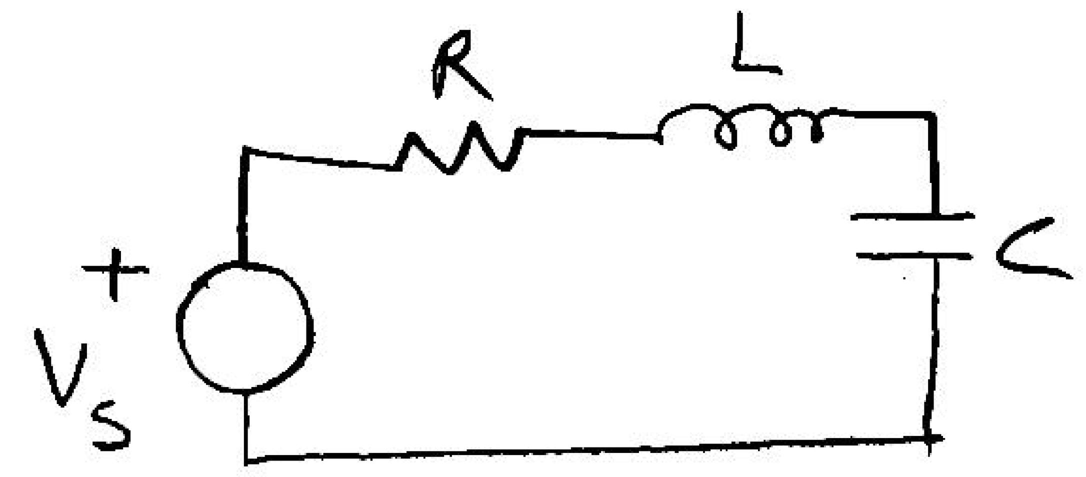
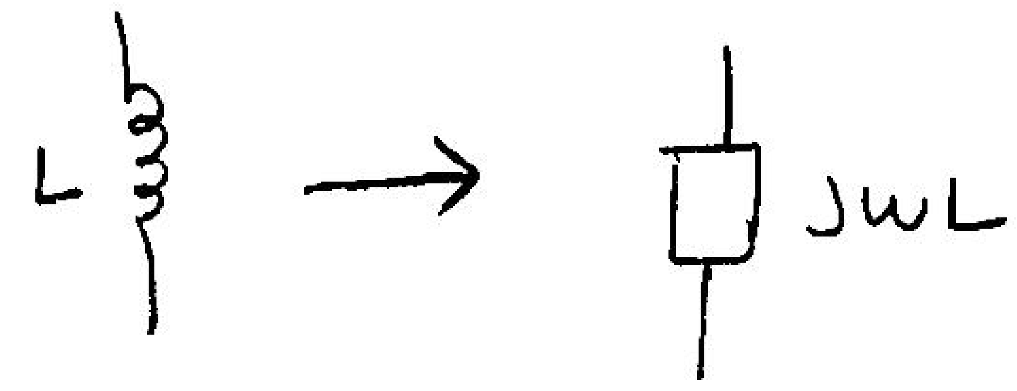
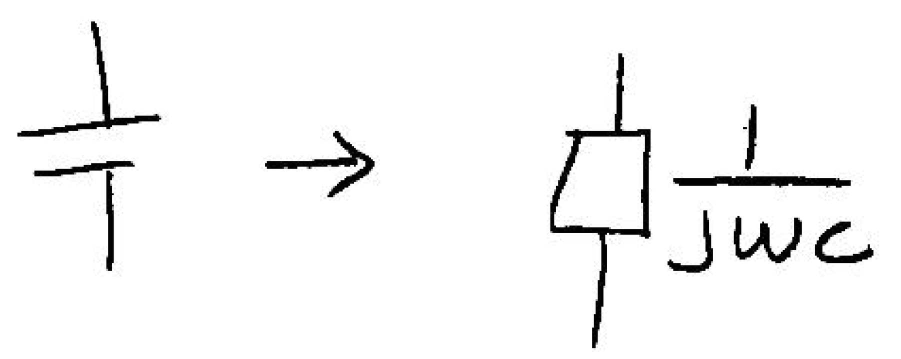
ee21

Sinusoidal steady state Analysis



$V_s = A\cos(\omega t + \phi)$





$A\cos(\omega t + \phi) \rightarrow A\,e^{J\phi}$ $\quad A\angle\phi$

EXAMPLE 9.6

A 90 Ω resistor, a 32 mH inductor, and a 5 μF capacitor are connected in series across the terminals of a sinusoidal voltage source, as shown in Fig. 9.15. The steady-state expression for the source voltage v_s is $750\cos(5000t + 30°)$ V.

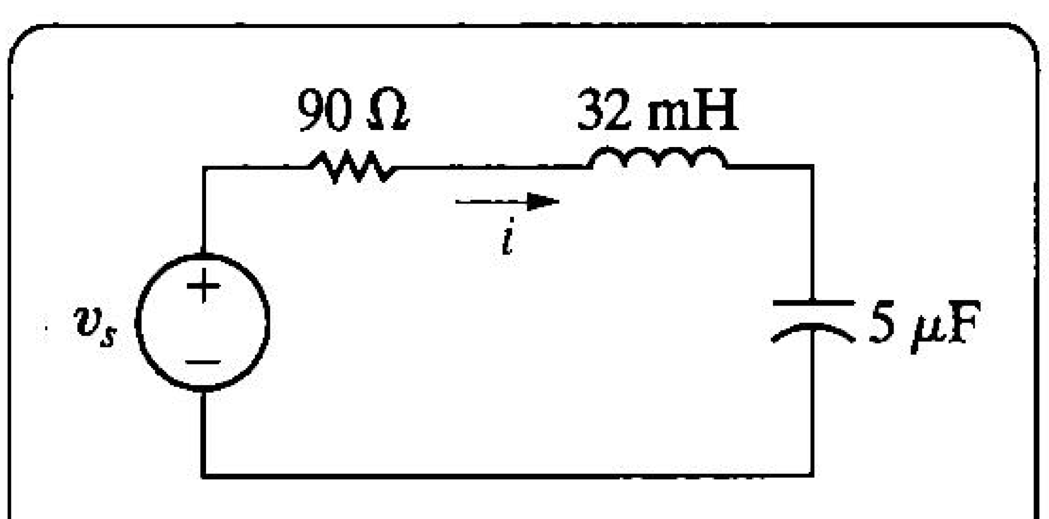


Figure 9.15 The circuit for Example 9.6.

a) Construct the frequency-domain equivalent circuit.

b) Calculate the steady-state current i by the phasor method.

SOLUTION

a) From the expression for v_s, we have $\omega = 5000$ rad/s. Therefore the impedance of the 32 mH inductor is

$$Z_L = j\omega L = j(5000)(32 \times 10^{-3}) = j160\ \Omega,$$

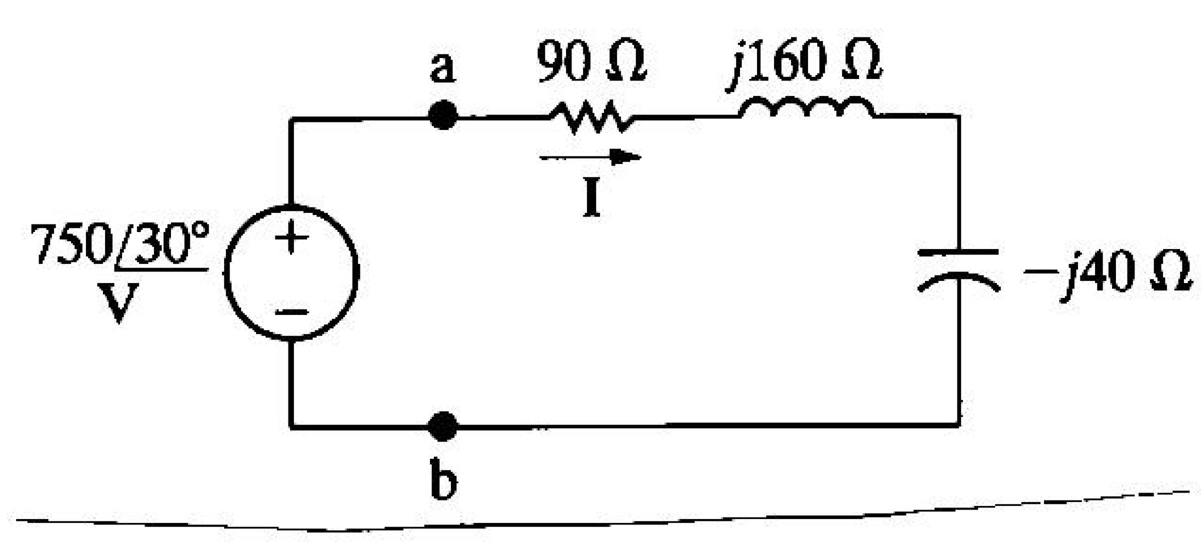
and the impedance of the capacitor is

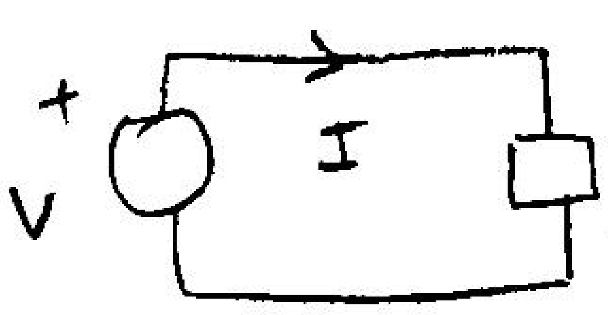
$$Z_C = j\frac{-1}{\omega C} = -j\frac{10^6}{(5000)(5)} = -j40\ \Omega.$$

The phasor transform of v_s is

$$\mathbf{V}_s = 750\,\angle 30°\ \text{V}.$$

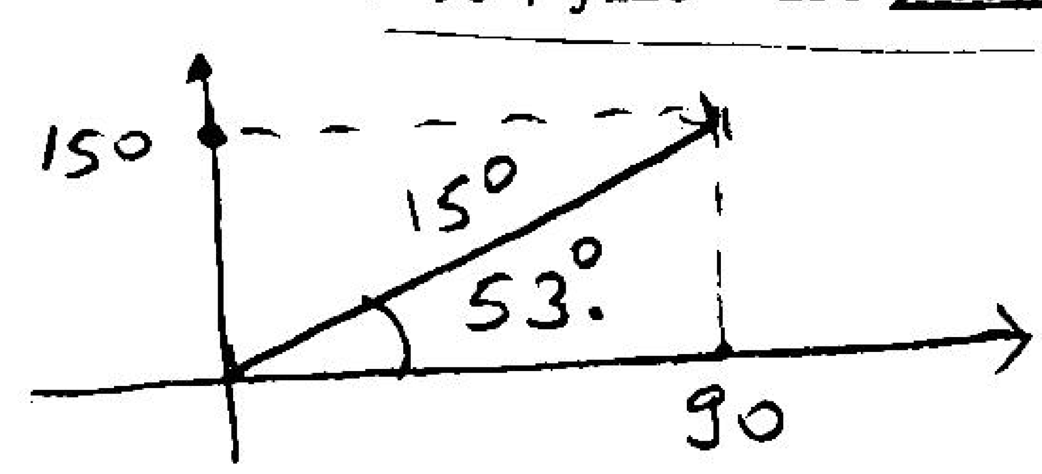
Figure 9.16 illustrates the frequency-domain equivalent circuit of the circuit shown in Fig. 9.15.





$$Z_{ab} = 90 + j160 - j40$$

$$= 90 + j120 = 150\,\angle 53.13°\ \Omega.$$



$$\mathbf{I} = \frac{750\,\angle 30°}{150\,\angle 53.13°} = 5\,\angle -23.13°\ \text{A}.$$

$$i = 5\cos(5000t - 23.13°)\ \text{A}.$$

EXAMPLE 9.7

The sinusoidal current source in the circuit shown in Fig. 9.18 produces the current $i_s = 8\cos 200{,}000t$ A.

a) Construct the frequency-domain equivalent circuit.

b) Find the steady-state expressions for v, i_1, i_2, and i_3.

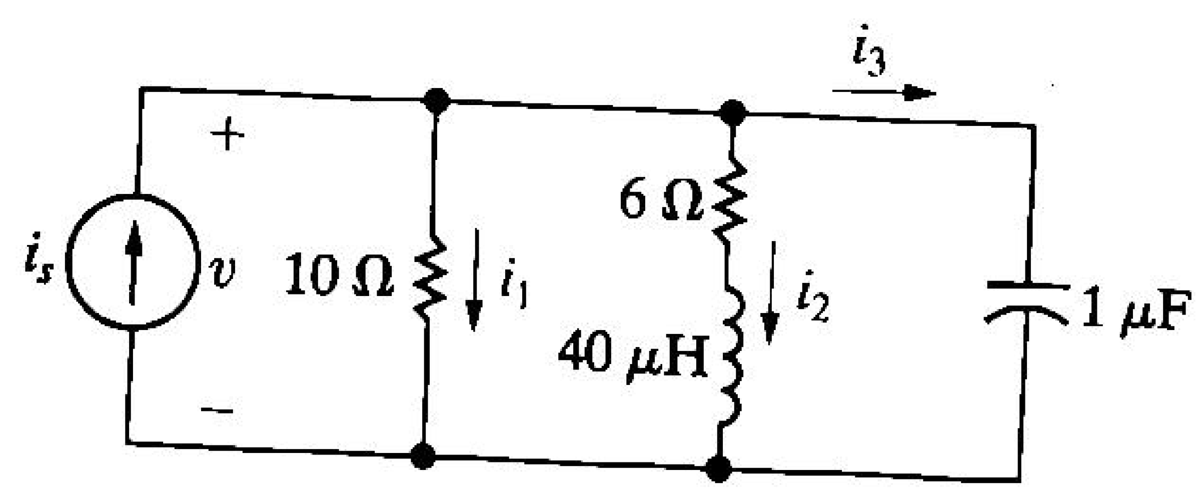


Figure 9.18 The circuit for Example 9.7.

