Simple Pole

f(z)= $\frac{z+1}{z+3}$, f(z) has a simple pole at z= −3.

f(z)= $\frac{z+1}{(z+3)(z-4)}$ f(z) has simple poles at z=−3 and z=4.

f(z)= $\frac{z+1}{(z^2-4)}$ f(z) has simple poles at z=−2 and z=2.

f(z)= $\frac{z+1}{(z^2+4)}$ f(z) has simple poles at z=−2i and z=2i.

f(z)= $\frac{z+1}{(z^2-2z+5)}$ f(z) has simple poles at z=1+2i, z=1-2i

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Multiple Pole

f(z)= $\frac{z+1}{(z+4)^2}$ f(z) has a **multiple** pole at z=−4

f(z)= $\frac{z+1}{(z+4)^2(z-3)}$ f(z) has a **multiple** pole at z=−4 and a simple pole at z=3

f(z)= $\frac{z+1}{(z+4)^5(z-3)}$ f(z) has a **multiple** pole at z=−4 and a simple pole at z=3

Residue: Residues are defined for poles**.**

If f(z) has a simple pole at z=a then residue at z=a is

$$\boxed{\mathrm{Res}(z=a)=\lim_{z\to a}(z-a)\,f(z)}$$

Example

f (z)= $\frac{z+5}{(z-4)(z+2)(z-3)}$.

f(z) has **3 poles** , and **3 residues** at z=4, z=-2, z=3

Residue at z=4 is

$$\mathrm{Res}(z=4)=\lim_{z\to 4}(z-4)\,f(z)=\lim_{z\to 4}(z-4)\frac{z+5}{(z-4)(z-3)(z+2)}$$

$$=\lim_{z\to 4}\frac{z+5}{(z-3)(z+2)}=\frac{4+5}{(4-3)(4+2)}=\frac{9}{6}=1.5$$

Residue at z=-2 is

$$\mathrm{Res}(z=-2)=\lim_{z\to -2}(z-(-2))\,f(z)=\lim_{z\to -2}(z+2)\frac{z+5}{(z-4)(z-3)(z+2)}$$

$$=\lim_{z\to -2}\frac{z+5}{(z-3)(z-4)}=\frac{-2+5}{(-2-3)(-2-4)}=\frac{3}{30}=0.1$$

Residue at z=3 is

$$\mathrm{Res}(z=3)=\lim_{z\to 3}(z-3)\,f(z)$$

If f(z) has a fractional form $f(z)=\frac{p(z)}{q(z)}$ then

$$\boxed{\mathrm{Re}\,s(z=a)=\lim_{z\to a}\frac{p(z)}{q'(z)}}$$

Example

$$\frac{2z+12}{z^2+2z+2}=\frac{2z+12}{[z-(-1+i)][z-(-1-i)]}=\frac{2z+12}{(z+1-i)(z+1+i)}$$

p(z)=2z+12, q(z)=z^2+2z+2, q'(z)=2z+2

$$\mathrm{Re}\,s(z=-1+i)=\lim_{z\to -1+i}\frac{p(z)}{q'(z)}=\frac{2z+12}{2z+2}=\frac{2(-1+i)+12}{2(-1+i)+2}$$

=1-5i

$$\mathrm{Re}\,s(z=-1-i)=\lim_{z\to -1-i}\frac{p(z)}{q'(z)}=\frac{2z+12}{2z+2}=\frac{2(-1-i)+12}{2(-1-i)+2}$$

=1+5i