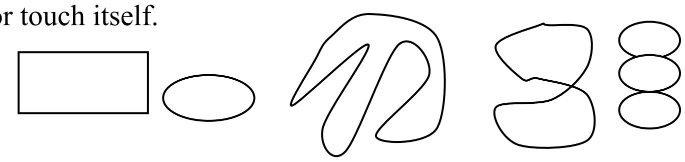
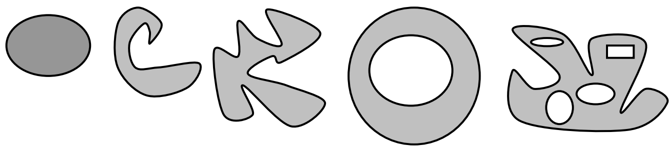
A simple closed path is a closed path that does not intersect or touch itself.



a) b) c) d) e)

a), b) c) are simple closed path, d), e) are not simple closed path.

A simple connected domain D in the complex plane is a domain such that every simple closed path in D encloses only point of D. A domain that is not simply connected is called **multiply connected**.



a) b) c) d) e)

a), b) c) simple connected domain d) e) multiply connected domain.

Example of Analytic Functions

f(z)= $\dfrac{1}{z+2}$ is analytic everywhere except z=-2

f(z)= $\dfrac{1}{(z-2)(z+3)}$ is analytic everywhere except z=2, z=-3

f(z)= $\dfrac{1}{(z^2+4)}$ is analytic everywhere except z=2i, z=-2i

f(z)=z^2 is analytic everwhere

f(z)= $3z^4 + 5z^3 - z^2 + 20$ is analytic everwhere

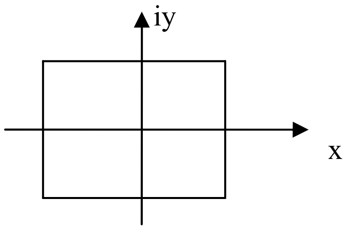
f(z)=cos(z)+sin(z) is analytic everwhere

f(z)=e^z is analytic everwhere

Cauch integral Theorem: If f(z) is **analytic** in a simple connected domain D, then every simple closed path C in domain D.

$$\oint_C f(z)dz = 0$$

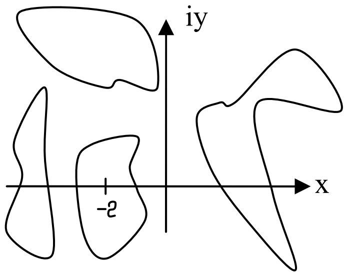
Example: (see problem AE-18 in notes page AEC 451)



Example:

$\oint_C z^2\, dz = 0$ C is any closed path.

$\oint_C (3z4 + 5z3 - z2 + 20)\, dz = 0$ C is any closed path.



$$\oint_{C1} \frac{1}{z+2} dz = 0$$

$$\oint_{C2} \frac{1}{z+2} dz = 0$$

$$\oint_{C3} \frac{1}{z+2} dz \neq 0$$

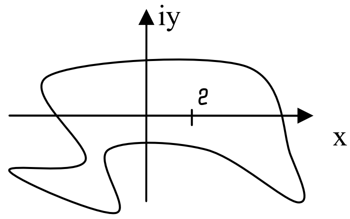
$$\oint_{C4} \frac{1}{z+2} dz = 0$$

Cauch integral Formula: If f(z) is **analytic** in a simple connected domain D, then for any point z_0 in D and any simple closed path C in D that encloses z_0

$$\oint_C \frac{f(z0)}{z - z_0} dz = 2\pi \text{ i } f(z_0)$$

The integration being taken counterclockwise

Example: Calculate $\oint_C \dfrac{z+3}{z-2} dz$ where C is shown below



Solution: It is clear that the term is not analytic at the point z_0=2. Select f(z)=z+3. and apply Cauchy integration formula.

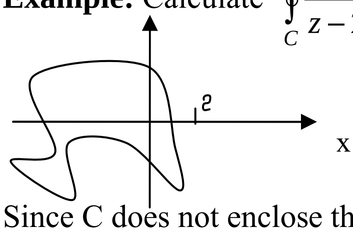
$$\oint_C \frac{f(z0)}{z - z_0} dz = 2\pi \text{ i } f(z_0)$$

Here f(z)=z+3. $f(z_0) = z_0+3 = 2+3=5$

$$\oint_C \frac{z+3}{z-2} dz = 2\pi \text{ i} f(z_0) = 2\pi \text{ i } 5 = 10\pi \text{ i} = 10\ \ 3.14 \text{ i} = 31.4\text{i}$$

$$\oint_C \frac{z+3}{z-2} dz = 31.4\text{i}$$

Example: Calculate $\oint_C \dfrac{z+3}{z-2} dz$ where C is shown below



Since C does not enclose the singular point z_0=2 the integration is zero. $\oint_C \dfrac{z+3}{z-2} dz = 0$