MAPPING

EXPONENTIAL FUNCTION

w=f(z)= e^z

The map of ***x=p*** **line**

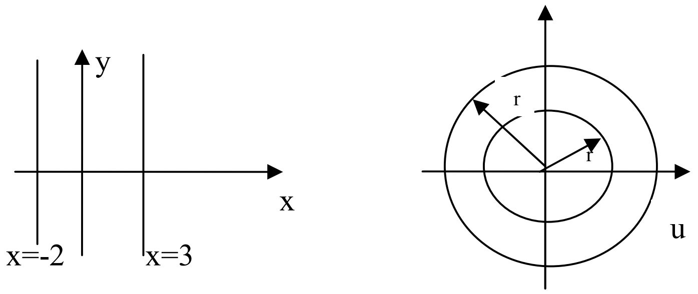
z=x+iy=p+i y (*p* constant)

w=f(z)= $e^z = e^{p+iy} = e^p e^{iy} = e^p(\cos y + i\sin y)$

The amplitude of $e^p(\cos y + i\sin y)$ is e^p (why ?)

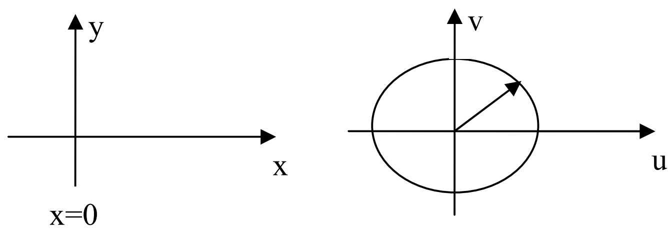
Since *p* is constant e^p is also constant. Thus magnitude of *w=f(z)=*e^z is constant if *x=p.*

Result: *x=p* line is mapped into *w=f(z)=*e^z domain as a circle. The radius of that circle is r=e^p



$r_1 = e^{-2} = 0.13, \quad r_2 = e^3 = 20.08$

Special case x=0 line is mapped as unit circle



Any number in the left half plane (x<0) is mapped inside the unit circle where the magnitude is less than 1.

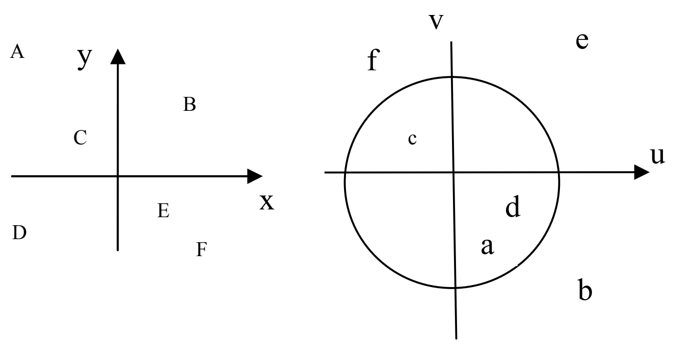
$|e^{-3+5i}| = |e^{-2}||e^{5i}| = 0.13 \;\; 1 = 0.13$

$|e^{-6+20i}| = |e^{-6}||e^{20i}| = 0.0025 \;\; 1 = 0.0025$

Any number in the right half plane (x>0) is mapped outside the unit circle where the magnitude is greater than 1.

$|e^{0.1+0.1i}| = |e^{0.1}||e^{0.1i}| = 1.105 \;\; 1 = 1.105$

$|e^{0.5-0.5i}| = |e^{0.5}||e^{-0.5i}| = 1.648 \;\; 1 = 1.648$



The map of ***y=q*** **line**

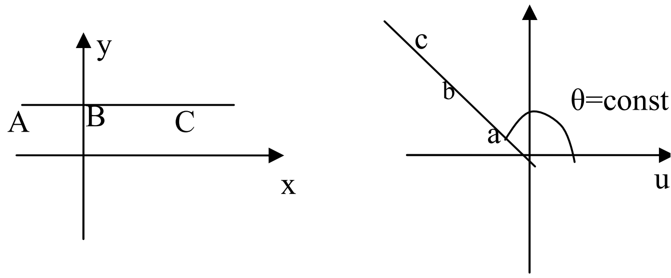
z=x+iy=x+iq (*q* constant)

w=f(z)=$e^z = e^{x+iy} = e^{x+iq} = e^x e^{iq}$

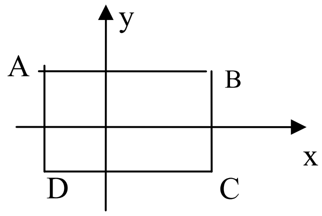
The amplitude is e^x

The angle is *q*

Thus angle is constant, but amplitude varies as *x* varies.



Example Problem: Find the image of the following rectangle in *f(z)=*e^z domain, where A=-2+3i, B=4+3i, C=4-1.5i, D=-2-1.5i .



Solution:

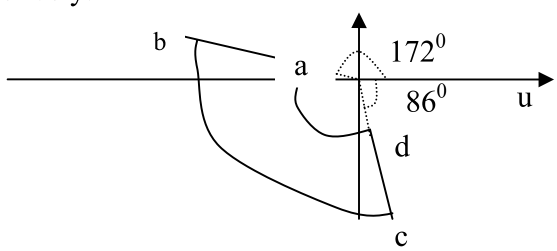
a=e^{-2+3i}=-0.13+0.019i=0.13 e^{3i} =0.13 $e^{172^\circ i}$

b=e^{4+3i}=-54.05+7.7i=54.6 e^{3i} =54.6 $e^{172^\circ i}$

c=$e^{4-1.5i}$=3.9-54.5i =54.6 $e^{-1.5i}$ =54.6 $e^{-86^\circ i}$

d=$e^{-2-1.5i}$=-0.01-0.13i =0.13 $e^{-1.5i}$ =0.13 $e^{-86^\circ i}$

Lines BC and AD are mapped as circles with radius e^4=54.59 and e^{-2}=0.13 respectively. Lines AB and DC are mapped as lines with slopes 3 radians=172^0 and -1.5 radians=-86^0 respectively.



w=f(z)= e^z is periodic with period 2πi

$e^{z+2\pi i} = e^z$

$e^{3+4i+2\pi i} = e^{3+4i}$