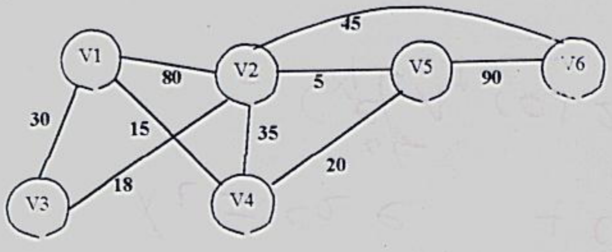
EE300 Analytical Methods in Engineering 3rd Major. 20.DEC.2004

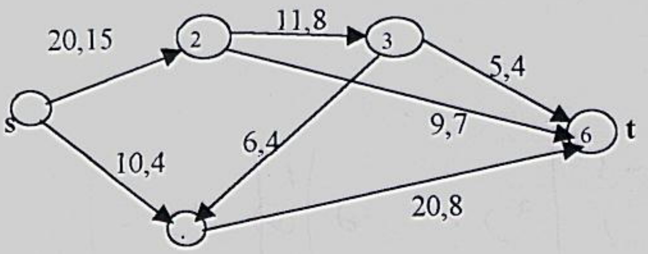
NO CALCULATOR (ANY TYPE OF CALCULATOR is NOT ALLOWED)

1) Expand the function $f(z)=e^{\cos(2z)}$ into Taylor series about $z_0=0$ (Macluren Series). Calculate 3 terms only.
[Calculate A,B,C in $f(z)=e^{\cos(2z)})=A+Bz+Cz^2$.]

2)Find the Find the shortest spanning tree in the following graph using Krustal Algorithm.



3) Try to obtain **maximum flow** from s to t, by changing the flows in each vertex in the following network. **Show your procedures step by step.**



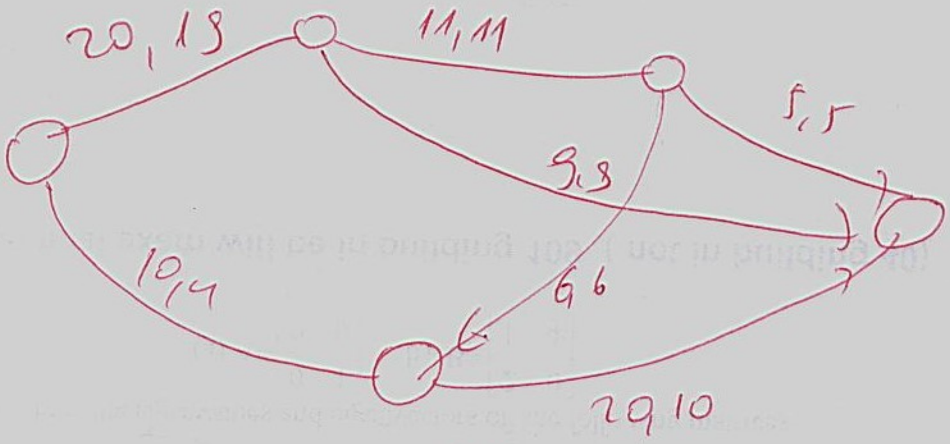
First number: capacity ($C_{i,j}$)
Second Number: given flow ($f_{i,j}$)
S: source **t:** target

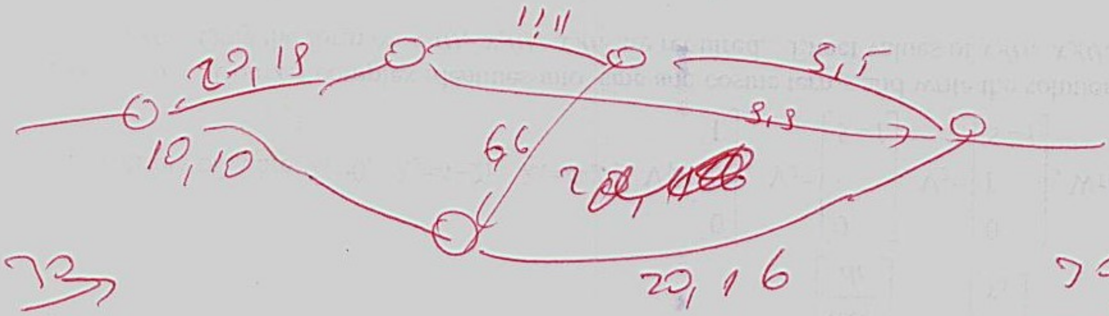
4)A Linear Matrix differential equation system is described by $\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} = [A] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and the eigenvalues and eigenvectors of A are $\lambda_1=0$, $\lambda_2=4+2i$, $\lambda_3=4-2i$, $V_1=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $V_2=\begin{bmatrix} 0 \\ 1 \\ 5-i \end{bmatrix}$, $V_3=\begin{bmatrix} 0 \\ 1 \\ 5-i \end{bmatrix}$, Write solutions $x_1(t)$, $x_2(t)$, $x_3(t)$. Convert complex identities into sine and cosine terms and write the solutions in terms of $x_1(t)$, $x_2(t)$, $x_3(t)$. Only the form of $x_1(t)$, $x_2(t)$, $x_3(t)$ are required. Exact values of $x_1(t)$, $x_2(t)$, $x_3(t)$ are **not** asked.

