Ford-Fulkerson Algorithm for Maximum Flow

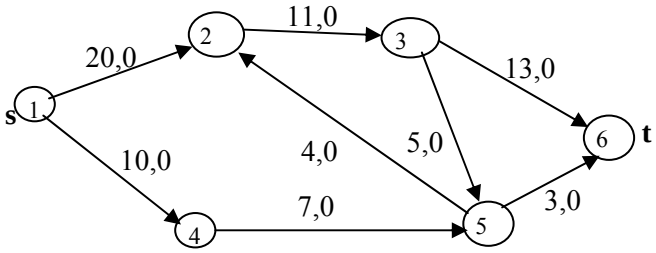
1. Assign an initial flow f_{ij} (for instance, $f_{ij}=0$) for all edges
2. Find all augmenting paths.

For path=1 to number_of_augmenting_paths

3. Select an augmenting path (Select all forward edges first)
4. Calculate the minimum additional flow to that path.
5. Add the founded additional flow to that path.

End

Example: Find the maximum flow from **s** to **t** in the following graph.



Solution

1) Set Initial flows

$f_{12}=f_{23}=f_{36}=f_{35}=f_{14}=f_{45}=f_{52}=f_{56}=0$,

$C_{12}=20$, $C_{23}=11$, $C_{36}=13$, $C_{35}=5$, $C_{14}=10$, $C_{45}=7$, $C_{52}=4$, $C_{56}=3$,

2) Augmenting Paths.

P1: 1-2-3-6 P2: 1-2-3-5-6
P3: 1-2-5-6 P4: 1-2-5-3-6
P5: 1-4-5-6 P6: 1-4-5-3-6
P7: 1-4-5-2-3-6

Note: P1,P2,P5,P7 have all forward edges
P3,P4,P6 have forward and backward edges

CYCLE 1

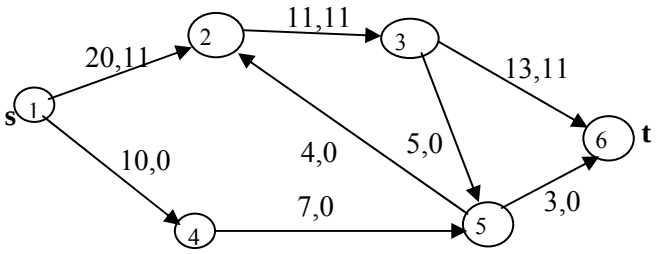
3) Select P1: 1-2-3-6

4) $\Delta_{12}= C_{12} - f_{12} = 20 - 0=20$
$\Delta_{23} = C_{23} - f_{23} = 11 - 0=11$
$\Delta_{36} = C_{36} - f_{36} = 13 - 0=13$

Minimum additional flow:
$\Delta t =\text{Min}(\Delta_{12}, \Delta_{23}, \Delta_{36})= \text{Min}(20,11,13)=11.$

Add this flow to path P1.

5) $f_{12}= f_{12}+ \Delta t$, $f_{23}= f_{23}+ \Delta t$, $f_{36}= f_{36}+ \Delta t$
$f_{12}=0+11=11$, $f_{23}=0+11=11$, $f_{36}=0+11=11$



CYCLE 2

3) Select P2: 1-2-3-5-6

4) $\Delta_{12}= C_{12} - f_{12} = 20 - 11=9$
$\Delta_{23} = C_{23} - f_{23} = 11 - 11=0$
$\Delta_{35} = C_{35} - f_{35} = 5 - 0=5$
$\Delta_{56} = C_{56} - f_{56} = 3 - 0=3$

$\Delta t =\text{Min}(\Delta_{12}, \Delta_{23}, \Delta_{35},\Delta_{56})= \text{Min}(9,0,5,3)=0$
$\Delta t =0$ No addition