SOLUTION

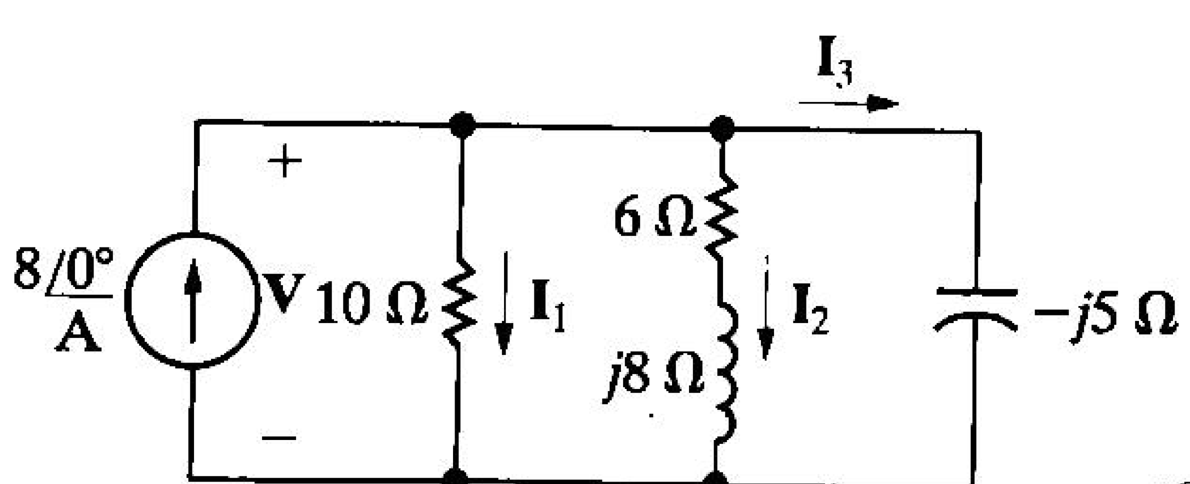
$\omega = 200\,000$

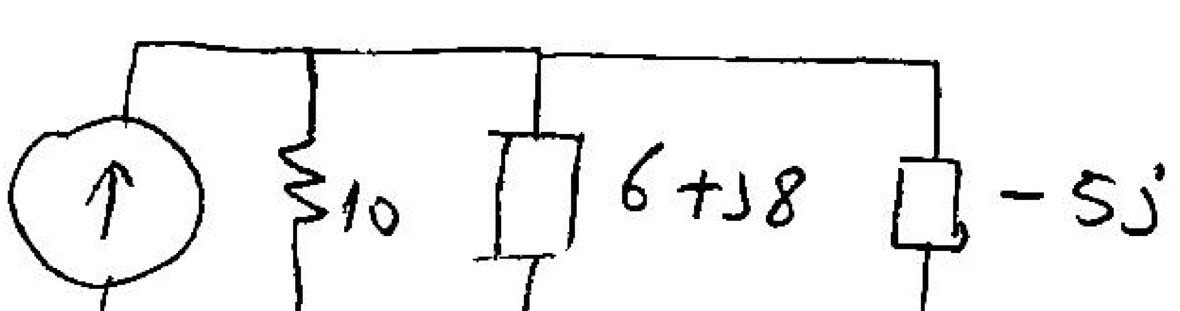
$40\mu H \rightarrow j\omega L = j \times 200\,000 \times 40\ 10^{-6}$
$= 8$

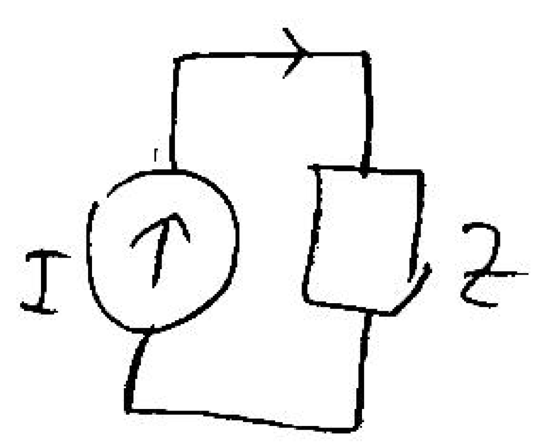
$1\mu F \rightarrow \frac{1}{j\omega c} = \frac{1}{j \times 200\,000 \times 1 \times 10^{-6}} = \frac{1}{j\,0{,}2}$

$= -5$

Note: $\frac{1}{j} = -j$







$\frac{1}{Z} = \frac{1}{10} + \frac{1}{6+j8} + \frac{1}{-j5}$

$= 0{,}1 + 0{,}06 - j0{,}08 + 0{,}2j$

$$\frac{1}{6+j8} = \frac{6-j8}{100} = 0.06 - j0.08$$

Note: $\frac{1}{a+bj} = \frac{a-bj}{a-bj}\,\frac{1}{a+bj} = \frac{a-bj}{a^2+b^2}$

$\frac{1}{Z} = 0.16 + 0.12j$

$= \sqrt{0.16^2 + 0.12^2}\ \angle \tan^{-1}\frac{0.12}{0.16}$

$= 0{,}2 \angle 36.8^0$

$Z = \frac{1}{0{,}2\angle 36{,}8} = \frac{1}{0{,}2} \angle -36.8$

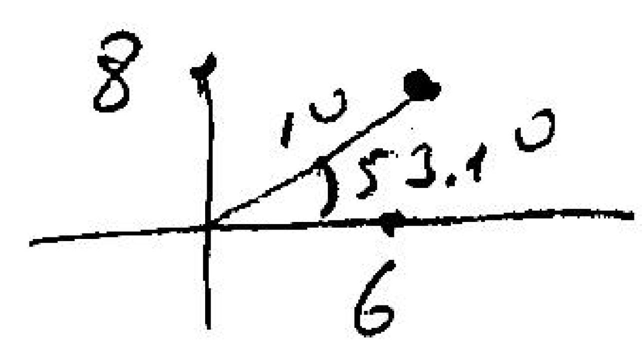
$= 5 \angle -36.8$

$V = Z\,I = 5\angle -36.8 \times 8$

$= 40 \angle -36{,}8$

$I_1 = \frac{V}{R} = \frac{40\angle -36{,}8}{10} = 4\angle -36{,}8$

$I_2 = \frac{V}{6+8j} = \frac{40\angle -36{,}8}{10\ \angle 53{,}1} = 4\angle -90$



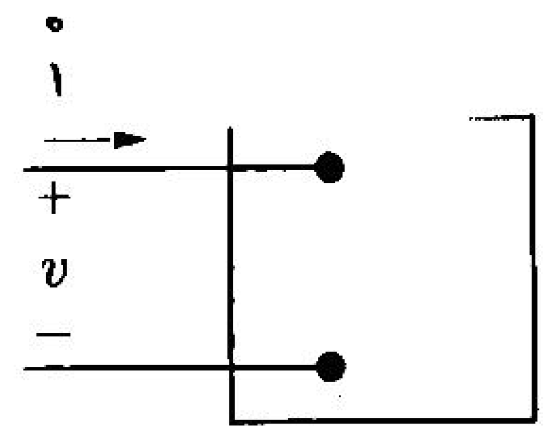
$I_3 = \frac{V}{-j5} = \frac{40\angle -36{,}8}{5\angle -90} = 8\angle 53{,}2$

The corresponding steady-state time-domain expressions are

$$v = 40\cos(200{,}000t - 36.87^\circ)\text{ V},$$
$$i_1 = 4\cos(200{,}000t - 36.87^\circ)\text{ A},$$
$$i_2 = 4\cos(200{,}000t - 90^\circ)\text{ A},$$
$$i_3 = 8\cos(200{,}000t + 53.13^\circ)\text{ A}.$$

EP1

Sinusoidal steady state Power



The black box representation of a circuit used for calculating power.

$p = v . i$

$$v = V_m \cos(\omega t + \theta_v),$$

$$i = I_m \cos(\omega t + \theta_i),$$

$$v = V_m \cos(\omega t + \theta_v - \theta_i),$$

$$i = I_m \cos \omega t.$$

$\theta_v - \theta_i$ = phase angle between V and i

$$p = V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos \omega t.$$

$$\cos\alpha \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i).$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(2\omega t + \theta_v - \theta_i) = \cos 2\omega t \cos(\theta_v - \theta_i) - \sin 2\omega t \sin(\theta_v - \theta_i)$$

$$p = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos 2\omega t - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin 2\omega t.$$

10.2 ◆ Average and Reactive Power

$$p = P + P\cos 2\omega t - Q\sin 2\omega t,$$

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i),$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i).$$

P is called the **average power**, and Q is called the **reactive power**

[redacted]

The two loads in the circuit shown in Fig. 10.14 can be described as follows: Load 1 absorbs an average power of 8 kW at a leading power factor of 0.8. Load 2 absorbs 20 kVA at a lagging power factor of 0.6.

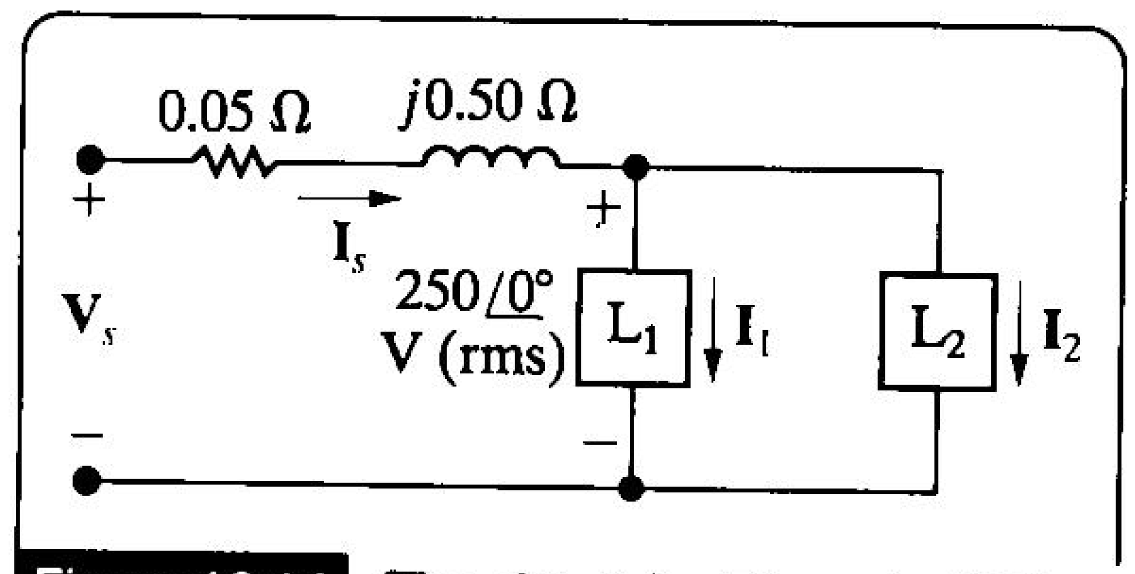


Figure 10.14 The circuit for Example 10.6.

a) Determine the power factor of the two loads in parallel.

b) Determine the apparent power required to supply the loads, the magnitude of the current, I_s, and the average power loss in the transmission line.

c) Given that the frequency of the source is 60 Hz, compute the value of the capacitor that would correct the power factor to 1 if placed in parallel with the two loads. Recompute the values in (b) for the load with the corrected power factor.

