It is given that $F(z) = \dfrac{z+1}{z^2+2z+2}$

a)Replace *z=iw* in *F(z)* and obtain *F(w), | F(w)|* and *<F(w)*

b)Calculate *F(w), | F(w)|*, and *<F(w)* for the following values of *w*. *w*=[0 0.5 1 1.1 1.2 2 5 10]

c)Plot the values of | *F(w)*| versus *w* and < *F(w)* versus *w*

d)determine the maximum of | *F(w)*|

e)For which value of *w*, | *F(w)*| is maximum

Solution

$$F(iw) = \frac{iw+1}{(iw)^2+2iw+2} = \frac{1+i\,w}{(2-w^2)+i\,2w} = \frac{\mathrm{N}(w)}{\mathrm{D}(w)}$$

for w=0. N(0)= 1+ i 0=1 D(0)=$(2-0^2)$+ i 2x0 = 2

F(0)=$\dfrac{\mathrm{N}(0)}{\mathrm{D}(0)} = \dfrac{1}{2}$ =0

for w=0.5 N(0.5)= 1+ i 0.5

D(0.5)= $(2-0.5^2)$+ i 2x0.5 =1.75+i

F(i0.5)=$\dfrac{\mathrm{N}(0.5)}{\mathrm{D}(0.5)} = \dfrac{1+0.5\,i}{1.75+i}$ = 0.553 - 0.03i

| F(i0.5) |= $\sqrt{0.553^2+0.03^2}$ =0.554

< F(i0.5)=$\tan^{-1}\left(\dfrac{-0.03}{0.553}\right) = -0.055^{\text{radian}} = -3.18^0$

Similarly

F(i)= 0.6−0.2i |F(1)|=0.6325, <F(1)= -18.4^0

F(1.1i)=0.587−0.24i |F(1.1)|=0.633, <F(1.1)= -22^0

F(1.2i)=0.56−0.28i |F(1.2)|=0.634, <F(1.2)= -26.6^0

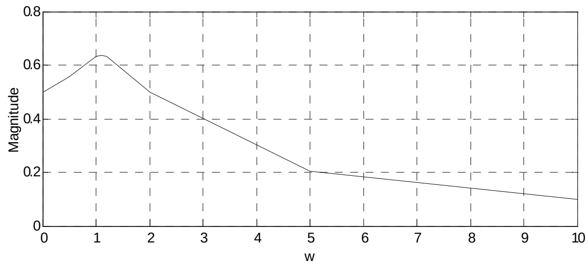
F(2i)= 0.3−0.4i |F(2)|=0.5, <F(1)= -53.1^0

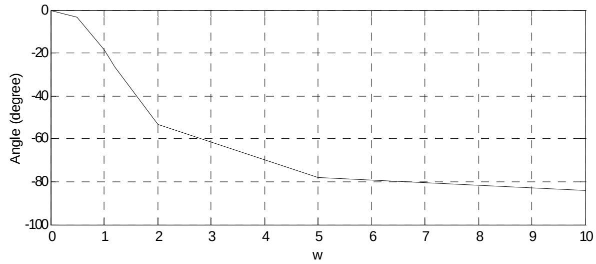
F(5i)= 0.042−0.198i |F(2)|=0.203, <F(5)= -77.8^0

F(10i)=0.01− -0.1i |F(10)|=0.1005, <F(5)= -84.1^0

F(∞)=0 − 0 i |F(∞)|=0, <F(∞)= -90^0

w	F_R	F_{IM}	*|F(iw)|*	*< F(iw)* degree
0	0.5	0	0.5	0
0.5	0.553	-0.03	0.554	-3.17
1	0.6	-0.2	0.632	-18.43
1.1	0.587	-0.243	0.63	-22.52
1.2	0.566	-0.284	0.633	-26.67
2	0.3	-0.4	0.5	-53.13
5	0.042	-0.198	0.203	-77.81
10	0.010	-0.1	0.1005	-84.17
∞	0	0	0	-90





From the table the maximum value of |*F(iw)*| is 0.636.

This maximum value is at *w*=1.1

(cos q+i sin q)^n = cos nq+i sin nq

(a+b)^3=a^3+3a^{2}b +3ab^2+b^3

(cos q+i sin q)^3 = cos 3q+i sin 3q

(cos q+i sin q)^3=(cos q)^3 + 3(cos q)^2(i sin q)

+3(cos q) (i sin q)^2+(i sin q)^3

=cos^3 q + i 3 cos^2 q sin q

+3 cos q i^2 sin^2 q + i^3 sin q^3

= cos^3 q + i 3 cos^2 q sin q

-3 cos q sin^2 q – i sin q^3

= cos^3 q -3 cos q sin^{2}q + i (3 cos^{2}q sin q – sin q^3)

Real Part {Z}= Real Part{Z}

cos 3q= cos^3 q -3 cos q sin^2 q

Imaginary part {Z}= Imaginary Part{Z}

sin 3q =3 cos^2 q sin q – sin q^3

---..

(a+b)^4=a^4+4a^{3}b+6a^{2}b^2+4ab^3+b^4

(a+b)^5=a^5+5a^{4}b+10a^{3}b^2+10a^{2}b^3+5ab^4+b^5

(a+b)^6=a^6+6a^{5}b+15a^{4}b^2+20a^{3}b^3+15a^{2}b^4+6ab^5+b^6

(a+b)^7=a^7+7a^{6}b+21a^{5}b^2+35a^{4}b^3+35a^{3}b^4+21a^{2}b^5+7ab^6 +b^7