Analytical Methods in Engineering (EE300) Final **Part I** 30-5-2005 21-4-1426

Fill the blanks ONLY.

1)It is given that $A=\begin{bmatrix}2&0\\0&3\end{bmatrix}$, $B=\begin{bmatrix}0&0\\0&0\end{bmatrix}$,

rank A= 2 rank B= 0 rank(AB)= 0

2) $A=\begin{bmatrix}2i&0&0\\0&4i&0\\0&0&1\end{bmatrix}$ $A^{-1}=\begin{bmatrix}-0.5i&0&0\\0&-0.25i&0\\0&0&1\end{bmatrix}$

$\frac{1}{2}i=-0.5i$ $\frac{1}{4i}=-0.25i$

3) (A,B,X are matrices)

$AX+BX=B$, $X=$ $[A+B]^{-1}B$

$[A+B]X=B$

$X=[A+B]^{-1}B$

4) $x=\begin{bmatrix}1\\2\\3\end{bmatrix}$, $Y=\begin{bmatrix}10\\20\\30\end{bmatrix}$ $y=10x$

x and y are linearly dependent

5) $A=\begin{bmatrix}a&b&c\\1&2&3\\x&y&z\end{bmatrix}$, det A= -3

$B=\begin{bmatrix}a&b&2c\\1&2&6\\3x&3y&6z\end{bmatrix}$, det B= $2\times3(-3)=-18$

$C3\to 2C3$

$r3\to 2r3$

6) $x=\begin{bmatrix}a\\b\\c\end{bmatrix}$, $y=\begin{bmatrix}p\\a\\r\end{bmatrix}$, $z=\begin{bmatrix}m\\n\\a\end{bmatrix}$

It is given that x,y,z are linearly **dependent**

a,b,c,p,r,m,n are all **nonzero** elements.

$A=\begin{bmatrix}a&p&m\\b&a&n\\c&r&a\end{bmatrix}$, $B=\begin{bmatrix}a&b&c\\p&a&r\\m&n&a\end{bmatrix}$

Write **True** *or* **False** .

a) det A = a^3 F

b) det B = a^3 F

c) det A= 0 T

d) det B= 0 T

e) A^{-1} = B F

f) B^{-1} =A F

g) Rank A=3 F

h) Rank A = Rank B T

7) a) $3i(5-2i)=$ $15i+6$

b) $\frac{1}{-1+i}=$

8) It is given that $<(a+bi)=20^{o}$

(< **means angle**)

calculate the angle of the following numbers

(write the results in **degrees**)

a) $<2(a+bi)=2\times20=40$

b) $<(-a+bi)=180-20=160$

c) $<(a-bi)=-20$

d) $<(-a-bi)=180+20=200$

e) $<(a+bi)^3=3\times20=60$

f) $<\frac{1}{a+bi}=0-20=-20$

9) It is given that $qe^{bi}=3+2i$

$qe^{-bi}=3-2i$

10) Find the magnitude of the following numbers

a) $|3+4i| = \sqrt{3^2+4^2} = 5$

b) $|-3-4i| = \quad = 5$

c) $|3i| = 3$

d) $|e^{i\pi}| = 1$

e) $|e^{-i\pi}| = 1$

f) $|e^{1+i\pi}| = |e^1||e^{i\pi}| = e = 2.71$

11) a) $i^{19} = i^{16}\, i^3 = i^3 = -i$

b) $i^{-19} = i^{-16}\, i^{-3} = i^{-3} = (i^3)^{-1} = (-i)^{-1} = i$

c) $i^{20} = 1$

d) $(-1)^{16751} = -1$

12) Complete the following formulas

a) $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$

b) $\cos x = \dfrac{e^{ix} + e^{ix}}{2}$

b) $\sinh x = \dfrac{e^{x} - e^{x}}{2}$

b) $\cosh x = \dfrac{e^{x} + e^{x}}{2}$

13) $\sinh 2 = 3.62$, $\sin 2i = i \sinh$

14) Maclauren series expansion of $f(z)=e^{z^2+i}$ is

$f(z) = e^i + (\ Q\ )z + (\ e^i\ )z^2 + \ldots\ldots$ $\quad f' = 2z\, e^{z^2+i}$

$Q = f'(z)\big|_{z=0} = 2z\, e^{z^2+i}\big|_{z=0} = 0$

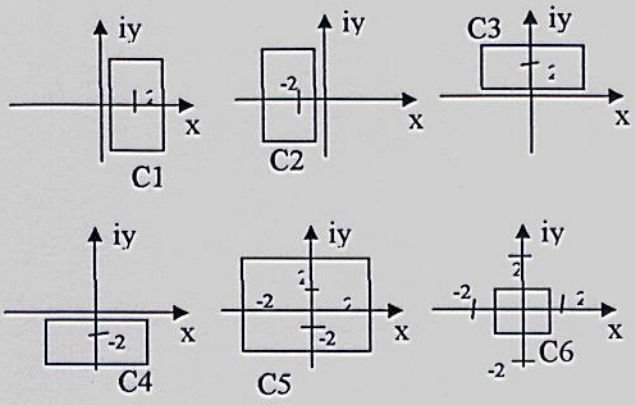
15) It is given that $\sin\Theta = 2$. Write **True** or **False**

a) Θ is a real number and $90^0 > \Theta > 180^0$ False

b) Θ is a real number and $180^0 \Theta > 360^0$ false

c) Θ is a complex number. True

16) Calculate the following integrals where paths are shown in the graphs.



a) $\oint_{C1} \dfrac{3}{z+2}dz =$

b) $\oint_{C1} \dfrac{3z}{z+2}dz =$

c) $\oint_{C2} \dfrac{3}{z+2}dz =$

d) $\oint_{C2} \dfrac{3z}{z+2}dz =$

e) $\oint_{C4} \dfrac{3z}{z+2i}dz =$

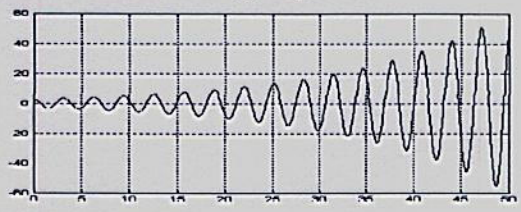
f) $\oint_{C5} \dfrac{3z}{z+2i}dz =$

17) State **True** or **False** (paths are shown above in)

g) $\oint_{C5} \dfrac{z^3+z^2+z}{(z^2+4)(z^2-4)}dz = 0$

h) $\oint_{C6} \dfrac{z^3+z^2+z}{(z^2+4)(z^2-4)}dz = 0$

18) A dynamic system has the linear differential equation $\dot{Q} = AQ$, Q is a **vector** A is a **matrix.** This system has the following response.



write **True** or **False**

b) This system has one real one complex eigenvalues F

a) This system has complex eigenvalues T

c) Real part of one eigenvalue is greater than zero T

d) Real part of all eigenvalues are less than zero F

e) $\lim_{t\to\infty} q(t) = 0$ F