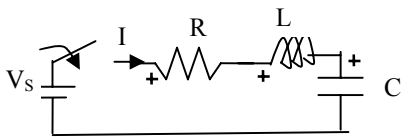
Seri RLC devresi (zorlanmis)



$V_R+V_L+V_C= V_S$ (1)

$V_R=R\,I$, $V_R=L\frac{dI}{dt}$, $I=C\frac{dV_C}{dt}$,

Bu tanimlarla (1) denklemini yeniden yazalim.

$$R\,I + L\frac{dI}{dt} + V_C= V_S \quad (2)$$

Kapasite tanim bagintisini yeniden yerine koyalim.

$$R\,C\frac{dV_C}{dt} + L\,\frac{d}{dt}\left(C\frac{dV_C}{dt}\right)+V_C=V_S \quad (3)$$

$$R\,C\frac{dV_C}{dt} + LC\,\frac{d^2V_C}{dt^2}+V_C= V_S \quad (4)$$

$$\frac{d^2V_C}{dt^2}+\frac{R}{L}\,\frac{dV_C}{dt}+\frac{1}{LC}V_C=\frac{1}{LC}V_S \quad (5)$$

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Ikinci dereceden lineer diferansiyel denklem.

1)Homojen cozum

$$\frac{d^2V_C}{dt^2}+\frac{R}{L}\,\frac{dV_C}{dt}+\frac{1}{LC}V_C=0 \quad (6)$$

2)Ozel cozum (zorlanmis cozum, forced solution)

IKinci tarafi saglayacak ozel cozum. Burada giris sabit bir degerdir. tahmini cozum (y_0=D , y_0=Ex+D, y_0= Fx^2+ Ex+ D) sekillerinde olabilir.

Homojen cozum oncekiornekte bulunmustu.

Karakteristik denklem

$\lambda^2+ (R/L)\,\lambda + (1/LC)=0$, kokleri λ_1, λ_2 olmak uzere

$V_C(t)= D_1\,e^{\lambda_1 t} + D_2\,e^{\lambda_2 t}$

zorlanmis cozum dif denklem cozumunden bulunabilir. tahmini cozum olarak $V_C(t) =At+B$ vererek bulunabilir. Ancak burada daha basit bir yol izleyebiliriz. Kaynak sabit oldugundan, bunun turevleri sifir olcaktir. (sabit sayinin turevi sifirdir.) Dolayisiyla dif denklemde turevlerein hepsine sifir koyalim.

$0+\frac{R}{L}\,0+\frac{1}{LC}V_C=\frac{1}{LC}V_S$ ==> $V_C=V_S$ bulunur.

Toplam cozum. $V_{Toplam}(t)= D_1\,e^{\lambda_1 t} + D_2\,e^{\lambda_2 t} + V_S$

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Ornek 1: R=8, L=1, C=1/12, $V_C(0)=5$, $I_L(0)=1$, V_S=20 icin cozumu bulun ve $V_C(t)$, $I_L(t)$ yi cizin .

Cozum:

Homojen cozum onceki ornekteki ile ayni

$\lambda^2+ (R/L)\,\lambda + (1/LC)=0$

$\lambda^2+ 8\,\lambda + 12=0$, $\lambda_1=-2$, $\lambda_2=-6$,

$V_C(t)= D_1\,e^{\lambda_1 t} + D_2\,e^{\lambda_2 t} = D_1\,e^{-2t} + D_2\,e^{-6t}$

zorlanmis cozum $V_C=V_S$ olarak bulunmustu.

