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Analytical Methods in Engineering (EE300) Final 21-5-2006 24-4-1427

1) $A=\begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 4 & -1 & 7 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & -1 & Q \end{bmatrix}$

It is known that det(A)=0, calculate Q

$0.5 R_3 + R_4 \rightarrow R_4$

$\begin{matrix} 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & Q+2 \end{matrix}$

$Q+2=0 \quad \boxed{Q=-2}$

2) A,B,C,D,E, X, are all matrices in the following equations. Solve X

a) X C + X D = E

b) $A X^{-1} + B X^{-1} = C$

$X(C+D)=E$

$X = E[C+D]^{-1}$

$(A+B)X^{-1}=C$

$(A+B)=CX$

$C^{-1}(A+B)=X$

$X=C^{-1}(A+B)$

3)Examine the following linear equations

x + 2 y=0

2 x +4 y=1

a)Write these linear equations in matrix form.

b)Calculate rank A, rank $\tilde{A}$ =

A is coefficient matrix. $\tilde{A}$ is augmenting matrix.

c)This system has ---------------- solution.
(write **unique**, **multiple**, or **no solution**)

$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \end{pmatrix} \quad -2R_1+R_2 \rightarrow R_2$

$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

rank A = 1

rank $\tilde{A}$ = 2

No solution

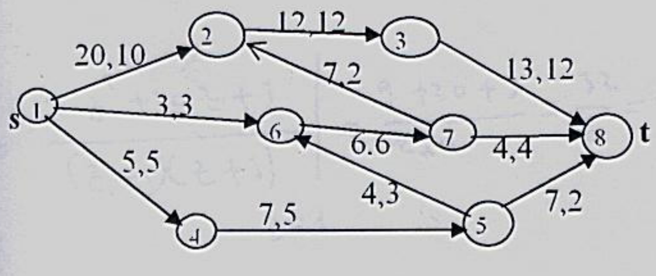
4)The vectors $P=\begin{bmatrix} \alpha \\ 2 \\ 1 \end{bmatrix}$ and $Q=\begin{bmatrix} 1 \\ -1 \\ \beta \end{bmatrix}$ are linearly dependent. Calculate α and β.

$2 \cdot m = -1 \quad m = -0.5$

$\alpha(-0.5)=1 \quad \boxed{\alpha=-2}$

$1(-0.5)=\beta \quad \boxed{\beta=-0.5}$

5) Obtain maximum flow (if possible) on path
1-2-7-6-5-8



1 - 2 - 7 - 6 - 5 - 8

$\Delta_{12} = 20 - 10 = 10$

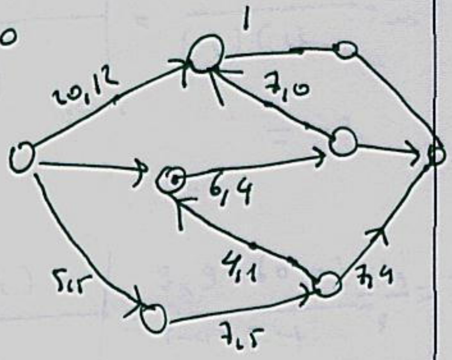
$\Delta_{27} = 2$

$\Delta_{76} = 6$

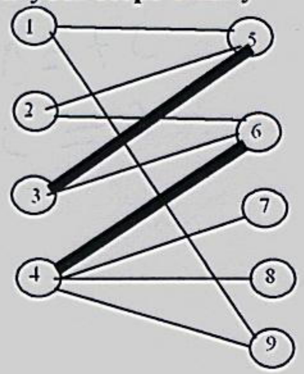
$\Delta_{65} = 3$

$\Delta_{58} = 7 - 2 = 5$

$\mathrm{MIN}(10, 2, 6, 3, 5) = 2$



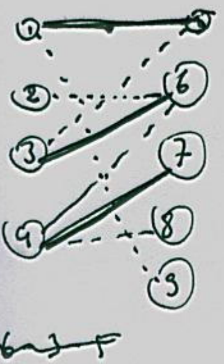
6) Obtain the best matching (maximum cardinality matching) for the following graph
Show your **steps** clearly