SOLUTION

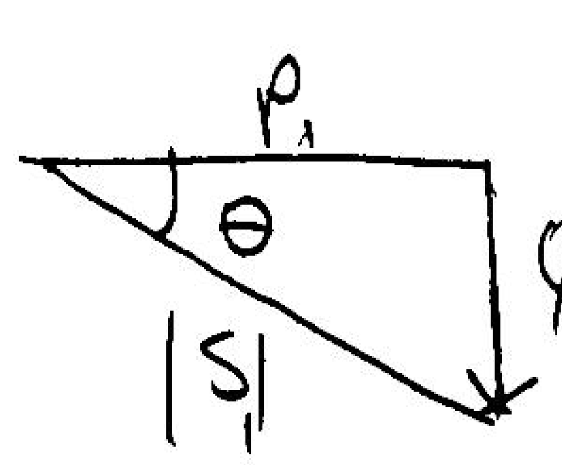
a) All voltage and current phasors in this problem are assumed to represent effective values.

$$\boxed{S = \frac{1}{2} \underline{V}\, \underline{I}^* = \underline{V}_{eff}\, \underline{I}_{eff}^*}$$

Load 1

average power $P = 8$ Kw $= 8000$ W

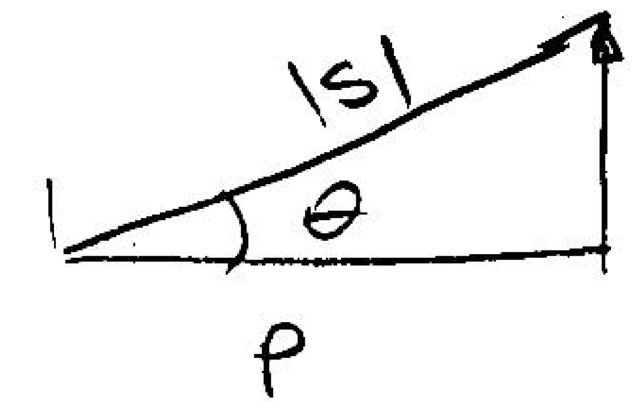
leading power factor 0.8



$\cos\theta = 0.8$

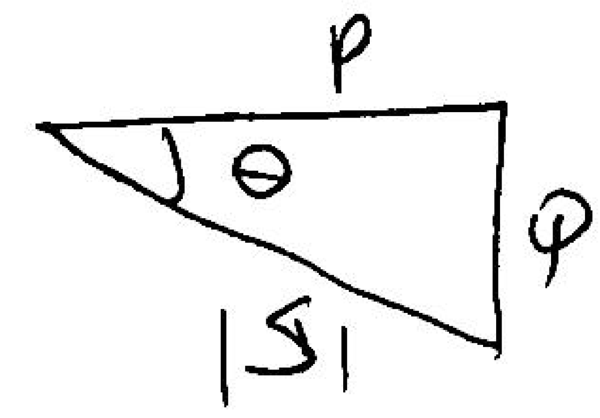
Q is minus

Notes:



lagging power factor

$\theta > 0 \quad Q > 0$



leading power factor

$\theta < 0 \quad Q < 0$

$\cos\theta = 0.8 \Rightarrow \theta = 36.8$

But $\theta < 0 \Rightarrow \theta = -36.8$

$\frac{Q_1}{P_1} = \tan\theta = \tan(-36.8)$
$= -0.75$

$\frac{Q_1}{8000} = -0.75$

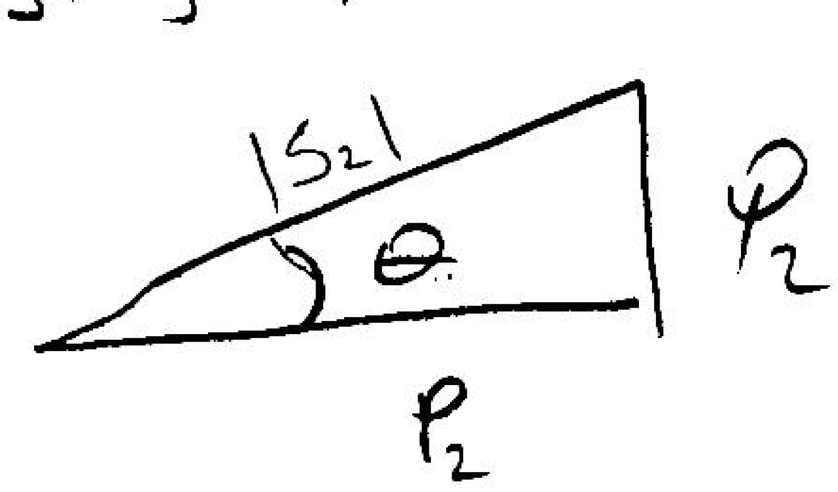
$Q_1 = 8000 \times (-0.75) = -6000$

$S_1 = P_1 + jQ_1 = 8000 - j\,6000$

load 2

absorbs 20 KVA $\Rightarrow |S|$ 20000

lagging power factor 0.6



$\cos\theta = 0.6 \Rightarrow \theta = 53.1 \Rightarrow$

$\sin\theta = 0.8$

$$\cos\theta = \frac{P_2}{|S_2|}$$

$$0.6 = \frac{P_2}{20000} \Rightarrow P_2 = 12000$$

$$\sin\theta = \frac{Q_2}{|S_2|}$$

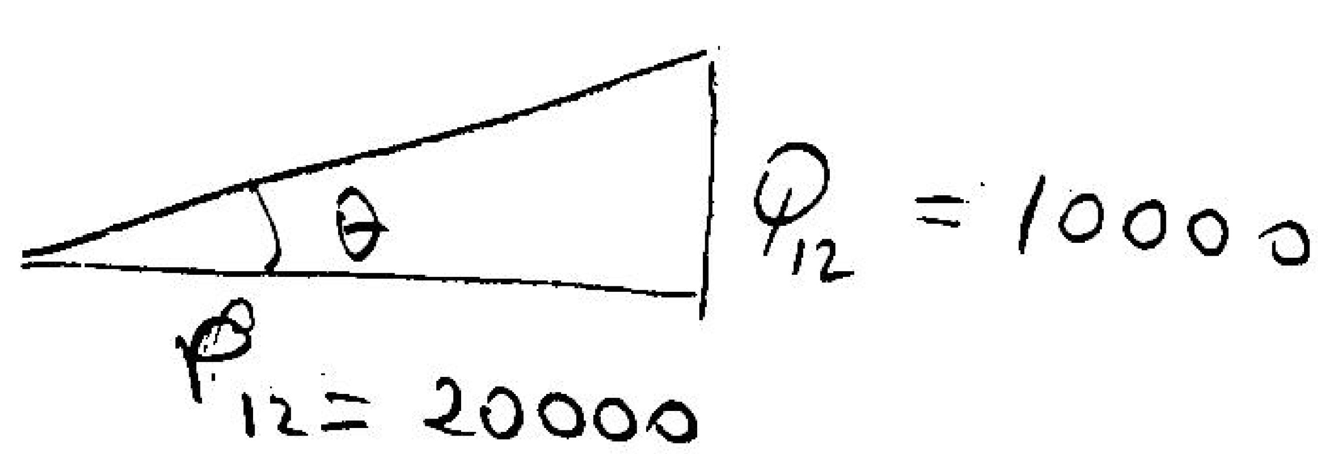
$$0.8 = \frac{Q_2}{20000} \Rightarrow Q_2 = 16000$$

$$S_2 = P_2 + Q_2 j = 12000 + j16000$$

$$S_1 = 8000 - j6000$$

$$S_{12} = S_1 + S_2 = 20000 + j10000$$

$$= P_{12} + j\,Q_{12}$$



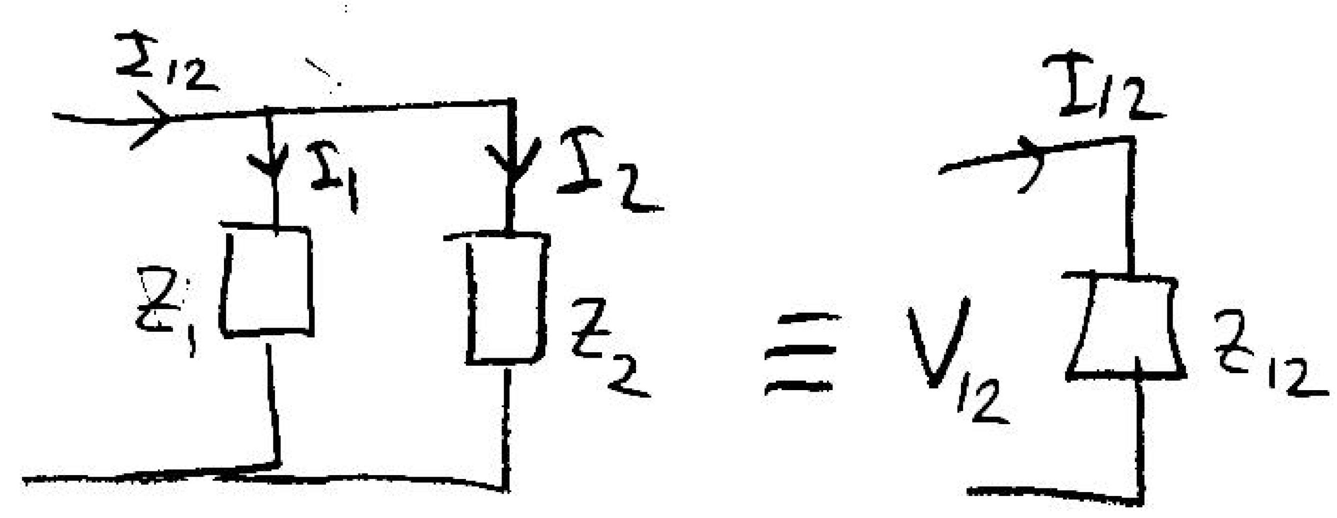
$$\tan\theta = \frac{Q_{12}}{P_{12}} = \frac{10000}{2000} = \frac{1}{2}$$

$$\theta = 26.56 \Rightarrow \cos\theta = 0.894$$

Since θ is positive

Power factor is <u>lagging</u>

b) The apparent power which must be supplied to these loads is



$$S_{12} = 20000 + j10000$$

$$|S_{12}| = \sqrt{20000^2 + 10000^2}$$

$$= 22360 \text{ (appearent power)}$$

$$V_{12} = 250\angle 0^\circ = 250 + j0$$

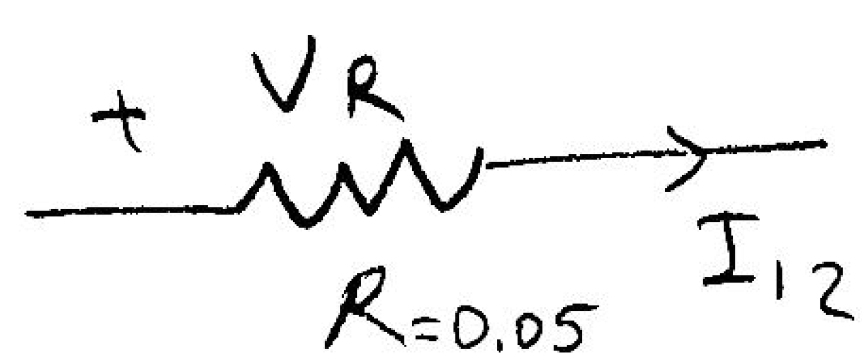
given in the figure

$$S_{12} = V_{12}\, I_{12}^*$$

$$I_{12}^* = \frac{S_{12}}{V_{12}} = \frac{20000 + j10000}{250}$$

$$I_{12}^* = 80 + j40$$

$$I_{12} = 80 - j40$$



$$V_R = R\, I_{12}$$

$$P_r = V_R\, I_{12}^*$$

Note: $P = \frac{1}{2} V_m I_m^* = V_{eff}\, I_{eff}^*$

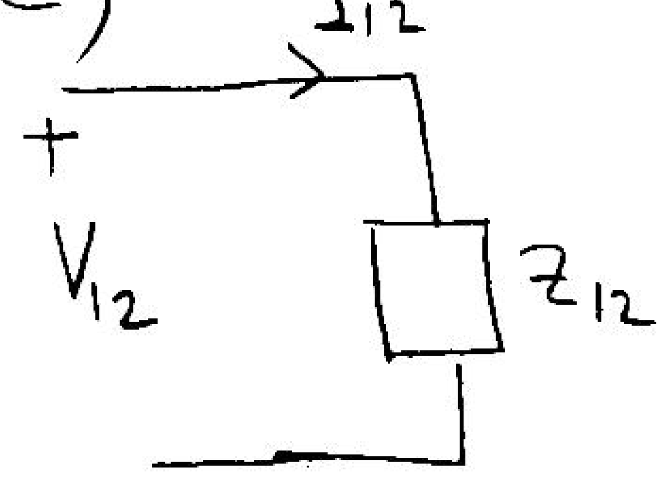
$$P_r = V_R \, I_{12}^*$$

$$= (R \, I_{12}) \, I_{12}^*$$

$$= R \, (80 - j40)(80 + j40)$$

$$= 0.05 \, (80^2 + 40^2) = 400 \text{ W}$$

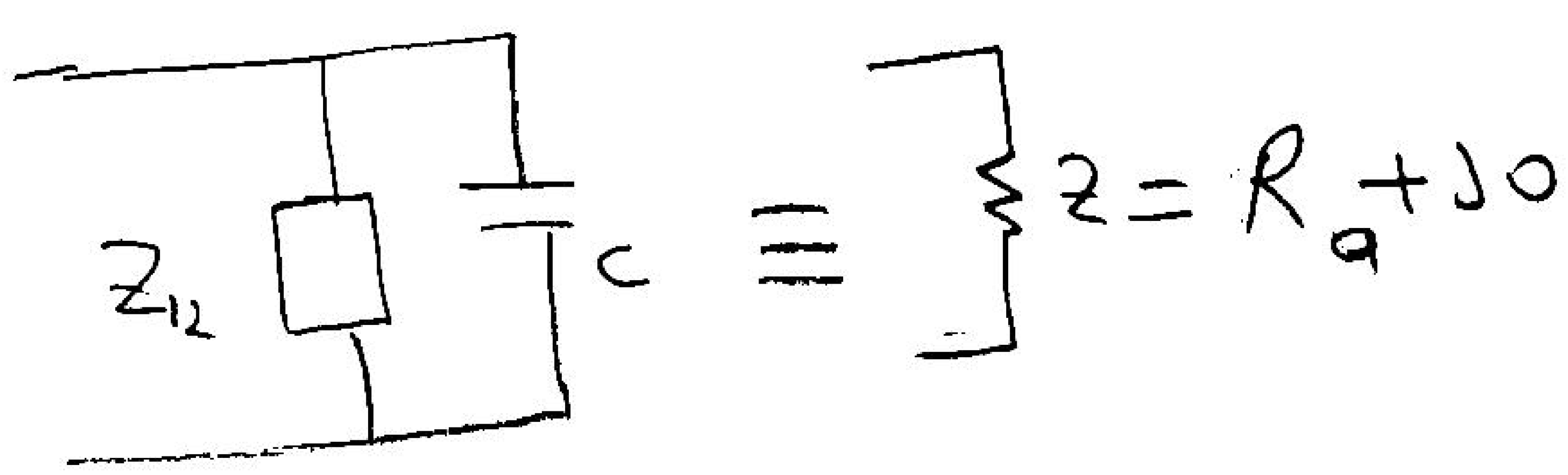
c)



