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Analytical Methods in Engineering (EE300) Major -2 9 APR 2005 30-2-1426

DO NOT USE CALCULATOR

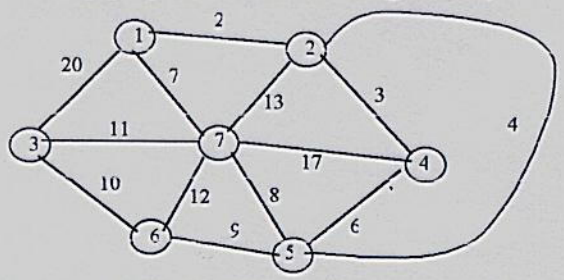
Any type of calculator is not allowed.

Warning: SHOW your calculations step by step. If you write the answer without any explanation, you may get zero from that question.

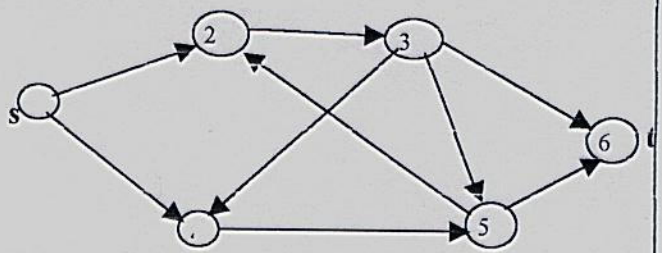
1) Draw the diagraph whose adjacency matrix is given below.

$$A=\begin{bmatrix}0 & 0 & 1 & 1\\ 0 & 0 & 0 & 0\\ 1 & 1 & 0 & 1\\ 0 & 1 & 1 & 0\end{bmatrix}$$

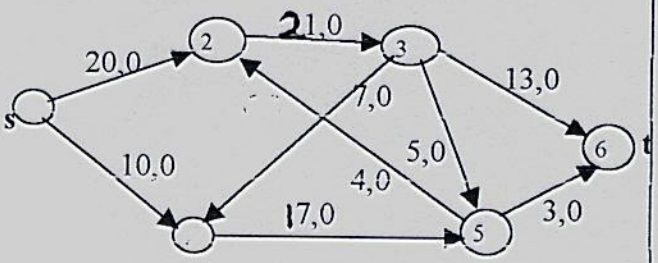
2)Find the shortest spanning tree for the following graph (Use Krustal Algorithm)



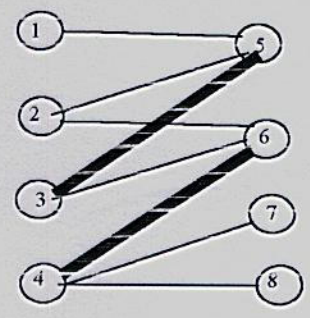
3)Show all flow augmenting paths in the following network.



4)Obtain Maximum flow in the following network.



5)Obtain the best matching (maximum cardinality matching) for the following graph



6)Find the magnitude and angle of the following complex numbers. Write the results in degrees)

a) $\dfrac{1}{-3+4i}$, b) $\dfrac{e^{-\pi i}}{-2i}$ c) $\dfrac{3-4i}{3+4i}$

($\tan 53^{\circ}=4/3$).

7)Calculate z in the following equation. (find the roots)

$z^3=-1$

8)It is given that a,b,c,d are all positive real numbers.

$\tan^{-1}(b/a)=30^{0}$, $\tan^{-1}(d/c)=50^{0}$

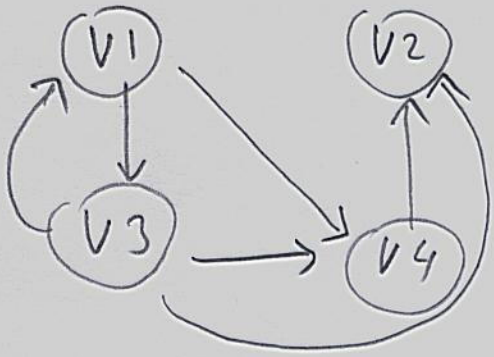
$\tan^{-1}(8/3)=\tan^{-1}(2.66)=70^{0}$

Calculate x and y in the following equation

$$\frac{(-a-bi)\ (c-di)\ (-3+8i)}{(a+bi)\ (c+di)\ (3+8i)}=x+yi$$

1)

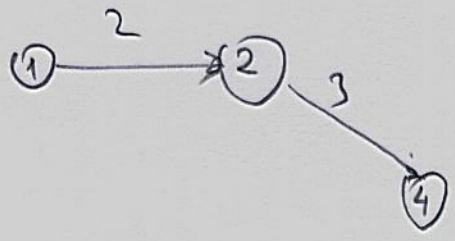
$$A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

