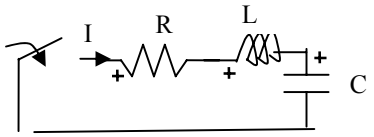
Seri RLC devresi



$V_R+V_L+V_C=0$ (1)

$V_R=R\,I$, $V_R=L\frac{dI}{dt}$, $I=C\frac{dV_C}{dt}$,

Bu tanimlarla (1) denklemini yeniden yazalim.

$$R\,I + L\frac{dI}{dt} + V_C=0 \quad (2)$$

Kapasite tanim bagintisini yeniden yerine koyalim.

$$R\,C\frac{dV_C}{dt} + L\,\frac{d}{dt}\left(C\frac{dV_C}{dt}\right)+V_C=0 \quad (3)$$

$$R\,C\frac{dV_C}{dt} + LC\,\frac{d^2V_C}{dt^2}+V_C=0 \quad (4)$$

$$\frac{d^2V_C}{dt^2}+\frac{R}{L}\,\frac{dV_C}{dt}+\frac{1}{LC}V_C=0 \quad (5)$$

---------- ---------- ---

Ikinci dereceden lineer diferansiyel denklem.
Karakteristik denklem

$\lambda^2+ (R/L)\,\lambda + (1/LC)=0$

Denklemin kokleri

$$= -\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\equiv -\alpha+\beta$$

$$= -\frac{R}{2L}-\sqrt{\left(\frac{R}{2L}\right)^2-\frac{1}{LC}}\equiv -\alpha-\beta$$

$$\alpha \equiv R/2L,\ \beta \equiv \sqrt{\alpha^2-\omega_0^2},$$

$$\omega_0 \equiv 1/\sqrt{LC}.$$

R,L,C nin degerlerine gore kokler reel cakisik, komleks olabilir.
Reel kok: Sonumlu, Osilasyon yok (overdamped)
Cakisik kok: Kritik Sonumlu, Osilasyon siniri (critically damped)
Kompleks kok: osilosyonlu durum. (underdamped)
Ornek:

---------- ---------- ----------

Ornek 41: R=8, L=1, C=1/12=0.083333, degerleri icini $V_C(t)$ yi hesaplayin.

Cozum:

$\alpha=R/(2L)=8/(2x1)=4$, $w_0= 1/\sqrt{1/12}$

$\beta = \sqrt{\alpha^2 - w_0^2} = \sqrt{4^2 - 12} = 2$

kokler $\lambda_1= -\alpha - \beta =-4-2=-6$. $\lambda_2= -\alpha + \beta =-4+2=-2$.

Cozum: $V_C(t)= D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} = D_1 e^{-2t} + D_2 e^{-6t}$

D_1 ve D_2 $V_C(0)$, ve $I_L(0)$ degerlerine bagli sabitlerdir.

---------- ----------

Cozumu bulmak icin α, w_0, β degerlerini bulmadan direk denklemin kokunu bularak da bulabiliriz.

$\lambda^2+ (R/L)\,\lambda + (1/LC)=0$

$\lambda^2+ (8/1)\,\lambda + (1/(1/12))=0$

$\lambda^2+ 8\,\lambda + 12=0$,

$\lambda_1=-2$, $\lambda_2=-6$,

$V_C(t)= D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} = D_1 e^{-2t} + D_2 e^{-6t}$

---------- ----------

Ornek 42:R=8, L=1, C=1/16=0.0625, degerleri icini $V_C(t)$ yi hesaplayin.

Cozum:

$\alpha=R/(2L)=8/(2x1)=4$, $w_0= 1/\sqrt{1/16} =4$

$\beta= \sqrt{\alpha^2 - w_0^2} = \sqrt{4^2 - 16} =0$

kokler $\lambda_1=-4$. $\lambda_1=-4$.

Cozum: $V_C(t)= D_1 e^{\lambda_1 t} + D_2\,t\ e^{\lambda_1 t} = D_1 e^{-2t} + D_2\,t\ e^{-4t}$

---------- ----------

Ornek 43:R=8, L=1, C=1/20=0.05 degerleri icini $V_C(t)$ yi hesaplayin.

Cozum:

$\alpha=R/(2L)=8/(2x1)=4$, $w_0= 1/\sqrt{1/20} =4.4721$

$\beta= \sqrt{\alpha^2 - w_0^2} = \sqrt{4^2 - 20} = \sqrt{-4} =2j$

kokler λ_1=-4+2j. λ_2=-4-2j.

Cozum: $V_C(t)= D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t}$

$= D_1 e^{(-4+2i)t} + D_2 e^{(-4-2i)t} = e^{-4t} (D_1 e^{-2it} + D2 e^{-2it})$

$=e^{-4t} (E \cos 2t + F \sin 2t)$

Ornek 51: R=8, L=1, C=1/12, $V_C(0)=5$, $I_L(0)=1$, icin cozumu bulun ve $V_C(t)$, $I_L(t)$ yi cizin .

Cozum:

$V_C(t)= D_1 e^{-2t} + D_2 e^{-6t}$ olarak bulunmustu.

t=0 icin $V_C(0)=5$, $I_L(0)=10$

$V_C(0)= D_1 e^0 + D_2 e^0 \Rightarrow 5=D_1+D_2$

$I_C(t)= I_L(t)=C \frac{dV_C}{dt} = C \frac{d}{dt} (D_1 e^{-2t} + D_2 e^{-6t})$

$= - 2 C D_1 e^{-2t} - 6 C D_2 e^{-6t}$

$I_L(0)= I_L(0)=1 = - 2 C D_1 e^0 - 6 C D_2 e^0$

$-2D_1 - 6D_2=1/C=12$

Iki denklem birlestirilip cozulurse

$D_1+D_2 =5$

$-2D_1-6D_2=12$

$D_1=10.5$ $D_2=-5.5$

$V_C(t)= 10.5 e^{-2t} - 5.5 e^{-6t}$

$I_L(t)=- 2 C D_1 e^{-2t} - 6 C D_2 e^{-6t}$

$= - 2 (1/12) (10.5) e^{-2t} - 6 (1/12) (-5.5) e^{-6t}$

$= - 1.75 e^{-2t} + 2.75 e^{-6t}$
