 3 8 10 3
```

## complex vectors

```
>>a=[ 3+4*j -6+9j 2+5j -7j 30]
```

```
>>w=abs(a)
```

```
5 10.81 5.38 7 30
```

$\sqrt{3^2 + 4^2} = 5, \quad \sqrt{6^2 + 9^2} = 10.81 \dots$

```
>>p=angle(a)
```

```
0.92 2.15 1.19 -1.57 0
```

```
>>s=angle(a)*180/pi
```

```
53.13 123.69 68.19 -90 0
```

$$\tan^{-1}\left(\frac{4}{3}\right)=0.92^{\text{radian}}=53.13^{\circ}$$

$$\tan^{-1}\left(\frac{9}{-6}\right)=2.15^{\text{radian}}=123.69^{\circ}$$

## MATRIX DEFINITION:

You can define matrices in MATLAB different ways.

If you write

```
>>a=[10 20 30; 40 50 60; 100 80 90];b=[ 1 2 3; 4 5
```

```
6; -2 8 9]; c=[15 25 35];
```

```
d=[1 2 5]';
```

then, the following matrices are defined:

$$a = \begin{bmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \\ 100 & 80 & 90 \end{bmatrix}, b = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -2 & 8 & 9 \end{bmatrix},$$

$$c = [15 \quad 25 \quad 35], d = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

The apostrophe sign ' is used to get the transpose of a matrix.

Multiplication, summation, subtraction are similar to scalars.

>> qq=a+b, ww=a-b; ee=a\*d; ff=a+j\*b

$$qq = \begin{bmatrix} 11 & 22 & 33 \\ 44 & 55 & 66 \\ -98 & 88 & 99 \end{bmatrix}, \quad ww = \begin{bmatrix} 9 & 18 & 27 \\ 36 & 45 & 54 \\ 102 & 72 & 81 \end{bmatrix},$$

$$ee = \begin{bmatrix} 200 \\ 440 \\ 710 \end{bmatrix}$$

$$ff = \begin{bmatrix} 10+j & 20+2j & 30+3j \\ 40+4j & 50+5j & 60+6j \\ 100-2j & 80+8j & 90+9j \end{bmatrix},$$

$$gg = [1+j \quad 2+3j \quad 4+5j; \quad 8+8j \quad 10 \quad 11]$$

$$gg = \begin{bmatrix} 1+j & 2+3j & 4+5j \\ 8+8j & 10 & 11 \end{bmatrix}$$

### Rows and columns of a matrix

Rows and columns of a matrix are processed as in the following example.

Write a matrix as:

>> G=[10 20 30 40; 210 220 230; 310 320 330 340; 410 420; 430; 440];

$$G = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 210 & 220 & 230 & 240 \\ 310 & 320 & 330 & 340 \\ 410 & 420 & 430 & 440 \end{bmatrix}$$

and write the following commands

>> h=G(:,1), k=G(:,2), m=G(:,4),  
n=G(1,:), p=G(2,:),

You will get the following submatrices.

$$h = \begin{bmatrix} 10 \\ 210 \\ 310 \\ 410 \end{bmatrix}, k = \begin{bmatrix} 20 \\ 220 \\ 320 \\ 420 \end{bmatrix}, m = \begin{bmatrix} 40 \\ 240 \\ 340 \\ 440 \end{bmatrix},$$

$$n = [10 \quad 20 \quad 30 \quad 40]$$

$$p = [410 \quad 420 \quad 430 \quad 440]$$

Futhermore, r=G(1:2,:), t=G(:,1:2), s=G(1:2,1:2) produce

$$r = \begin{bmatrix} 10 & 20 & 30 & 40 \\ 210 & 220 & 230 & 240 \end{bmatrix}, t = \begin{bmatrix} 10 & 20 \\ 210 & 220 \\ 310 & 320 \\ 410 & 420 \end{bmatrix},$$

$$s = \begin{bmatrix} 10 & 20 \\ 210 & 220 \end{bmatrix},$$

>> aa=1:10 →

$$aa = [1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10]$$

>> aa=1:7, bb=sin(aa), → bb=[sin(1) sin(2) sin(3) sin(4) sin(5) sin(6) sin(7)];

Matrices can be nested into each other. Examine the following examples.

>> a=[1 2 3]; b=[10 100 200]; c=[11 22 33]; d=[a; b; c]; e=[a b c];

$$d = \begin{bmatrix} 1 & 2 & 3 \\ 10 & 100 & 200 \\ 11 & 22 & 33 \end{bmatrix},$$

$$e = [1 \quad 2 \quad 3 \quad 10 \quad 100 \quad 200 \quad 11 \quad 22 \quad 33]$$

>> a=[1 2; 3 4]; b=[a [10 20]'; 7 8 9]

$$a = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, b = \begin{bmatrix} 1 & 2 & 10 \\ 3 & 4 & 20 \\ 7 & 8 & 9 \end{bmatrix}$$

### Built-in defined matrices

**zeros(n,m)** an  $n \times m$  matrix that, all elements are zero.

**ones(n,m)** an  $n \times m$  matrix that, all elements are 1.

**eye(n)** an  $n \times n$  unit matrix (diagonal elements are 1, all other elements are zero)

**size(qq)** size of the matrix qq (number of rows and number of columns)

**qq'** Transpose of the matrix qq

**inv(qq)** inverse of the matrix qq

**diag(qq)** diagonal of the matrix qq

**sum(qq)** sum of the elements of columns of matrix qq

**all(qq), any(qq)** To test whether any elements of the matrix qq is zero.

**det(qq)** determinant of the matrix qq.

### Example 1)

```
>> ww=ones(2,3), ff=zeros(3,4), gg=eye(3),
```

$$ww = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad ff = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$gg = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### Example 2)

```
>> [wrow wcolumn]=size(ww),  
    wrow=2    wcolumn=3
```

```
>> [frow fcolumn]=size(ff),  
    frow=3    frow=4
```

### Example 3)

```
>> q=[1 2; 3 4], p=[10 20; 30 40];
```

```
r=[ [q zeros(2,2)] [ones(2,2) p]]
```

$$q = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad q = \begin{bmatrix} 10 & 20 \\ 30 & 40 \end{bmatrix},$$

$$r = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 & 1 & 10 & 20 \\ 3 & 4 & 0 & 0 & 1 & 1 & 30 & 40 \end{bmatrix}$$

### Example 4)

```
>> e = [ zeros(1,4) ones(1,3) ]  
        0 0 0 0 1 1 1
```

```
>> e = [ zeros(1,4) ones(1,3) ]  
        0 0 0 0 1 1 1
```

```
>> f = [ ones(1,3) 10*ones(1,4) ]  
        1 1 1 10 10 10 10
```

```
>> g = 10*[1:3]  
        10 20 30
```

```
>> h = [ ones(1,3) 10:2:20]  
        1 1 1 10 12 14 16 18 20
```

```
>> k = [ 10*ones(1,3) 17 10:3:19 ]  
        10 20 30 17 10 13 16 19
```

```
>> f = [ ones(1,3) 10*ones(1,4) ]  
        1 1 1 10 10 10 10
```

```
>> g = 10*[1:3]  
        10 20 30
```

```
>> h = [ ones(1,3) 10:2:20]  
        1 1 1 10 12 14 16 18 20
```

```
>> k = [ 10*ones(1,3) 17 10:3:19 ]  
        10 20 30 17 10 13 16 19
```

### Example 5)

```
>> aa=sum(r)  
        aa=[4 6 0 0 2 2 40 60]  
        Elements in each column are added.  
>> bb=sum(aa)  
        bb=114  
        All the elements of vector is added.
```

### Example 6)

Most built-in functions (sin,cos,tan, exp.. ) also works for matrices.

```
>> a=[1 2; 3 4];
```

```
>> b=sin(a);
```

$$b = \begin{bmatrix} \sin(1) & \sin(2) \\ \sin(3) & \sin(4) \end{bmatrix} = \begin{bmatrix} 0.841 & 0.909 \\ 0.141 & -0.756 \end{bmatrix},$$

```
>> c=exp(a);
```

$$c = \begin{bmatrix} e^1 & e^2 \\ e^3 & e^4 \end{bmatrix} = \begin{bmatrix} 2.718 & 7.389 \\ 20.08 & 54.59 \end{bmatrix}$$

```
>> d=a.^2;
```

$$d = \begin{bmatrix} 1^2 & 2^2 \\ 3^2 & 4^2 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 16 \end{bmatrix},$$

```
>> g=a^2
```

$$g = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

Notice the **difference between  $a.^2$  and  $a^2$**

## VECTOR MULTIPLICATION AND DIVISION

```
>> a=[15 16 12 20], b=[10 4 6 5]
```

```
>> x=a*b
```

**??? Error using ==> mtimes**

**Inner matrix dimensions must agree.**

**Correct form is below**

```
>> a=[15 16 12 20], b=[10 4 6 5]
```

```
>> x=a.*b
```

150 64 72 100

$$15 * 10 = 150$$

$$16 * 4 = 64$$

$$12 * 6 = 72$$

$$20 * 5 = 100$$

```
>> a=[15 16 12 20], b=[10 4 6 5]
```

```
>> y=a/b
```

2.18

**( This is not you wanted !!!!! )**

**Correct form is below**

```
>> a=[15 16 12 20], b=[10 4 6 5]
```

```
>> y=a./b
```

1.5 4 2 4

$$15 / 10 = 1.5$$

$$16 / 4 = 4$$

$$12 / 6 = 2$$

$$20 / 5 = 4$$

## DIMENSION ERROR

```
>> a=[2 5 4], b=[8 3 12 5]
```

```
>> x=a+b
```

**?? Error using ==> plus**

**Matrix dimensions must agree.**

a and b must be same size. Here a has 3 elements b has 4 elements. **We cannot add a and b**

$x = a*b$ ,  $y = a.*b$ ,  $z = a/b$   $w = a./b$

**all of them have dimension error**

## VECTOR SQUARE and POWER

```
>> a=[2 5 7 -8], b=a^2
```

**??? Error using ==> mpower**

**Matrix must be square.**

Correct form is below

```
>> a=[2 5 7 -8], b=a.^2
```

4 25 49 64

$$2^2 = 4 \quad 5^2 = 25 \quad 7^2 = 49 \quad (-)^8^2 = 64$$

```
>> a=[2 5 7 -8], b=a.^3
```

8 125 343 -512

**Problem:**  $y = 3x^2 + e^{0.1x} - 20 \sin(x)$

Calculate y for  $x=0$ ,  $x=0.5$ ,  $x=1$ , and  $x=2$

**Long method:**

```
>> x=0, y=3*x^2 + exp(0.1*x) -20*sin(x)
```

1

```
>> x=0.5, y=3*x^2 + exp(0.1*x) -20*sin(x)
```

-7.78

```
>> x=1, y=3*x^2 + exp(0.1*x) -20*sin(x)
```

-12.72

**Short method**

```
>>x=[0 0.5 1 2],
>>y=3*x.^2+exp(0.1*x)-20*sin(x)
1 -7.78 -12.72 -4.96
```

Notice **the dot .** in  $x.^2$

```
for
>> for kk=1:4, aa(kk)=kk^3; end;

aa=[1^3 2^3 3^3 4^3 ]

aa=[ 1 8 27 64]
```

-----

Example 431 : Calculate  $y=3x^2+5x+7$   
for  $x=[0 \ 1 \ 8 \ 5]$ , Show in a table.

**Method 1.**

```
a1=0; b1=3*a1^2 + 5*a1 +7
```

```
a2=1; b2=3*a2^2 + 5*a2 +7
```

```
a3=8; b3=3*a3^2 + 5*a3 +7
```

```
a4=5; b4=3*a4^2 + 5*a4 +7
```

```
tt=[a1 b1; a2 b2; a3 b3; a4 b4]
tt=
0 7
1 15
8 239
5 107
```

-----

**Method 2.**

```
aa=[0 1 8 5]
```

```
aa_Length=length(aa);
```

```
for kk=1:aa_Length,
```

```
bb(kk)= 3* aa(kk) ^2 + 5*aa(kk) +7
end;
```

```
bb=[7 15 239 107]
```

```
tt=[aa' bb']
```

```
tt=
0 7
1 15
8 239
5 107
```

-----

**Method 3.**

```
aa=[0 1 8 5]
```

```
bb=3*aa.^2 + 5*aa +7
```

-----

**Method 4.**

```
aa=[0 1 8 5]
pol_coef =[3 5 7]
```

```
b1=polyval(pol_coef,0)
```

```
b2=polyval(pol_coef,0)
```

```
b3=polyval(pol_coef,0)
```

```
b4=polyval(pol_coef,0)
bb=[ b1 b2 b3 b4]
```

-----

**Method 5.**

```
aa=[0 1 8 5]
```

```
pol_coef =[3 5 7]
```

```
bb=polyval(pol_coef,aa)
```

-----

**Exercise: 11)**

$A=[1 \ 2 \ 3; 4 \ 5 \ 6; 7 \ 8 \ 8]; \ b=[10 \ 20 \ 30]'$ ;

a)find  $x = A^{-1}b$

b)Calculate  $Ax$ . Is  $Ax=b$  ?

b)write  $C=A^2$  and  $D=A \bullet^2$

why C and D are different.

c)Find  $Ab$

d) Calculate determinat of A

**Exercise 12)**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Calculate determinat and inverse of A.

**Exercise 3)**

```
>>A=[1 2 3; 10 20 30; 40 50 60],
```

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 10 & 20 & 30 \\ 40 & 50 & 60 \end{bmatrix}$$

Obtain matrix B from A by exchanging the rows of A.

Obtain matrix C from A by exchanging the columns of A.

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 40 & 50 & 60 \\ 10 & 20 & 30 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 3 & 20 \\ 40 & 60 & 50 \\ 10 & 30 & 20 \end{bmatrix}$$

Exercise 4) Obtain the following matrices by MATLAB commands

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 100 & 100 \\ 100 & 100 \end{bmatrix},$$

$$\begin{bmatrix} \overbrace{1 \quad 1 \quad \dots \quad 1}^n \quad 1 \\ \text{q} \left\{ \begin{array}{l} 1 \quad 1 \quad \dots \quad 1 \quad 1 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ 1 \quad 1 \quad \dots \quad 1 \quad 1 \end{array} \right. \\ \text{m} \left\{ \begin{array}{l} 0 \quad 0 \quad \dots \quad 0 \quad 0 \\ 0 \quad 0 \quad \dots \quad 0 \quad 0 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ 0 \quad 0 \quad \dots \quad 0 \quad \dots \quad 0 \end{array} \right. \\ \text{k} \left\{ \begin{array}{l} 1 \quad 0 \quad \dots \quad 0 \quad \dots \quad 0 \\ 0 \quad 1 \quad \dots \quad 0 \quad \dots \quad 0 \\ \dots \quad \dots \quad \dots \quad 0 \quad \dots \quad 0 \\ 0 \quad 0 \quad \dots \quad 0 \quad 1 \quad \dots \quad 0 \end{array} \right. \\ \text{p} \left\{ \begin{array}{l} 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad \dots \quad 0 \end{array} \right. \end{bmatrix}$$

C)

Exercise 5) Obtain the following matrices

$$\text{q} \left\{ \begin{array}{l} \overbrace{1 \quad 1 \quad \dots \quad 1 \quad 1}^n \quad \overbrace{0 \quad 0 \quad \dots \quad 0}^m \\ 1 \quad 1 \quad \dots \quad 1 \quad 1 \quad 0 \quad 0 \quad \dots \quad 0 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ 1 \quad 1 \quad \dots \quad 1 \quad 1 \quad 0 \quad 0 \quad \dots \quad 0 \end{array} \right. \quad \text{a)}$$

$$\text{q} \left\{ \begin{array}{l} \overbrace{1 \quad 1 \quad \dots \quad 1 \quad 1}^n \quad \overbrace{0 \quad 0 \quad \dots \quad 0}^m \quad \overbrace{2 \quad 2 \quad \dots \quad 2}^k \\ 1 \quad 1 \quad \dots \quad 1 \quad 1 \quad 0 \quad 0 \quad \dots \quad 0 \quad 2 \quad 2 \quad \dots \quad 2 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ 1 \quad 1 \quad \dots \quad 1 \quad 1 \quad 0 \quad 0 \quad \dots \quad 0 \quad 2 \quad 2 \quad \dots \quad 2 \end{array} \right. \quad \text{b)}$$

$$\begin{bmatrix} \overbrace{1 \quad 1 \quad \dots \quad 1 \quad 1}^n \quad \overbrace{0 \quad 0 \quad \dots \quad 0}^m \quad \overbrace{0 \quad \dots \quad 0}^k \\ \text{p} \left\{ \begin{array}{l} 1 \quad 1 \quad \dots \quad 1 \quad 1 \quad 0 \quad 0 \quad \dots \quad 0 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ 1 \quad 1 \quad \dots \quad 1 \quad 1 \quad 0 \quad 0 \quad \dots \quad 0 \end{array} \right. \\ \text{q} \left\{ \begin{array}{l} 4 \quad 4 \quad \dots \quad 4 \quad 4 \quad 1 \quad 0 \quad \dots \quad 0 \\ 4 \quad 4 \quad \dots \quad 4 \quad 4 \quad 0 \quad 1 \quad \dots \quad 0 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ 4 \quad 4 \quad \dots \quad 4 \quad 4 \quad 0 \quad 0 \quad \dots \quad 1 \end{array} \right. \\ \text{r} \left\{ \begin{array}{l} 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad 0 \quad 0 \quad \dots \quad 0 \\ \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \\ 0 \quad 0 \quad \dots \quad 0 \quad 0 \quad \dots \quad 0 \quad \dots \quad 0 \end{array} \right. \end{bmatrix} \quad \text{d)}$$

$$A = \begin{bmatrix} 15 & 2 & 3 & 4 & 5 & 6 \\ 30 & 3 & 2 & 0 & 0 & 3 \\ 11 & 0 & 0 & 3 & 8 & 6 \\ 7 & 1 & 3 & 0 & 3 & 12 \\ 7.5 & 0 & 3 & 4 & 0 & 6 \end{bmatrix}$$

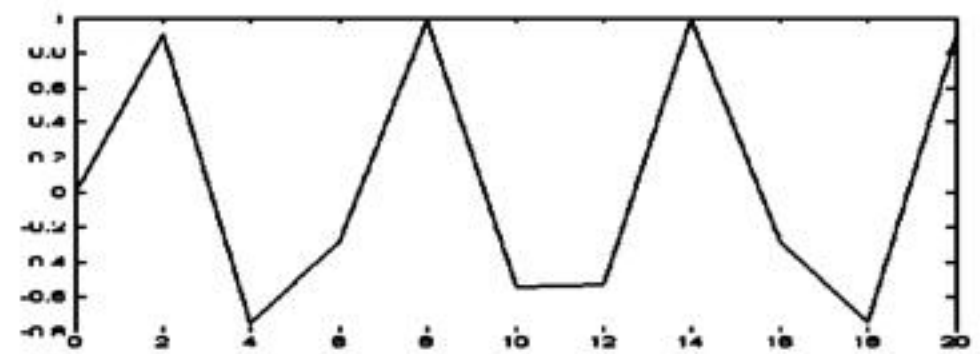
$$A(2,:) = A(2,:) - A(1,:) * A(2,1) / A(1,1)$$

