# Analytical Methods in Engineering (EE300) Final 5 DEC 2003 12-11-1424

**1)**Given the following matrices, calculate
a)AB, b)BA, c)$A^TB^T$, d) $B^TA^T$

$$A=\begin{bmatrix}2 & 3 & 5\\ 3 & 6 & 2\\ 1 & a & 8\end{bmatrix},\quad B=\begin{bmatrix}3 & 0 & 0\\ 0 & 6 & 0\\ 0 & 0 & b\end{bmatrix}$$

**2)** Calculate the inverse of the matrix.

$$\mathbf{A}=\begin{bmatrix}\mathrm{i} & \mathrm{i} & \mathrm{i}+1\\ 0 & 2 & \mathrm{i}\\ 0 & -\mathrm{i} & 3\end{bmatrix},\qquad (\mathrm{i}=\sqrt{-1})$$

**3)**Examine the following linear equations.

$$\begin{bmatrix}1 & 2 & 3\\ 3 & -3 & 0\\ 2 & 4 & p\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}0\\ 0\\ q\end{bmatrix},$$

a) Find a value for p, so that the system has **unique solution**.
b)Find values of p and q, so that the system has **multiple solution**
b)Find values of p and q, so that the system has **no solution**

**4)**Calculate magnitude and angle of the following values
a) $e^{i+1}$ b)-2 c)3-4i d)$-\pi i$

**5)**Find the roots of the following polynomials
a)$z^5-32=0$, b)$z^3-8i=0$

**6)**a)Calculate Taylor series for the function $f(z)=\sqrt{z+1}$ (calculate two terms only )
b)Calculate $\sqrt{1+i}$ using the formula you have found in part a).

**7)a)**Calculate A,B,C,D in the following fraction.

$$\frac{10z^2+20}{z^4-3z^2-4}=\frac{10z^2+20}{(z+i)(z-i)(z+2)(z-2)}$$
$$=\frac{A}{(z-i)}+\frac{B}{(z+i)}+\frac{C}{(z+2)}+\frac{D}{(z-2)}$$

b)Calculate the integral

$$\oint_C \frac{10z^2+20}{z^4-3z^2-4}dz \quad \text{where C is } |z|=1.5 \text{ circle.}$$

A circle whose center is origin and radius is 1.5.

c)Calculate the integral

$$\oint_C \frac{10z^2+20}{z^4-3z^2-4}dz \quad \text{where C is } |z|=10 \text{ circle.}$$

A circle whose center is origin and radius is 10

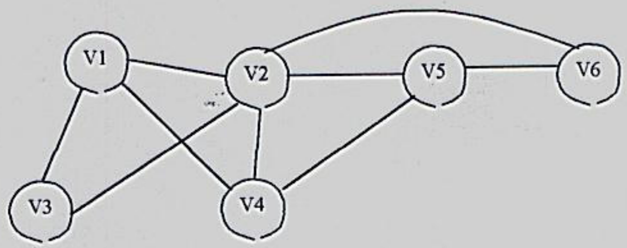
**8)**Calculate the eigenvalues of the following matrix $A=\begin{bmatrix}4 & 3\\ -3 & 4\end{bmatrix}$

**9)** One of the eigenvalues of the matrix

$$A=\begin{bmatrix}6 & 14 & -8\\ -15 & 71 & -35.\\ -24 & 104 & -50.\end{bmatrix} \quad \text{is } \lambda_1=6$$

Calculate the eigenvector of this eigenvalue. (you are required to calculate **only one eigenvector**)

**10)**A simple graph is shown below. Write the adjacency matrix of this graph.



2) $$\left[\begin{array}{ccc|ccc} i & i & i+1 & 1 & 0 & 0 \\ 0 & 2 & i & 0 & 1 & 0 \\ 0 & -i & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\frac{i}{2} \times R_2 + R_3 = R_3$$

$$\left[\begin{array}{ccc|ccc} i & i & i+1 & 1 & 0 & 0 \\ 0 & 2 & i & 0 & 1 & 0 \\ 0 & 0 & 2.5 & 0 & 0.5i & 1 \end{array}\right]$$

$$\frac{R_3}{2.5} \rightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} i & i & i+1 & 1 & 0 & 0 \\ 0 & 2 & i & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0.2 & 0.4 \end{array}\right]$$

$$-R_3 * i + R_2 \rightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} i & i & i+1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1.2 & -0.4i \\ 0 & 0 & 1 & 0 & 0.2i & 0.4 \end{array}\right]$$

$$\frac{R_2}{2} \rightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} i & i & i+1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0.6 & -0.2i \\ 0 & 0 & 1 & 0 & 0.2i & 0.4 \end{array}\right]$$

$-R_2 i + R_1 \to R_1$

$-(i+1) R_3 + R_1 \to R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -i & -0.8-0.2i & -0.4+0.6i \\ 0 & 1 & 0 & 0 & 0.6 & -0.2i \\ 0 & 0 & 1 & 0 & 0.2i & 0.4 \end{array}\right]$$

3) $$\tilde{A} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 3 & -3 & 0 & 0 \\ 2 & 4 & p & q \end{bmatrix}$$