$$Z_{12} = \frac{V_{12}}{I_{12}}$$

$$Z_{12} = \frac{250}{80 - j40} = \frac{250(80 + j40)}{80^2 + 40^2}$$

$$Z_{12} = 2.5 + j\,1.25 = a + jb$$



$$Z_{12} = a + bj \qquad Z_c = \frac{1}{jwc}$$

$$\frac{1}{Z_{12}} + \frac{1}{Z_c} = \frac{1}{R_a} + j0$$

$$\frac{1}{a + bj} + \frac{1}{\frac{1}{jwc}} = \frac{1}{R_a} + j0$$

$$\frac{a - bj}{a^2 + b^2} + jwc = \frac{1}{R_a} + j0$$

$$\frac{a}{a^2 + b^2} - j\frac{b}{a^2 + b^2} + jwc = \frac{1}{R_a} + j0$$

$$\frac{a}{a^2 + b^2} + j\left(wc - \frac{b}{a^2 + b^2}\right) = \frac{1}{R_a} + j0$$

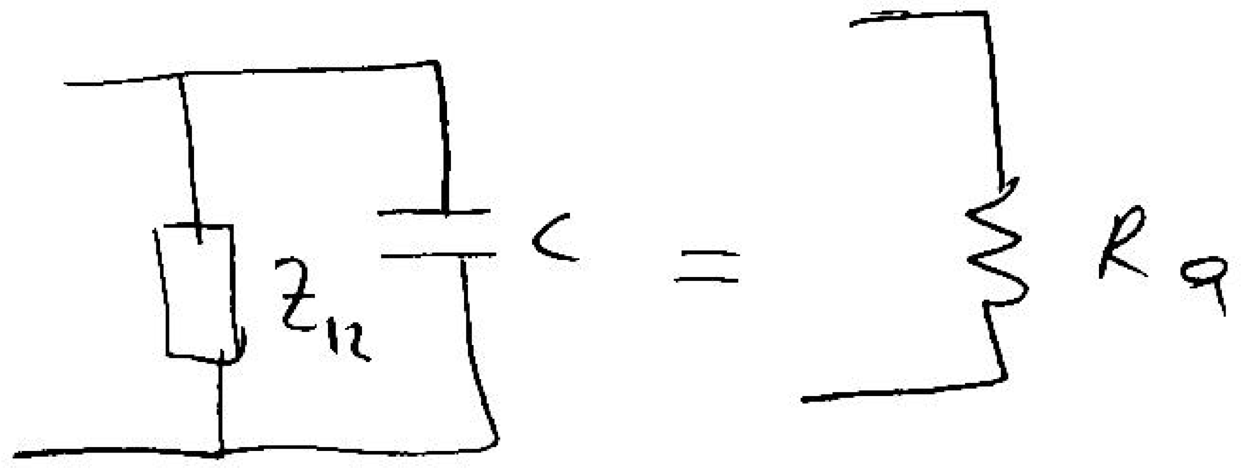
$$wc - \frac{b}{a^2 + b^2} = 0$$

$$wc = \frac{b}{a^2 + b^2}$$

$$C = \frac{b}{w(a^2 + b^2)} = \frac{1.25}{2\pi \times 60\,(2.5^2 + 1.25^2)}$$

$$C = 4.246 \; 10^{-4} \text{ F}$$

$$= 424 \; 10^{-6} \text{ F} = 424 \; \mu\text{F}$$



$$\frac{1}{Z_{12}} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{R_a}$$

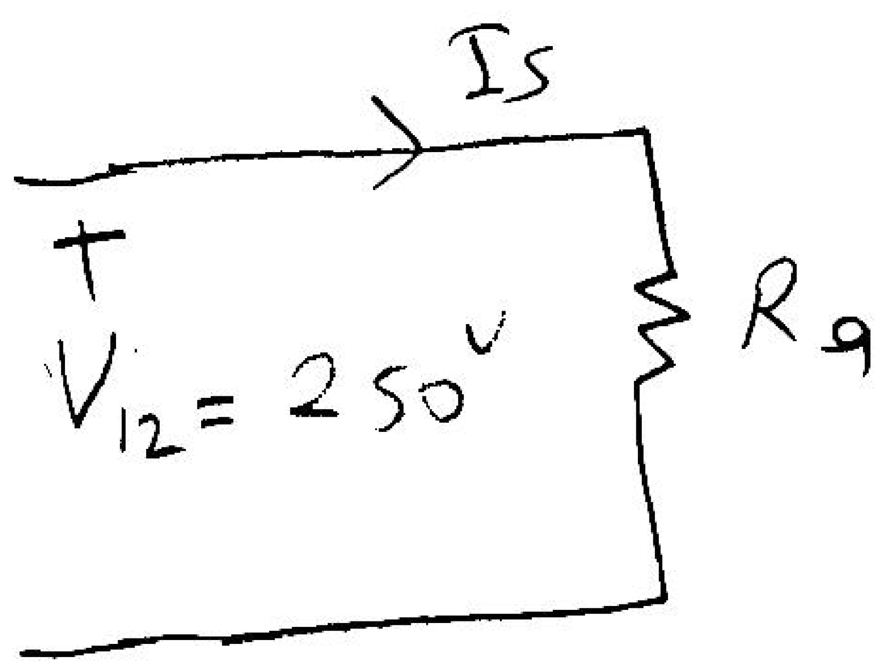
$$\frac{1}{2{,}5 + j1{,}25} + j\, 2\pi\, 60 \times 4.24\ 10^{-6} =$$

$$\frac{2{,}5 - j1.25}{2{,}5^2 + 1{,}25^2} + j\ 0.1597$$

$$0.32 - j\,0.16 \quad + j\,0{,}159 =$$

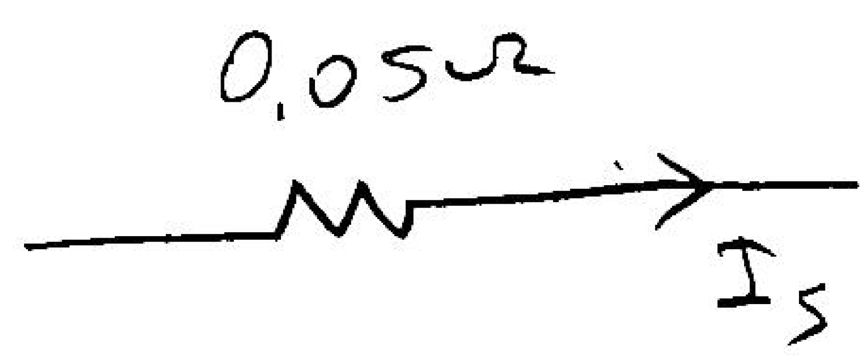
$$0.32 + j\,0 = \frac{1}{R_a}$$

$$R_a = 3.125\ \Omega$$



$$V_{12} = R_a\ I_s$$

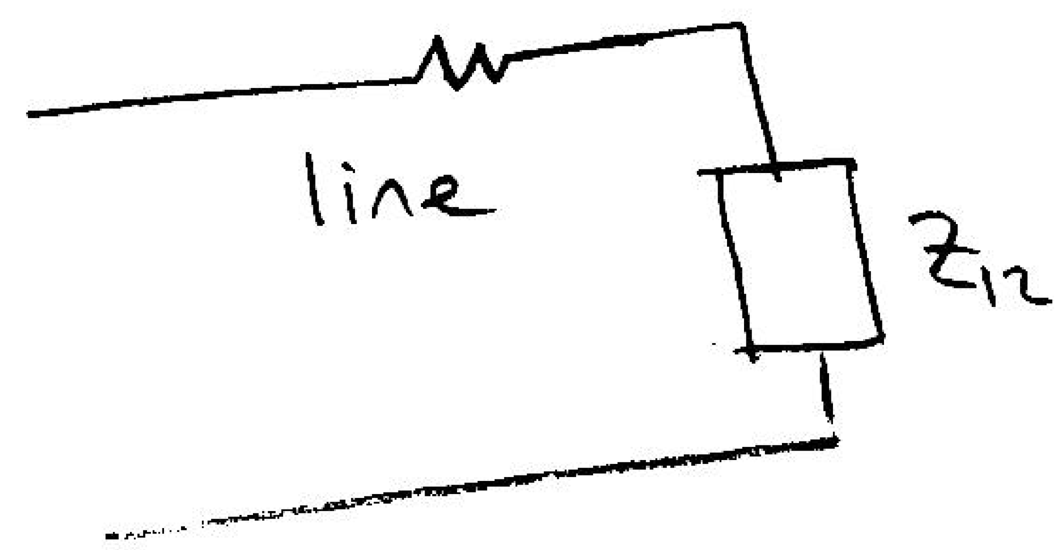
$$I_s = \frac{V_{12}}{R_a} = \frac{250}{3{,}125} = 80_A$$



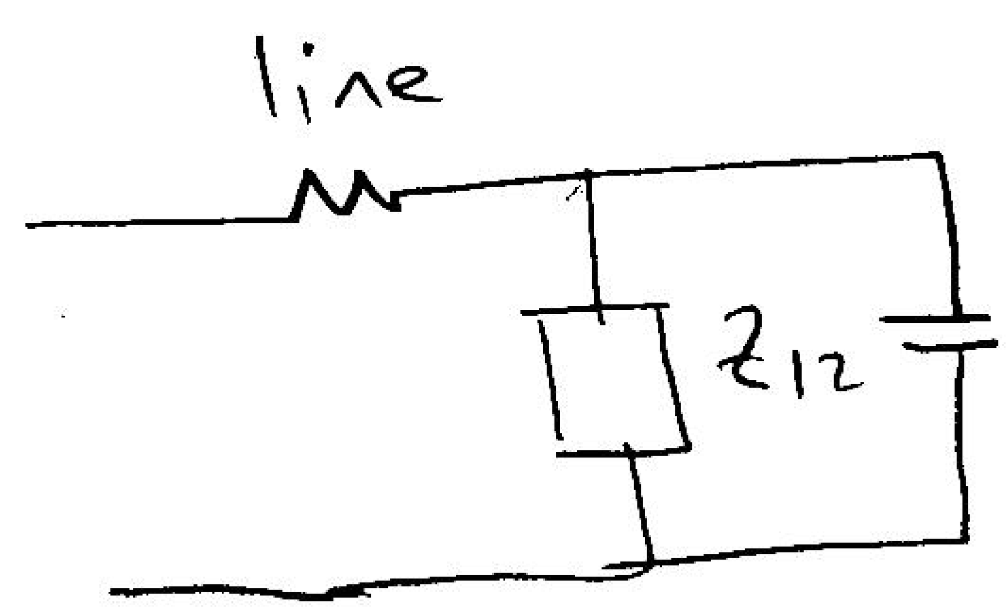
$$P = R\, I_s^2 = 0{,}05 \times 80^2$$

$$= 320\ \text{watt}$$

Note:



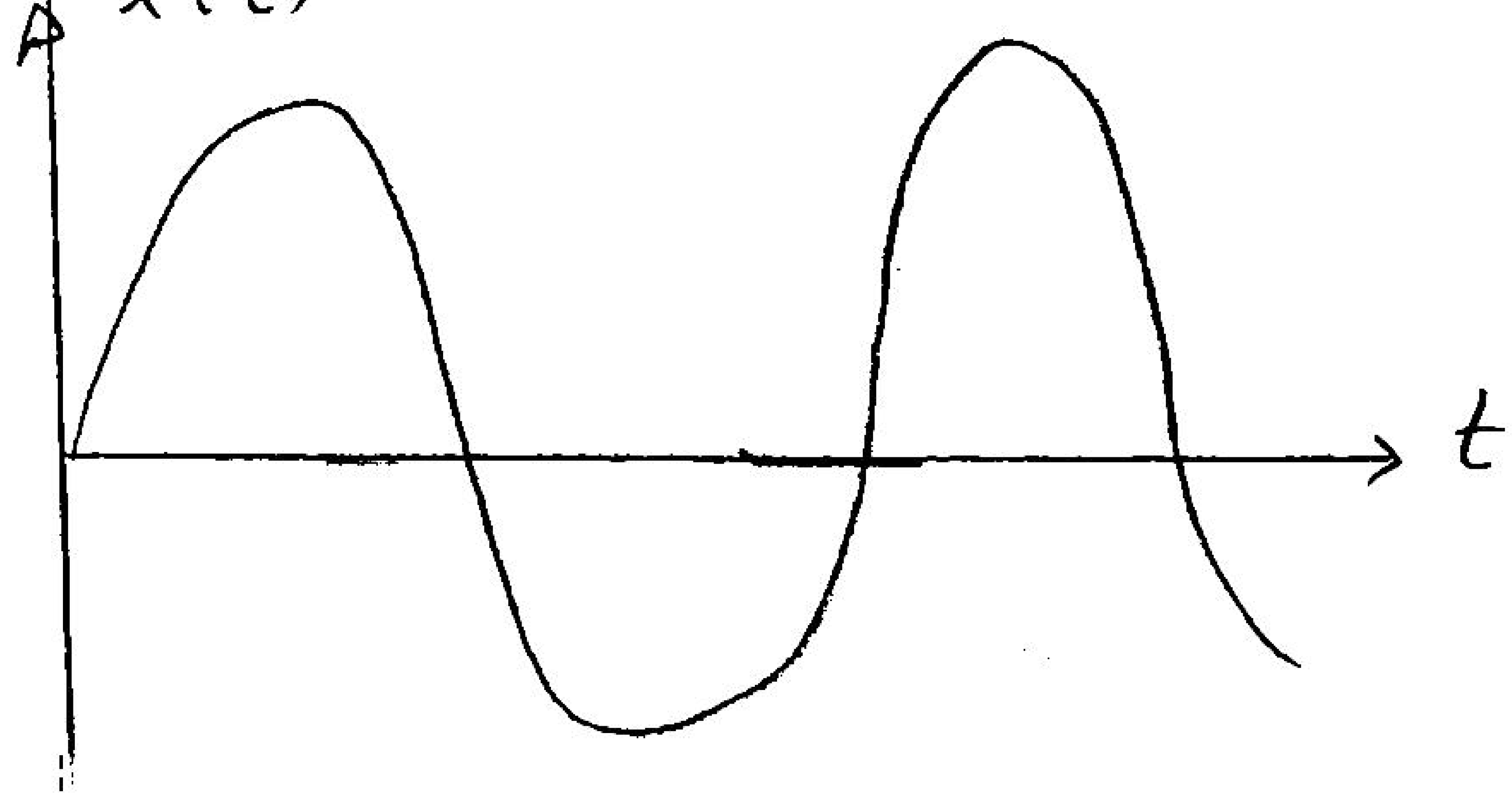
$$P_{line} = 400\ W$$



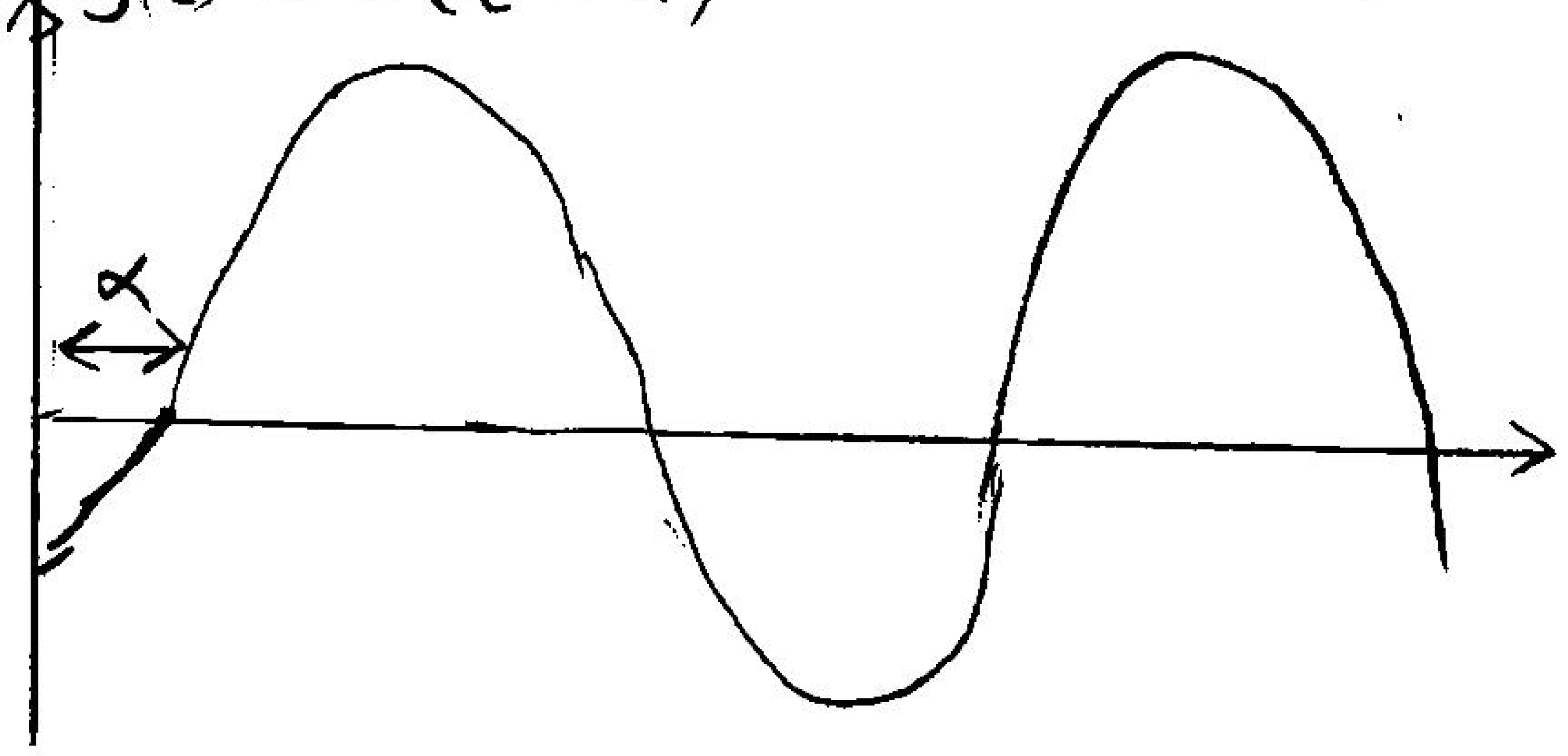
$$P_{line} = 320\ W$$

line power is reduced when capacitor is connected.