```
A(3,:)= A(3,:)- A(1,:)*A(3,1)/A(1,1)
A(4,:)= A(4,:)- A(1,:)*A(4,1)/A(1,1)
A(5,:)= A(5,:)- A(1,:)*A(5,1)/A(1,1)
```

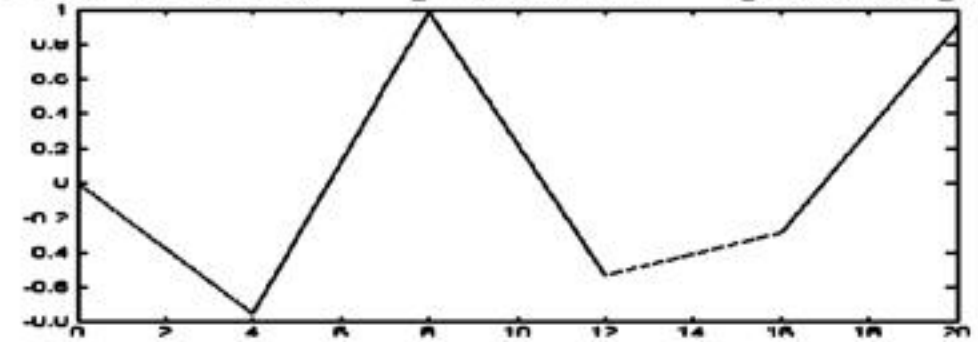
```
A(3,:)= A(3,:)- A(2,:)*A(3,2)/A(2,2)
A(4,:)= A(4,:)- A(2,:)*A(4,2)/A(2,2)
A(5,:)= A(5,:)- A(2,:)*A(5,2)/A(2,2)
```

```
A(4,:)= A(4,:)- A(3,:)*A(4,3)/A(3,3)
A(5,:)= A(5,:)- A(3,:)*A(5,3)/A(3,3)
```

```
A(5,:)= A(5,:)- A(4,:)*A(5,4)/A(4,4)
```



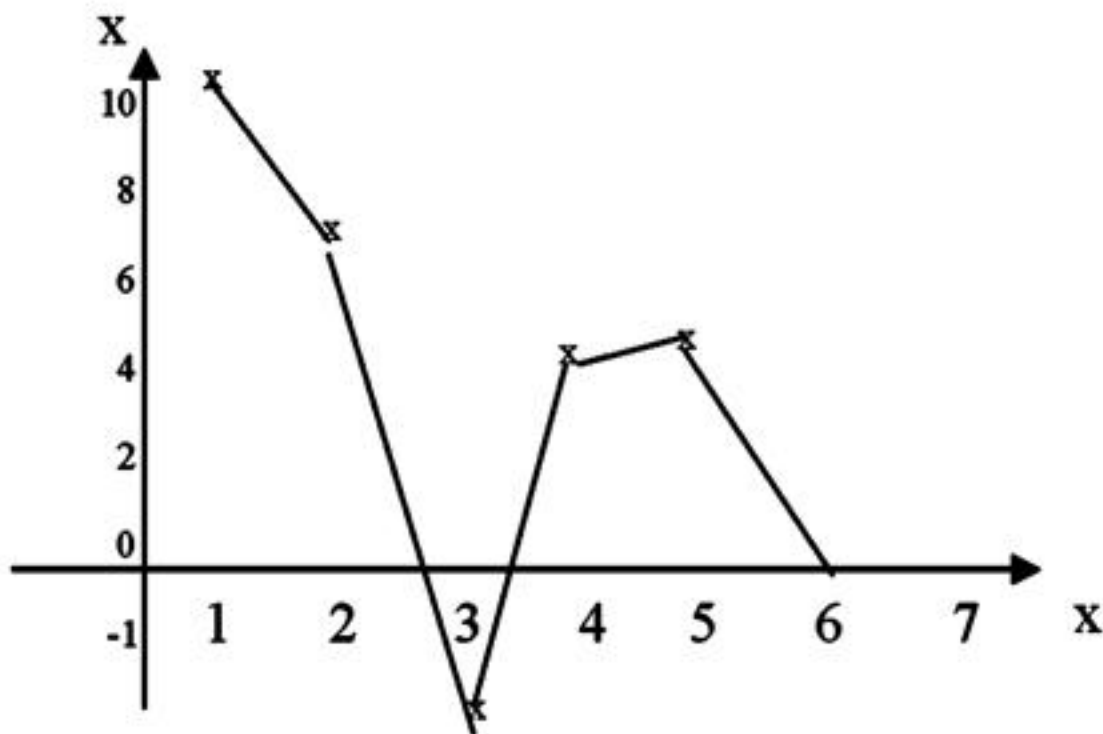
```
>> x=0:4:20; y=sin(x); plot(x,y),
```



```
>> x=0:0.1:20; y=sin(x); plot(y,x),
```

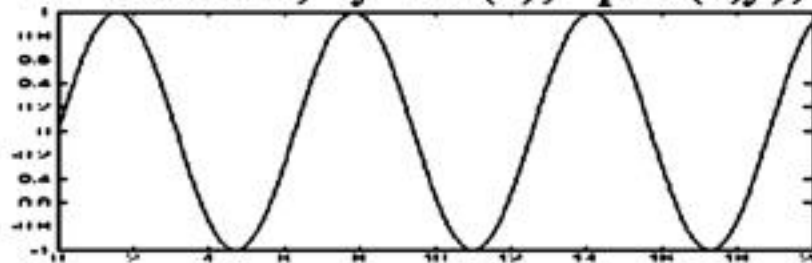
## MATLAB GRAPHICS

$x=[1 \ 2 \ 3 \ 4 \ 5 \ 6]$ ;  $y=[10 \ 7 \ -1 \ 5 \ 6 \ 0]$ ;  
If we plot  $y$  versus  $x$  we get the following



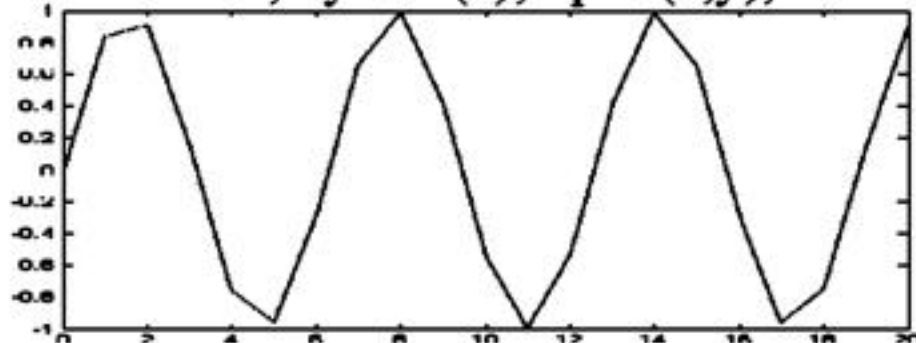
Matlab command to plot  $y$  versus  $x$  is  
`plot(x,y)`

```
>>x=0:0.1:20; y=sin(x); plot(x,y),
```

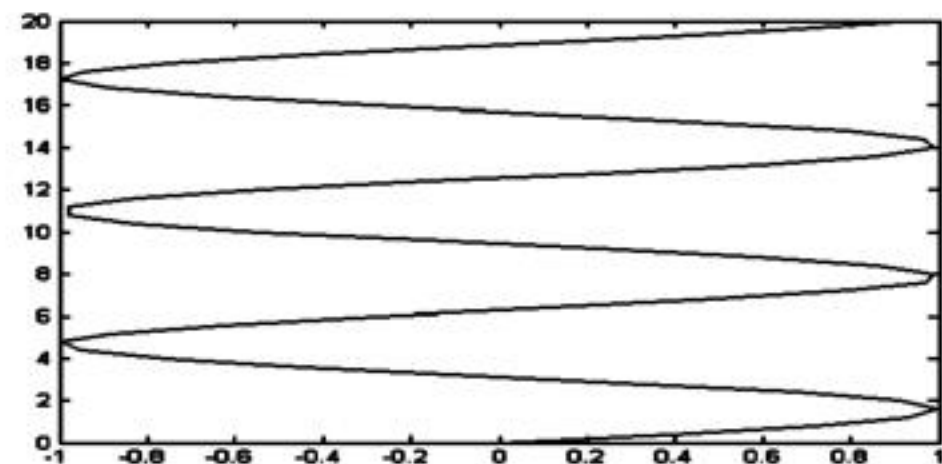


## Graphic Resolution

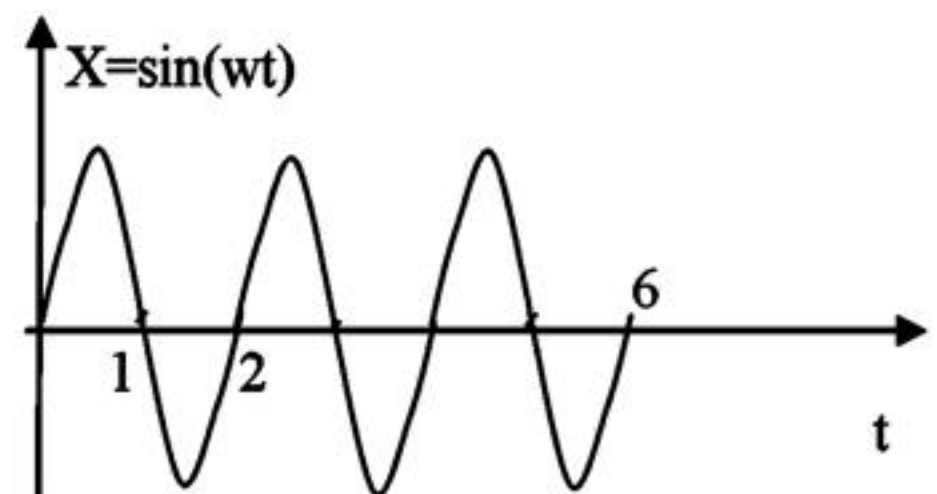
```
>>x=0:1:20; y=sin(x); plot(x,y),
```



```
>>x=0:2:20; y=sin(x); plot(x,y),
```



**Problem 32:** Draw the following graph



**Solution**

```
t=0:0.1:6;
```

```
TT=2;
```

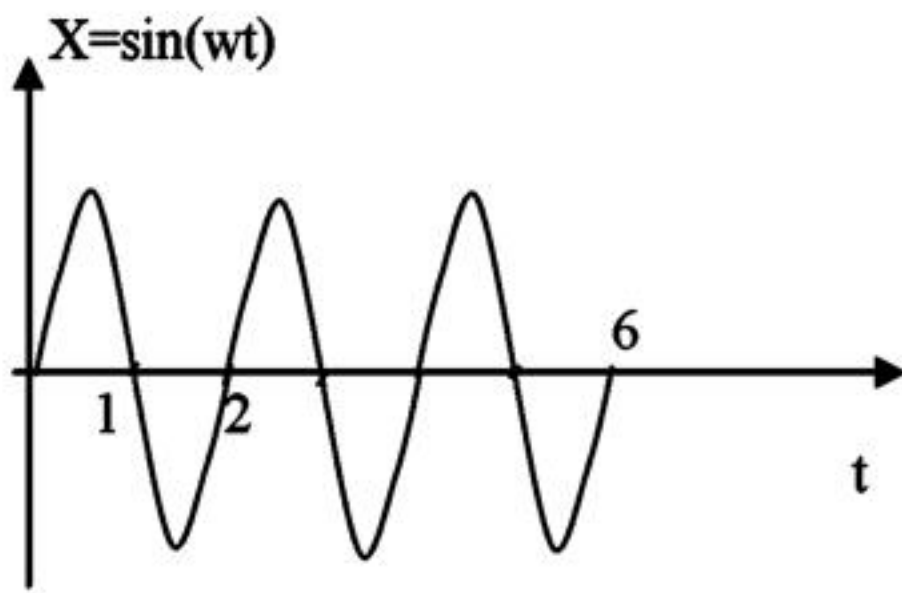
```
w=2*pi/TT;
```

```
x=sin(w*t);
```

```
plot(t,x);
```

```
t=0:0.1:6; TT=2; w=2*pi/TT; x=sin(w*t);plot(t,x);
```

**Problem 33:** Draw the following graph

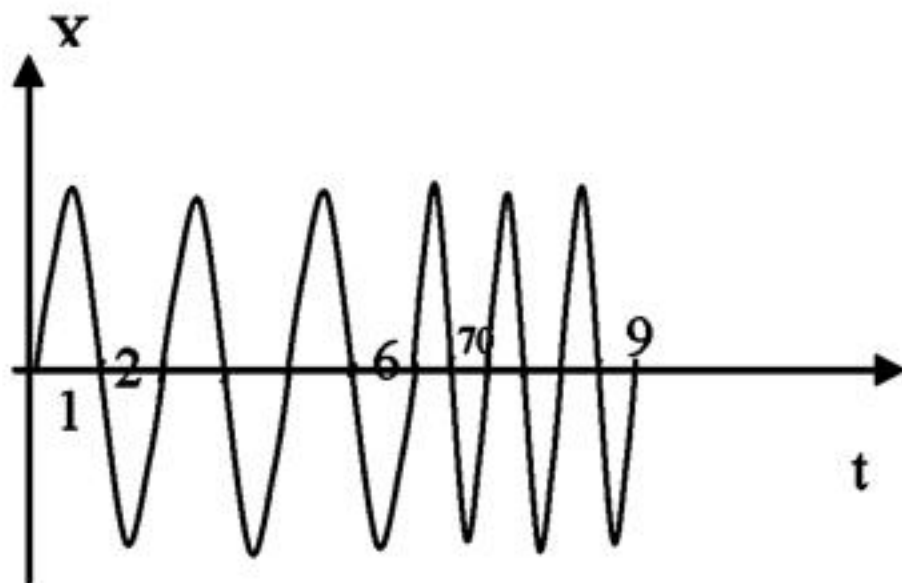


### Solution

```
t=0:0.1:60; TT=20; w=2*pi/TT; x=sin(w*t); plot(t,x);
```

### Problem 34: Draw the following graph.

Note: frequency is doubled from  $t=60$  to  $t=90$



### Solution

