

$$w^n = z \quad z = r e^{i\theta} \quad w = R e^{i\alpha}$$

$$R = \sqrt[n]{r} \quad \alpha = \frac{\theta + 2k\pi}{n} \quad w = \sqrt[n]{r} e^{i\alpha}$$

Example CR1- $w^3 = -2+5i$, $w_1 = ?$, $w_2 = ?$, $w_3 = ?$

Solution: $n=3$, $z=-2+5i$ $r = \sqrt{2^2 + 5^2} = 5.3852$,

$$\Theta = \tan^{-1} \frac{5}{-2} = 111.8^\circ = 1.95 \text{ radian}$$

$$R = \sqrt[3]{5.3852} = 1.75 \quad \alpha = \frac{111.8 + 2k180}{3} \quad w = 1.75 e^{i\alpha}$$

$$\alpha_0 = \frac{111.8 + 2 \cdot 0 \cdot 180}{3} = 37.26$$

$$\alpha_1 = \frac{111.8 + 2 \cdot 1 \cdot 180}{3} = 157.26^\circ$$

$$\alpha_2 = \frac{111.8 + 2 \cdot 2 \cdot 180}{3} = 277.26^\circ$$

$$\alpha_3 = \frac{111.8 + 2 \cdot 3 \cdot 180}{3} = 397.26^\circ = 397.26^\circ - 360^\circ = 37.26^\circ = \alpha_0$$

similarly $\alpha_4 = \alpha_1$, $\alpha_5 = \alpha_2$, $\alpha_6 = \alpha_3$,

$$w_0 = R e^{i\alpha_0} = 1.75 (\cos \alpha_0 + i \sin \alpha_0) =$$

$$1.75 (\cos 37.26 + i \sin 37.26) = 1.39 + 1.05i$$

$$w_1 = R e^{i\alpha_1} = 1.75 (\cos \alpha_1 + i \sin \alpha_1) =$$

$$1.75 (\cos 157.26 + i \sin 157.26) = -1.61 + 0.67i$$

$$w_2 = R e^{i\alpha_2} = 1.75 (\cos \alpha_2 + i \sin \alpha_2) =$$

$$1.75 (\cos 277.26 + i \sin 277.2) = 0.22 - 1.73i$$

$$w_3 = R e^{i\alpha_3} = 1.75 (\cos \alpha_3 + i \sin \alpha_3) =$$

$$1.75 (\cos 397.26 + i \sin 397.26) = 1.39 + 1.05i$$

$$w_3 = w_0, \quad w_4 = w_1, \quad w_5 = w_2 \dots$$

Example CR2- $w^4 = 1$, $w_1 = ?$, $w_2 = ?$, $w_3 = ?$ $w_4 = ?$

Solution: $n=4$, $z=1+j0$ $r=1$, $\Theta=0$.

$$R = \sqrt[4]{1} = 1 \quad \alpha = \frac{0 + 2k\pi}{4} \quad w = 1 e^{i\alpha} = e^{i\alpha}$$

$$\alpha_0 = \frac{0 + 2 \cdot 0 \cdot \pi}{4} = 0$$

$$\alpha_1 = \frac{0 + 2 \cdot 1 \cdot \pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2} = 1.5708 \text{ radian} = 90^\circ$$

$$\alpha_2 = \frac{0 + 2 \cdot 2 \cdot \pi}{4} = \frac{4\pi}{4} = \pi = 3.1415 \text{ radian} = 180^\circ \dots$$

$$\alpha_3 = \frac{0 + 2 \cdot 3 \cdot \pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2} = 4.7124 \text{ radian} = 270^\circ$$

$$\alpha_4 = \frac{0 + 2 \cdot 4 \cdot \pi}{4} = \frac{8\pi}{4} = 2\pi = 6.2832 \text{ radian} = 360^\circ = 0^\circ = \alpha_0$$

$$\alpha_5 = \frac{0 + 2 \cdot 5 \cdot \pi}{4} = \frac{10\pi}{4} = \frac{5\pi}{2} = 7.854 \text{ rad} = 450^\circ = 90^\circ = \alpha_1$$