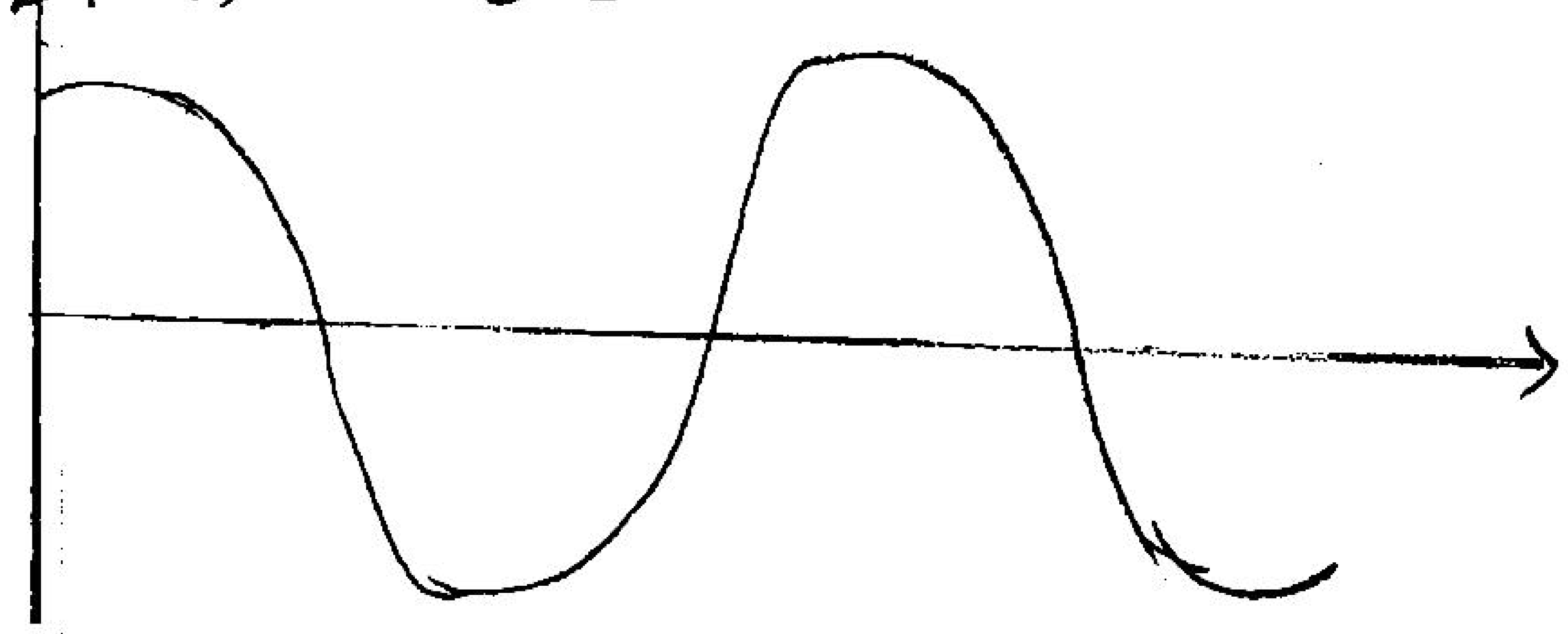
$x(t) = \sin t$



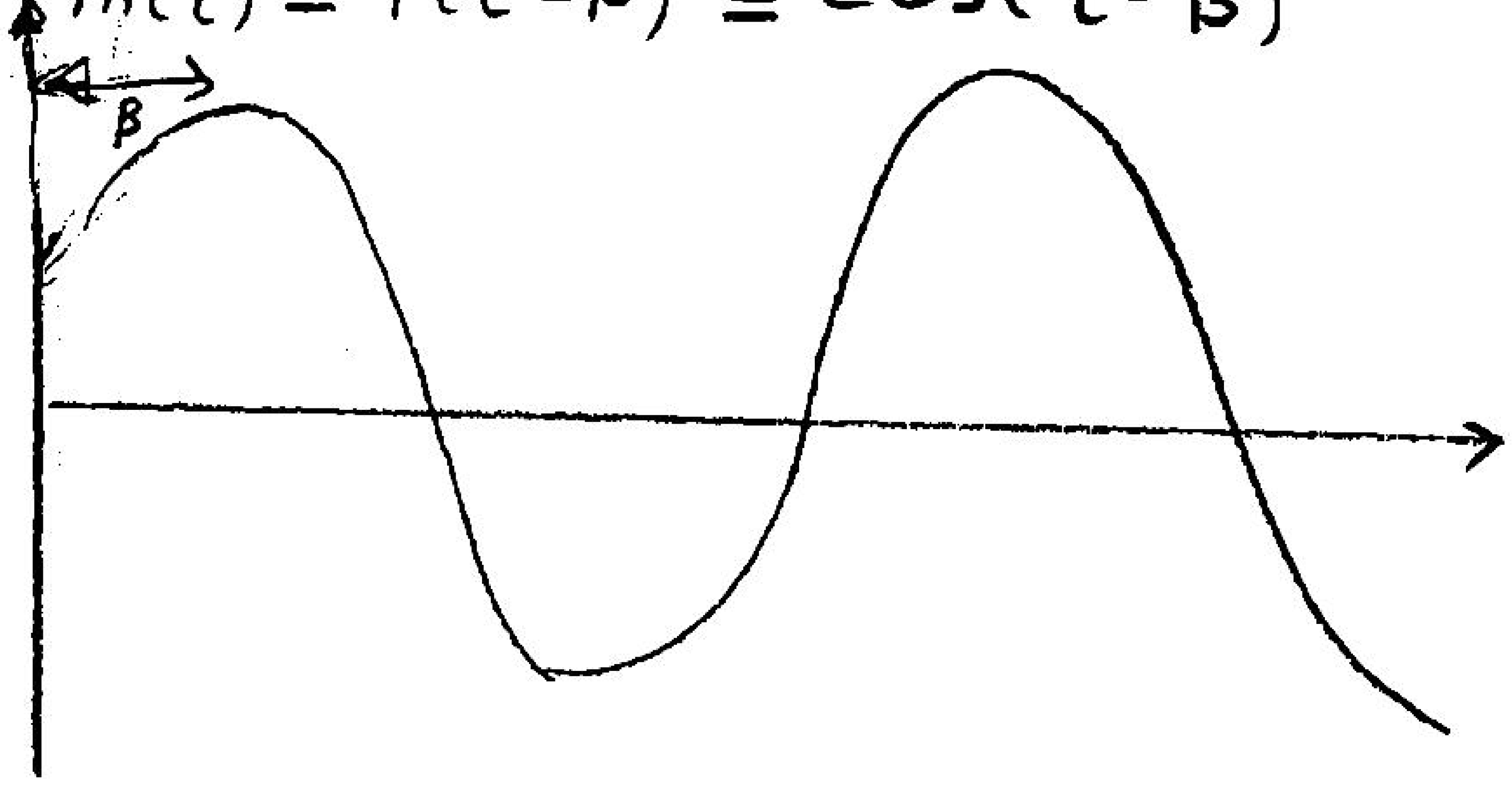
$y(t) = x(t-\alpha) = \sin(t-\alpha)$

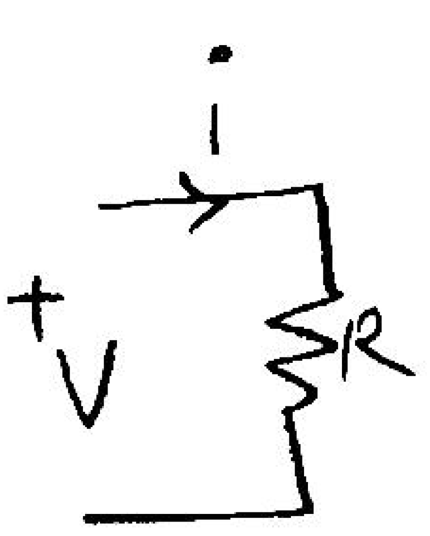


$p(t) = \cos t$



$m(t) = p(t-\beta) = \cos(t-\beta)$

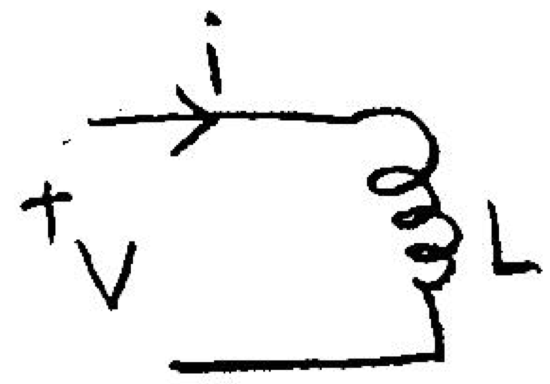




$v(t) = \cos t$

$i(t) = \frac{V}{R} = \frac{1}{R}\cos t$

phase difference between $v(t)$ and $i(t)$ is <u>zero</u>



$V = jwLI$

$i(t) = \cos t$

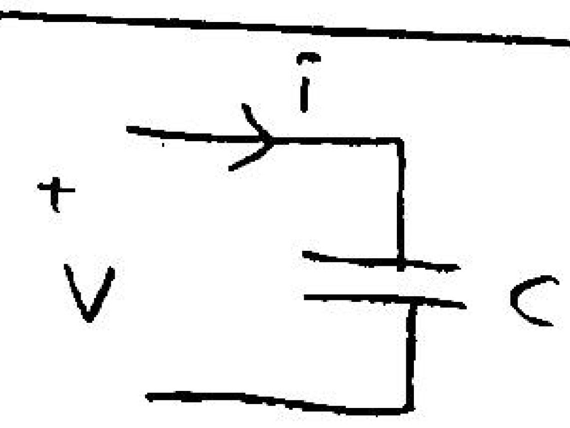
$v(t) = \cos(t+90°)$

In an inductor

Voltage leads the current

Current lags the current

phase difference between voltage and current is 90°.



$V = \frac{1}{jwc} I = -j\frac{1}{wc} I$

$i(t) = \cos t$

$v(t) = \cos(t-90°)$

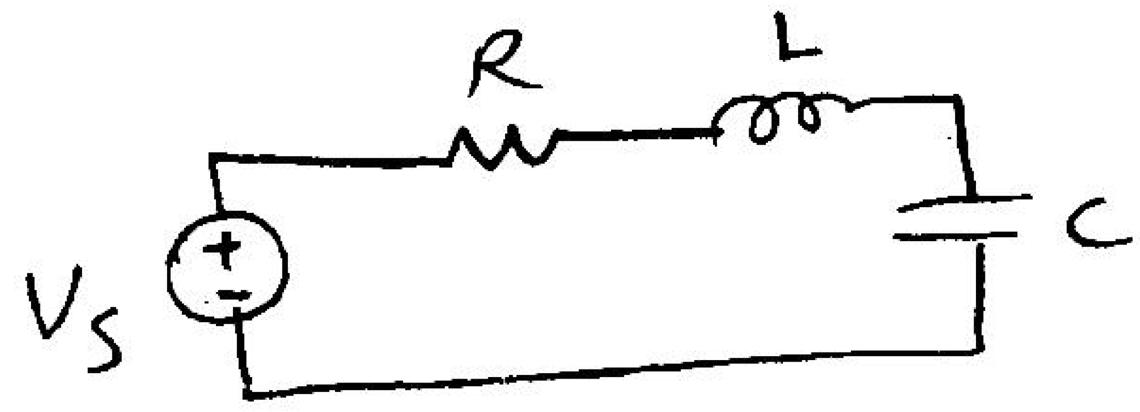
In a capacitor

Voltage lags the current

Current leads the current

phase difference between Voltage and current is $-90°$

Example problem: Calculate power for R, L, C elements.



$V_s = 750 \cos(5000t + 30)$

$R = 90\,\Omega$

$L = 32\,mH = 32 \times 10^{-3}\,H$

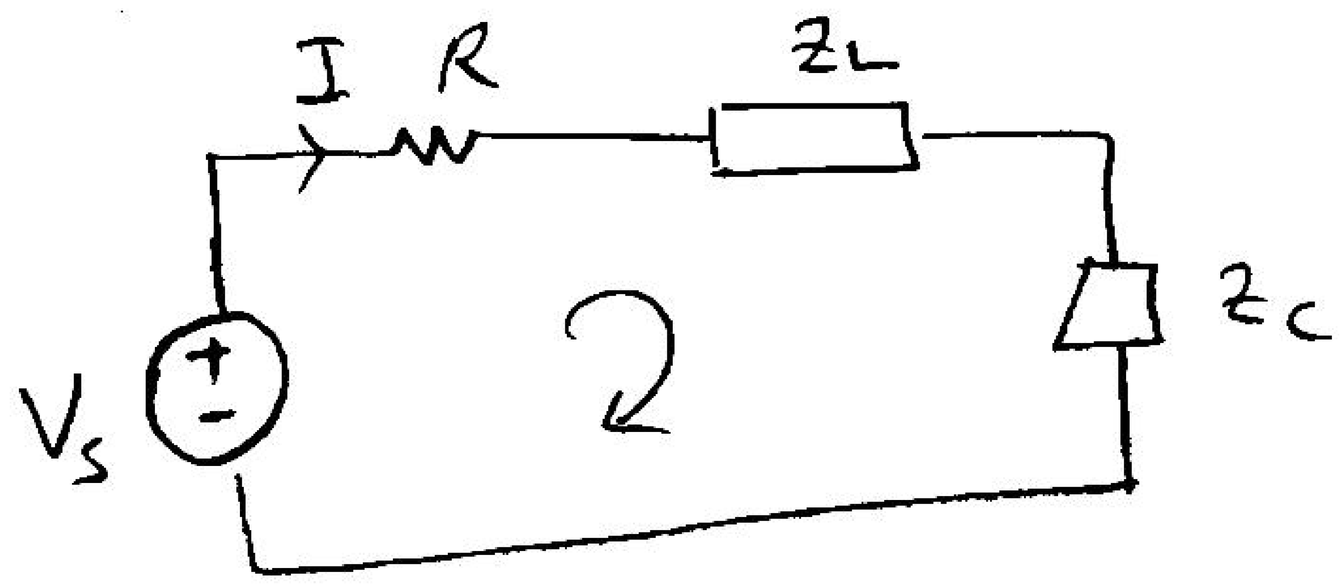
$C = 5\,\mu F = 5 \times 10^{-6}\,F$

Solution: We must solve the circuit first

$jwL = j \times 5000 \times 32 \times 10^{-3} = j160\,\Omega = Z_L$

$$Z_C = \frac{1}{jwC} = \frac{1}{j \times 5000 \times 5 \times 10^{-6}} = -j40\,\Omega$$

$V_s = 750 \cos(5000t + 30) \Rightarrow \underline{V_s} = 750\angle 30 = 750\, e^{j30}$



$-V_s + RI + Z_L I + Z_C I = 0$

$$I = \frac{V_s}{R + Z_L + Z_C} = \frac{750\, e^{j30}}{90 + j160 - j40}$$

$90 + j160 - j40 = 90 + j120 = \sqrt{90^2 + 120^2}\, e^{j\theta} = 150\, e^{j\theta}$

$\theta = \tan^{-1}\frac{120}{90} = 53.1^\circ$

$$I = \frac{750\, e^{j30}}{150\, e^{j53.1}} = \frac{750}{150}\, e^{j(30-53.1)} = 5\, e^{-23.1j}$$

Power for R. $P_R = \frac{1}{2} V_R I_R^*$ $\quad V_R = R I_R \quad \boxed{I_R = I}$

$P_R = \frac{1}{2} V_R I_R^* = \frac{1}{2} R I_R I_R^* = \frac{1}{2} R |I_R|^2 = \frac{1}{2} 90\, 5^2 = 1125$

Note $|a+bj| = \sqrt{a^2+b^2}$ $\qquad \angle a+bj = \tan^{-1}\frac{b}{a}$ $\qquad$ (epzz

$$[a+bj][a+bj]^* = [a+bj][a-bj] = a^2+b^2 = |a+bj|^2$$

$$I\, I^* = |I|^2$$

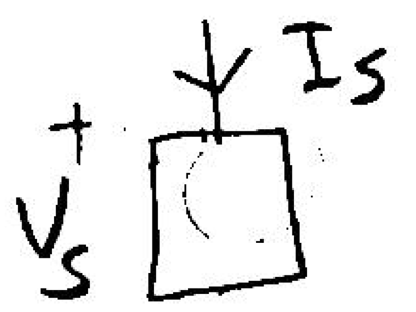
Power for L $\qquad P_L = \frac{1}{2} V_L I_L^* \qquad V_L = Z_L I_L$

$$P_L = \frac{1}{2} V_L I_L = \frac{1}{2} Z_L I_L I_L^* = \frac{1}{2} Z_L |I_L|^2 = \frac{1}{2} j160\ 5^2 = 2000j$$

Power for C $\qquad P_c = \frac{1}{2} V_c I_c^* \qquad V_c = Z_c I_c$

$$P_c = \frac{1}{2} V_c I_c^* = \frac{1}{2} Z_c I_c I_c^* = \frac{1}{2} Z_c |I_c|^2 = \frac{1}{2}(-j40)\ 5^2 = -500j$$

Power for the voltage source



$$P = \frac{1}{2} V_s I_s^* = \frac{1}{2} V_s (-I)^* = \frac{1}{2} 750 e^{j30} (-5 e^{-j23.1})^*$$

$$= \frac{1}{2} 750 e^{j30} (-5 e^{j23.1}) = -\frac{750 \times 5}{2} e^{j(30+23.1)}$$

$$= -1875 e^{j53.1} = -1875[\cos 53.1 + j \sin 53.1]$$

$$= -(1125 + j1500)$$

Total power of R, L, C

$$P_R + P_L + P_c = 1125 + 2000j - 500j = 1125 + 1500j$$

Voltage source power $\qquad P_s = -1125 - j1500j$

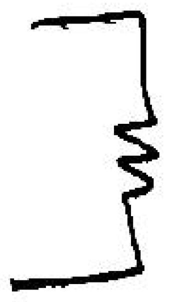
$$\boxed{(P_R + P_L + P_c) + P_s = 0}$$

$$P_s = -(P_R + P_L + P_c)$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

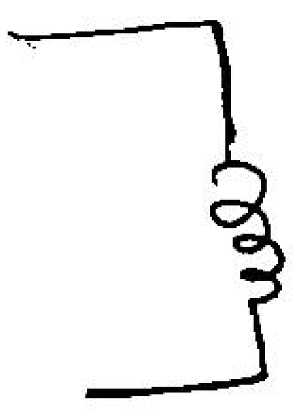
EP3

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$



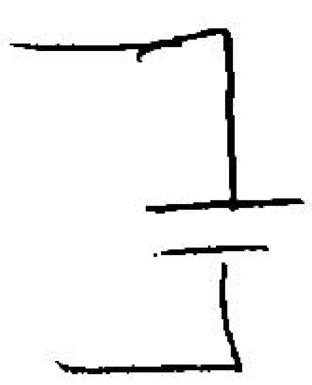
$$\theta_v - \theta_i = 0 \Rightarrow \cos(\theta_v - \theta_i) = 1 \qquad P = \frac{V_m I_m}{2}$$

$$\sin(\theta_v - \theta_i) = 0 \qquad Q = 0$$



$$\theta_v - \theta_i = 90^\circ \Rightarrow \cos 90^\circ = 0 \qquad P = 0$$

$$\sin 90^\circ = 1 \qquad Q = \frac{V_m I_m}{2}$$



$$\theta_v - \theta_i = -90^\circ \Rightarrow \cos(-90) = 0 \qquad P = 0$$

$$\sin(-90) = -1 \qquad Q = -\frac{V_m I_m}{2}$$

EXAMPLE 10.1

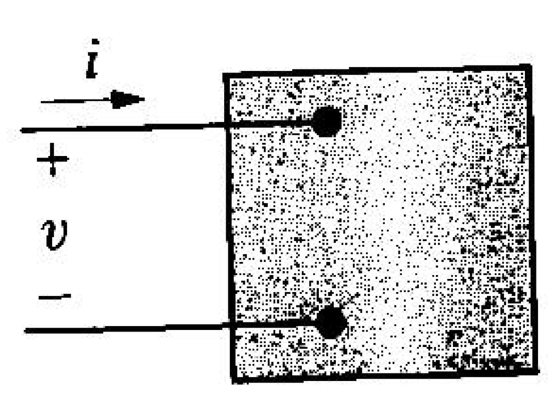
a) Calculate the average power and the reactive power at the terminals of the network shown in Fig. 10.6 if

$$v = 100\cos(\omega t + 15^\circ)\ \text{V},$$

$$i = 4\sin(\omega t - 15^\circ)\ \text{A}.$$

b) State whether the network inside the box is absorbing or delivering average power.

c) State whether the network inside the box is absorbing or supplying magnetizing vars.



