Dijkstra's Algorithm for Shortest Path

1. Initial step

Vertex l gets PL: $L_1 = 0$.

Vertex j (= 2, ….., n) gets TL: $\bar{L}_j = \bar{L}_{ij}$

{=∞ if there is no edge (l,j)

Set PL={1}, TL ={2,3,…n}

2. Fixing a permanent label

2.a Find a k in TL for which $\bar{L}_K$ is minimum,

2.b Set $L_K = \bar{L}_K$. (Take the smallest if equal)

2.c Transfer k from TL into PL.

If TL is empty then Stop

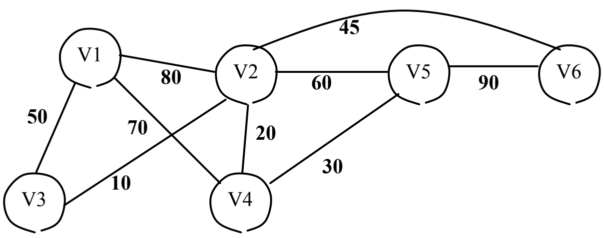
Else continue (that is, go to Step 3).

3. Updating temporary labels

For all j in TL set $\bar{L}_j = \min(\bar{L}_j, \bar{L}_K + \bar{L}_{Kj})$

Go to Step 2.

Find the shortest lengths of the following graph using Dijkstra's Algortihm



Set initial lengths and initial paths.

$L_1=0$

$\bar{L}_2=80$ (the length between V1 and V2)

$\bar{L}_3=50$ (the length between V1 and V3)

$\bar{L}_4=70$ (the length between V1 and V4)

$\bar{L}_5=\infty$ (No connection between V1 and V5)

$\bar{L}_6=\infty$ (No connection between V1 and V6)

PL:Permanent length **TL**:Temporary length

PL={1} TL={2,3,4,5,6}

$FV_2=1$, $FV_3=1$, $FV_4=1$, $FV_5=1$, $FV_6=1$

$FV_2=1$: **Initial assumption**: the shortest path between V_2 and V_1 is the direct path. V_1—V_2

$FV_3=1$: **Initial assumption**: the shortest path between V_3 and V_1 is the direct path V_1—V_3

Step 1.1 Find the minimum of $\{\bar{L}_2, \bar{L}_3, \bar{L}_4, \bar{L}_5, \bar{L}_6\}$

$\min\{\bar{L}_2, \bar{L}_3, \bar{L}_4, \bar{L}_5, \bar{L}_6\}=$

$\min\{80,50,70,\infty,\infty\}=50$

Step 1.2 Find k. k=3 → $L_3=\bar{L}_3=50$

Step 1.3 Transfer $\mathbf{L_3}$ from **PL** to **TL**.

PL={1,3} TL={2,4,5,6}

Step 2: Calculate new values for $\bar{L}_2, \bar{L}_4, \bar{L}_5, \bar{L}_6$

$\bar{L}_2 = \min\{\bar{L}_2, L_3 + L_{32}\}$

$= \min\{80,50+10\}=60$

$\bar{L}_2 > L_3 + L_{32}$ → **$FV_2=3$**

$\bar{L}_4 = \min\{\bar{L}_4, L_3 + L_{34}\}$

$= \min\{70,50+\infty\}=70$

$\bar{L}_4 < L_3 + L_{34}$ → **No change**

$\bar{L}_5 = \min\{\bar{L}_5, L_3 + L_{35}\}$

$= \min\{\infty,50+\infty\}=\infty$

$L_3 + L_{35} = \infty$ → **No change**

$\bar{L}_6 = \min\{\bar{L}_6, L_3 + L_{36}\}$

$= \min\{\infty,50+\infty\}=\infty$

$L_3 + L_{36} = \infty$ → **No change**

Result :

$L_3 = 50$, $\bar{L}_2 = 60$, $\bar{L}_4 = 70$, $\bar{L}_5=\infty$, $\bar{L}_6=\infty$