$$\alpha_6 = \alpha_2, \quad \alpha_7 = \alpha_3 \quad \alpha_8 = \alpha_4, \dots$$

$$w_0 = e^{i\alpha_0} = \cos \alpha_0 + i \sin \alpha_0 = \cos 0 + i \sin 0 = 1$$

$$w_1 = e^{i\alpha_1} = \cos \alpha_1 + i \sin \alpha_1 = \cos 90 + i \sin 90 = i$$

$$w_2 = e^{i\alpha_2} = \cos \alpha_2 + i \sin \alpha_2 = \cos 180 + i \sin 180 = -1$$

$$w_3 = e^{i\alpha_3} = \cos \alpha_3 + i \sin \alpha_3 = \cos 270 + i \sin 270 = -i$$

$$w_4 = e^{i\alpha_4} = \cos \alpha_4 + i \sin \alpha_4 = \cos 360 + i \sin 360 = 1 = w_0$$

$$w_5 = w_1 \quad w_6 = w_2 \quad w_7 = w_3 \dots$$

Example CR3- $w^7 = -4$, $w_1 = ?$, $w_2 = ?$, $w_3 = ? \dots$

Solution: $n=7$, $z=-4$ $r=4$, $\Theta = 180^\circ = \pi$ radian

$$R = \sqrt[7]{4} = 1.22 \quad \alpha = \frac{180 + 2k180}{7} \quad w = 1.22 e^{i\alpha}$$

$$\alpha_0 = 25.7, \quad \alpha_1 = 77.1, \quad \alpha_2 = 128.5, \quad \alpha_3 = 180, \quad \alpha_4 = 231.4,$$

$$\alpha_5 = 282.8, \quad \alpha_6 = 334.2, \quad \alpha_7 = 385.7, \quad \alpha_8 = 437.1,$$

$$w_0 = 1.1 + 0.53i, \quad w_1 = 0.27 + 1.19i, \quad w_3 = -0.7 + 0.95i,$$

$$w_3 = -1.22, \quad w_4 = -0.76 - 0.95i, \quad w_5 = 0.27 - 1.19i$$

$$w_6 = 1.1 - 0.53i, \quad w_7 = w_0, \quad w_8 = w_1, \quad w_9 = w_2$$

Example CR4- $w^8 = 1$, $w_1 = ?$, $w_2 = ?$, $w_3 = ? \dots$

Solution: $n=8$, $z=1$ $r=8$, $\Theta=0$

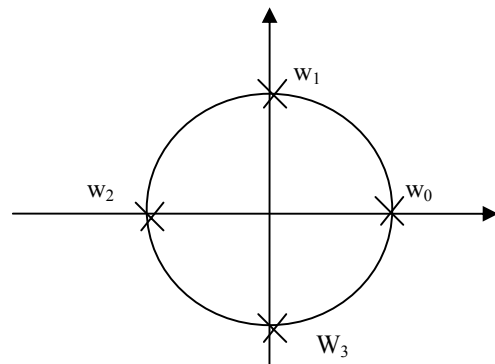
$$\alpha_0 = 0, \quad \alpha_1 = 45, \quad \alpha_2 = 90, \quad \alpha_3 = 135, \quad \alpha_4 = 180 \quad \alpha_5 = 225$$

$$\alpha_6 = 270, \quad \alpha_7 = 315$$

$$w_0 = 1, \quad w_1 = 0.707 + 0.707i, \quad w_2 = i, \quad w_3 = -0.707 + 0.707i,$$

$$w_4 = -1, \quad w_5 = -0.707 - 0.707i, \quad w_6 = -i, \quad w_7 = 0.707 - 0.707i,$$

CR5 $w^4 = 1$ $w_0 = 1$, $w_1 = i$, $w_2 = -1$, $w_3 = -i$,



CR6 $w^8 = 1$, $w_0 = 1$, $w_1 = 0.7 + 0.7i$, $w_2 = i$,

$$w_3 = -0.7 + 0.7i, \quad w_4 = -1, \quad w_5 = -0.7 - 0.7i, \quad w_6 = -i, \quad w_7 = 0.7 - 0.7i$$

