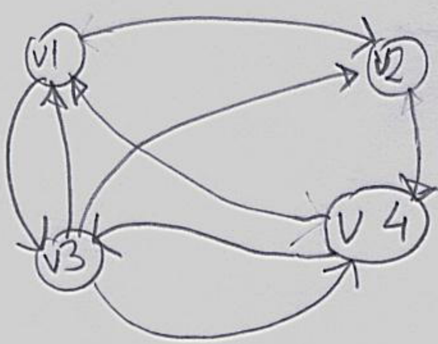
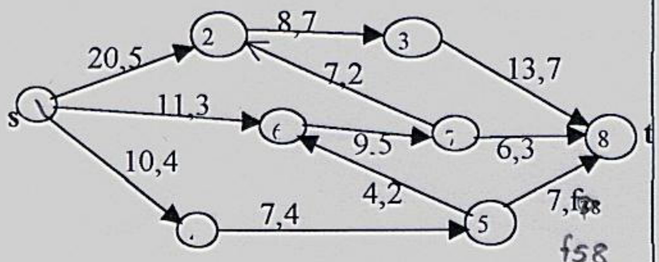
Analytical Methods in Engineering **(EE300)** Major -2 3 DEC 2005 1-11-1426

1) Draw the diagraph whose adjacency matrix is given below.

$$\begin{array}{cc} & \text{from} \\ \text{To} \downarrow & A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



2)a) Calculate f_{38} in the following network.
b)Write all flow augmenting paths in the network.



c)Obtain maximum flow on path 1-6-5-8
d)Obtain maximum flow on path 1-2-7-8

a) $4-2-f_{58}=0$ $\boxed{f_{58}=4-2=2}$

b)

1-2-3-8	1-6-5-8
1-2-7-8	1-4-5-8
1-2-7-6-5-8	1-4-7-8
1-6-7-8	1-4-2-3-8
1-6-7-2-3-8	Total 9

Path 1-6-5-8

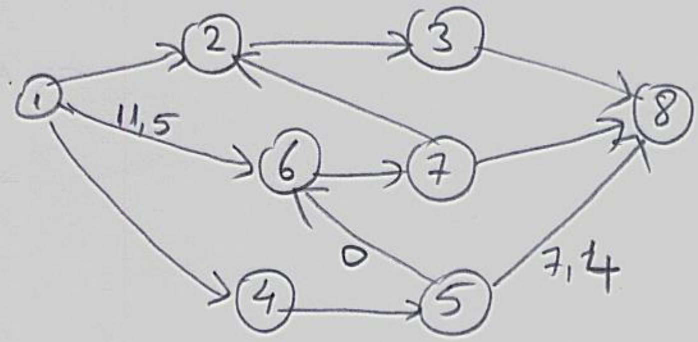
$\Delta_{16} = c_{16} - f_{16} = 11 - 3 = 8$

$\Delta_{65} = 2$ (reverse direction)

$\Delta_{58} = c_{58} - f_{58} = 7 - 2 = 5$

$\min(\Delta_{16}, \Delta_{65}, \Delta_{58}) = \min(8, 2, 5)$
$= 2$

$\Delta t = 2$



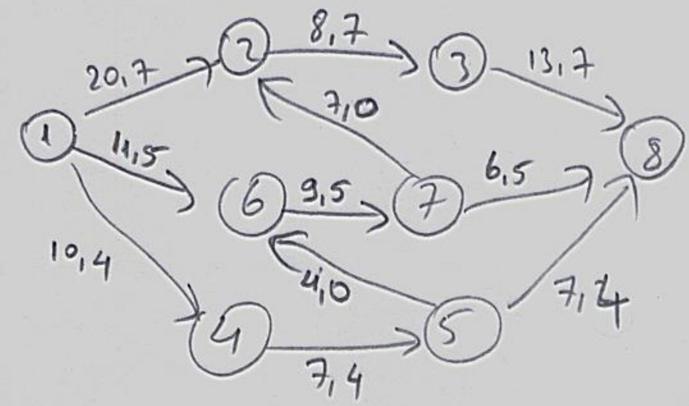
Path 1-2-7 8

$\Delta_{12} = c_{12} - f_{12} = 20 - 5 = 15$

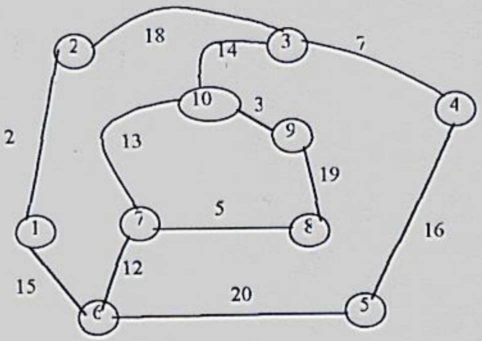
$\Delta_{27} = 2$ (reverse direction)