Toplam cozum. $V_{Toplam}(t)= D_1\,e^{-2t} + D_2\,e^{-6t} + V_S$

$V_C(0)=5$, $I_L(0)=1$,

$V_{Toplam}(0)= D_1\,e^0+ D_2\,e^0+ V_S$

$5= D_1+D_2+ 20$ ----> $D_1 + D_2= -15$

$I_C(t)= I_L(t)=C\frac{dV_{Toplam}}{dt} =C\frac{d}{dt}(\,D_1\,e^{-2t} + D_2\,e^{-6t} + V_S)$

$= -\,2\,C\,D_1\,e^{-2t} - 6\,C\,D_2\,e^{-6t} +0$

$I_L(0)= I_L(0)=1 = -\,2\,C\,D_1\,e^0- 6\,C\,D_2\,e^0$

$-2D_1-6D_2=12$

Iki denklem birlestirilip cozulurse

$D_1+D_2 =-15$

$-2D_1-6D_2=12$

$D_1=-19.5$ $D_2=4.5$

$V_C(t)= -19.5\,e^{-2t} + 4.5\,e^{-6t} +20$

$I_L(t)=-\,2\,C\,D_1\,e^{-2t} - 6\,C\,D_2\,e^{-6t}$

$= -\,2\,(1/12)\,(-19.5)\,e^{-2t} - 6\,(1/12)\,(4.5)\,e^{-6t}$

$= 3.25\,e^{-2t} + 2.25\,e^{-6t}$

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Cozum: $V_C(t)= D_1\,e^{\lambda_1 t} + D_2\,t\,e^{\lambda_1 t} = D_1\,e^{-2t} + D_2\,t\,e^{-4t}$

Zorlanmis cozum. $V_C=V_S=20$

Toplam cozum:

$V_{Toplam}(t)= e^{\lambda_1 t} = D_1\,e^{-2t} + D_2\,t\,e^{-4t} + 20$

Ornek 43:R=8, L=1, C=1/20=0.05
degerleri icini $V_C(t)$ yi hesaplayin.

Cozum:

$\alpha=R/(2L)=8/(2x1)=4$, $w_0= 1/\sqrt{1/20} =4.4721$

$\beta= \sqrt{\alpha^2 - w_0^2} = \sqrt{4^2 - 20} = \sqrt{-4} =2j$

kokler $\lambda_1=-4+2j$. $\lambda_2=-4-2j$.

Cozum: $V_C(t)= D_1 e^{\lambda_1 t} + D_2 t\ e^{\lambda_2 t}$

$= D_1 e^{(-4+2i)t} + D_2\ e^{(-4-2i)t} = e^{-4t} (D_1 e^{-2it} + D2\ e^{-2it})$

$=e^{-4t} (E \cos 2t + F \sin 2t)$

Ornek 51: R=8, L=1, C=1/12, $V_C(0)=5$, $I_L(0)=1$, icin cozumu bulun ve $V_C(t)$, $I_L(t)$ yi cizin .

Cozum:
$V_C(t)= D_1 e^{-2t} + D_2 e^{-6t}$ olarak bulunmustu.

t=0 icin $V_C(0)=5$, $I_L(0)=10$

$V_C(0)= D_1 e^0 + D_2 e^0$ ==> $5=D_1+D_2$

$I_C(t)= I_L(t)=C\frac{dV_C}{dt} =C\frac{d}{dt}(D_1 e^{-2t} + D_2 e^{-6t})$

$= - 2 C D_1 e^{-2t} - 6 C D_2 e^{-6t}$

$I_L(0)= I_L(0)=1 = - 2 C D_1 e^0 - 6 C D_2\ e^0$

$-2D_1 - 6D_2=1/C=12$

Iki denklem birlestirilip cozulurse

$D_1+D_2 =5$

$-2D_1-6D_2=12$

$D_1=10.5$ $D_2=-5.5$

$V_C(t)= 10.5\ e^{-2t} - 5.5\ e^{-6t}$

$I_L(t)=- 2 C D_1 e^{-2t} - 6 C D_2 e^{-6t}$

$= - 2 (1/12) (10.5) e^{-2t} - 6 (1/12) (-5.5) e^{-6t}$

$= - 1.75 e^{-2t} + 2.75 e^{-6t}$