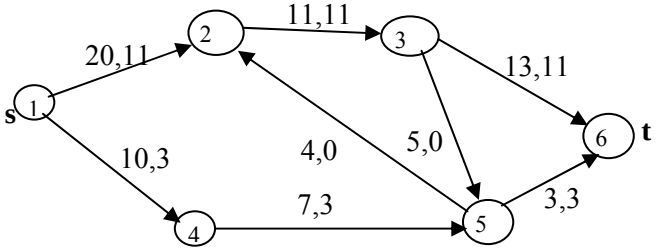
CYCLE 3

3) P5: 1-4-5-6

4) $\Delta_{14}= C_{14} - f_{14} = 10 - 0=10$
$\Delta_{45} = C_{45} - f_{45} = 7 - 0=7$
$\Delta_{56} = C_{56} - f_{56} = 3 - 0=3$

$\Delta t =\text{Min}(\Delta_{14}, \Delta_{45},\Delta_{56})= \text{Min}(10,7,3)=3$
$\Delta t =3$

5) $f_{14}= f_{14}+ \Delta t$, $f_{45}= f_{45}+ \Delta t$, $f_{56}= f_{56}+ \Delta t$
$f_{14}=0+3=3$, $f_{45}=0+3=3$, $f_{56}=0+3=3$



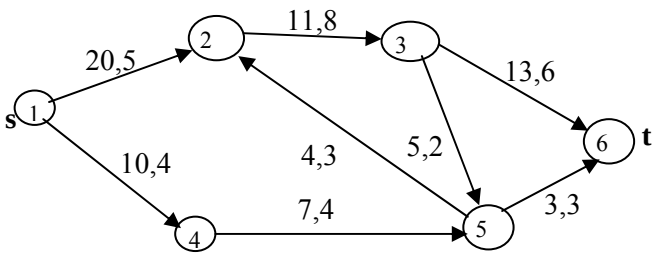
CYCLE 4

3) P7: 1-4-5-2-3-6

4) $\Delta_{14}= C_{14} - f_{14} = 10 - 3=7$
$\Delta_{45} = C_{45} - f_{45} = 7 - 3=4$
$\Delta_{52} = C_{52} - f_{52} = 4 - 0=4$
$\Delta_{23} = C_{23} - f_{23} = 11 - 11=0$
$\Delta_{36} = C_{36} - f_{36} = 13 - 11=2$

$\Delta t =\text{Min}(7,4,4,0,3)=0$
$\Delta t =0$ No addition

Example: Find the maximum flow from **s** to **t** in the following graph.



Solution

Initial flows are given

C_{12}=20, C_{23}=11, C_{36}=13, C_{35}=5, C_{14}=10, C_{45}=7, C_{52}=4, C_{56}=3,

f_{12}=5, f_{23}=8, f_{36}=6, f_{35}=2, f_{14}=4, f_{45}=4, f_{52}=3, f_{56}=3,

2) Augmenting Paths.

P1: 1-2-3-6 P2: 1-2-3-5-6

P3: 1-2-5-6 P4: 1-2-5-3-6

P5: 1-4-5-6 P6: 1-4-5-3-6

P7: 1-4-5-2-3-6

Note:P1,P2,P5,P7 have all forward edges

P3,P4,P6 have forward and backward edges

CYCLE 1

3) Select P1: 1-2-3-6

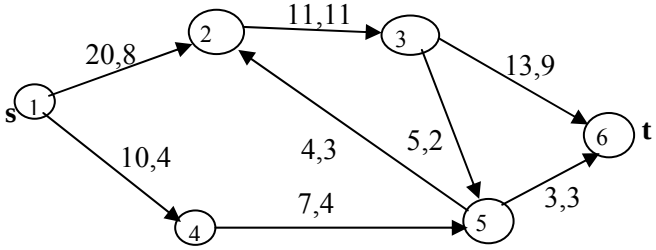
4) $\Delta_{12} = C_{12} - f_{12} = 20 - 5 = 8$

$\Delta_{23} = C_{23} - f_{23} = 11 - 8 = 3$

$\Delta_{36} = C_{36} - f_{36} = 13 - 6 = 5$

Minimum additional flow:

$\Delta t = \text{Min}(8,3,5) = 3$.