A pair of terminals used for calculating power.

SOLUTION

$$\sin(x) = \cos(x - 90)$$

$$\sin(\omega t - 15) = \cos(\omega t - 15 - 90)$$

$$i = 4\cos(\omega t - 105^\circ)\ \text{A}.$$

$$P = \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

$$Q = \frac{V_m I_m}{2} \sin(\theta_v - \theta_i)$$

$$\theta_v = 15^\circ \qquad \theta_i = -105^\circ$$

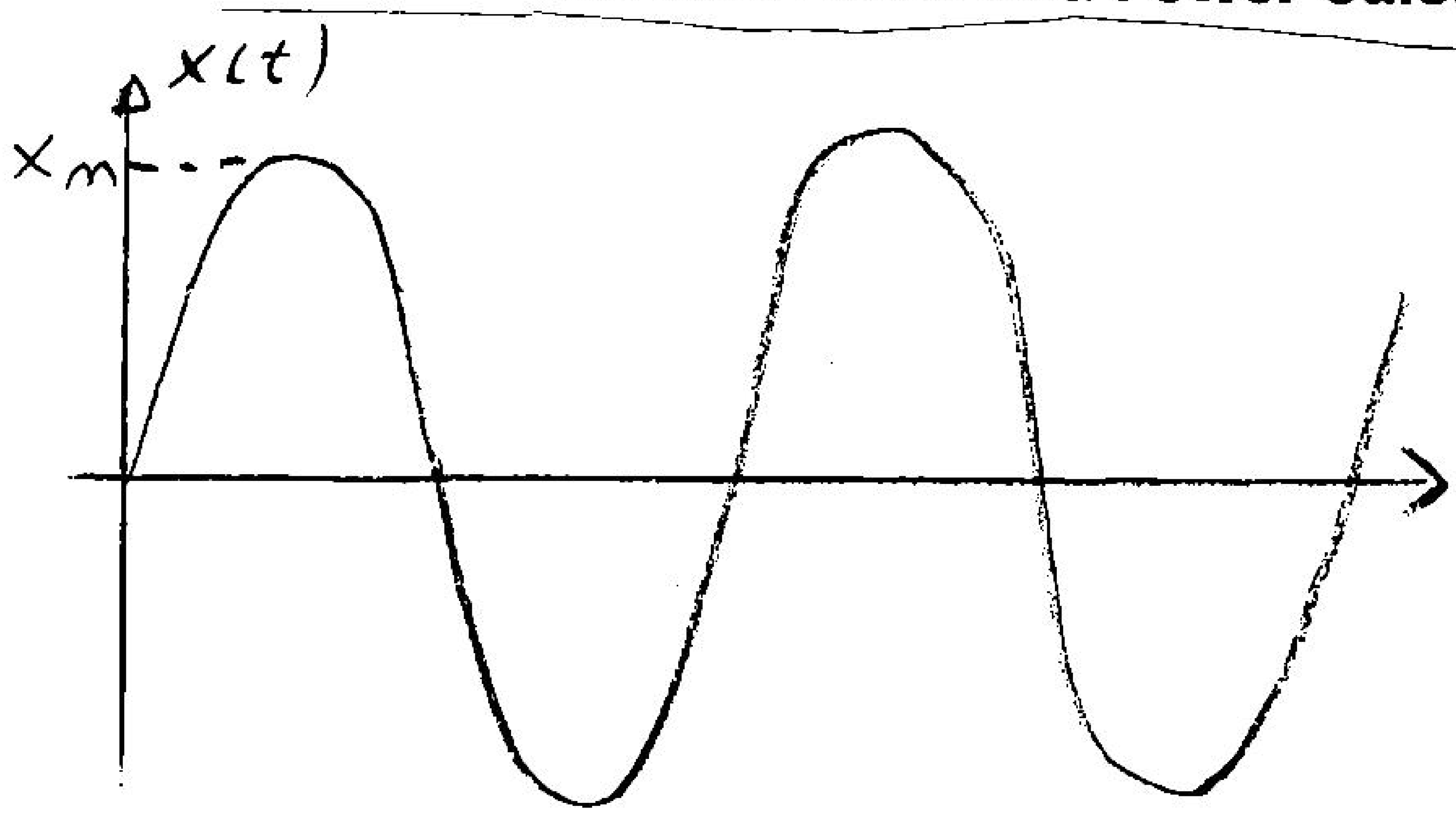
$$P = \frac{1}{2}(100)(4)\cos[15 - (-105)] = -100\ \text{W},$$

$$Q = \frac{1}{2}100(4)\sin[15 - (-105)] = 173.21\ \text{VAR}.$$

b) Note from Fig. 10.6 the use of the passive sign convention. Because of this, the negative value of −100 W means that the network inside the box is delivering average power to the terminals.

c) The passive sign convention means that, because Q is positive, the network inside the box is absorbing magnetizing vars at its terminals.

EP4



$x(t) = X_m \cos(\omega t + \theta)$

X_m = maximum value

$X_{ef} = \frac{X_m}{\sqrt{2}}$ effective Value

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

$$= V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i); \qquad (10.21)$$

and, by similar manipulation,

$$Q = V_{\text{eff}} I_{\text{eff}} \sin(\theta_v - \theta_i). \qquad (10.22)$$

XAMPLE 10.3

a) A sinusoidal voltage having a maximum amplitude of 625 V is applied to the terminals of a 50 Ω resistor. Find the average power delivered to the resistor.

b) Repeat (a) by first finding the current in the resistor.

SOLUTION

a) The rms value of the sinusoidal voltage is $625/\sqrt{2}$, or approximately 441.94 V. From

$v(t) = V_m \cos(\omega t + \alpha)$

$i(t) = \frac{V_m}{R}\cos(\omega t + \alpha)$

$\theta_v - \theta_i = \alpha - \alpha = 0$

$P = \frac{V_m I_m}{2} = \frac{V_m V_m/R}{2} = \frac{625 \times 625}{2} \cdot \frac{1}{50} = 3906.25$ W

$P = \frac{V_m}{\sqrt{2}}\frac{V_m}{\sqrt{2}}\frac{1}{R} =$

$= V_{eff} \cdot V_{eff} \frac{1}{R} = (441.94)^2 \frac{1}{50} = 3906.25$ W

EPS

$$S = P + jQ. \tag{10.23}$$

TABLE 10.2 Three Power Quantities and Their Units

QUANTITY	UNITS
Complex power	volt-amps
Average power	watts
Reactive power	vars

Another advantage of using complex power is the geometric interpretation it provides. When working with Eq. 10.23, think of P, Q, and $|S|$ as the sides of a right triangle, as shown in Fig. 10.9. It is easy to show that

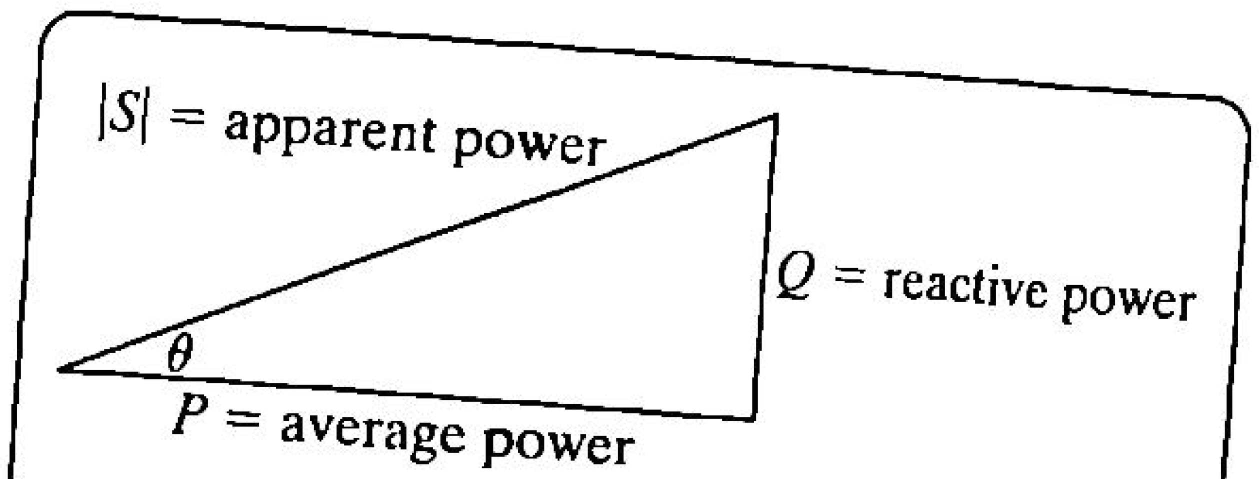


Figure 10.9 A power triangle.

$$\cos\theta = \frac{P}{|S|}$$

$$\sin\theta = \frac{Q}{|S|}$$

$$\tan\theta = \frac{Q}{P}.$$

$$\frac{Q}{P} = \frac{(V_m I_m/2)\sin(\theta_v - \theta_i)}{(V_m I_m/2)\cos(\theta_v - \theta_i)} = \tan(\theta_v - \theta_i).$$

The magnitude of complex power is referred to as **apparent power**. Specifically,

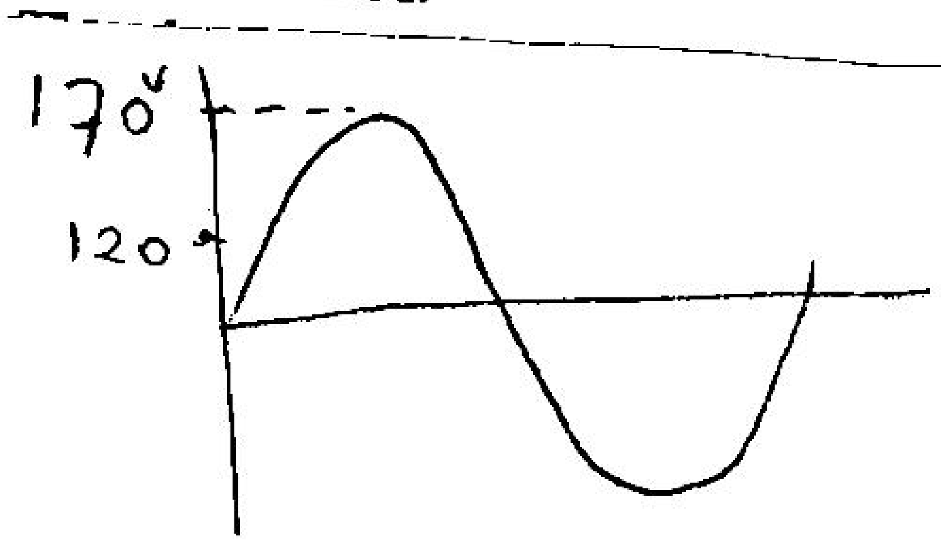
APPARENT POWER

$$|S| = \sqrt{P^2 + Q^2}.$$

$\cos\theta$ = power factor

rms value = effective value = $\frac{\text{maximum}}{\sqrt{2}}$ (sinuzoidal)

Appliances such as electric lamps, irons, and toasters all carry rms ratings on their nameplates. For example, a 120 V, 100 W lamp has a resistance of $120^2/100$, or 144 Ω, and draws an rms current of 120/144, or 0.833 A. The peak value of the lamp current is $0.833\sqrt{2}$, or 1.18 A.



$V_{max} = 170^V$

$V_{rms} = V_{eff} = 120^V$

EP 6

EXAMPLE 10.4

An electrical load operates at 240 V rms. The load absorbs an average power of 8 kW at a lagging power factor of 0.8.

a) Calculate the complex power of the load.

b) Calculate the impedance of the load.