$FV_2=$**3**, $FV_3=1$, $FV_4=1$, $FV_5=1$, $FV_6=1$

CYCLE 2

Step 1: Find the minimum of $\{\bar{L}_2, \bar{L}_4, \bar{L}_5, \bar{L}_6\}$

$\min\{\bar{L}_2, \bar{L}_4, \bar{L}_5, \bar{L}_6\}=$

$\min\{60,70,\infty,\infty\}=60$

k=2

$L_2=\bar{L}_2=60$

PL={1,3,2} TL={4,5,6}

Step 2:

$\bar{L}_4 = \min\{\bar{L}_4, L_2 + L_{24}\}$

$= \min\{70,60+20\}=70$

$\bar{L}_4 < L_2 + L_{24}$ → **No change**

$\bar{L}_5 = \min\{\bar{L}_5, L_2 + L_{25}\}$

$= \min\{\infty,60+60\}=120$

$\bar{L}_5 > L_2 + L_{25}$ → **$FV_5= 2$**

$\bar{L}_6 = \min\{\bar{L}_6, L_2 + L_{26}\}$

$= \min\{\infty,60+45\}=105$

$\bar{L}_6 > L_2 + L_{26}$ → **$FV_6=2$**

Result :

$L_3 = 50$, $L_2 = 60$, $\bar{L}_4 = 70$, $\bar{L}_5=120$, $\bar{L}_6=105$

$FV_2=$**3**, $FV_3=1$, $FV_4=1$, $FV_5=$**2**, $FV_6=$**2**

CYCLE 3

Step 1:Find the minimum of $\{\overline{L}_4, \overline{L}_5, \overline{L}_6\}$

$\min\{\overline{L}_4, L_5, L_6\}=$
$\min\{70,120, 105\}=70$

$k=4$ and $L_4=\overline{L}_4=70$

$PL=\{1,3,2,4\}$ $TL=\{5,6\}$

Step 2:

$\overline{L}_5 = \min\{\overline{L}_5, L_4+ L_{45}\}$
$= \min\{120,70+30\}=100$

$\overline{L}_5 > L_4+ L_{45}$ → **$FV_5=4$**

$\overline{L}_6 = \min\{\overline{L}_6, L_4+ L_{46}\}$
$= \min\{105,70+\infty\}=105$

$\overline{L}_6 < L_5+ L_{56}$ → **No change**

Result :

$L_3 = 50$, $L_2 = 60$, $L_4 = 70$, $\overline{L}_5=100$, $\overline{L}_6=105$

$FV_2=3$, $FV_3=1$, $FV_4=1$, $FV_5=4$, $FV_6=2$

CYCLE 4

Step 1: $\min\{\overline{L}_5, \overline{L}_6\}=$
$\min\{100,120\}=100$,

$k=5$, and $L_5=\overline{L}_5=100$

$PL=\{1,3,2,4,5\}$ $TL=\{6\}$

Step 1:

$\overline{L}_6 = \min\{\overline{L}_6, L_5+ L_{56}\}=$
$\min\{105,100+90\}=105$

$\overline{L}_6 < L_5+ L_{56}$ → **No change**

Last Result :

$L_3 = 50$, $L_2 = 60$, $L_4 = 70$, $L_5=100$, $L_6=105$

$FV_2=3$, $FV_3=1$, $FV_4=1$, $FV_5=4$, $FV_6=2$

The Shortest Paths

$FV_3=1$: Vertex 3 follows vertex 1.
Vertex 3 is after vertex 1.
The shortest length from 1 to 3 is via the direct connection between vertex 1 and vertex 3.

V1 → V3 ($L_3=50$)

$FV_4=1$: Vertex 4 follows vertex 1.
Vertex 4 is after vertex 1.
The shortest length from 1 to 4 is via the direct connection between vertex 1 and vertex 4.