$\Delta_{78} = c_{78} - f_{78} = 6 - 3 = 3$

$\min(15, 2, 3) = 2$ $\boxed{\Delta t = 2}$

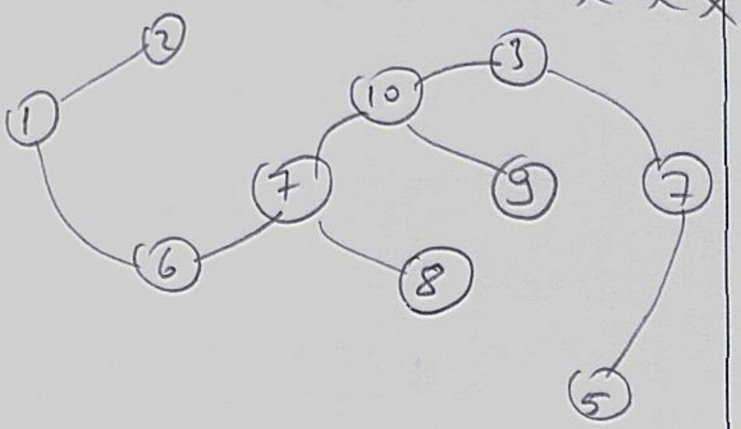


3)Find the shortest spanning tree for the following graph (Use Krustal Algorithm)

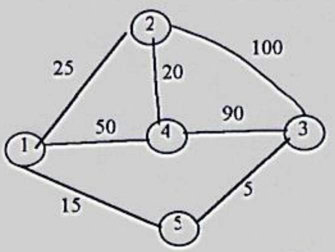


2, 3, 5, 7, 12, 13, 14, 15, 16, 18, 19, 20

× × ×



4) Using Dijkstra's Algorithm Calculate the shortest paths from **vertex 4** to all other vertices. (**Use Dijkstra's Algorithm.)**



$TL=\{1,2,3,5\}$

$FV_1=FV_2=FV_3=4$

$FV_5=4$

$\bar L_1=50 \quad \bar L_2=50 \quad \bar L_3=90 \quad \bar L_5=\infty$

$\min(\bar L_1,\bar L_2,\bar L_3,\bar L_5)=20=\bar L_2=L_2$

$K=2$

$TL=\{1,3,5\}$

$\bar L_1=\min(\bar L_1, L_2+L_{21})$

$=\min(50, 20+25)=45$

$\boxed{FV_1=2}$

$\bar L_3=\min(\bar L_3, L_2+L_{23})$

$=\min(90, 20+100)=90$

$\bar L_5=\min(\bar L_5, L_2+L_{25})=$

$=\min(\infty, 20+\infty)=\infty$

$\min(\bar L_1,\bar L_3, L_5)=45=\bar L_1=L_1$

$K=1 \quad TL=\{3,5\}$

$\bar L_3=\min(\bar L_3, L_1+L_{13})=90$

$\bar L_5=\min(\bar L_5, L_5+L_{15})=$

$=\min(\infty, 45+15)=60$

$\boxed{FV_5=1}$

$\min(\bar L_3,\bar L_5)=\min(90,60)=60$

$=\bar L_5=L_5$

$K=5$

$TL=\{3\}$

$\bar L_3=\min(\bar L_3, L_5+L_{53})$

$=\min(90, 60+5)=65$

$FV_3=5$

$TL=\{\}$

$FV_1=2 \quad 1\to 2$

$FV_5=1 \quad 5\to 1$

$FV_3=5 \quad 3\to 5$

$FV_2=4 \quad 2\to 4$

$2\to 4 \Rightarrow 20$

$1\to 2\to 4 \Rightarrow 25+20=45$

$5\to 1\to 2\to 4 \Rightarrow 15+25+20=60$

$3\to 5\to 1\to 2\to 4 \Rightarrow 5+60=65$

5)It is given that a,b,c,d are **real** numbers. (a,b,c,d may be negative or positive). $<(a+bi)=20^0$, $<(c+di)=220^0$.
Calculate the angle of the following numbers.