```
t1=0:0.1:60; TT1=20; w1=2*pi/TT1; x1=sin(w1*t1);
```

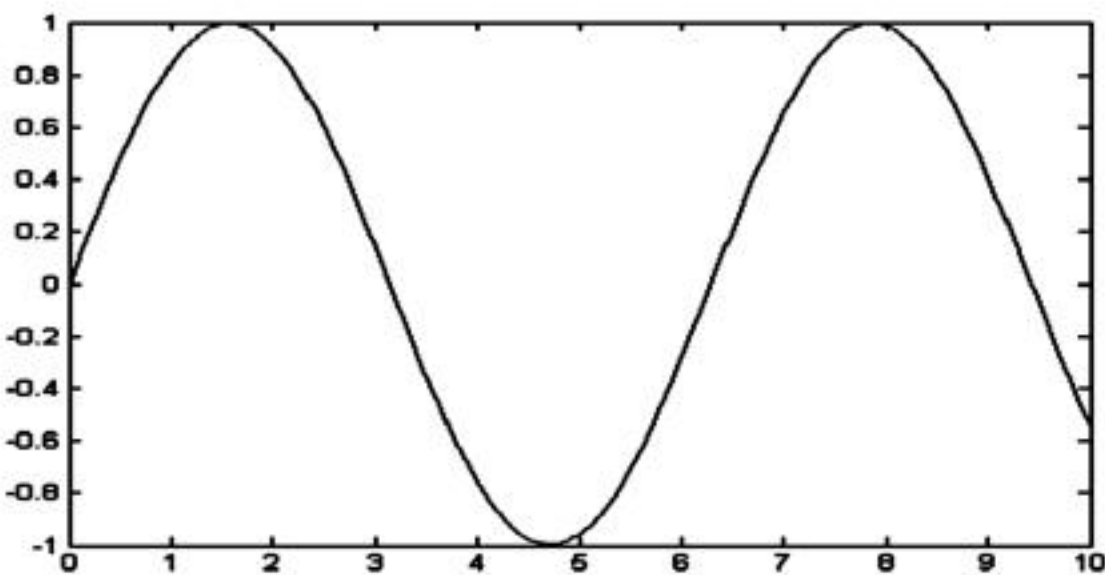
```
t2=60:0.1:90; TT2=10; w2=2*pi/TT2; x2=sin(w2*t2);
```

```
tTotal=[t1 t2]; xTotal=[x1 x2]; plot(tTotal,xTotal);
```

### Problem 35: Draw $x=\sin(t)$ $t=0$ to 10

#### Solution:

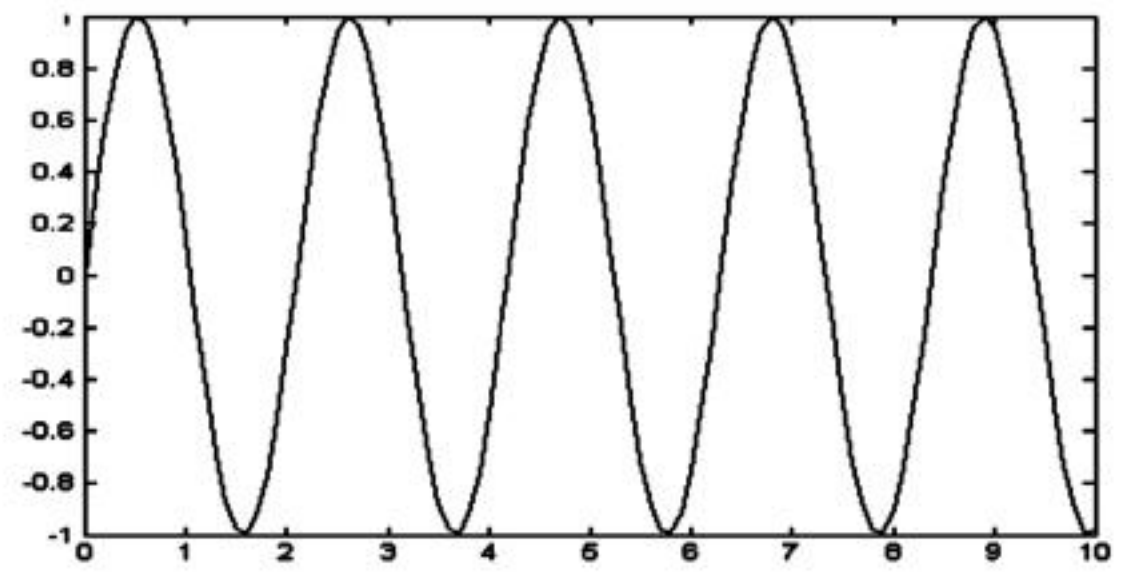
```
t=0:0.1:10; w1=1; x1=sin(w1*t); plot(t,x1);
```



### Problem 36: Draw $x=\sin(3t)$ $t=0$ to 10

#### Solution:

```
t=0:0.1:10; w1=3; x1=sin(w1*t); plot(t,x1);
```

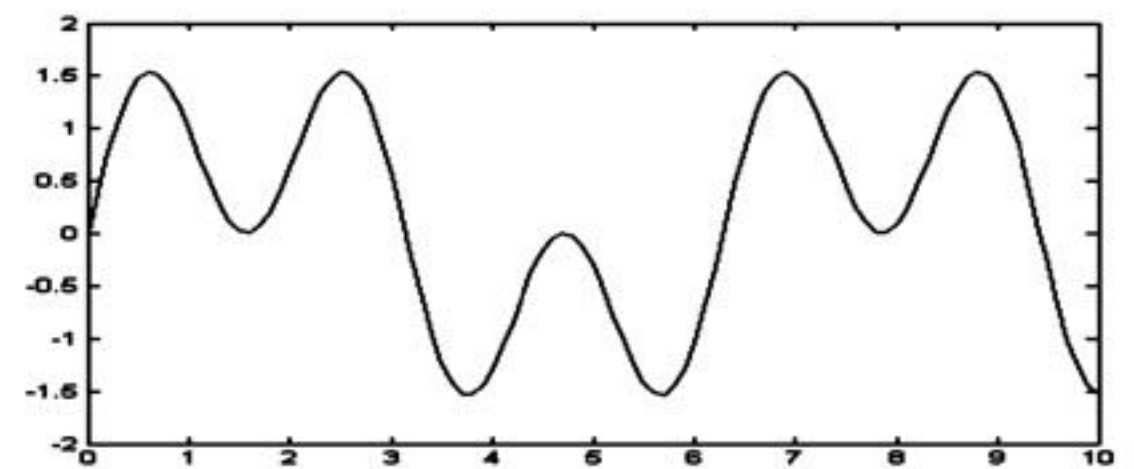


### Problem 37: Draw $x=\sin(t) + \sin(3t)$ $t=0$ to 10

#### Solution:

```
t=0:0.1:10; w1=1; w2=3;
```

```
x1=[sin(w1*t)+sin(w2*t)]; plot(t,x);
```



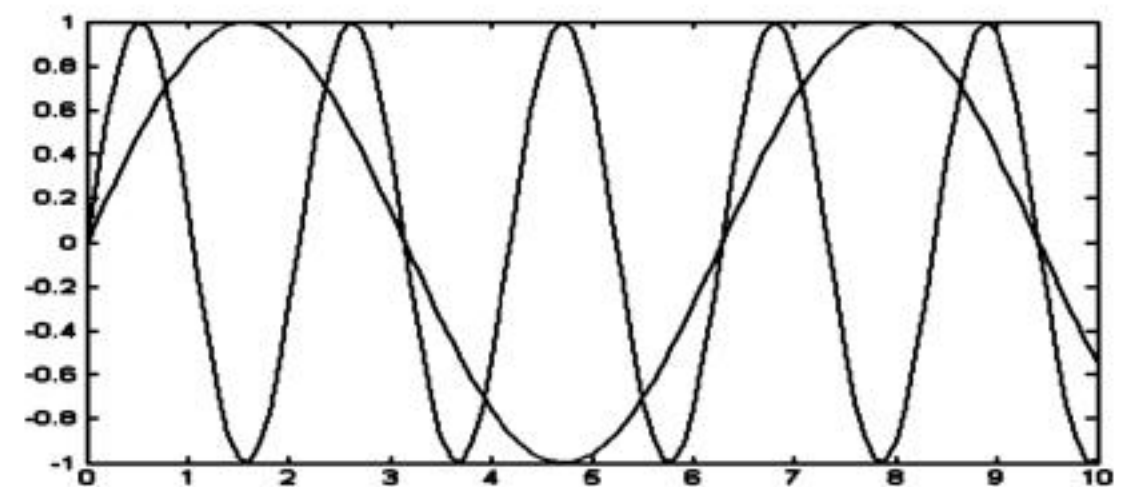
### Problem 38: Draw $x1=\sin(t)$ and $x2=\sin(3t)$ $t=0$ to 10

#### Solution

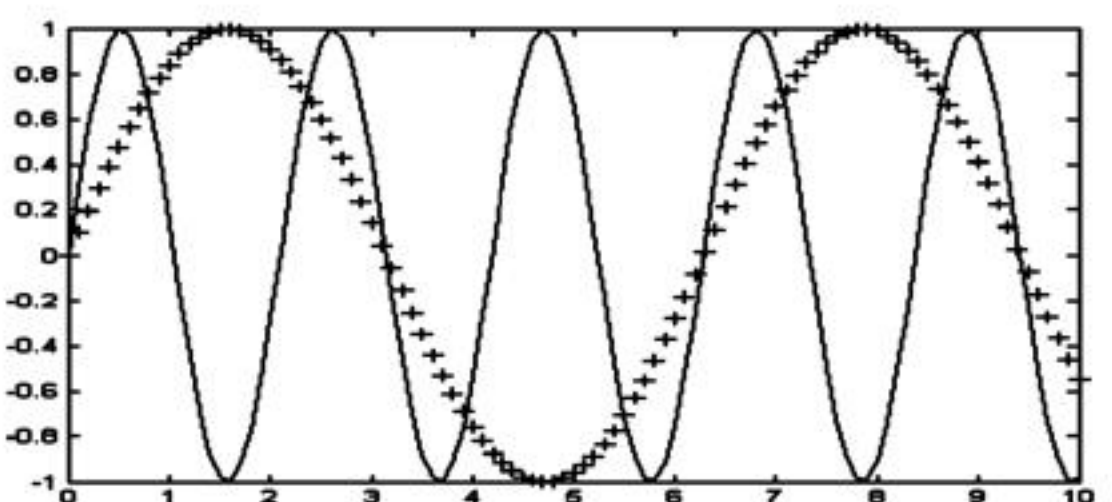
```
t=0:0.1:10; w1=1; w2=3;
```

```
x1=[sin(w1*t)]; x2=[sin(w2*t)];
```

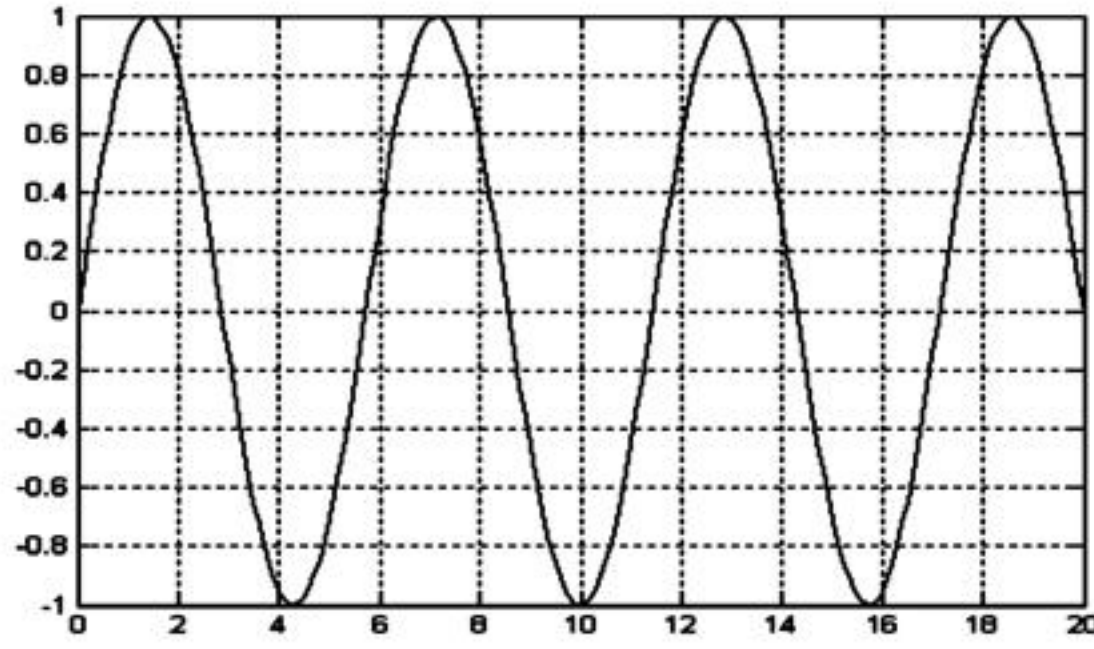
```
plot( t, x1 , 't', x2 );
```