V1 → V4 ($L_4=70$)

$FV_2=3$: Vertex 2 follows vertex 3.
Vertex 3 follows vertex 1

V1 → V3 → V2 ($L_2=50+10=60$)

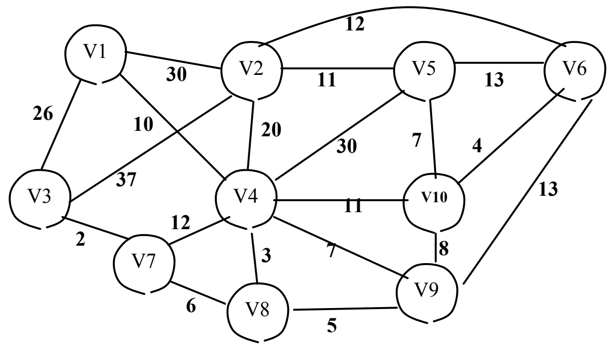
$FV_5=4$: Vertex 5 follows vertex 4.
Vertex 4 follows vertex 1.

V1 →V 4 →V 5 ($L_5=70+30=100$)

$FV_6=2$: Vertex 6 follows vertex 2.
Vertex 2 follows vertex 3
Vertex 3 follows vertex 1

V1 → V3 → V2→ V6 ($L_6=50+10+45=105$)

Example 2.
Find the shortest lengths from v3 to all vertices in the following graph using Dijkstra's Algortihm



Set initial lengths and initial paths.

$\overline{L}_1=26$, (the length between V3 and V1)

$\overline{L}_2=37$, $\overline{L}_3=0$, $\overline{L}_4=\overline{L}_5=\overline{L}_6=\overline{L}_8=\overline{L}_9=\overline{L}_{10}=\infty$, $\overline{L}_7=2$

$PL=\{3\}$ $TL=\{1,2,4,5,6,7,8,9,10\}$

$FV_1=FV_2=FV_4=FV_5=FV_6=FV_7=FV_8=FV_9=FV_{10}=3$,

CYCLE 1

$\min\{\overline{L}_1,\overline{L}_2, \overline{L}_4, \overline{L}_5, \overline{L}_6, \overline{L}_7, \overline{L}_8,\overline{L}_9,\overline{L}_{10}\}=2$

$k=7$, $PL=\{3,7\}$ $TL=\{1,2,4,5,6,8,9,10\}$, $\overline{L}_7=L_7=2$

$\overline{L}_1 = \min\{\overline{L}_1, L_7+ L_{71}\}=\min\{26,2+\infty\}=26$

$\overline{L}_2 = \min\{\overline{L}_2, L_7+ L_{72}\}=\min\{37,2+\infty\}=37$

$\overline{L}_4 = \min\{\overline{L}_4, L_7+ L_{74}\}=\min\{\infty,2+12\}=14$, **$FV_4=7$**

$\overline{L}_5 = \min\{\overline{L}_5, L_7+ L_{75}\}=\min\{\infty,2+\infty\}=\infty$

$\overline{L}_6 = \min\{\overline{L}_6, L_7+ L_{76}\}=\min\{\infty,2+\infty\}=\infty$

$\overline{L}_8 = \min\{\overline{L}_8, L_7+ L_{78}\}=\min\{\infty,2+6\}=8$ **$FV_8=7$**

$\overline{L}_9 = \min\{\overline{L}_9, L_7+ L_{79}\}=\min\{\infty,2+\infty\}=\infty$

$\overline{L}_{10} =\min\{\overline{L}_{10}, L_7+ L_{7,10}\}=\min\{\infty,2+\infty\}=\infty$

$FV_1=FV_2=FV_5=FV_6=FV_7 =FV_9=FV_{10}=3$, $FV_4=7,FV_8=7$

CYCLE 2

$\min\{\overline{L}_1,\overline{L}_2, \overline{L}_4, \overline{L}_5, \overline{L}_6,\overline{L}_8,\overline{L}_9,\overline{L}_{10}\}=8$

$k=8$, $PL=\{3,7,8\}$ $TL=\{1,2,4,5,6,9,10\}$, $\overline{L}_8=L_8=8$

$\overline{L}_1 = \min\{\overline{L}_1, L_8+ L_{81}\}=\min\{26,8+\infty\}=26$

$\overline{L}_2 = \min\{\overline{L}_2, L_8+ L_{82}\}=\min\{37,8+\infty\}=37$

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