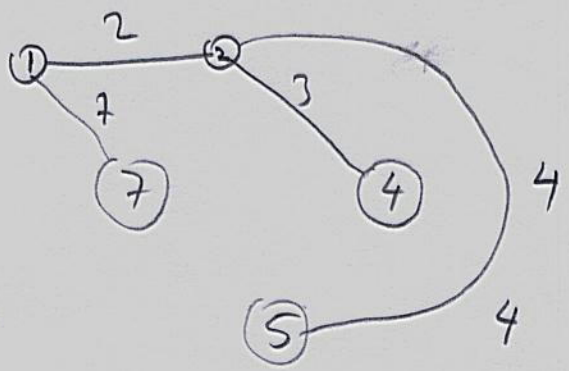


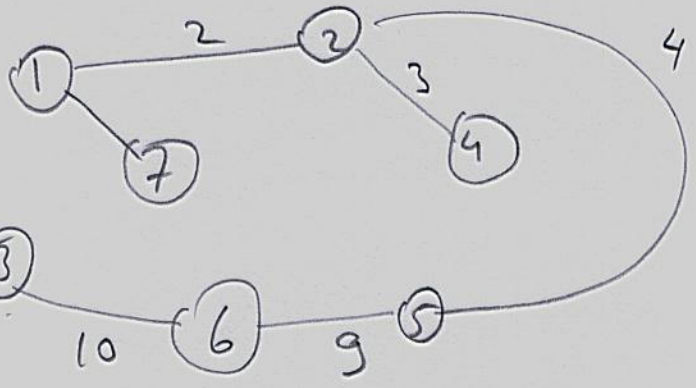
2)

2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 17, 20

1-2, 2-4, 2-5, 3-6, 1-7, 7-5, 5-6, 3-6, 3-7, 6-7, 2-7, 4-7, 1-3







(3)

1-2-3-6

1-2-5-6

1-2-3-5-6

1-2-3-4-5-6

1-2-4-5-6

1-2-5-3-6

1-4-5-6

1-4-5-2-3-6

1-4-3-6

1-4-3-5-6

1-4-3

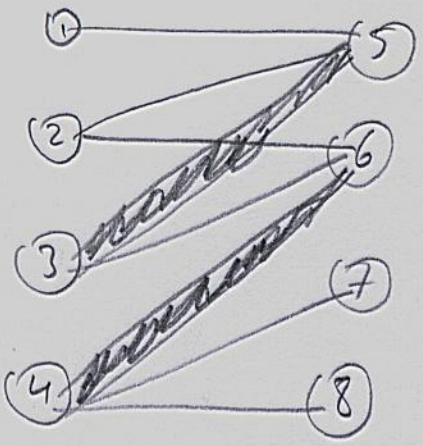
(4) $P1 = 1-2-3-6 \quad \Delta t = 13$

$P2 = 1-4-5-6 \quad \Delta t = 3$

then max capacity is reached.

Note: You must give explanations !!!

(5)



1-5-3-6-4-8 (P1)

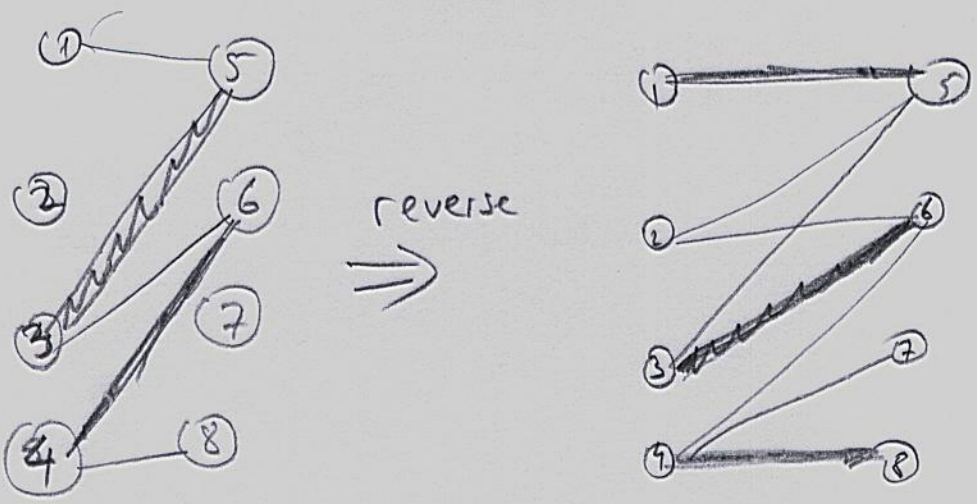
or

1-5-3-6-4-7 (P2)

