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Analytical Methods in Engineering (EE300) Final **Part I** 2-1-2006 2-12-1426

1) $A=\begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 4 & -1 & 7 \\ 0 & 0 & i & 1+i \\ 0 & 0 & -1 & Q \end{bmatrix}$ $-(R_3 \times i) + R_4 \to R_4$

It is known that det(A)=0, calculate Q

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 4 & -1 & 7 \\ 0 & 0 & i & 1+i \\ 0 & 0 & 0 & -i(1+i)+Q \end{bmatrix}$$

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$-i(1+i)+Q=0$

$Q=(1+i)i = i+i^2 = -1+i$

$2 \times 4 \begin{vmatrix} i & i+1 \\ -1 & Q \end{vmatrix} = 0$

$8(iQ-(-1)(i+1)) = 0$

$Q = \frac{-(i+1)}{i} = \frac{-i}{i} + \frac{-1}{i} = -1+i$

2) A,B,C,D,E, X, are all matrices in the following equations. Solve X

a) X C + X D = E

b) $A X^{-1} + B X^{-1} = C$

$X(C+D)=E$

$X = E[C+D]^{-1}$

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$[A+B]X^{-1} = C$

$[A+B] = CX$

$X = C^{-1}[A+B]$

$X^{-1} = [A+B]^{-1}C$

$X = [\ ]^{-1} = C^{-1}[A+B]$

3) Examine the following linear equations

$x + (1-i)y = 0$

$(1+i)x + 2y = 1$

a) Write these linear equations in matrix form.

b) Calculate rank A, rank $\tilde{A}$ = 2 (1)

A is coefficient matrix. $\tilde{A}$ is augmenting matrix.

c) This system has No ---------------- solution.

(write **unique**, **multiple**, or **no solution**)

$$\begin{bmatrix} 1 & 1-i \\ 1+i & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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$-(1+i) \times R_1 + R_2 \to R_1$

$$\begin{bmatrix} 1 & 1-i & 0 \\ 0 & -(1+i)(1-i)+2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1-i & 0 \\ 0 & -(1^2+1^2)+2 & 1 \end{bmatrix}$$

$$\begin{matrix} 1 & 1-i & 0 \\ 0 & 0 & 1 \end{matrix}$$

No solution

rank A = 1

rank $\tilde{A}$ = 2

4) A and B are square matrices and AB=0. State true or false

a) det A = 0 ---True---

b) det B = 0 ---T---

c) A **must be** a zero matrix ---F---

d) A^{-1} = B ---F---

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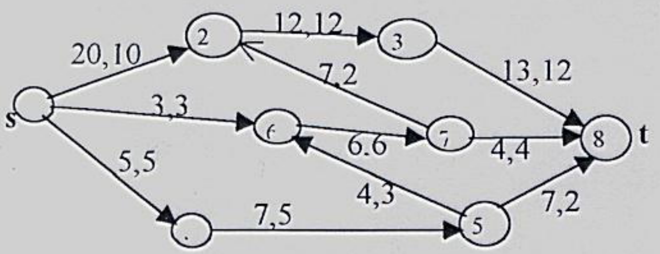
5) Obtain maximum flow (if possible) on path 1-2-7-6-5-8



1 - 2 - 7 - 6 - 5 - 8

$\Delta_{12} = 20-10 = 10$

$\Delta_{27} = 2$

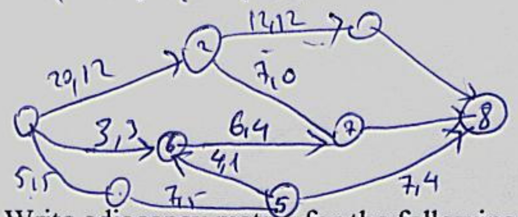
$\Delta_{76} = 6$

$\Delta_{65} = 3$

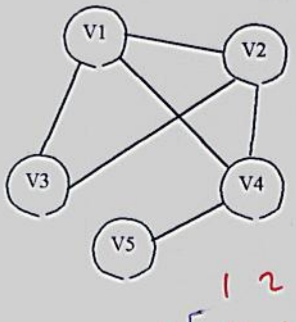
$\Delta_{58} = 7-2 = 5$

$\min(10, 2, 6, 3, 5) = 2$

(8)



6) Write adjacency matrix for the following graph.



(5)

	1	2	3	4	5
1	0	1	1	1	0
2	1	0	1	1	0
3	1	1	0	0	0
4	1	1	0	0	1
5	0	0	0	1	0

7) It is given that $\angle Z = 180^0$ (< means angle)
State true or false (**write explanations**)

a) Re{Z}=0 ----F----------
b) Im{Z}=0 ------T-------
c) Z=0 --------F------

(6) (10)

d) Re{Z}>0 ------f--------
e) $Z=\bar{Z}$ -------T--------

($\bar{Z}$: complex conjugate of)
Re{Z}: real parts of Z
Im{Z}: Imaginary parts of Z

(Explanation required)
(شرح)

8) Calculate the inverse of the following matrix by Gaus elimination technique

$$A=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

(10)

$$\left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccccccc} 0 & 0 & 4 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 & 0 & 0 & 1 & 0 \end{array}\right]$$ $4R_3 + R_4 \to R_4$,

$$\left[\begin{array}{cccc|cccc} 2 & & & & 1 & 0 & 0 & 0 \\ & 4 & & & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2. & 0 & 0 & 1 & 0.25 \end{array}\right]$$

$$\left[\begin{array}{cccc} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & \frac{1}{2.} & \frac{0.25}{2.} \end{array}\right]$$

9) Fill the following table.

	magnitude	Angle Degree	Angle Radian
-1 - i	$\sqrt{2}$=1.41	225(-135)	3.92(-2.35)
$e^{i\pi}$	1	180 (-180)	3.14 (-3.14)
$e^{i\pi+1}$	e	180	3.14 (-3.14)
πi	π	90	1.57 (-4.71)

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10) *It is given that* $<Z = 170^0$

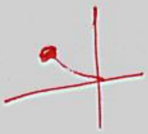
State true or false (**write explanations**)

a) $Re\{Z\}>0$ ---F

b) $Im\{Z\}>0$ ...T.........

c) $|e^Z|>1$ ....F........

d) $|e^{-Z}|>1$ ....T...........