```
plot( t, x1 , '+', t, x2 );
```



**Exercise 57.** Find the period and frequency of the following sinewave



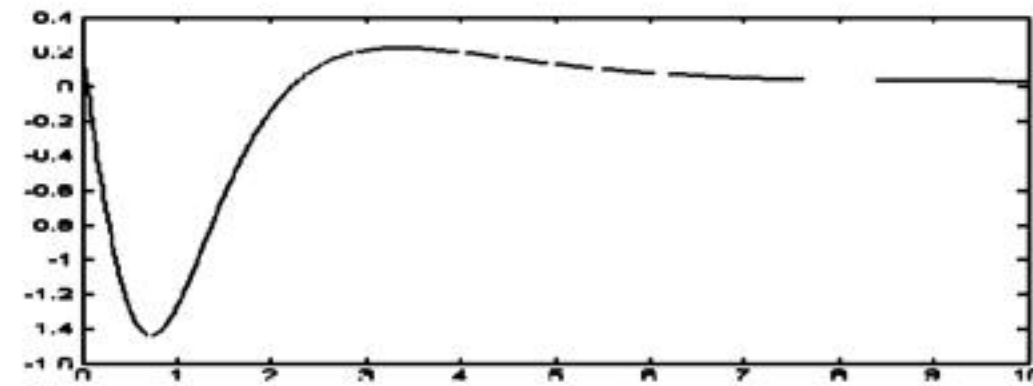
T= f= w=

$$T=17/3=5.666 \text{ ???}$$

**Problem 41:**  $y = \frac{3x^2 + e^{0.1x} - 20\sin(x)}{x^4 + 4x^2 + 5}$

plot x,y graph for x=0 to x=10

x=0:0.1:10; aa=3\*x.^2+exp(0.1\*x)-20\*sin(x);  
bb=x.^4+4\*x.^2+5; yy=aa./bb; plot(x,yy)



**Problem 413:**

$F(z) = \frac{s+1}{s^2+2s+2}$  fonksiyonunda  $s=jw$  koyup

genlik ve faz spektrumunu cizin. Genligin maximum oldugu frekansi bulun.

a) s yerine  $s=jw$  koyun.

b)  $F(w)$ ,  $|F(w)|$ , ve  $\angle F(w)$  yi asagidaki w degerlei icin hesaplayin.  $w=[0 \ 0.5 \ 1 \ 1.1 \ 1.2 \ 2 \ 5 \ 10]$

c) x eksenini w y eksenini  $|F(w)|$  olacak sekilde  $|F(w)|$  ve  $\angle F(w)$  yi cizin.

d)  $|F(w)|$  nin maximum degerini grafikten bulun.

**Cozum**

$$F(iw) = \frac{iw+1}{(iw)^2+2iw+2} = \frac{1+iw}{(2-w^2)+i2w} = \frac{N(w)}{D(w)}$$

for  $w=0$ .  $N(0)=1+i0=1$   $D(0)=(2-0^2)+i2 \times 0=2$

$$F(0) = \frac{N(0)}{D(0)} = \frac{1}{2} = 0$$

for  $w=0.5$   $N(0.5)=1+i0.5$

$$D(0.5)=(2-0.5^2)+i2 \times 0.5=1.75+i$$

$$F(i0.5) = \frac{N(0.5)}{D(0.5)} = \frac{1+0.5i}{1.75+i} = 0.553 - 0.03i$$

$$|F(i0.5)| = \sqrt{0.553^2 + 0.03^2} = 0.554$$

$$\angle F(i0.5) = \tan^{-1}\left(\frac{-0.03}{0.553}\right) = -0.055 \text{ radian} = -3.18^\circ$$

Benzer sekilde

$$F(i) = 0.6 - 0.2i \quad |F(1)| = 0.6325, \quad \angle F(1) = -18.4^\circ$$

$$F(1.1i) = 0.587 - 0.24i \quad |F(1.1)| = 0.633, \quad \angle F(1.1) = -22^\circ$$

$$F(1.2i) = 0.56 - 0.28i \quad |F(1.2)| = 0.634, \quad \angle F(1.2) = -26.6^\circ$$

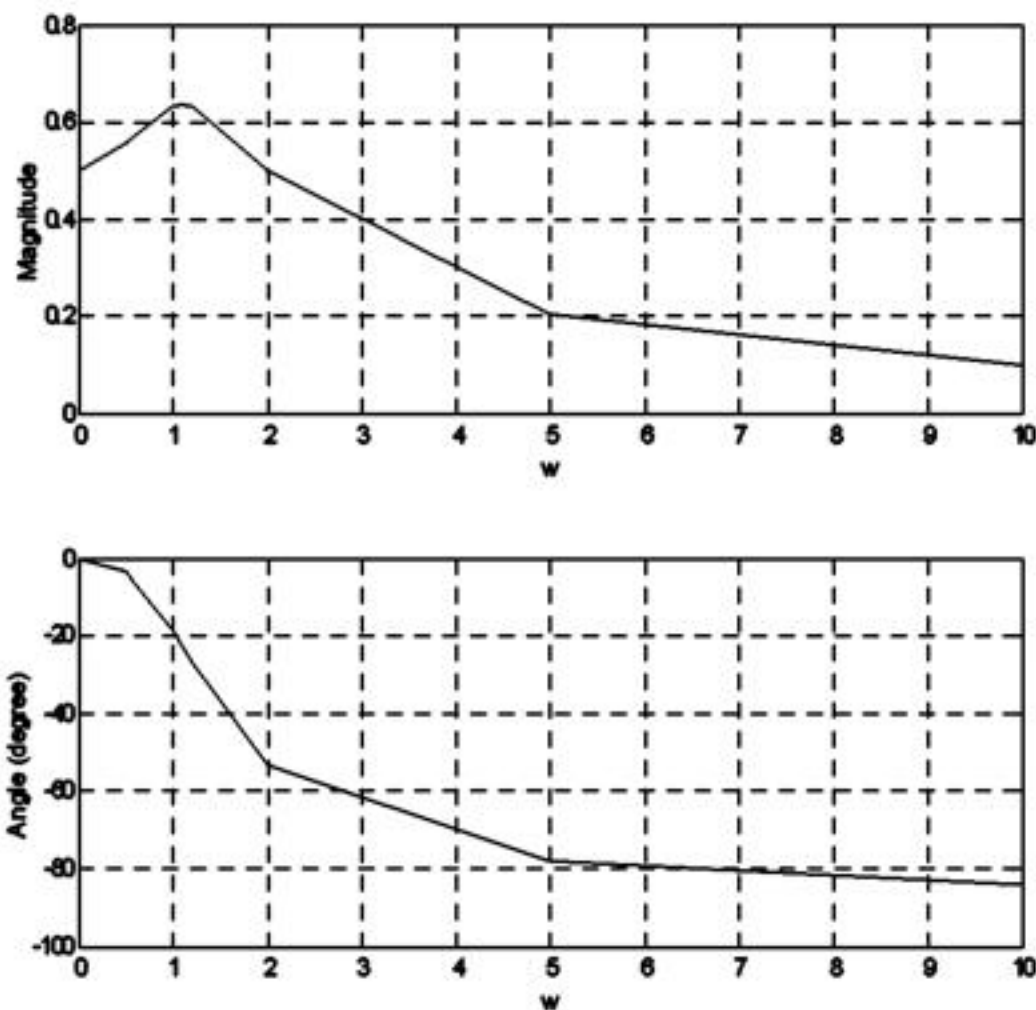
$$F(2i) = 0.3 - 0.4i \quad |F(2)| = 0.5, \quad \angle F(2) = -53.1^\circ$$

$$F(5i) = 0.042 - 0.198i \quad |F(5)| = 0.203, \quad \angle F(5) = -77.8^\circ$$

$$F(10i) = 0.01 - 0.1i \quad |F(10)| = 0.1005, \quad \angle F(10) = -84.1^\circ$$

$$F(\infty) = 0 - 0i \quad |F(\infty)| = 0, \quad \angle F(\infty) = -90^\circ$$

| w        | $F_R$ | $F_{IM}$ | $ F(iw) $ | $\angle F(iw)$<br>degree |
|----------|-------|----------|-----------|--------------------------|
| 0        | 0.5   | 0        | 0.5       | 0                        |
| 0.5      | 0.553 | -0.03    | 0.554     | -3.17                    |
| 1        | 0.6   | -0.2     | 0.632     | -18.43                   |
| 1.1      | 0.587 | -0.243   | 0.63      | -22.52                   |
| 1.2      | 0.566 | -0.284   | 0.633     | -26.67                   |
| 2        | 0.3   | -0.4     | 0.5       | -53.13                   |
| 5        | 0.042 | -0.198   | 0.203     | -77.81                   |
| 10       | 0.010 | -0.1     | 0.1005    | -84.17                   |
| $\infty$ | 0     | 0        | 0         | -90                      |



Grafikten gorulecegi gibi naximum deger  $|F(iw)| = 0.636$ . This maximum deger  $w=1.1$  de meydana gelmektedir.

#### Example 424:

Draw amplitude and phase spectrum of the transfer function

$$H(s) = \frac{10(s+1)}{s^2 + 4s + 13}$$

```
>> w=0; s=j*w; hh=10*(s+1)/(s^2+4*s+13);
amp=abs(hh), pp=phase(hh)
```

```
amp = 0.769
pp = 0
```

```
>> w=0.1; s=j*w; hh=10*(s+1)/(s^2+4*s+13);
amp=abs(hh), pp=phase(hh)
```

```
amp = 0.773
pp = 0.689
```

```
>> w=0.2; s=j*w; hh=10*(s+1)/(s^2+4*s+13);
amp=abs(hh), pp=phase(hh)
```

```
amp = 0.785
pp = 0.1357
```

```
>> w=0.3; s=j*w; hh=10*(s+1)/(s^2+4*s+13);
amp=abs(hh), pp=phase(hh)
```

```
amp = 0.805
pp = 0.1988
```

```
>> w=1; s=j*w; hh=10*(s+1)/(s^2+4*s+13);
amp=abs(hh), pp=phase(hh)
```

```
amp = 1.118
pp = 0.4636
```

Make an array

```
>> ww = [ 0 0.1 0.2 0.3 1
]
>> total_amp=[0.769 0.773 0.785 0.805
1.118 ]
>> total_ph=[ 0 0.689 0.1357 0.198
0.463 ]
```

```
>> plot(ww,total_amp), figure, plot(ww,total_ph)
```

#### KISA YOL

```
>> ww=0:0.1:1; s=j*w; num=(s+1), den=s.^2+.4.*
s+13, hh=num./den, amp=abs(hh), >>
ph=phase(hh)
>> plot(ww,amp), figure, plot(ww,ph)
```

**Note:**  $x=[3+4j \ 2+5j \ 5+3j \ -8j \ -7]$   
 $y=abs(x)$

```
y=[ 5 5.38 5.831 8. 7. ]
```

#### COK KISA YOL

```
pay=10*[1 1];
payda=[1 4 13];
ww=0:0.01:30;
hh=freqs(pay,payda,ww);
plot(ww,abs(hh));
plot(ww,180*angle(hh)/pi);
```

#### COK COK KISA YOL

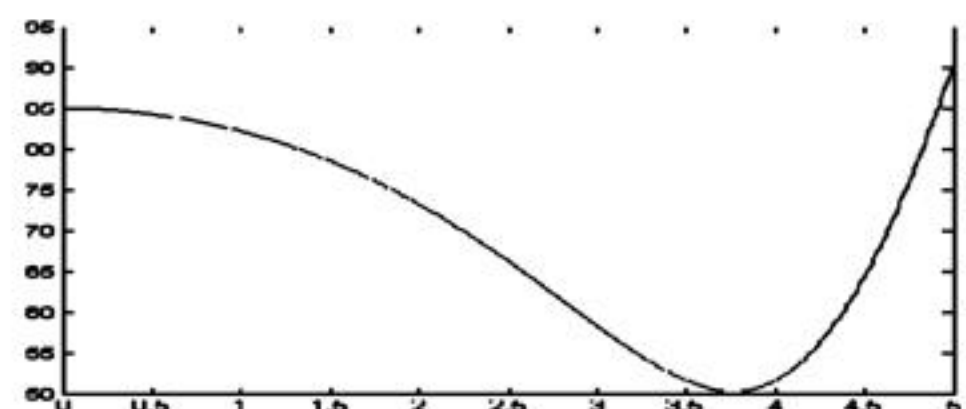
```
pay=10*[1 1];
payda=[1 4 13];
freqs(pay,payda);
```

**Problem.**  $H(jw) = (jw)^3 + 5(jw)^2 + 10(jw) + 8$

Draw amplitude and phase spectrum for  $w=0$  to  $w=5$

#### Solution:

```
w=0:0.01:5; s=j*w;
hh=1*s.^3+7*s.^2+27*s+85;
plot(w, abs(hh))
```

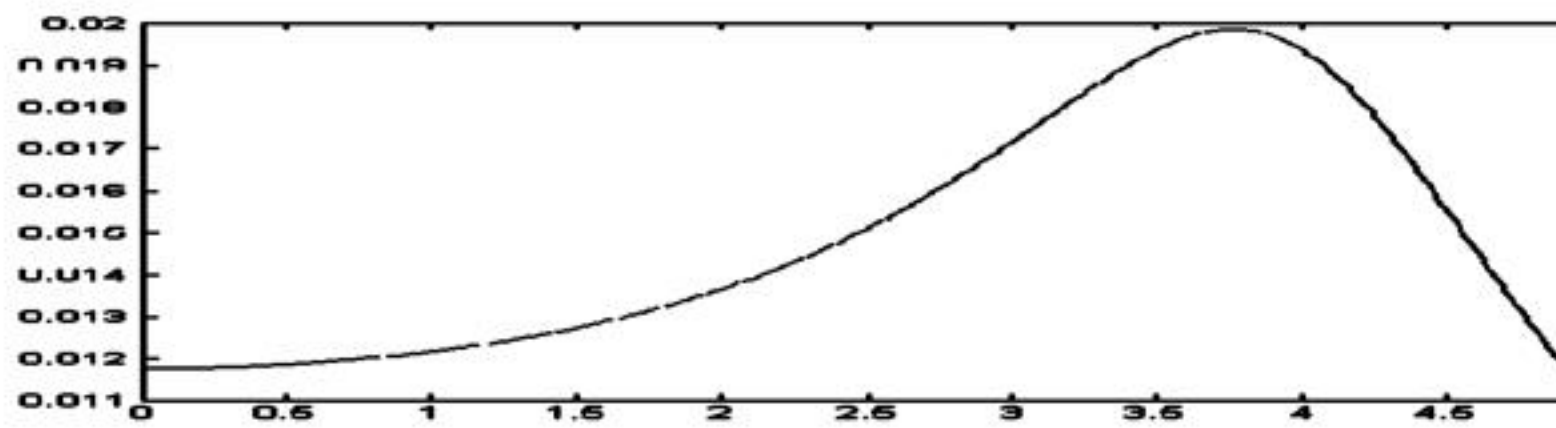


**Problem 42:**  $H(jw) = \frac{1}{(jw)^3 + 5(jw) + 10(jw) + 8}$

Draw amplitude and phase spectrum for  $w=0$  to  $w=5$

#### Solution:

```
w=0:0.01:5; s=j*w;
hh =1*s.^3+7*s.^2+27*s+85;
hh=1 ./ hh; plot(w, abs(hh) )
```



### Exercise 54

Draw amplitude and phase spectrum of

a). 
$$H(s) = \frac{s^2 + 3s + 4}{s^3 + 5s^2 + 10s + 8}$$

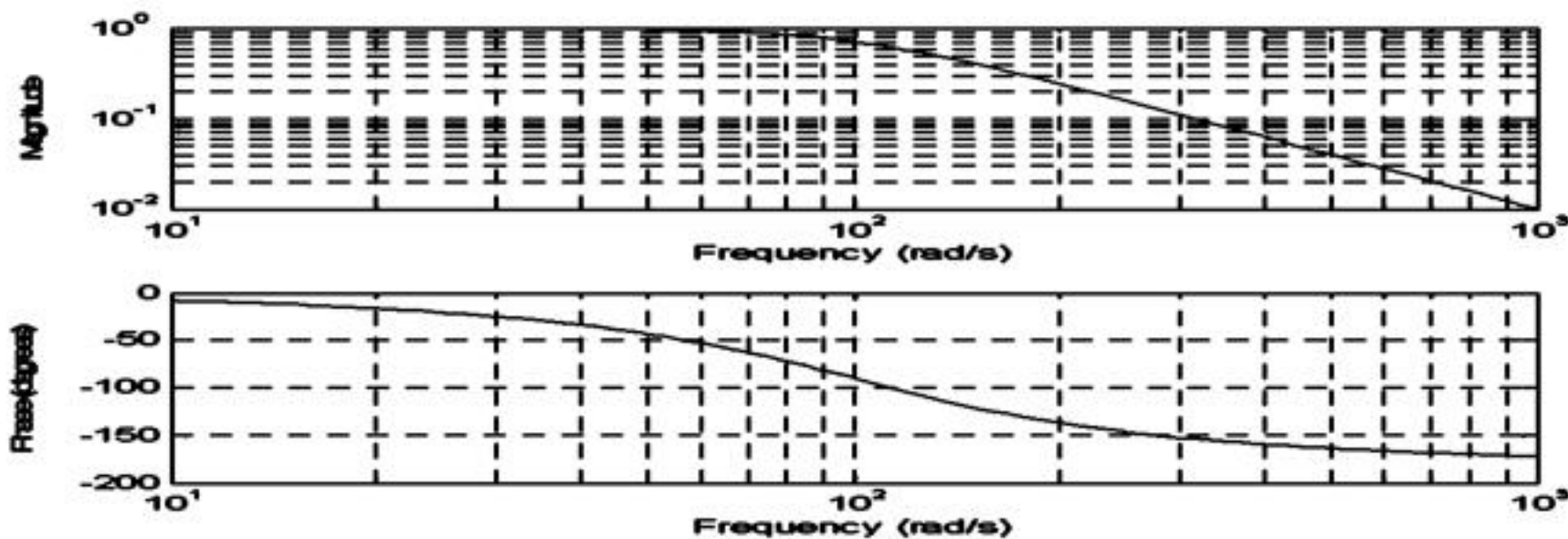
b)

$$H(s) = \frac{10000}{s^2 + 141s + 10000}$$

### Example 411

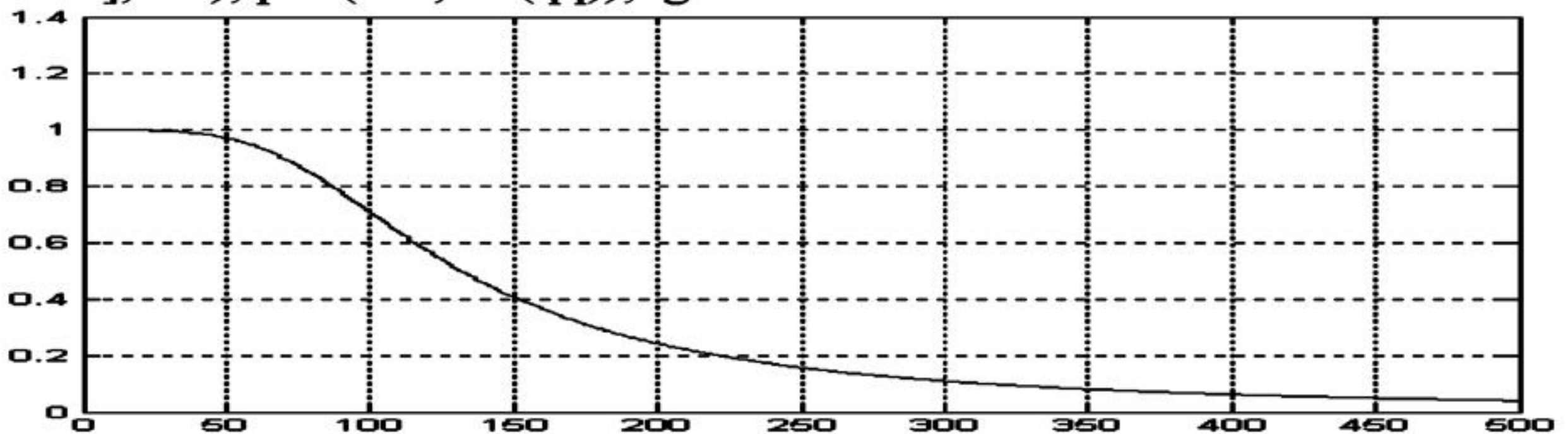
plot the frequency response