$-3R_1 + R_2 \to R_2$

$-2R_1 + R_3 \to R_3$

$$\tilde{A} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -9 & -9 & 0 \\ 0 & 0 & p-6 & q \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -9 & -9 \\ 0 & 0 & p-6 \end{bmatrix}$$

if $p = 6$ and $q = 0$ multiple solution

if $p \neq 6$ unique solution

if $p = 6$ and $q \neq 0$ No solution

4) $|e^{i+1}| = |e^{i}\, e^{1}| = \underbrace{|e^{i}|}_{1}\ \underbrace{|e^{1}|}_{2.71} = 2.71$

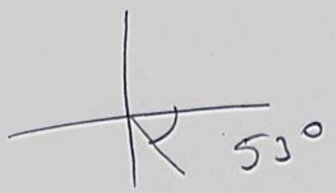
$\angle e^{i+1} = \angle [e^{i}\, e^{1}] = \underbrace{\angle e^{i}}_{1 \text{ radian}} + \underbrace{\angle e^{1}}_{0} = 1 \text{ radian}$

$= 57.3^{\circ}$

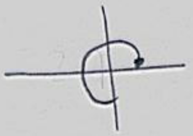
b) $|-2| = 2$

$\angle -2 = \pi$ radian $= 180^{\circ}$

c) $|3-4i| = \sqrt{3^2+4^2} = 5$




d) $|-\pi i| = \pi = 3.14$

$\angle -\pi i =$  $270^{\circ} = \frac{3\pi}{2} = 4.71$ radian

e)

5

a) $z^5 = 32 \qquad \theta = 0 \qquad R = \sqrt[5]{32} = 2$

$$\alpha = \frac{2k\pi + \theta}{5}$$

$\alpha_0 = 0 \qquad \alpha_1 = 72^\circ \qquad \alpha_2 = 144 \qquad \alpha_3 = 216 \qquad \alpha_4 = 288$

$w_0 = 2(\cos 0 + i \sin 0) = 2$

$w_1 = 2(\cos 72 + i \sin 72) = 0.6 + 1.9i$

$w_2 = \qquad = -1.6 + 1.17i$

$= -1.6 - 1.17i$

$= 0.6 - 1.9i$

b) $z^3 = 8i \qquad \theta = 90^\circ \qquad R = \sqrt[3]{8} = 2$

$\alpha_0 = \frac{90}{3} = 30 \qquad \alpha_1 = \frac{90+360}{3} = 150^\circ \qquad \alpha_2 = 270^\circ$

$w_0 = -1.73 + i \qquad w_2 = -1.73 + i \qquad w_3 = -2i$

6) $f(z) = \sqrt{z+1} \qquad f'(z) = \frac{1}{2\sqrt{z+1}}$

$f(0) = 1 \qquad f'(0) = \frac{1}{2}$

$L(z) = f(0) + f'(0) \cdot z = \underline{1 + \frac{1}{2} z}$

$\sqrt{1+i} = 1 + \frac{1}{2}(1+i) = 1.5 + 0.5i$

(7)

$$\frac{10z^2+20}{z^4-3z^2-4} = \frac{A}{z-i} + \frac{B}{z+i} + \frac{C}{z+2} + \frac{D}{z-2}$$

second residue formula.

$$\frac{10z^2+20}{\varphi'(z)} = \frac{10z^2+20}{4z^3+6z}$$

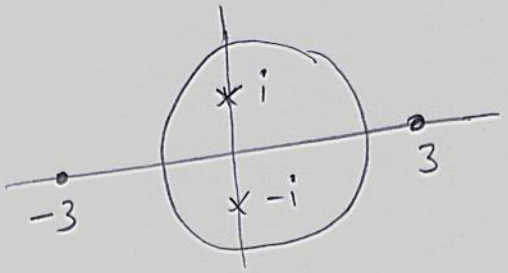
$$A = \frac{10i^2+20}{4i^3+6i} = i$$

$$B = \frac{10(-i)^2+20}{4(-i)^3+6(-i)} = -i$$

$$C = \frac{10(-2)^2+20}{4(-2)^3+6(-2)} = -3$$

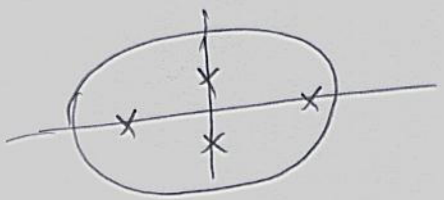
$$D = \frac{10(2)^2+20}{4(2)^3+6(2)} = 3$$

b)




$2\pi i(A+B) = 0$

c)



$2\pi i(A+B+C+D) = 0$

8) $A = \begin{bmatrix} 4 & 3 \\ -3 & 4 \end{bmatrix}$

$$\det \begin{bmatrix} 4-\lambda & 3 \\ -3 & 4-\lambda \end{bmatrix} = (4-\lambda)(4-\lambda) - (3)(-3)$$

$$= \lambda^2 - 8\lambda + 25$$

$$\lambda_1 = 4 + 3i$$

$$\lambda_2 = 4 - 3i$$

9) $Ax = \lambda x$ or $\{A - \lambda I\} x = 0$

$$\begin{bmatrix} 6 & 14 & -8 \\ -15 & 71 & -35 \\ -24 & 104 & -50 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = 6 \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\begin{bmatrix} 6-6 & 14 & -8 \\ -15 & 71-6 & -35 \\ -24 & 104 & -50-6 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

rank $= 2$ $\quad n = 3 \quad 3 - 2 = 1$ variable free

Set $p = 1$ calculate $q$ and $r$

$q =$ $\qquad r =$