CYCLE 2

Select P2: 1-2-3-6

$\Delta t = 0$ No addition (Δ_{23}=11-11=0)

CYCLE 3

Select P5: 1-4-5-6 (Δ_{56}=3 -3=0)

$\Delta t = 0$ No addition

CYCLE 4

Select P7: 1-4-5-2-3-6 (Δ_{23}=11-11=0)

$\Delta t = 0$ No addition

CYCLE 5

Select P3: 1-2-5-6

$\Delta t = 0$ No addition (Δ_{56}=3 -3=0)

CYCLE 6

Select P4: 1-2-5-3-6

$\Delta_{12} = C_{12} - f_{12} = 20 - 8 = 12$

$\Delta_{25} = f_{25} = 3$ **(Backward edge!!!!)**

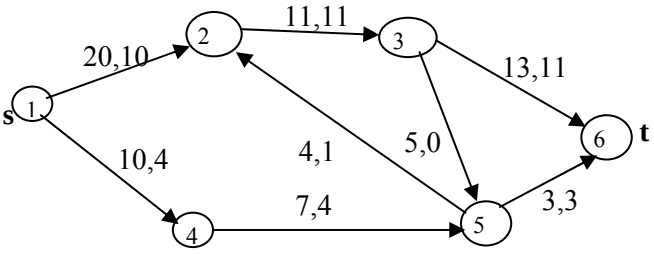
$\Delta_{53} = f_{53} = 2$ **(Backward edge!!!!)**

$\Delta_{36} = C_{36} - f_{36} = 13 - 11 = 2$

Minimum additional flow:

$\Delta t = \text{Min}(\Delta_{12}, \Delta_{25}, \Delta_{53}, \Delta_{36}) = \text{Min}(12,3,2,2) = 2$

Add 2 to the path P4



Note: $f_{12} = f_{12} + \Delta t = 8+2 = 10$

$f_{25} = f_{25} - \Delta t = 3-2 = 1$

$f_{53} = f_{53} - \Delta t = 2-2 = 0$

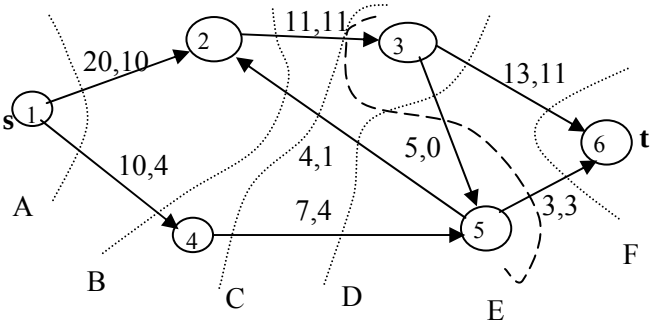
$f_{36} = f_{36} + \Delta t = 9+2 = 11$

CYCLE 7

Select P6: 1- 4-5-3-6

$\Delta t = 0$ No addition (Δ_{53}=0)

CUT SET



Cut Set A. $C_{12}+C_{14}$=20+10=30 $f_{12}+f_{14}$=10+4=14

Cut Set B. $C_{23}+C_{14}$ =11+10=21 $f_{23}-f_{52}+f_{14}$=14

Cut Set C. $C_{23}+C_{45}$=11+7=18 $f_{23}-f_{52}+f_{45}$=14

Cut Set D. $C_{36}+C_{53}+C_{45}$=13+5+7=25

$f_{36}+f_{53}-f_{52}+f_{45}$=14

Cut Set E. $C_{23}+C_{56}$=11+ 3 =14 $f_{23}-C_{35}+f_{56}$=14

Cut Set F. $C_{36}+C_{56}$=13+ 3 =16 $f_{36}+C_{56}$=11+3=14

Min(30,21,18,25,14,16)=14.

Possible Max flow=14

Current flow=14 (Maximum flow is obtained)