7-4-6-2 (P3)

alternating paths

Select 1-5-3-6-4-8



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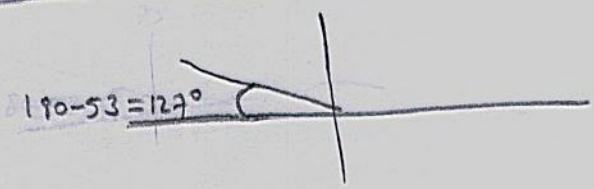
(6) $\angle -3+4i = 127°$



$$\angle \frac{1}{-3+4i} = \angle 1 - \angle(-3+4i) = 0-(127) = -127°$$

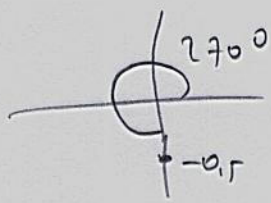
$$\left| \frac{1}{-3+4i} \right| = \frac{1}{|-3+4i|} = \frac{1}{5} = 0,2$$

b)

$$e^{-\pi i} = \cos\pi - i\sin\pi = -1$$

$$\frac{e^{-\pi i}}{-2i} = \frac{-1}{-2i} = \frac{-1}{-2}\,\frac{1}{i} = 0.5(-i) = -0.5\,i$$

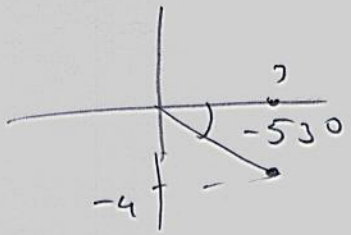
$$\angle -0.5i = 270^\circ$$



$$|-0.5i| = |0-0.5i| = \sqrt{0^2+0.5^2} = 0.5$$

c)

$$\angle 3-4i = -53^\circ$$



$$\angle\frac{3-4i}{3+4i} = \angle(3-4i) - \angle(3+4i)$$

$$= -53^\circ - 53^\circ = -106^\circ$$

$z^3 = -1$ $\quad \theta = 180^\circ$ $\quad \alpha_0 = \frac{180^\circ}{3} = 60^\circ$

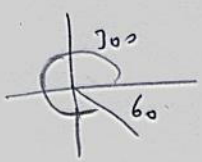
$$\alpha_1 = \frac{180+360}{3} = 180^\circ$$

$$\alpha_2 = \frac{180+720}{3} = 300^\circ$$

$\cos 60 = 0.5$ $\quad \sin 60 = 0.86$

$\cos 180 = -1$ $\quad \sin 180 = 0$

$\cos 300 = ?$



$\cos 300 = \cos(-60)$

$\cos(-60) = \cos 60 = 0.5$

Sin 300 = ? $\sin 300 = \sin(-60) = -\sin 60^\circ$

$= -0.86$

$z_0 = 1.(\cos 60 + i \sin 60) = 0.5 + 0.86i$

$z_1 = 1(\cos 180 + i \sin 180) = -1$

$z_2 = 1(\cos 300 + i \sin 300) =$

$= \cos(-60) + i \sin(-60)$

$= 0.5 - i\ 0.86$

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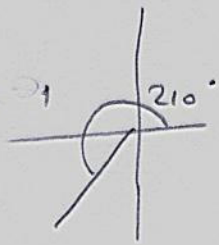
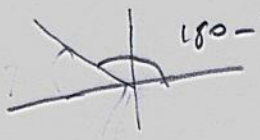
(8) $\angle a - bi = 180 + 30 = 210^\circ$ 

$\theta_1 = 210^\circ$

$\angle c - di = -50^\circ$  $\theta_2 = -50^\circ$

$\angle -3 + 8i = 110^\circ$ 

$\theta_3 = 110^\circ$

$$\frac{(-a-bi)(c-di)(-3+8i)}{(a+bi)(c+di)(3+8i)} = \frac{\sqrt{a^2+b^2}\sqrt{c^2+d^2}\sqrt{3^2+8^2}\ e^{(\theta_1+\theta_2+\theta_3)i}}{\sqrt{a^2+b^2}\sqrt{c^2+d^2}\sqrt{3^2+8^2}\ e^{(\theta_4+\theta_5+\theta_6)i}}$$

$$= \frac{e^{(210^\circ+(-50^\circ)+110^\circ)i}}{e^{(30^\circ+50^\circ+70^\circ)i}} = e^{(270-150)i} = e^{120i} = -0.5 + 0.86$$