a) $\dfrac{1}{a+bi}$ b) $\dfrac{a-bi}{c+di}$ c) $\dfrac{1}{c-di}$ d) $\dfrac{(a+bi)^3}{(c+di)^2}$

e) $[2(a+bi)]\,[3(c+di)]$

a) $\angle \dfrac{1}{a+bi} = \angle 1 - \angle a+bi$

$= 0 - 20 = -20^\circ$

b) $\angle a-bi = -20^\circ$

$\angle \dfrac{a-bi}{c+di} = (-20^\circ) - (220^\circ) = -240^\circ$

$-240^\circ = 120^\circ$

c) $\angle c-di = -220^\circ$

$\dfrac{1}{c-di} = \angle 1 - \angle c-di$

$= 0 - (-220) = +220^\circ$

d) $\angle (a+bi)^3 = 3 \times 20 = 60^\circ$

e) $2(a+bi)\,3\,(c+di)$

$= 6(a+bi)(c+di)$

$\angle 6(a+bi)(c+di) = \angle 6 + \angle a+bi + \angle c+di$

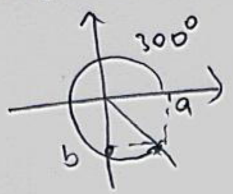
$= 0 + 20 + 220$

$= 240^\circ$

6)a,b are real numbers, and $<(a+bi)=300^0$
State true or false

a) $|e^{a+bi}| > 1$ True
b) $|e^{a+bi}| = 1$ false
c) $|e^{a+bi}| < 1$ false

(write explanation if necessary)



$a > 0$
$b < 0$

$|e^{a+bi}| = |e^{a}|\,|e^{bi}|$

$\downarrow \quad \downarrow_1$

$a > 0 \Rightarrow |e^{a}| > 1$

$|e^{bi}| = 1$ always

7)cosh(z)=0.5. calculate z. (z is a complex number)

method 1

$\cosh(z) = \cos(iz)$

$\cos(iz) = 0.5$

$iz = \cos^{-1}(0.5) = 1.047$ radian

$iz = 1.047 \Rightarrow z = \dfrac{1.047}{i} = -1.047i$

method 2

$\cosh(z) = \dfrac{e^z + e^{-z}}{2} = 0.5 \Rightarrow e^z + e^{-z} = 1$

define $m = e^z \Rightarrow e^{-z} = m^{-1} = \dfrac{1}{m}$

$m + \dfrac{1}{m} = 1 \Rightarrow m^2 + 1 = m \Rightarrow m^2 - m + 1 = 0$

roots $m_{1,2} = \dfrac{1 \mp \sqrt{1-4}}{2} = 0.5 \mp 0.866i$

$m = e^z \Rightarrow z = \ln(m) = \ln(0.5 + 0.866i)$

$\ln(0.5 + 0.866i) = \text{Ln}\left(\sqrt{0.5^2 + 0.866^2}\right) + \theta i$

$= 0 + 1.04i$

$\ln(0.5 - 0.866i) = 0 - 1.04i$

8)Calculate a) $\sqrt{-i}$ b) $i^{\sqrt{-i}}$

(Hint for part a, use $z^2 = -i$)

(use **principle** value, if you use **logarithm**)

$z^2 = -i \qquad \theta = \angle -i = \angle -90^\circ = 270^\circ$

$\alpha = \dfrac{\theta + 2k\pi}{2} = \dfrac{270 + 2 \times 180 \times k}{2}$

$k = 0 \qquad \alpha_1 = \frac{270^\circ}{2} = 135^\circ$

$k = 1 \qquad \alpha_2 = 315^\circ$

$z_1 = r e^{i\alpha_1} = 1 e^{i135} = \cos 135 + i \sin 135$

$= -0.707 + i\, 0.707$

$z_2 = r e^{\alpha_2} = 0.707 - i\, 0.707$

$i^{\sqrt{-i}} = i^{-0.707 + 0.707 i} = e^{(-0.707 + i\, 0.707) \ln i}$

$\ln i = \ln 1 + i \frac{\pi}{2} = \frac{\pi}{2} i = 1.57 i$

$i^{\sqrt{-i}} = e^{(-0.707 + i\, 0.707)\, 1.57 i}$

$= e^{-1.11 - 1.11 i} = e^{-1.11} e^{-1.11 i}$

$= 0{,}3296 (\cos \underset{\text{radian}}{1.11} + i \sin (1.11))$

$= 0.1465 - 0.2952 i$

$i^{0.707 - 0.707 i} = e^{(0.7 - 0.7 i)\, 1.57 i}$

$= 1.3\ \cancel{4}8 + 2.72 i$

P.O