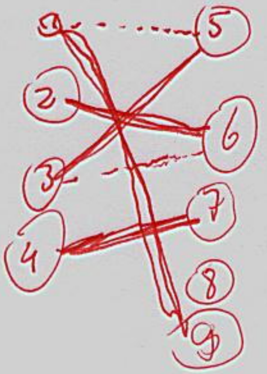


1 - 5 - 3 - 6 - 4 - 7

~~9 - 1 - 5 - 3 - 6 - 4 - 8~~

2 - 6 - 3 - 5 - 1 - 8





$$\left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 & 0 & 0 & 1 & 0 \end{array}\right] \quad R_4 + \frac{1}{4}R_3 \to R_4$$

$$\left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 & 1 & \frac{1}{4} \end{array}\right]$$

$$\begin{bmatrix} 1/2 & 0 & 0 & 0 \\ 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 1/4 \\ 0 & 0 & \frac{1}{2} & 1/8 \end{bmatrix}$$

7) Calculate the inverse of the following matrix by Gaus elimination technique

$$A = \left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad 4R_3 + R_4 \to R_4$$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & -1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 4 & 1 \end{array}\right] \quad -\frac{R_4}{4} + R_3 \to R_3$$

$$\left[\begin{array}{cccc|cccc} 0 & 0 & -1 & 0 & 0 & 0 & 0 & -\frac{1}{4} \\ 0 & 0 & 0 & 8 & 0 & 0 & 4 & 1 \end{array}\right]$$

$\frac{R_1}{2} \to R_1$

$\frac{R_2}{4} \to R_2$

$\frac{R_3}{-1} = R_3$

$\frac{R_4}{8} = R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0.25 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & +0.25 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0.5 & 0.125 \end{array}\right]$$

$$\begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & 0 & +\frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{1}{8} \end{bmatrix}$$

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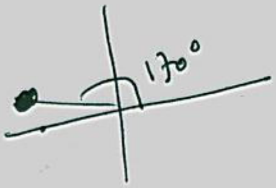
8) Fill the following table.

	magnitude	Angle Degree	Angle Radian
-1 - i	$\sqrt{2}=1.41$	$225=-135$	$3.92=-2.35$
$e^{i\pi}$	1	180°	3.14
$e^{i\pi+1}$	$e=2.71$	180°	3.14
πi	$\pi=3.14$	90°	1.57

→R4

9) It is given that $<Z = 170^0$
State true or false (**write explanations**)
a) $Re\{Z\}>0$ --- F
b) $Im\{Z\}>0$ ...T.........
c) $|e^Z|>1$ ...F..........
d) $|e^{-Z}|>1$ ...T.............

→R3



R1

→R2

R3

10) Calculate 2^{-i} (**use principle value for Logarithm.**)

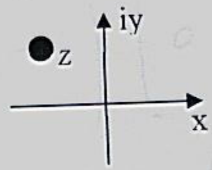
$$2^{-i} = e^{-i\ln 2} = e^{-0.69i} =$$

$\cos 0.69 - i \sin 0.69$.

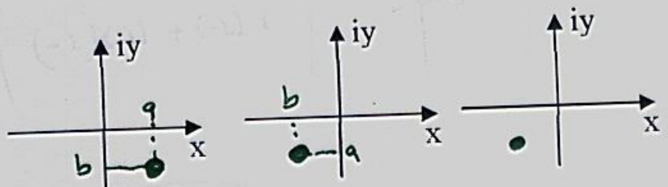
$= 0.769 - i\,0.63$

R4

11) The complex number Z is shown in the Cartesian coordinates



Show the following numbers in the Cartesian coordinates. a) $-Z$ b) Zi c) $\bar{Z}$ ($\bar{Z}$: conjugate)



a) $-Z$ b) Zi c) $\bar{Z}$

$z = -a + bi$

$-z = a - bi$

$zi = -ai + bi^2$
$= -b - ai$

$\bar{z} = -a - bi$

12) $\cos(Z)=3$. Calculate Z. (Z is a complex number)