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11) Calculate 2^{-i} (**use principle value only**.)

$a^b = e^{b \ln a}$

$2^{-i} = e^{-i \ln 2} = e^{-i\, 0.693} = \cos 0.693 - i \sin(0.693)$

$= 0.769 - i\ 0.638$

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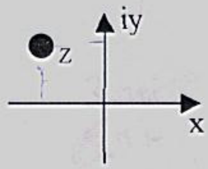
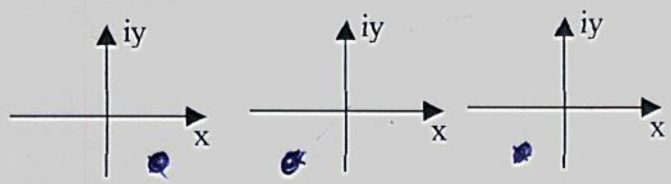
12) The complex number Z is shown in the Cartesian coordinates



Show the following numbers in the Cartesian coordinates. a) –Z b)Zi c) $\bar{Z}$ ($\bar{Z}$: conjugate)



a)-Z b)Zi c) $\bar{Z}$

$|zi| = |z|\,|i| = z$

$< zi = <z + <i = <z + 90°$

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13) cos (Z) =2. Calculate Z. (Z is a complex number)

$\cos z = \cosh iz = 2$

$\cosh q = 2 \quad q = 1.316$

$q = iz = 1.316$

$z = -1.316\, i$

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14)Calculate A,B,C in the following equation.

$$\frac{9z^2+20z+3}{z^3+3z^2-z-3}=\frac{A}{z-1}+\frac{B}{z+1}+\frac{C}{z+3}$$

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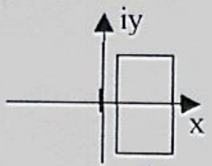
$\varphi' = 3z^2 + 6z - 1$

$A = \frac{3(1)^2 + 20(1) + 3}{3(1)^2 + 6(1) - 1} = \frac{32}{8} = 4$

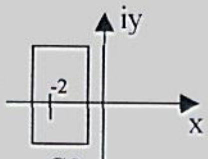
$B = \frac{3(-1)^2 + 20(-1) + 3}{3(-1)^2 + 6(-1) - 1} = \frac{-8}{-4} = 2$

$C = \quad = 3$

15) Calculate the following integrals where paths are shown in the graphs.



C1



C2

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a) $\oint_{C1} \frac{3}{z+2}dz$, b) $\oint_{C2} \frac{3}{z+2}dz$, c) $\oint_{C2} \frac{3z}{(z+2)^2}dz$

a) 0

b) $3 \times 2\pi i$ = $6\pi i$ = 18.8 i

c) $3 \times 2\pi i$ = 18.8 i

16)A dynamic system has the linear differential equation

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix} = \begin{bmatrix} & & & \\ & A & & \\ & & & \end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q3\\ q4\end{bmatrix}$$

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The matrix A has the following eigenvalues $\lambda_1=3+2i$, $\lambda_2=3-2i$, $\lambda_3=-4-5i$, $\lambda_4=-4+5i$.
The solution $q_1(t)$ is in the form of

$q_1(t)= e^{3t}(A \cos 2t + B \sin 2t) + e^{-4t}(C \cos 5t + D \sin 5t)$

fill the blanks.

17) State **True** or **False. (write explanation)**

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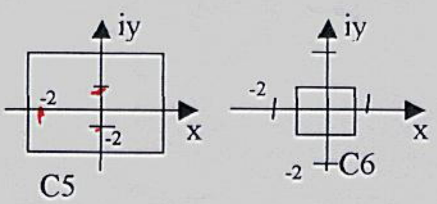

a) $\oint_{C5} \frac{z^3 + z^2 + z}{(z^2 + 4)(z^2 - 4)} dz = 0$ ......false..........

b) $\oint_{C6} \frac{z^3 + z^2 + z}{(z^2 + 4)(z^2 - 4)} dz = 0$ ......True..........

18) A dynamic system has the following differential equation

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} & M & \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

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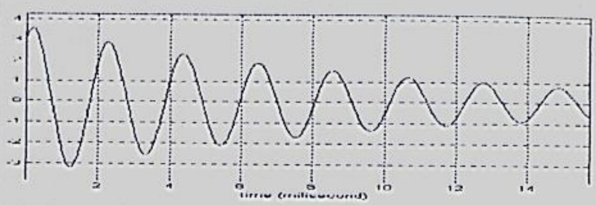

The solution of $q_1(t)$ is shown in the graphics.
One eigenvalue of M is $\lambda_1 = -0.1 + bi$, Calculate b.

$14 - 12 = 2 = T = 2$ second

$w = \frac{2\pi}{T} = \frac{2 \times 3.14}{2} = 3.14 = b$