Analytical Methods in Engineering (EE300) Final 27-12-2004 15-11-1424

1) Given the following matrices, calculate

a) AB, b) A^{-1} (Note: $(i=\sqrt{-1})$

$$A=\begin{bmatrix}1-i & 0 & 2i\\ 0 & 2i & 0\\ 0 & 0 & i\end{bmatrix},\qquad B=\begin{bmatrix}0 & 1 & 0\\ 5i & 0 & -1\\ 0 & i-1 & -i\end{bmatrix}$$

2) Find the value of p such that the following vectors x, y, z are linearly **dependent**.

$$x=\begin{bmatrix}1\\0\\3\end{bmatrix},\quad y=\begin{bmatrix}1\\2\\2\end{bmatrix},\quad z=\begin{bmatrix}2+2i\\4\\p\end{bmatrix}$$

(Hint: Calculate the rank of $\begin{bmatrix}x\\y\\z\end{bmatrix}$

3) It is given that $\tan^{-1}(\frac{B}{A})=50^\circ$, $A^2+B^2=4$ and $\tan^{-1}(\frac{D}{C})=40^\circ$, $C^2+D^2=1$ Calculate the **magnitude** and the **angle** of the following expressions

a) $\dfrac{A+Bi}{C-Di}$ b) $\dfrac{A-Bi}{-C+Di}$ c) $\dfrac{-i}{(-C+Di)(A-Bi)}$

4) a) Calculate Taylor series for the function $f(z)=e^{z^2+3z+1}$ about $z_0=0$ (Maclauren series) (calculate three terms only)

5) a) Calculate A,B,C,D in the following fraction.

$$\frac{10z^2+20z+20}{z^4-3z^2-4}=\frac{10z^2+20z+20}{(z+i)(z-i)(z+2)(z-2)}$$

$$=\frac{A}{(z-i)}+\frac{B}{(z+i)}+\frac{C}{(z+2)}+\frac{D}{(z-2)}$$

Using theses results calculate the integral

$$\oint_C \frac{10z^2+20z+20}{z^4-3z^2-4}\,dz \quad \text{where C is}$$

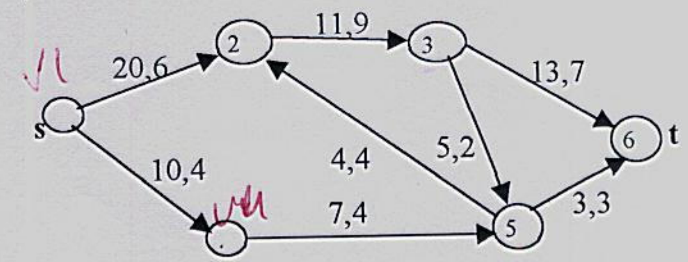
b) $|z|=1.5$ circle. A circle whose center is origin and radius is 1.5.

c) $|z|=10$ circle. A circle whose center is origin and radius is 10

6) The adjacency matrix of a simple graph is given below. Draw this graph.

$$\begin{array}{c|c} & \begin{matrix}V1 & V2 & V3 & V4 & V5\end{matrix}\\ \begin{matrix}V1\\V2\\V3\\V4\\V5\end{matrix} & \begin{bmatrix}0 & 1 & 1 & 1 & 0\\ 1 & 0 & 1 & 0 & 1\\ 1 & 1 & 0 & 1 & 0\\ 1 & 0 & 1 & 0 & 1\\ 0 & 1 & 0 & 1 & 0\end{bmatrix}\end{array}$$

7) Examine the following figure



a) Write node equations for all vertices and **check** whether all node equations are satisfied or not satisfied?

State true or false for the following sentences Write an explanation if necessary.

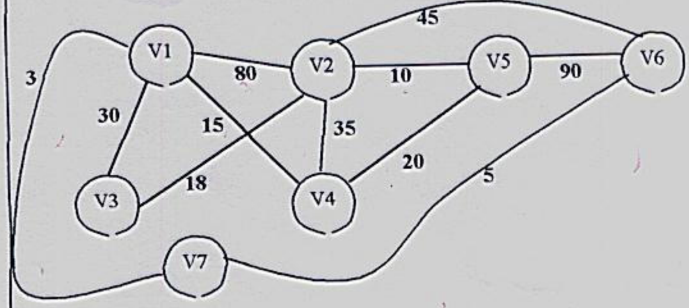
b) This figure represents a graph

c) This figure represents a diagraph

d) This figure represents a complete graph

e) This figure represents a network

8) Find the shortest **spanning tree** in the following graph using Krustal Algorithm.



(Note: it is not shortest length)

2) $\begin{bmatrix} 1 & 0 & 3 \\ 1 & 2 & 2 \\ 2+2i & 4 & P \end{bmatrix}$ $(-2+2i)R_1 + R_3 \to R_3$ $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 4 & P-3(2+2i) \end{bmatrix}$

$-2R_2 + R_3 \to R_3$ $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & P-(4+6i) \end{bmatrix}$

$$P-(4+6i) = 0$$

$$P = 4+6i$$

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3) $\tan^{-1} \frac{B}{A} = 50^\circ$

$\angle A+Bi = 50^\circ$ $\qquad \angle A-Bi = -50^\circ = 310^\circ$

$\angle C+Di = 40^\circ$ $\qquad \angle C-Di = -40^\circ$

$\angle -C+Di = 180^\circ - 40^\circ = 140^\circ$

$$\angle \frac{A+Bi}{C+Di} = 50-(-40) = 90^\circ$$

$$\angle \frac{A-Bi}{-C+Di} = -50-(140) = -190^\circ$$

$$\angle \frac{-i}{(-C+Di)(A-Bi)} = -90 - [140 + (-50)] = 180^\circ$$

4) $f(z) = e^{z^2+3z+1} = e^{g(z)}$

$$f'(z) = g'(z)\, e^{g(z)} = p\, q$$

$$f''(z) = g'' z\, e^{g(z)} + g'(z)\, g'(z)\, e^{g(z)}$$

$$p'\, q + p\, q'$$

$g(z) = z^2 + 3z + 1 \qquad g(0) = 1$

$g'(z) = 2z + 3 \qquad g'(0) = 3$

$g''(z) = 2 \qquad g''(0) = 2$

$f(0) = e^{g(0)} = e$

$$f'(0) = g'(0)\, e^{g(0)} = 3\, e^1 = 3e$$

$$f''(0) = g''(0)\, e^{g(0)} + g'(0)\, g'(0)\, e^{g(0)}$$

$$= 2e + 3\, 3\, e^1 = 11e$$

$$f(z) = f(0) + f'(0)z + \frac{f''(0)}{2!} z^2$$

$$= e + 3ez + \frac{11e}{2!} z^2$$