5) Find the eigenvalues and eigenvectors of the following matrices

(a) $A=\begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix}$ b) $B=\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

The final exam will be in building 109 (not in building 40)





73 $\Rightarrow$ 70 $\rightarrow$

4

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e^{\lambda_1 t} + c_2 \begin{bmatrix} 0 \\ 1 \\ 5-i \end{bmatrix} e^{\lambda_2 t} + c_3 \begin{bmatrix} 0 \\ 1 \\ 5+i \end{bmatrix} e^{\lambda_3 t}$$

$x_1 = c_1 \cdot 0 \qquad \underline{x_1 = 0}$

$$x_2 = c_2 e^{\lambda_2 t} + c_3 e^{\lambda_3 t}$$

$$= c_{4} e^{4t}(A_1 \cos 2t + B_1 \sin 2t)$$

$$x_3 = c_1 e^{0 \cdot t} + c_{2}(A_2 \cos 2t + B_2 \sin 2t)$$

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$\begin{bmatrix} -\lambda & 1 \\ 9 & -\lambda \end{bmatrix}$ $\qquad \lambda^2 - 9 = 0$, $\lambda_1 = 3$, $\lambda_2 = -3$

$\begin{bmatrix} -3 & 1 \\ & \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix}$ $\qquad -3p + q = 0$, $p = 1$, $q = 3$

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $\qquad \lambda = -3$: $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

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$$① \quad f(z) = e^{\cos 2z} = f(0) + f'(0)\, z + \frac{f''(0)}{2!} z^2 + \ldots$$

$$f(0) = e^{\cos 0} = e^1 = e$$

$$f'(z) = (\cos 2z)'\, e^{\cos 2z} = -2 \sin 2z\, e^{\cos 2z}$$

$$f'(0) = -2 \sin(2\times 0)\, e^{\cos 2\times 0} = 0$$

$$(pq)' = p'q + q'p$$

$$f''(z) = -2\left[(\sin 2z)'\, e^{\cos 2z} + \sin 2z \left(e^{\cos 2z}\right)'\right]$$

$$= -2\left[2 \cos 2z\, e^{\cos 2z} + \sin 2z\left(-2 \sin 2z\, e^{\cos 2z}\right)\right]$$

$$f''(0) = -2\left[2 \underbrace{\cos 2\times 0}_{1}\, e^{\cos 2\times 0} + \underbrace{\sin 2\times 0}_{0}(\ldots\ldots)\right]$$

$$= -2\times 2\, e^1 = -4e$$

$$f(z) = f(0) + f'(0)\, z + \frac{f''(0)}{2!} z^2$$

$$= 1 + 0 + \frac{-4e}{2!} z^2$$

$$= 1 - \frac{4e}{2} z^2 = 1 - \frac{4\times 2.71}{2} z^2 = 1 - 5.43\, z^2$$

4) $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} e^{0t} + c_2 \begin{bmatrix} 0 \\ 1 \\ 5-i \end{bmatrix} e^{(4+2i)t} + c_3 \begin{bmatrix} 0 \\ 1 \\ 5+i \end{bmatrix} e^{(4-2i)t}$$

$$x_1 = 0$$

$$x_2 = c_2 (5-i) e^{(4+2i)t} + c_3 (5+i) e^{(4-2i)t}$$

$$= e^{4t}(A \cos 2t + B \sin 2t)$$

5) $$A = \begin{bmatrix} 0 & 1 \\ 9 & 0 \end{bmatrix} \qquad (A - \lambda I) = \begin{bmatrix} -\lambda & 1 \\ 9 & -\lambda \end{bmatrix}$$

$$\lambda^2 - 9 = 0 \qquad \lambda = 3 \qquad \lambda_2 = -3$$

$$\begin{bmatrix} -\lambda & 1 \\ 9 & -\lambda \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\lambda = 3$ $$\begin{bmatrix} -3 & 1 \\ 9 & -3 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$p = 1$ (free)

$-3p + q = 0$

$q = 3p = 3$

$$X_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$\lambda = -3$ $$\begin{bmatrix} 3 & 1 \\ 9 & 3 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$p = 1$

$3p + q = 0$

$q = -3$

$$X_2 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

b) $\lambda_1 = 2$ $\lambda_2 = 4$ $X_1 = [\ ]$ $X_2 = [\ ]$