Analytical Methods in Engineering (EE300) Final 24-5- 2004 5-4-1425

1)a)Reduce the following matrix into echolen form.

b)Calculate the determinat of this matrix.

c)What is the the rank of this matrix.

$$A = \begin{bmatrix} 1 & 3 & 2 & 4i \\ 1.5 & 1 & 7 & 6i \\ 2 & 6 & 9 & 10i+8 \\ 4 & 12 & 8 & 2i+5 \end{bmatrix},$$

2)Find the values of p and q such that the following vectors are linearly dependent

$$\begin{bmatrix} i \\ i-1 \\ q \end{bmatrix}, \quad \begin{bmatrix} i+1 \\ p \\ 2 \end{bmatrix},$$

3) Calculate the following expressions. Use the principal value of the logarithm function. The result must be in **cartesian** form

a) $(1-2i)^{-6+8i}$ **b)** 2^{6-8i} **c)** $2^{(-6-8i)}$

4)Find all the values of z that satisfies the following equation. The result must be in **cartesian** form.

$Z^3=6+8i$

5)It is given that the angle of Z is 120^0

$<Z=120^0$

State **TRUE, FALSE**, or **UNKNOWN** for the following statements. Give explaination for your answer.

A)Magnitude of e^Z is less than 1.

B)Magnitude of e^Z is greater than 1.

C)Magnitude of e^{iZ} is less than 1.

D)Magnitude of e^{-Z} is less than 1.

E)Angle of Z^2 is -120^0

F)Angle of Z^{-2} is -120^0

6)Calculate the Taylor series expansion of the function

$f(\theta)=\cos(q(\theta)) = A+B\,\theta + C\,\theta^2 + \ldots$

where derivatives of $q(\theta)$ are given as:

$q(\theta)=4;$ $q'(\theta)=-3;$ $q''(\theta)=2;$

(only three terms are required, Calculate A,B,C only)

7)Calculate the following complex integrals

a) $\displaystyle\oint_{C1} \frac{z+2}{z+0.5}dz$ b) $\displaystyle\oint_{C1} \frac{z^2+2}{(z+0.5)^2}dz$

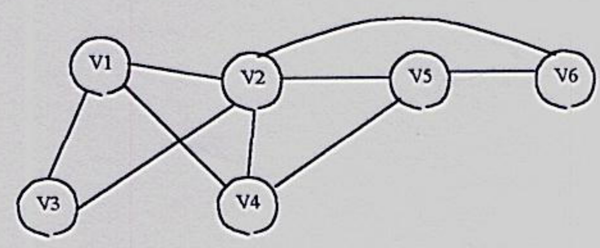
c) $\displaystyle\oint_{C2} \frac{z+2}{(z+1)(z+4)}dz$

C1 is the unit circle and C2 is the circle with radius 3 and center 5+2i

8)Calculate the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1+3i & 7 \\ 0 & 0 & 1-3i \end{bmatrix}$$

9)A simple graph is shown below. Write the adjacency matrix of this graph.



(1) $\det = -87.5 \cancel{+} 245i$

$$\begin{bmatrix} 1 & 3 & 2 & 4i \\ 1.5 & 1 & 7 & 6i \\ 2 & 6 & 9 & 10i+8 \\ 4 & 12 & 8 & 2i+5 \end{bmatrix} \begin{array}{l} -1.5R_1 + R_2 \to R_2 \\ -2R_1 + R_3 \to R_3 \\ -4R_1 + R_4 \to R_4 \end{array} \begin{bmatrix} 1 & 3 & 2 & 4i \\ 0 & -3.5 & 4 & 0 \\ 0 & 0 & 5 & 8+2i \\ 0 & 0 & 0 & 5-14i \end{bmatrix}$$

(2)

$$\begin{bmatrix} i & i-1 & q \\ i+1 & p & 2 \end{bmatrix} \quad -\frac{i+1}{i} R_1 + R_2 \to R_2$$

$$-\frac{i+1}{i}(i-1) + p = 0 \qquad \boxed{p = 2i} \qquad \boxed{q = 1+i}$$

(3) $$(1-2i)^{-6+8i} = e^{(-6+8i)\,\mathrm{Ln}(1-2i)} \qquad \mathrm{Ln}(1-2i) = 0.804 - 1.104i$$

$$= e^{4+13i} = 56(0.907 + 0.42i) = 50.81 + 23.6i$$

$$48.92 + 27.6i$$

$\mathrm{Ln}\, 2 = 0.693$

$$\boxed{2^{6-8i} = 47.3 + 43i} \qquad \boxed{2^{-6-8i} = 0.0116 + 0.01i}$$

(4) $z^3 = 6+8i \qquad \theta = \tan^{-1}\frac{8}{6} = 0.927 = 53.13^\circ \qquad r = \sqrt[3]{10} = 2.154$

$\alpha_0 = \frac{\theta}{3} = 0.309 = 17.7^\circ \qquad z_0 = 2.05 + 0.65i$

$\alpha_1 = \quad = 2.4 = 137^\circ \qquad z_1 = -1.59 + 1.44i$

$\alpha_2 = \quad = 4.49 = 257^\circ \qquad z_2 = -0.45 - 2.1i$

(5) A) $|e^z| < 1$ TRUE B) FALSE c) $e^{iz} = e^{-b+ai}$ $|e^{iz}| < 1$ TRUE

D) $|e^{-z}| > 1$ FALSE E) $z^2 = 2^2 e^{240}$ $240 = -110$ TRUE f) $z^2 = 2^2 e^{-240}$ FALSE

(6) $f'(0) = -\sin[g(0)] \times g'(0) = -\sin 4\,(-3) = -2.27$

$f''(0) = -\{\cos[g(0)]\, g'(0)\, g'(0) + \sin g(0)\, g''(0)\} =$

$= -\{-0.65\,(-3)(-3) + (-0.75)\,2\} = 7.39$

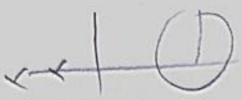
$f(0) = \cos[g(0)] = \cos(4) = -0.65$

$A = -0.65$

$B = -2.27$

$C = \frac{7.39}{2!} = 3.69$

(7) a) $\displaystyle\int \frac{z+2}{z+0.5}\,dz = (-0.5+2)\,2\pi i = 9.42i$ b) $\displaystyle\int \frac{z^2+2}{(z+0.5)^2} = 2\times(-0.5)\,2\pi i = -6.28i$

c)  integration is zero

(8) $\lambda_1 = 2 \qquad \lambda_2 = 1+3i \qquad \lambda_3 = 1-3i \qquad V_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \qquad V_2 = \begin{bmatrix} -ib \\ b \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ ai \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ i \\ 0 \end{bmatrix} = \begin{bmatrix} -i \\ 1 \\ 0 \end{bmatrix}$

$$V_3 = \begin{bmatrix} 1 \\ -0.43 - 0.75i \\ -0.64 + 0.36i \end{bmatrix} = \begin{bmatrix} -0.37 - i \\ 1 \\ -0.85i \end{bmatrix} = \begin{bmatrix} -1.16 - 0.66i \\ 1.16i \\ 1 \end{bmatrix}$$

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 \\ 4 & 1 & 1 & 0 & 0 & 1 & 0 \\ 5 & 0 & 1 & 0 & 1 & 0 & 1 \\ 6 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}$$