```
freqs( [10000], [1 141 10000])
```



### Example 76)

```
ww=0:1:500; qq=freqs( [10000], [1 141
10000],ww); plot(ww,abs(qq)); grid
```



## FAST FOURIER TRANSFORM

Fast Fourier Transform (FFT) is discrete Fourier Transform. It is a **fast algorithm** to calculate discrete Fourier Transform (DFT)

MATLAB command `fft` calculates FFT.

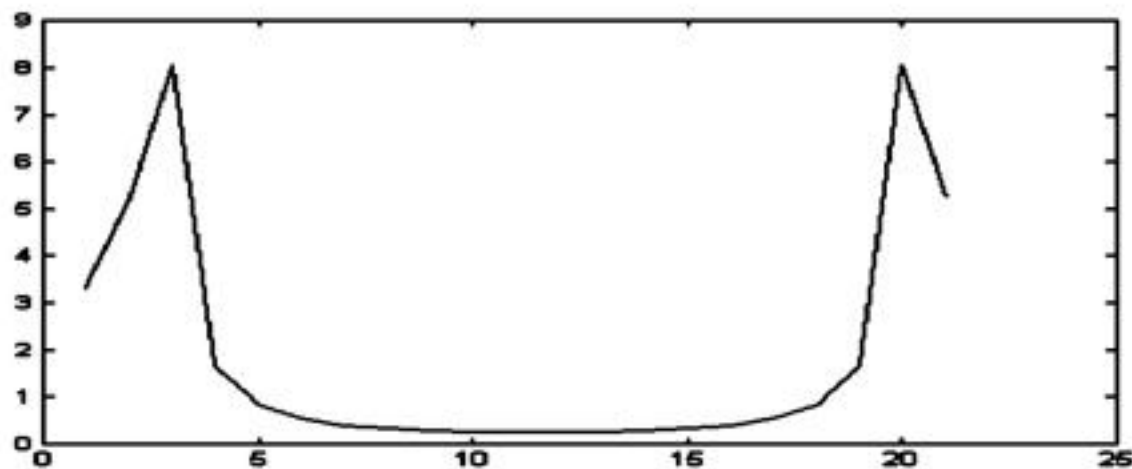
**Example.**  $x(t)=\sin(t)$ ; This is an analog (continuous) signal. We want to see the amplitude and phase spectrum of this signal.

**Sampling:**

```
t = [0 0.5 1 1.5 2 ..... 9.5 10]
x(n) = [ sin(0) sin(0.5) sin(1) ..... sin(9.5) sin(10) ]
x(n)=[0 0.48 0.84 ..... -0.075 -0.54];
```

MATLAB format is

```
t=0:0.5:10; x=sin(t);
frx = fft(x); framp=abs(frx); plot(framp);
```

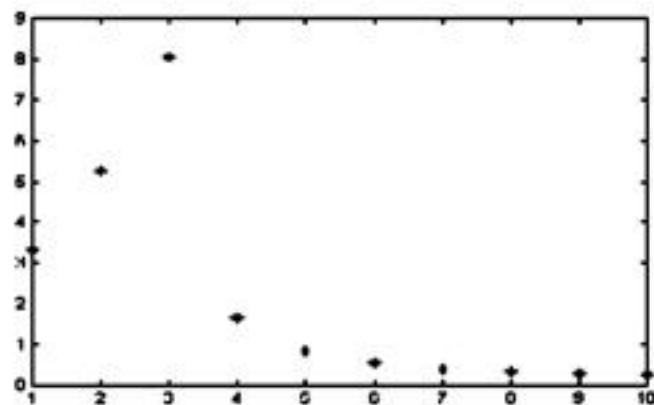


Since its symmetrical we draw only the half.

```
>>plot(framp(1:end/2));
```

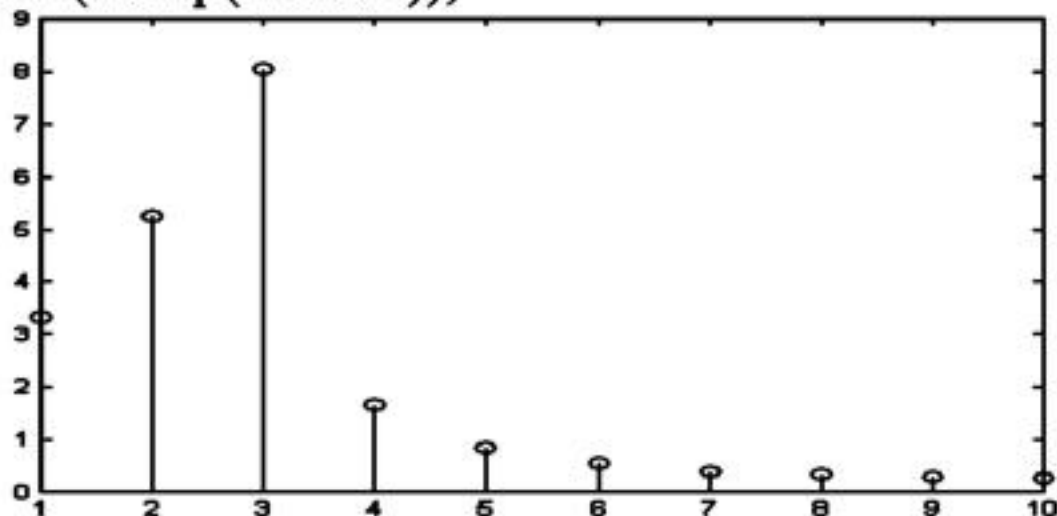
It may be a good idea to see only the values.

```
plot(framp(1:end/2));
```



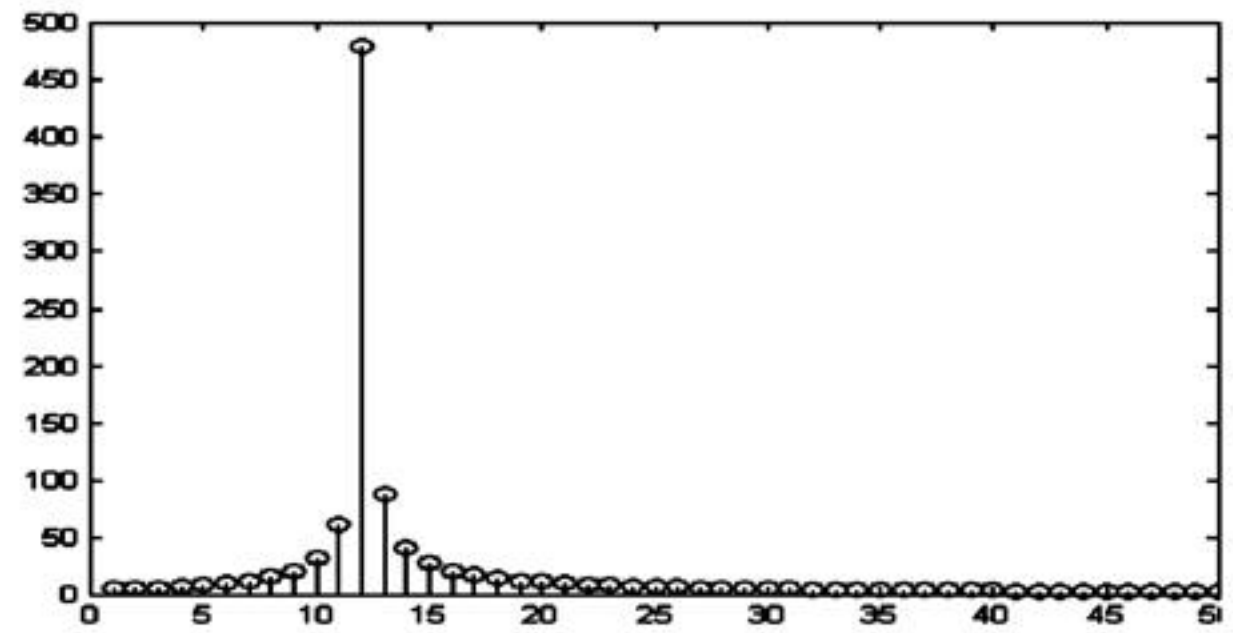
Or more clearly

```
stem(framp(1:end/2));
```



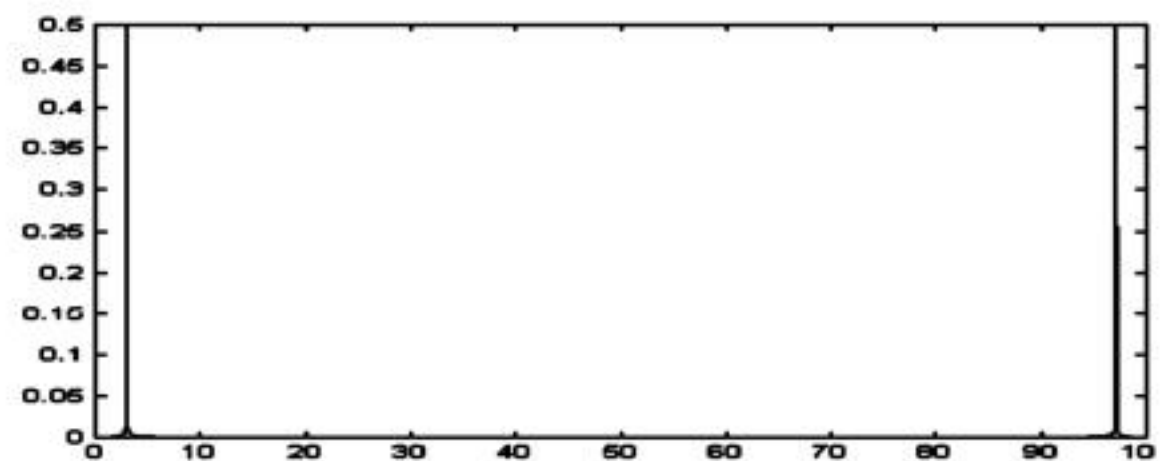
Try the followings

```
t=0:0.01:10; x=sin(7*t);
frx = fft(x); framp=abs(frx); stem(framp);
```



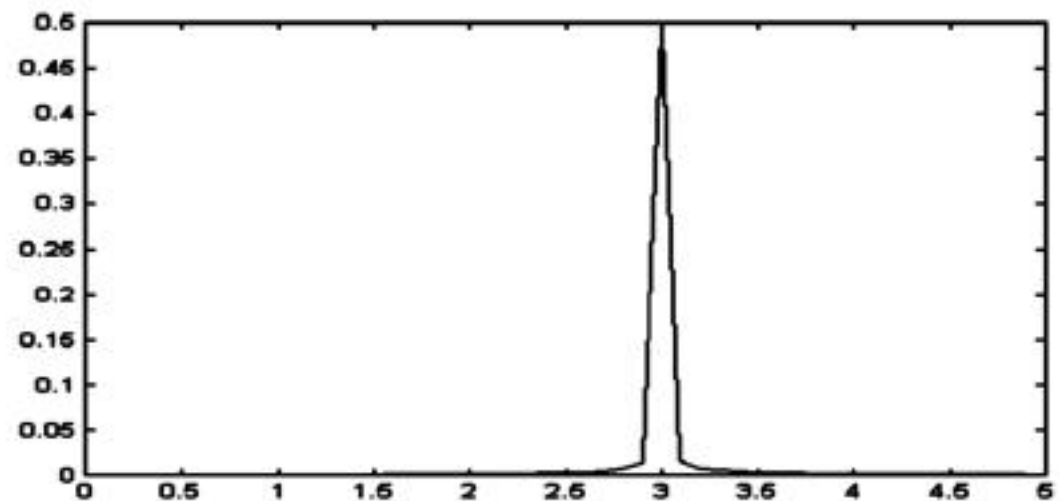
## SCALING THE FREQUENCY AXIS

```
t=0:0.01:10; x=sin(2*pi*3*t);
total_time=10;
NN=length(t);
frr=fft(x); framp=abs(frr)/NN;
Ts=total_time/NN; fs=1/Ts;
freq_axes=fs*[0:NN-1]/NN;
plot(freq_axes,framp);
```

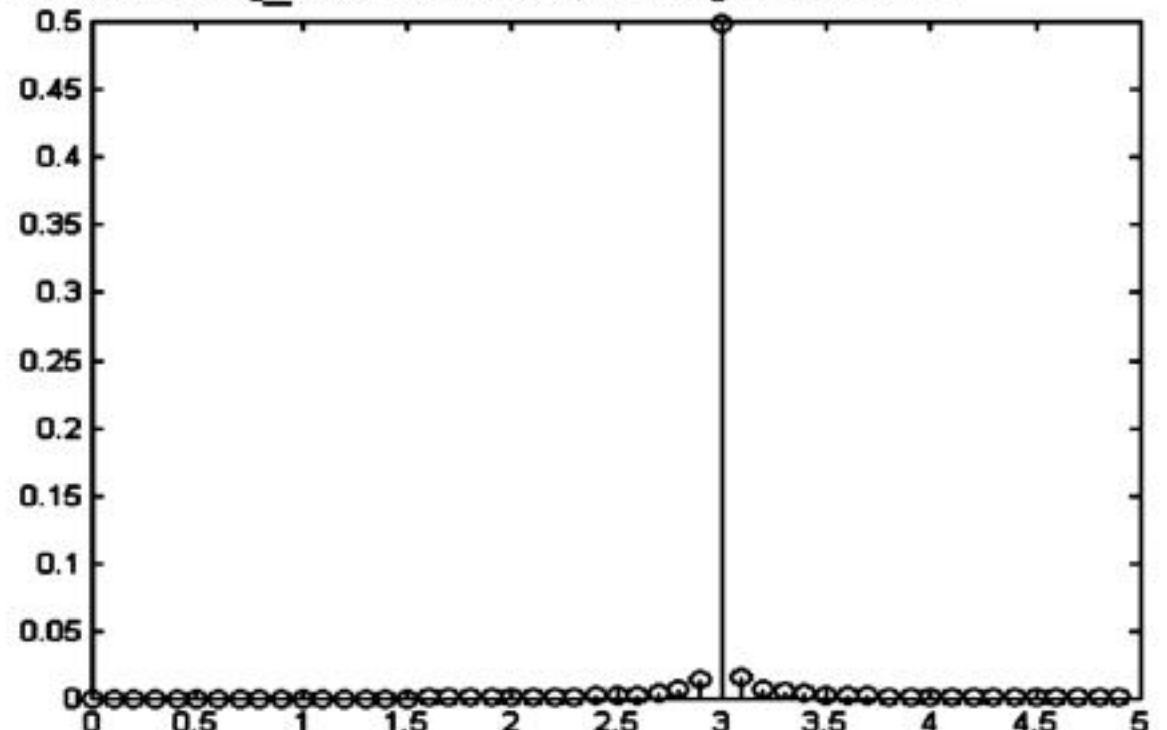


To see clearly

