

$$e^{ix} = \cos x + i \sin x \quad e^{-ix} = \cos x - i \sin x$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}, \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cos z = \cos(x+iy) = \cos x \cosh y - i \sin x \sinh y$$

$$\sin z = \sin(x+iy) = \sin x \cosh y + i \cos x \sinh y$$

set  $x=0$

$$\cos iy = \cosh y \quad \sin(iy) = i \sinh y$$

set  $y=iq$

$$\cosh iq = \cos q \quad \sinh iq = i \sin(q)$$

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$$\ln z = \ln |z| + i(\Theta + 2k\pi) \quad \Theta = \text{angle of } z.$$


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### Example CL1

$$\ln(3+4i) = ?$$

$$|3+4i| = \sqrt{3^2 + 4^2} = 5 \quad \Theta = \tan^{-1} \frac{4}{3} = 0.927 \text{ radian}$$

$$\ln(3+4i) = \ln 5 + i(0.927 + 2k\pi)$$

$$= 1.6 + i(0.927 + 2k \cdot 3.14)$$

$$k=0 \rightarrow 1.6 + 0.927i$$

$$k=1 \rightarrow 1.6 + 7.21i$$

$$k=2 \rightarrow 1.6 + 13.49i$$

$$k=3 \rightarrow 1.6 + 19.7i$$

.....

$$k=10 \rightarrow 1.6 + 63.75i$$

### Example CL2

$$\ln(1) = ?$$

$$\ln(1) = \ln 1 + i(0 + 2k\pi)$$

$$= 0 + i \cdot 2k \cdot 3.14$$

$$k=0 \rightarrow 0$$

$$k=1 \rightarrow 6.28i$$

$$k=2 \rightarrow 12.56i$$

$$k=3 \rightarrow 18.84i$$

### Example CL3

$$\ln(-1) = ?$$

$$\ln(-1) = \ln 1 + i(\pi + 2k\pi)$$

$$= 0 + i(2k+1)\pi$$

$$k=0 \rightarrow 3.14i$$

$$k=1 \rightarrow 9.42i$$

$$k=2 \rightarrow 15.7i$$

### Example CL4

$$\ln(i) = ?$$

$$\ln(i) = \ln 1 + i\left(\frac{\pi}{2} + 2k\pi\right)$$

$$= 0 + i\left(2k + \frac{1}{2}\right)\pi$$

$$k=0 \rightarrow 1.57i$$

$$k=1 \rightarrow 7.85i$$

$$k=2 \rightarrow 14.13i$$

.....

$$Z^C = e^{C \ln z}$$

### Example CP6

$$(1+i)^{(3+4i)} = ?$$

#### Solution:

$$(1+i)^{(3+4i)} = e^{(3+4i) \ln(1+i)}$$

$$\ln(1+i) = 0.35 + 0.785i$$

$$(3+4i)(0.35 + 0.785i) = -2.09 + 3.75i$$

$$e^{-2.09+3.75i} = e^{-2.09} [\cos(3.75) + i \sin(3.75)]$$

$$= 0.12 [-0.82 - i 0.57]$$

$$= -0.1 - 0.069i$$

$$(1+i)^{(3+4i)} = -0.1 - 0.069i$$

### Example CP7

$$i^i = ?$$

#### Solution:

$$i^i = e^{i \ln i} = e^{i(1.57i)} = e^{-1.57} = 0.2$$

### Example CP8

$$i^{1.7} = ?$$

#### Solution:

$$i^{1.7} = e^{1.7 \ln i} = e^{1.7(1.57i)} = e^{2.67i} = \cos(2.67) + i \sin(2.67)$$

$$= -0.89 + 0.45i$$

### Example CP9

$$i^2 = e^{2 \ln i} = e^{2(1.57i)} = e^{3.14i} = \cos(3.14) + i \sin(3.14)$$

$$= -1 + 0.0i = -1$$

### Example CP10

$$i^{-1} = e^{-1 \ln i} = e^{-(1.57i)} = e^{-1.57i} = \cos(1.57) - i \sin(1.57)$$

$$= 0 - 1i = -i$$

### Example CP11

$$1^i = e^{i \ln 1} = e^0 = 1$$

### Example CP12

$$2^i = e^{i \ln 2} = e^{i 0.69} = \cos(0.69) + i \sin(0.69) = 0.77 + 0.64i$$

### Example CP13

$$1^{1/3} = e^{1/3 \ln 1} = e^{1/3(2k\pi i)} = \cos\left(\frac{2}{3}k\pi\right) + i \sin\left(\frac{2}{3}k\pi\right)$$

The values for  $k=0, k=1, k=2, k=3, k=4 \dots$

$$\{1, -0.5 + 0.86i, -0.5 - 0.86i, 1.0000 \dots\}$$