$\cos z = \cosh(iz)$

$z = ip$

$\cos ip = \cosh(i\,ip) = \cosh(-p)$

$\cos ip = \cosh(-p) = 3$

$\cosh^{-1} 3 = p = 1.762$

$z = ip = 1.762\,i$

or $z = i(-p) = -ip = -1.762i$

1.76

13)Calculate A,B,C in the following equation.

$$\frac{9z^2+20z+3}{z^3+3z^2-z-3}=\frac{A}{z-1}+\frac{B}{z+1}+\frac{C}{z+3}$$

$$A=\left.\frac{9z^2+20z+3}{(z+1)(z+3)}\right|_{z=1}=\frac{9+20+3}{2\times4}=\frac{32}{8}=4$$

$$B=\left.\frac{9z^2+20z+3}{(z-1)(z+3)}\right|_{z=-1}=\frac{9-20+3}{(-2)(2)}=\frac{-8}{-4}=2$$

$$=-2$$

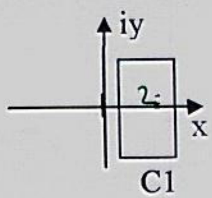
$$C=\left.\frac{9z^2+20z+3}{(z-1)(z+1)}\right|_{z=-3}=\frac{9\times9-60+3}{(-4)(-2)}=\frac{24}{8}=3$$

$A=4$

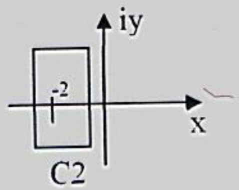
$B=+2$

$C=3$

14) Calculate the following integrals where paths are shown in the graphs.



C1



C2

a) $\oint_{C1}\frac{3}{z+2}dz$, b) $\oint_{C2}\frac{3}{z+2}dz$,

a) $\oint_{C_1}=0$

$$\int_{C_2}=3\times2\pi i=6\pi i$$

$$=18.84\,i$$

15)A dynamic system has the linear differential equation

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix}=\begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q3\\ q4\end{bmatrix}$$

The matrix A has the following eigenvalues $\lambda_1=3+2i$, $\lambda_2=3-2i$, $\lambda_3=-4-5i$, $\lambda_4=-4+5i$.

The solution $q_1(t)$ is in the form of

$q_1(t)= e^{3t}(A\cos 2t + B\sin 2t) + e^{-4t}(C\cos 5t + D\sin 5t)$

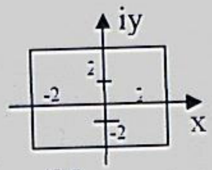
fill the blanks. (A,B,C,D are real constant numbers)

16)

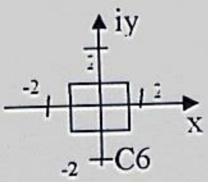
a) $\oint_{C5}$

b) $\oint_{C6}$

16) State **True** or **False. (write explanation)**



C5



a) $\oint_{C5} \frac{z^3+z^2+z}{(z^2+4)(z^2-4)} dz = 0$ ......False......

b) $\oint_{C6} \frac{z^3+z^2+z}{(z^2+4)(z^2-4)} dz = 0$ ......True......

17)The matrix A has the following eigenvector. Calculate eigenvalue for this eigenvector.

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 3 & 4 & 2 \\ -2 & -2 & -1 \end{bmatrix}, \quad X_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$$

Hint: Use the eigenvalue formula $A\,X_i = \lambda_i\,X_i$

$$\begin{pmatrix} 3 & 2 & 4 \\ 3 & 4 & 2 \\ -2 & -2 & -1 \end{pmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} =$$

$$\begin{pmatrix} -3+2 \\ -3+4 \\ (-2)(-1)+(-2)1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\lambda = 1$$