```
plot(freq_axes(1:50),framp(1:50));
```



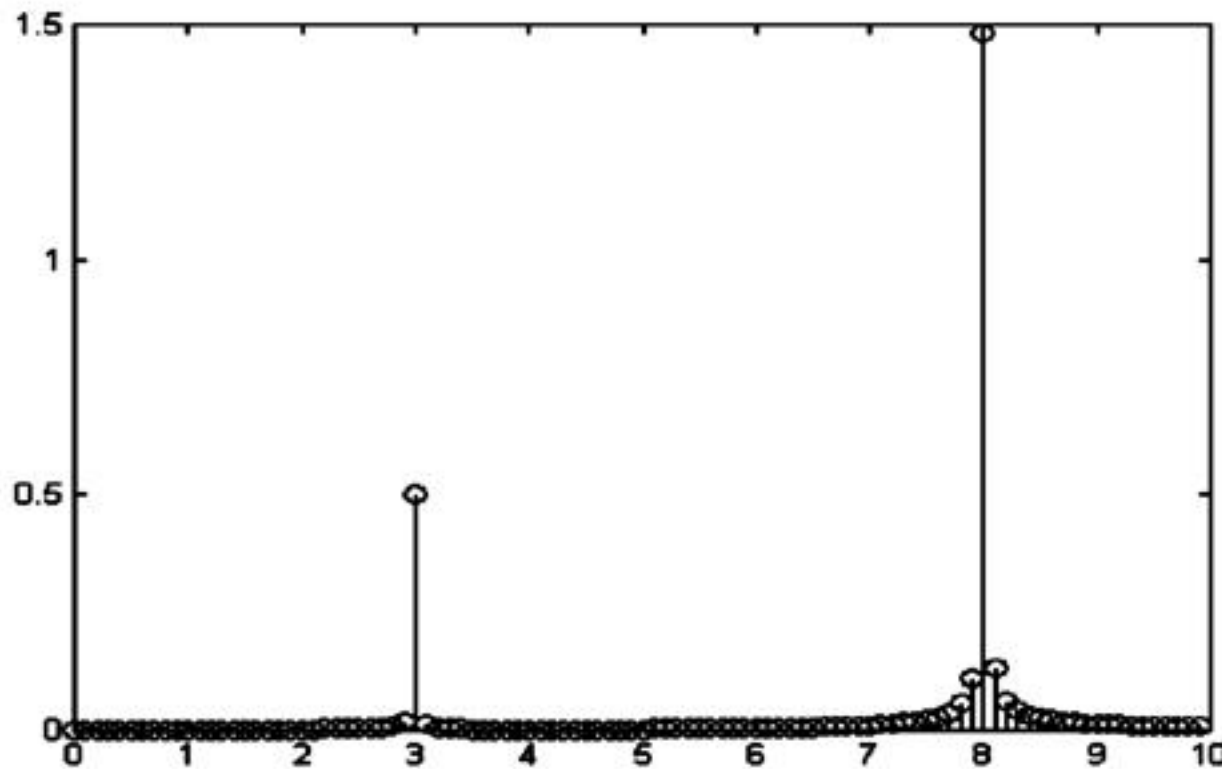
```
stem(freq_axes(1:50),framp(1:50));
```



```

x=sin(2*pi*3*t);
-----
t:0.01:10;
x=sin(2*pi*3*t)+ 3*sin(2*pi*8*t);
total_time=10;
NN=length(t);
frr=fft(x);    framp=abs(frr)/NN;
Ts=total_time/NN;
fs=1/Ts;
freq_axes=fs*[0:NN-1]/NN;
stem(freq_axes(1:100),framp(1:100),'k');

```

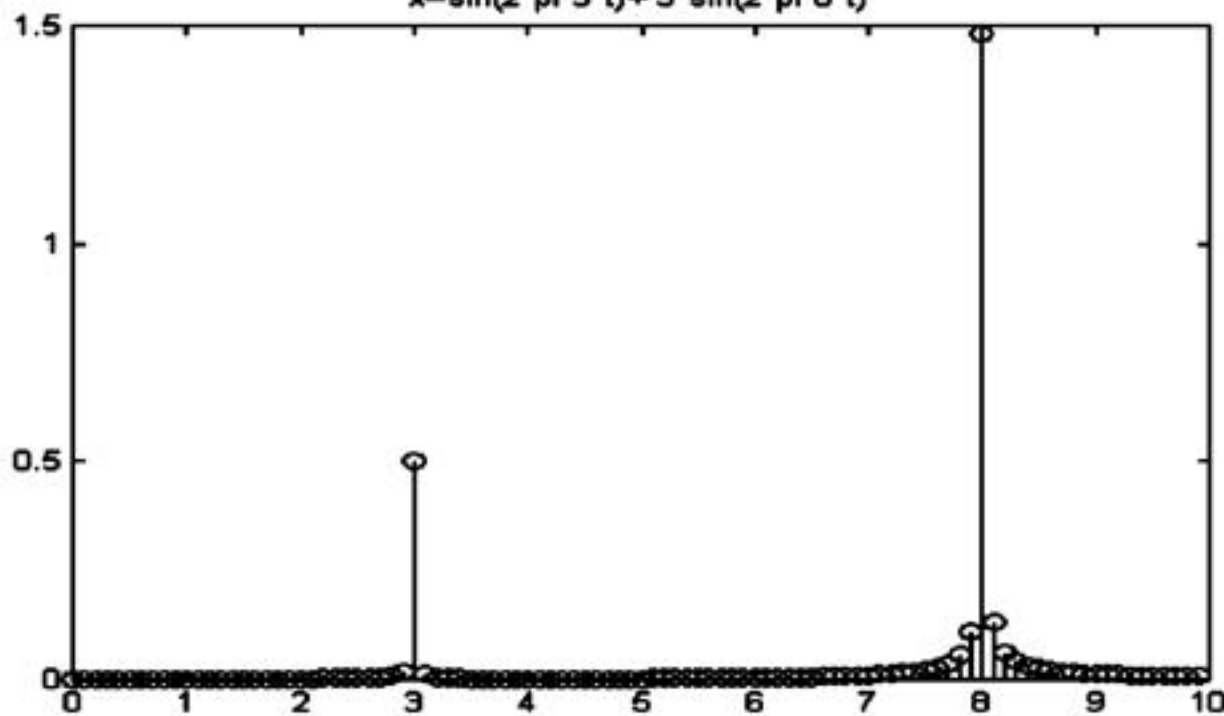


Put a title on the figure

```

title('x=sin(2*pi*3*t)+ 3*sin(2*pi*8*t)');
x=sin(2*pi*3*t)+ 3*sin(2*pi*8*t)

```



## CASE STUDIES

### Example 1.

Run the file. See the figure. Listen to the voice.

```

total_T=1;
NN=16000;
t_axis=([0:NN-1]/NN)*total_T;
xx= sin(2*pi*1000*t_axis);
Ts=total_T/NN; fs=1/Ts;
f_axis=([0:NN-1]/NN)*fs;
yy=abs(fft(xx));
subplot(211); plot(t_axis,xx); grid;
title('Time response ');
subplot(212); plot(f_axis,yy); grid;
title('Amplitude spectrum ');
wavplay(xx,16000)

```

Note: [0:5] ==> 0 1 2 3 4 5  
 [0:5]/5 ==> 0 0.2 0.4 0.6 0.8 1  
 30\*[0:5]/5 ==> 0 6 12 18 24 30  
 [0:NN-1]/NN)\* total\_T from zero to  
 total time. step size is total\_T/NN .

**Example 2.** Change frequency 400,300,200 and listen to the voice

```
sin(2*pi*400*t_axis);
```

**Example 3.** Change frequency 4000,5000,7000 and listen to the voice

```
sin(2*pi*7000*t_axis);
```

**Example 4.** Add two frequency listen to the voice see the spectrum

```

xx= sin(2*pi*500*t_axis)+
    sin(2*pi*7000*t_axis);

```

**Example 5.** Add three or more frequency listen to the voice see the spectrum

```

xx= sin(2*pi*500*t_axis)+
    sin(2*pi*2000*t_axis)+
    sin(2*pi*3000*t_axis) +
    sin(2*pi*7000*t_axis);

```

### Example 3

```

x= sin(2*pi*10*t)+ sin(2*pi*20*t);
x= 0.3*sin(2*pi*10*t)+ sin(2*pi*20*t);
x= sin(2*pi*10*t)+ 0.3*sin(2*pi*20*t);

```

```

x= sin(2*pi*10*t) + sin(2*pi*20*t)+
    sin(2*pi*30*t) + sin(2*pi*40*t);

```

sampling\_period = total\_time/Number\_of\_sampling;  
 sampling\_frequency = 1/sampling\_period;  
 freq\_axes=sampling\_frequency\*[1:NN]/NN;

```
>> plot(freq_axes,framp); grid;
```

```
>> plot(freq_axes(1:15),framp(1:15)); grid;
```

```
>> stem(freq_axes(1:15),framp(1:15)); grid;
```

```
t=0:0.5:10; x=sin(t); Ts=1; fs=1/0.5=2
```

time axes has 20 elements. Frequency axes has 20 elements.

t varies from zero to 10. (step size 0.5)

f varies from zero to 2. (step size is 0.1)  
freq\_axes=[ 0 0.1 0.2 0.3 0.4 ... 1.9 2]

Odev:

qq=[.....] seklinde bir dizi verilsin.

İki data arasındaki zaman 0.2milisaniye olarak veriliyor.

a)İsaretin frekans spektrumunu cizin.

b)İsaretin içindeki genliği en büyük olan frekans bileşenini bulun.

Program başlangıcında qq=[ 1 2 3 4 5 6 7] varsayarak programı yazın.

```
total_T=1;
NN=16000;
t_axis=( [0:NN-1]/NN)*total_T;
xx= sin(2*pi*1000*t_axis);
Ts=total_T/NN; fs=1/Ts;
f_axis=( [0:NN-1]/NN)*fs;
yy=abs(fft(xx));
subplot(211);
plot(t_axis(1:end/50),xx(1:end/50)); grid;
title('Time response ');
subplot(212); plot(f_axis,yy); grid;
title('Amplitude spectrum ');
wavplay(xx,16000)
```

```
clear all
total_T=0.5;
NN=16000;
tt=( [0:NN-1]/NN)*total_T;
w1=300; w2=w1*2; w3=w1*3; w4=w1*4;
w5=w1*5;
xa= [ sin(2*pi*w1*tt) sin(2*pi*1000*tt)
sin(2*pi*w2*tt) sin(2*pi*w3*tt)
sin(2*pi*w4*tt) sin(2*pi*w5*tt) ] ;

w1=800; w2=w1*2; w3=w1*3; w4=w1*4;
w5=w1*5;
xb= [ sin(2*pi*w1*tt) sin(2*pi*1000*tt)
sin(2*pi*w2*tt) sin(2*pi*w3*tt)
sin(2*pi*w4*tt) sin(2*pi*w5*tt) ] ;
xc=[xa xb];
wavplay(xc,16000)
```

## ANALOG FILTER DESIGN

```
[b,a] = butter(2,100,'s');
```

2<sup>nd</sup> order Lowpass Butterworth filter cutoff frequency 100 rad/sec=100\*2\* $\pi$ =314Hz

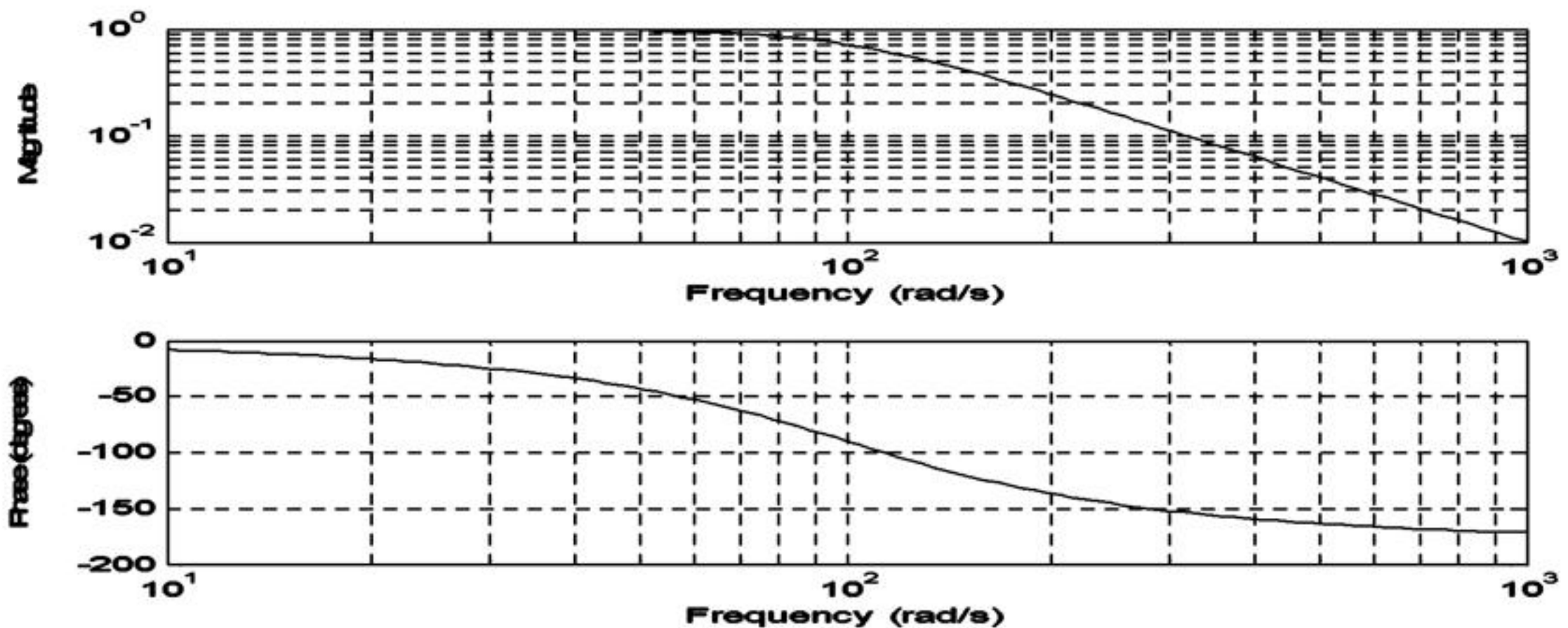
```
b =  
10000
```

```
a =  
1 141 10000
```

$$H(s) = \frac{10000}{s^2 + 141s + 10000}$$

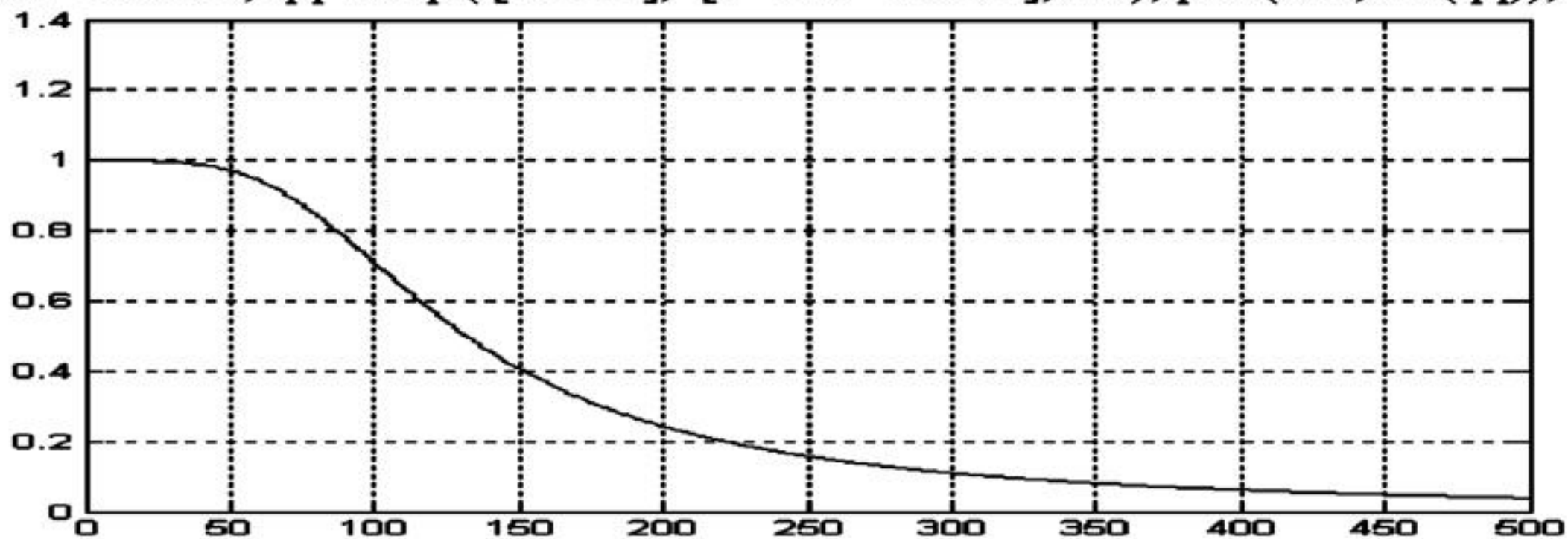
plot the frequency response

```
freqs([10000], [1 141 10000])
```



### Example 76)

```
ww=0:1:500; qq=freqs([10000], [1 141 10000],ww); plot(ww,abs(qq)); grid
```



At  $w=100$   $|H(jw)|=0.707$

### Example 77)

```
[b,a] = butter(2,100,'high','s')
```

```
b =  
1 0 0
```

```
a =  
1 141 10000
```

$$H(s) = \frac{s^2}{s^2 + 141s + 10000}$$

```
ww=0:1:500; qq=freqs( [1 0 0 ], [1 141 10000],ww); plot(ww,abs(qq)); grid
```



```
[b,a] = cheby1(4,1,[100 200],'s'); % Bandpass Chebyshev Type I
[b,a] = cheby2(6,60,[100],'high','s'); % Highpass Chebyshev Type II
[b,a] = ellip(3,1,60,[100 150],'stop','s'); % Bandstop elliptic
```

Butterworth  $[n, W_n] = \text{buttord}(W_p, W_s, R_p, R_s)$

Chebyshev Type I  $[n, W_n] = \text{cheb1ord}(W_p, W_s, R_p, R_s)$

Chebyshev Type II  $[n, W_n] = \text{cheb2ord}(W_p, W_s, R_p, R_s)$

Elliptic  $[n, W_n] = \text{ellipord}(W_p, W_s, R_p, R_s)$

## The relationship between LAPLACE, Z FOURIER, DISCRETE FOURIER TRANSFORM

$$\mathcal{L}\{g(t)\}=G(s) \quad \mathcal{L}\{f(nT)\}=F(z) \text{ or } \mathcal{Z}\{f(nT)\}=F(z)$$

$g(t) \rightarrow$  continuous (analog signal)

$f(nT) \rightarrow$  discrete (digital) signal.

$$\mathcal{F}\{g(t)\}=G(j\omega) \quad \mathcal{F}\{f(nT)\}=F(e^{j\omega T})$$

If all poles of  $G(s)$  are on the left plane then  $G(j\omega)=G(s)|_{s=j\omega}$

If all poles of  $F(z)$  are inside unit circle then  $F(e^{j\omega})=F(z)|_{z=e^{j\omega}}$

### Examples

$$H(s)=\frac{10(s+2)}{(s+7)} \rightarrow H(j\omega)=\frac{10(j\omega+2)}{(j\omega+7)}$$

$$H(s)=\frac{10(s+2)}{(s-7)} \rightarrow H(j\omega) \text{ does not exist}$$

$$H(s)=\frac{10(s+2)}{(s-7)(s+3)} \rightarrow H(j\omega) \text{ does not exist}$$

$$h(t)=u(t) \rightarrow H(s)=\frac{1}{s} \quad H(j\omega) \text{ exists but } H(j\omega) \neq G(s)|_{s=j\omega} \quad H(j\omega)=\frac{1}{j\omega} + \delta(j\omega)$$

$$H(s)=\frac{10(s+2)}{(s^2+4)} \rightarrow H(j\omega) \text{ exists but } H(j\omega) \neq G(s)|_{s=j\omega}$$

### POLES and ZEROS of transfer functions

$$H(s)=\frac{10(s+2)}{(s+7)} \quad \text{This function has a zero at } s=-2 \text{ and has a pole at } s=-7,$$

If  $s=-2$  then  $H(s)=0$  (zero at  $s=-2$ )

if  $s=-7$   $H(s)=\infty$  (pole at  $s=-7$ )

$$H(s)=\frac{(s+1)(s-2)(s+3)}{(s+4)(s-5)(s-6)(s+7)} \quad \text{This function has three zeros at } s=-1, s=+2, s=-3,$$

and has four poles at  $s=-4, s=+5, s=+6, s=-7$

$$H(s)=\frac{(s+5+9j)(s-8-15j)}{(s+2-j3)(s+6-11j)} \quad \text{This function has two zeros at } s=-5-9j, s=+8+15j,$$

and has two poles at  $s=-2+3j, s=-6+11j$

$H(s) = \frac{(s+5+9j)(s+5-9j)}{(s-2+j3)(s-2-3j)}$  This function has two zeros at  $s=-5-9j$ ,  $s=-5+9j$ ,  
and has two poles at  $s=2-3j$ ,  $s=2+3j$

$H(s) = \frac{s^2 + 4s + 13}{s^2 + 5s + 6}$ , This function has two zeros at  $s=-2+3j$ ,  $s=-2-3j$ ,  
and has two poles at  $s=-2$ ,  $s=-3$

$H(s) = \frac{s^4 + 6s^3 + 23s^2 + 34s + 26}{s^5 + 10s^4 + 51s^3 + 156s^2 + 302s + 260}$ , This function has four zeros at  $-1+j$   $-1-j$   $-2+3j$   $-2-3j$   
and has five poles at  $-1+3j$ ,  $-1-3j$ ,  $-3+2j$ ,  $-3-2j$ ,  $-2$

**MATLAB commands**

>>roots([1 4 13])  
will find the roots of  $s^2 + 4s + 13$ ,  $s=-2+3j$ ,  $s=-2-3j$ ,

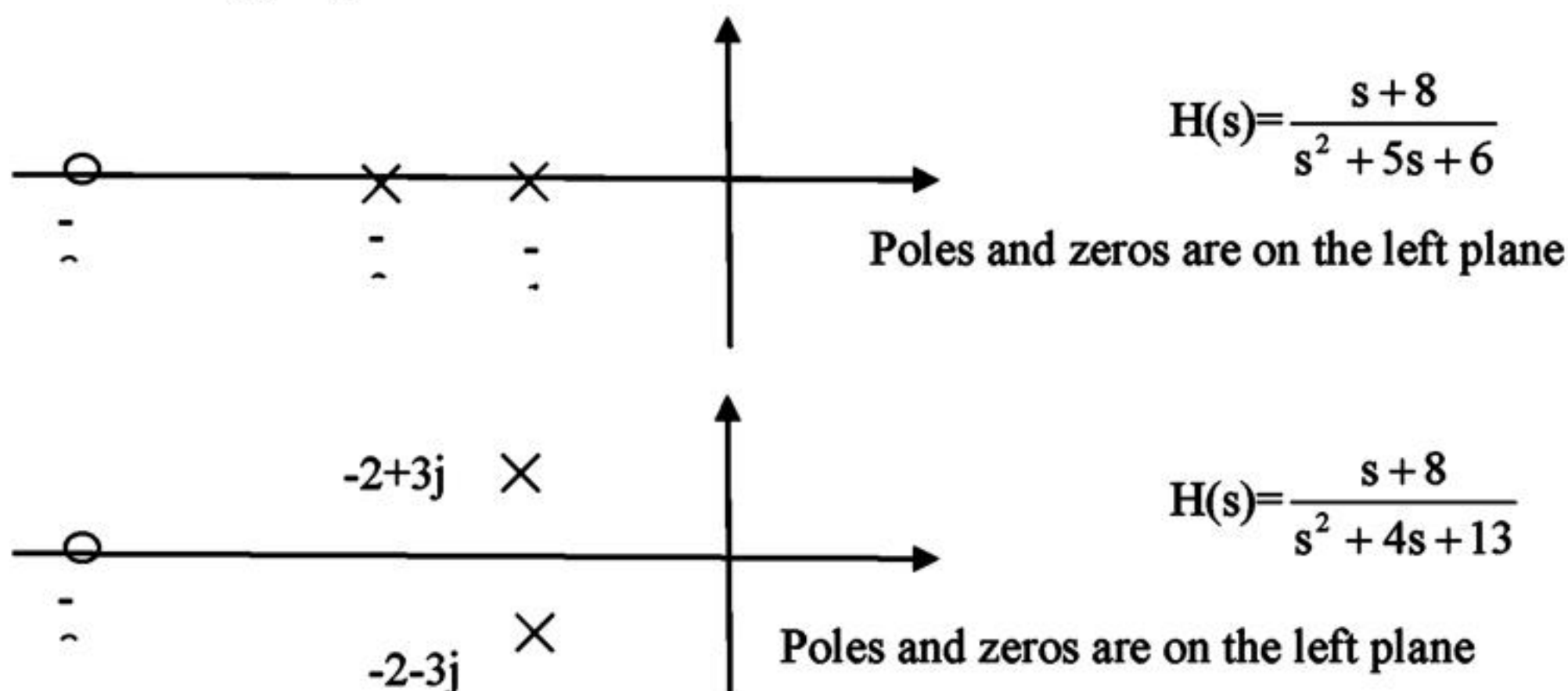
>>roots([1 10 51 156 302 260])  
will find the roots of  $s^5 + 10s^4 + 51s^3 + 156s^2 + 302s + 260$   
 $-1+3j$ ,  $-1-3j$ ,  $-3+2j$ ,  $-3-2j$ ,  $-2$

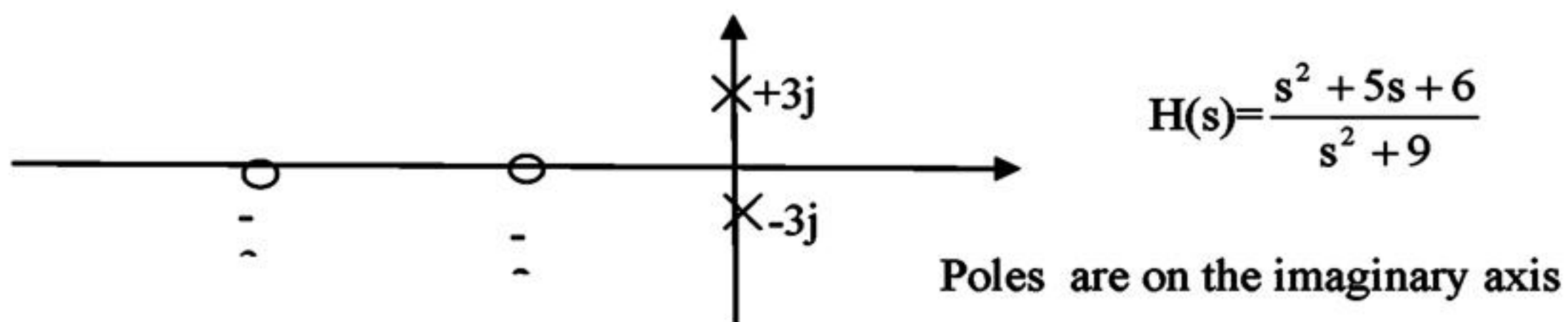
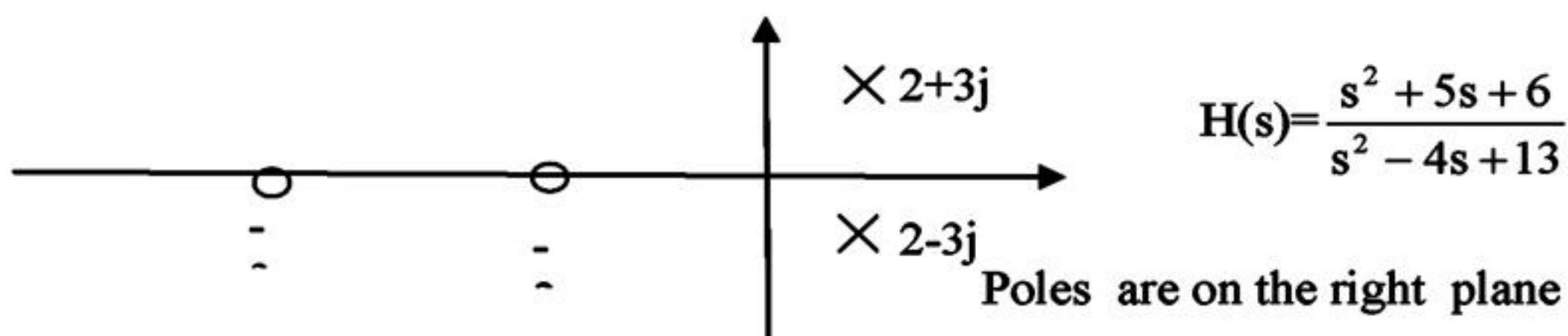
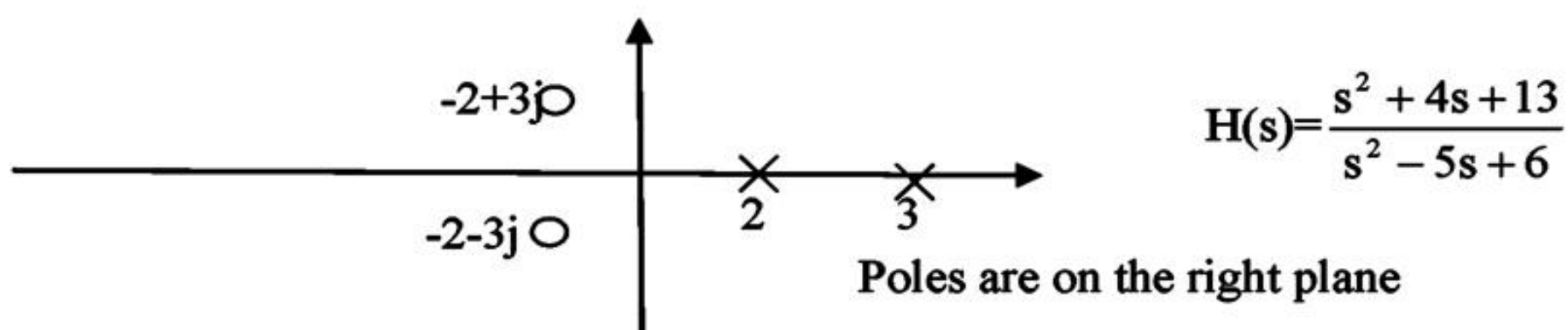
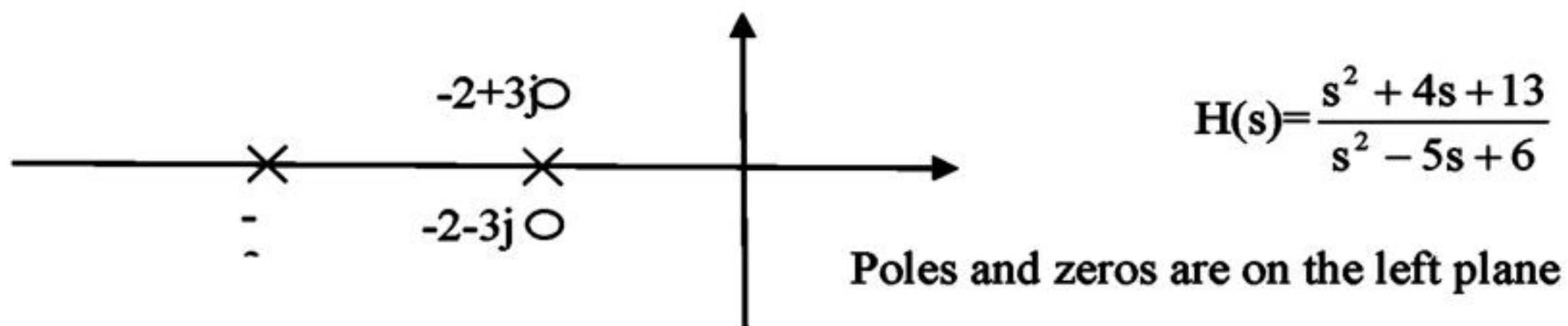
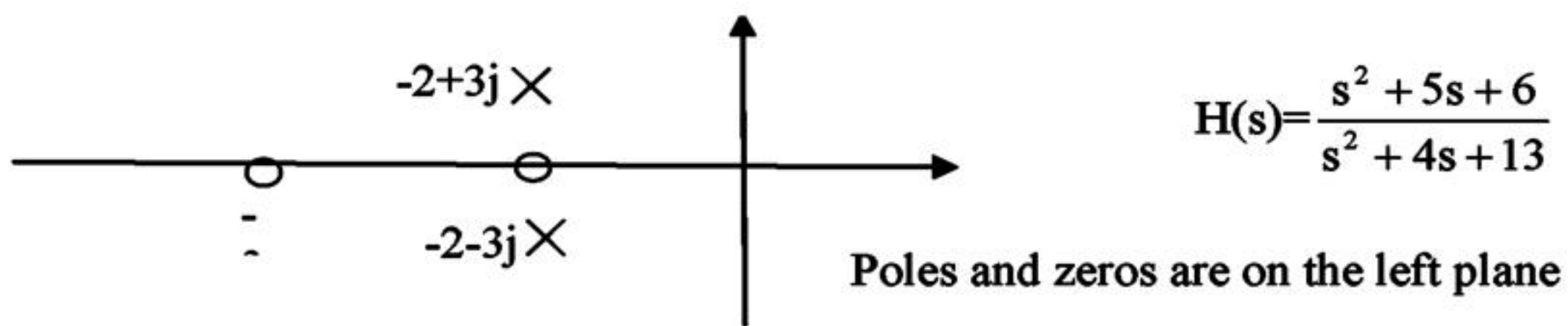
>>poly([-2 -3])  
Will produce the polynomial,  
1 5 6  $\rightarrow s^2 + 5s + 6$

>>poly([-2+3j -2-3j])  
Will produce the polynomial,  
1 4 13  $\rightarrow s^2 + 4s + 13$

>>poly([-1+3j -1-3j -3+2j -3-2j -2])  
Will produce the polynomial,  
1 10 51 156 302 260  $\rightarrow s^5 + 10s^4 + 51s^3 + 156s^2 + 302s + 260$

## Pole zero graphics





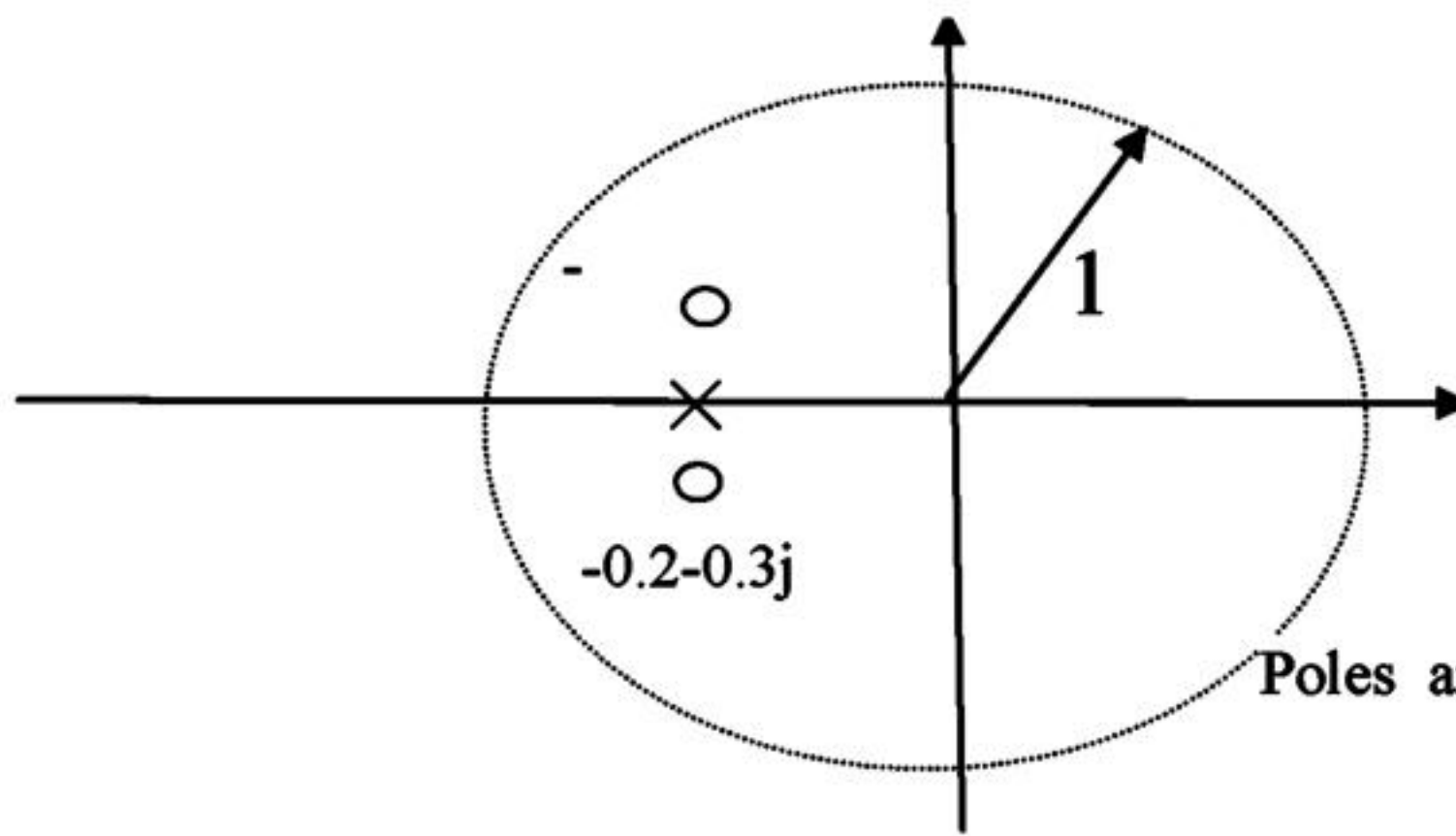
$$H(s) = \frac{(s+1)(s-2)(s+3)}{(s+4)(s-5)(s+6)(s+7)}$$

This function has one pole on the right half plane ( $s=5$ ).

$$H(s) = \frac{s^4 + 6s^3 + 23s^2 + 34s + 26}{s^5 + 10s^4 + 51s^3 + 156s^2 + 302s + 260}$$

This function has all poles are on the left plane

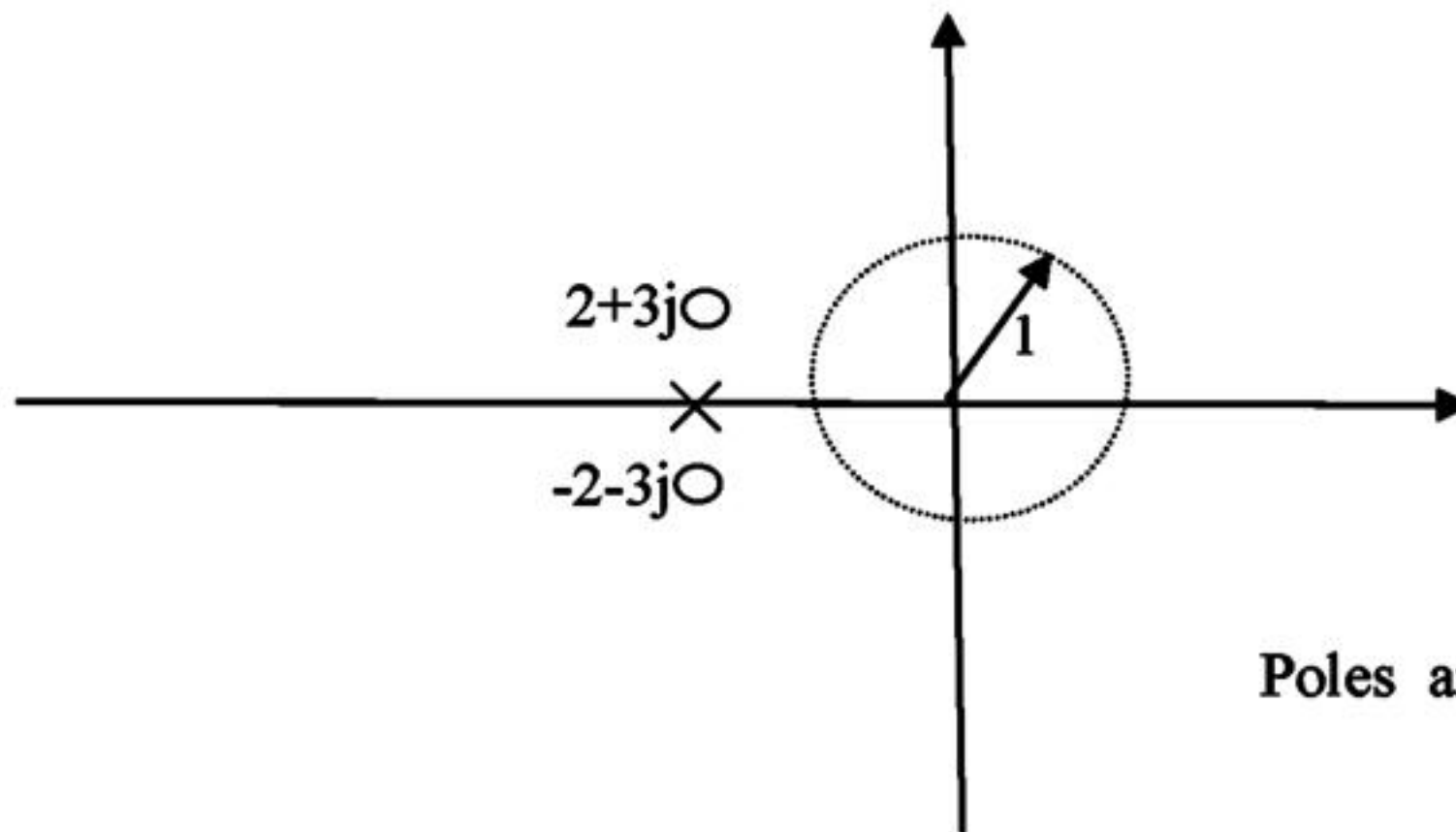
(poles  $-1+3j$ ,  $-1-3j$ ,  $-3+2j$ ,  $-3-2j$ ,  $-2$ )



$$H(s) = \frac{s^2 + 0.4s + 0.13}{s + 0.2}$$

Poles and zeros are inside unit circle

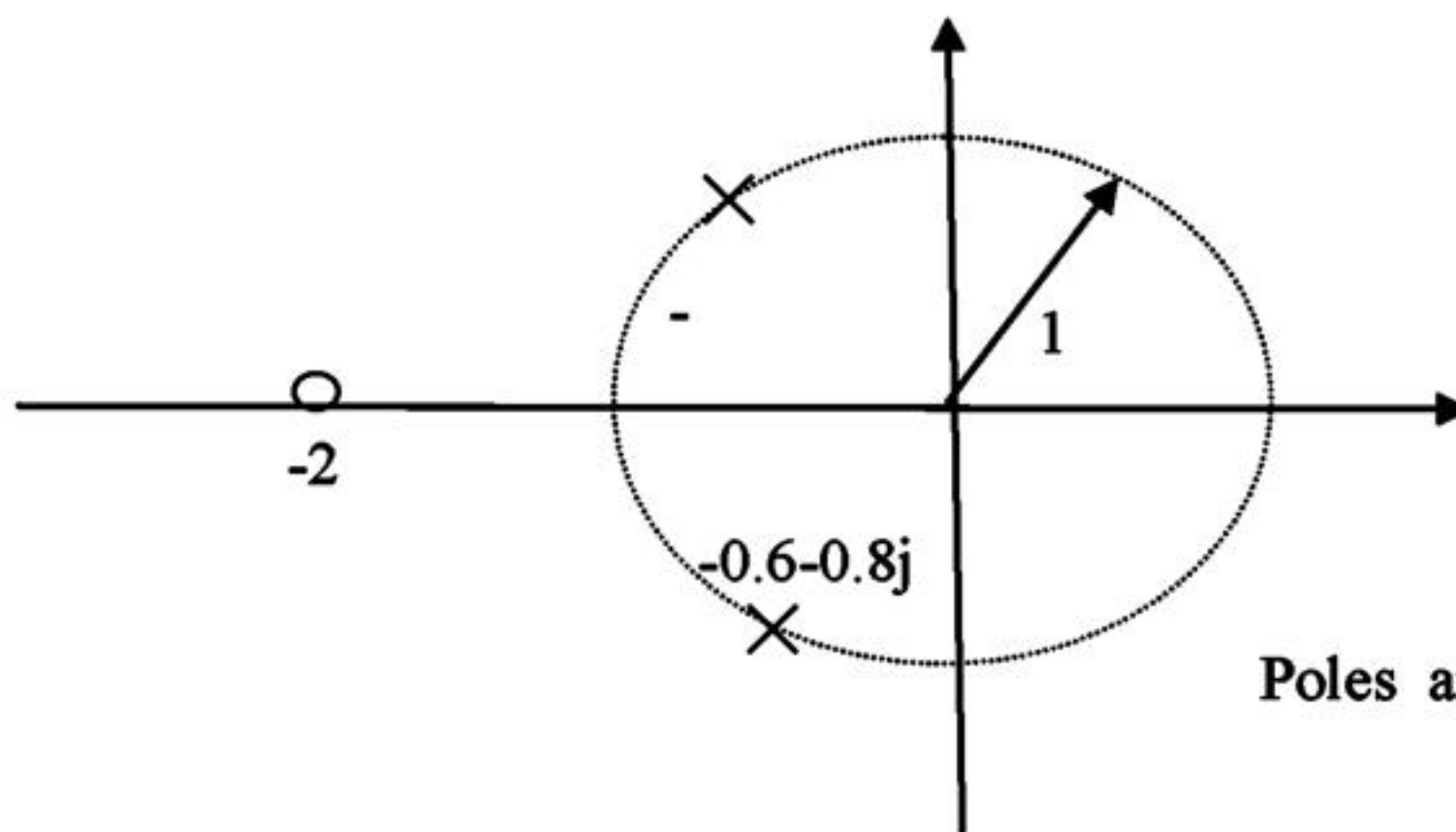
$$\sqrt{0.2^2 + 0.3^2} = 0.36 < 1 \text{ inside unit circle}$$



$$H(s) = \frac{s + 3}{s^2 + 4s + 13}$$

Poles and zeros are outside of the unit circle

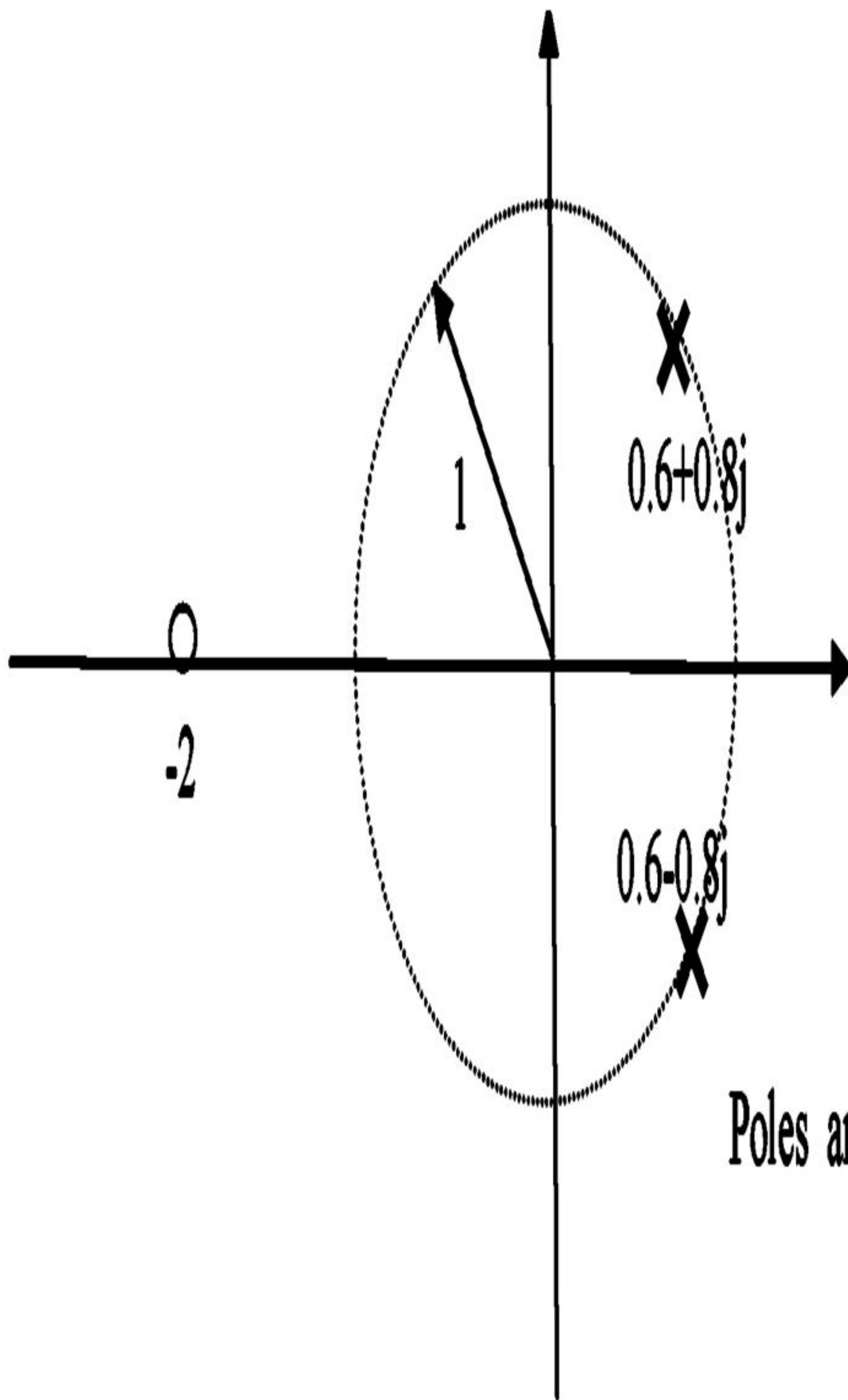
$$\sqrt{2^2 + 3^2} = 3.6 > 1 \text{ outside unit circle}$$



$$H(s) = \frac{s + 2}{s^2 + 1.2s + 1}$$

Poles and on the unit circle

$$\sqrt{0.6^2 + 0.8^2} = 1 \text{ on the unit circle}$$



$$H(s) = \frac{s+2}{s^2 + 1.2s + 1}$$

Poles and on the unit circle

$$\sqrt{0.6^2 + 0.8^2} = 1 \text{ on the unit circle}$$