SOLUTION

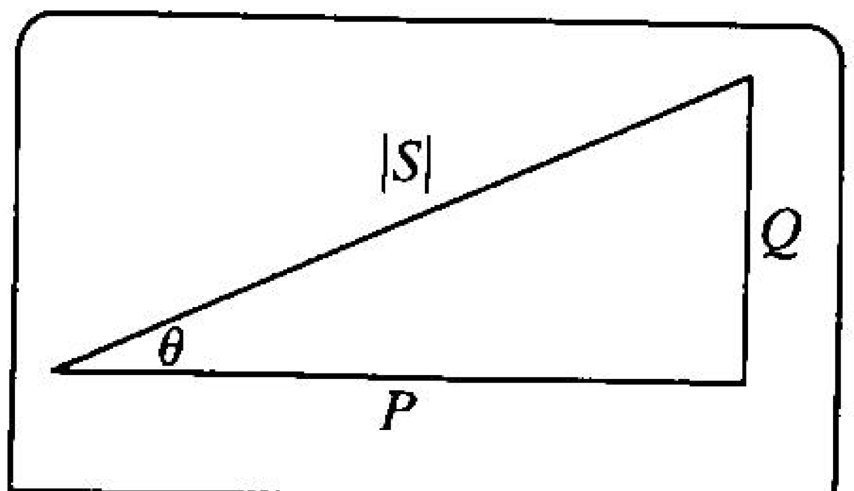
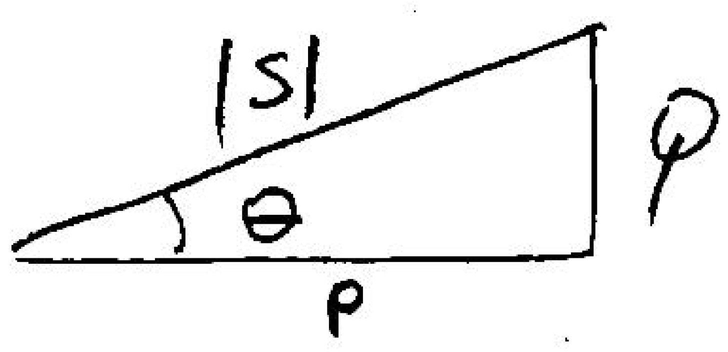


Figure 10.10 A power triangle.

240^V rms $\Rightarrow$ Veff = 240

Average power P = 8kw = 8000 w

lagging power factor 0.8



$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-0.8^2} = 0.6$

a)

$$P = |S|\cos\theta,$$

$$Q = |S|\sin\theta.$$

Now, because $\cos\theta = 0.8$, $\sin\theta = 0.6$. Therefore

$$|S| = \frac{P}{\cos\theta} = \frac{8\text{ kW}}{0.8} = 10\text{ kVA},$$

$$Q = 10\sin\theta = 6\text{ kVAR},$$

and

$$S = 8 + j6\text{ kVA}.$$

b) From the computation of the complex power of the load, we see that $P = 8$ kW. Using Eq. 10.21,

$$P = V_{\text{eff}} I_{\text{eff}} \cos(\theta_v - \theta_i)$$

$$= (240) I_{\text{eff}} (0.8)$$

$$= 8000\text{ W}.$$

$I_{eff} = \dfrac{8000}{240 \times 0.8} = 41.67$

$$\theta = \cos^{-1}(0.8) = 36.87°.$$

$V = Z \cdot I$

$|V| = |Z|\,|I|$

$V_{eff} = |Z|\; I_{eff}$

$240 = |Z|\; 41.67$

$|Z| = \dfrac{240}{41.67} = 5.76$

$Z = R + jX = |R + jX| \angle \tan^{-1}\dfrac{X}{R}$

$Z = 5.76 \angle 36.87°$

$Z = 5.76(\cos 36.87 + j\sin 36.87)$

$= 5.76(0.8 + j\,0.6)$

$Z = 4.608 + j\,3.45$

Power Calculations

$$S = P + jQ$$

$$S = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + j\frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2}[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

$$= \frac{V_m I_m}{2}e^{j(\theta_v - \theta_i)} = \frac{1}{2}V_m I_m \underline{/(\theta_v - \theta_i)}. = \frac{1}{2} V_m I_m e^{j(\theta_v - \theta_i)}$$

$$= \frac{1}{2}\underbrace{V_m e^{j\theta_v}}_{\underline{V}} \underbrace{I_m e^{-j\theta_i}}_{\underline{I}^*} = \frac{1}{2}\underline{V}\,\underline{I}^*$$

$$(a + bj)^* = a - bj \qquad (e^{jx})^* = e^{-jx}$$

Example $V = 100\cos(\omega t + 15)$ $\quad I = 4\cos(\omega t - 105)$

$$\underline{V} = 100\, e^{j15} \qquad \underline{I} = 4\, e^{-j105} \qquad I^* = 4\, e^{+j105}$$

$$S = \frac{1}{2}\underline{V}\,\underline{I} = \frac{1}{2}100\, e^{j15}\, 4\, e^{j105} = \frac{1}{2}400\, e^{j120}$$

$$= 200\, e^{j120} = 200\left[\cos 120 + j\sin 120\right]$$

$$= 200[-0.5 + j\,0.866]$$

$$= -100 + j\,173.2$$

$P = -100$ Watt (active power, average power)

$Q = 173.2$ VAR (reactive power)

$S = -100 + j\,173.2$ (Complex power)

$|S| = \sqrt{100^2 + 173.2^2} = 200 \longrightarrow$ appearent power

EXAMPLE 10.5

In the circuit shown in Fig. 10.13, a load having an impedance of $39 + j26\ \Omega$ is fed from a voltage source through a line having an impedance of $1 + j4\ \Omega$. The effective, or rms, value of the source voltage is 250 V.

a) Calculate the load current $\mathbf{I_L}$ and voltage $\mathbf{V_L}$.

b) Calculate the average and reactive power delivered to the load.

c) Calculate the average and reactive power delivered to the line.

d) Calculate the average and reactive power supplied by the source.

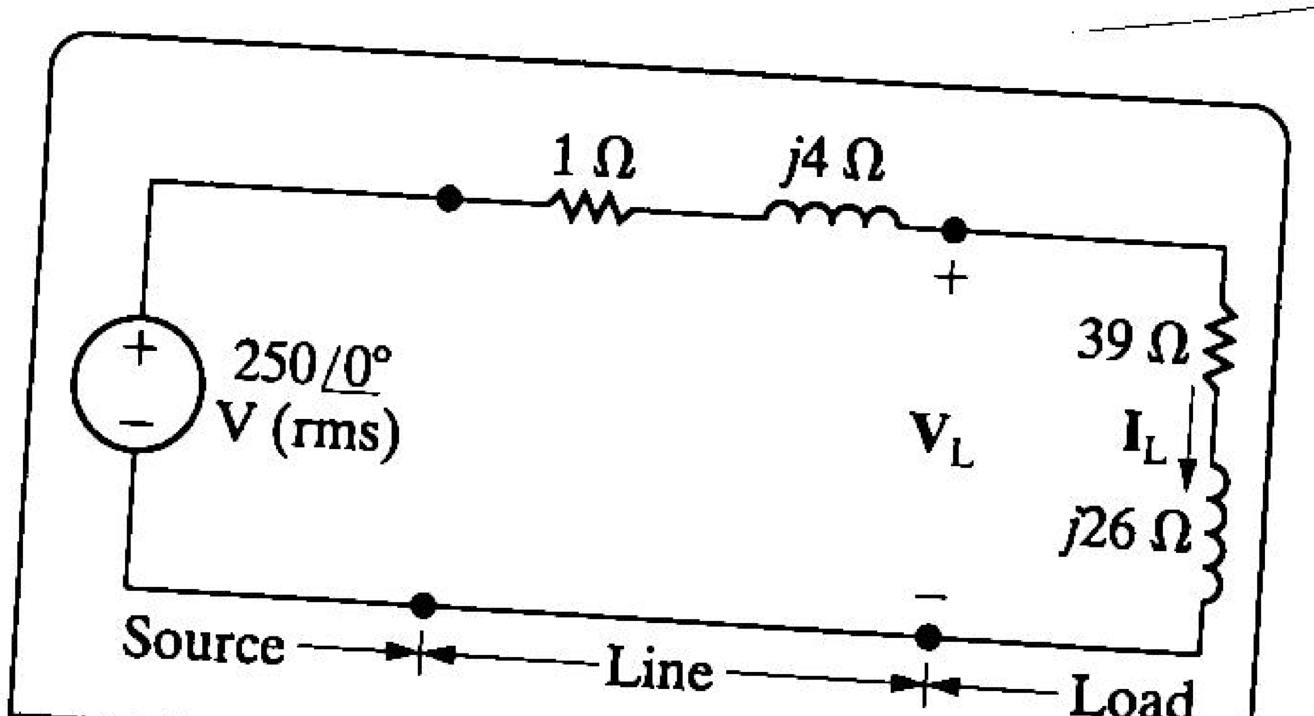
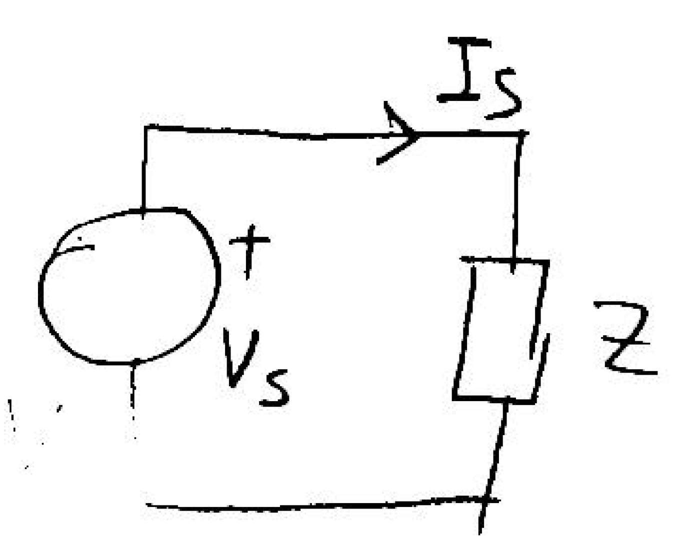


Figure 10.13 The circuit for Example 10.5.

SOLUTION



$-250\sqrt{2} = 354$

$\underline{V_s} = 354\cos(\omega t + 0)$

$= 354\, e^{j0}$

$Z = (1+j4) + (39+j26)$

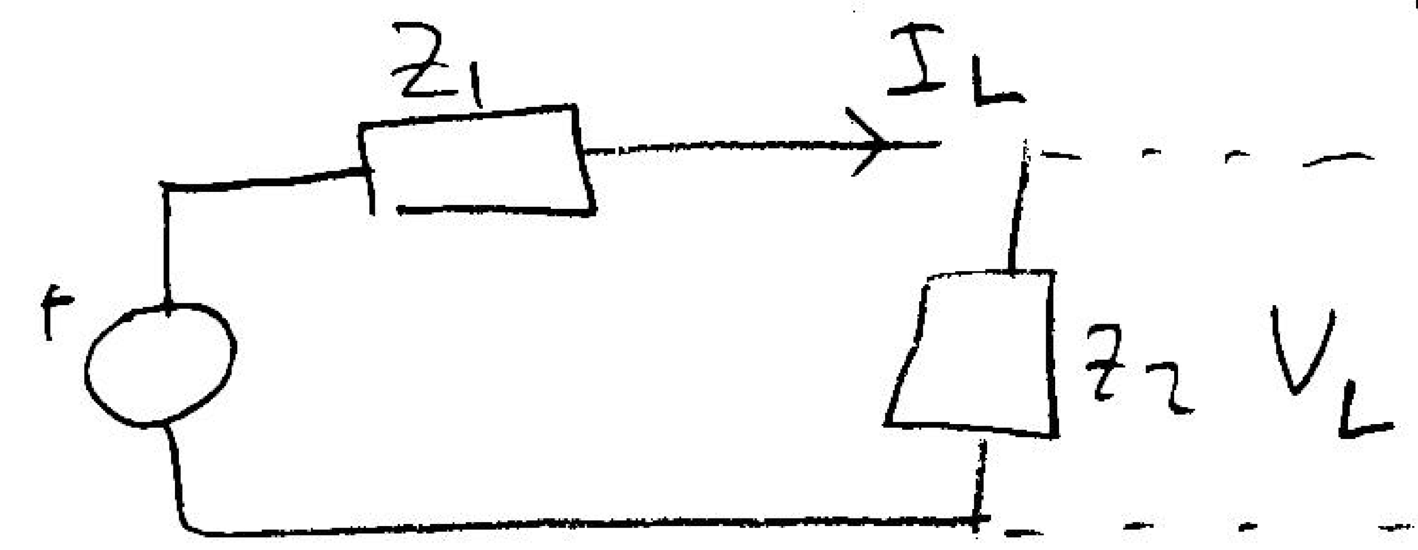
$= 40 + j30$

$$\underline{I_L} = \frac{V_s}{Z} = \frac{354}{40+j30} = \frac{354(40-j30)}{40^2+30^2}$$

$$= \frac{14160 - j10620}{2500}$$

$$= 5.664 - j4.248$$

EP8



$\underline{V_L} = Z_2\, \underline{I_s} = (39+j26)(5.66-j4.24)$

Power of Z_2 is

$S = \frac{1}{2} \underline{V_L}\, \underline{I_L^*}$

$= \frac{1}{2} \underbrace{[(39+j26)(5.66-j4.24)]}_{V_L} \underbrace{(5.66+j4.24)}_{I_L^*}$

$= \frac{1}{2}(39+j26)(5.66^2+4.24^2)$

$= \frac{1}{2}(39+j26)\ 50$

$= 975 + j650$

$P = 975$ watt

$Q = 650$ VAR

Power of Z_1 is

$S = \frac{1}{2} V_1 I_L^* = \frac{1}{2} \underbrace{(1+j4)\, I_L}_{V_1} I_L^*$

$I_L I_L^* = 50$

$S = \frac{1}{2}(1+j4)\ 50 = 25 + j100$

$P = 25$ watt

$Q = 100$ VAR

Note:

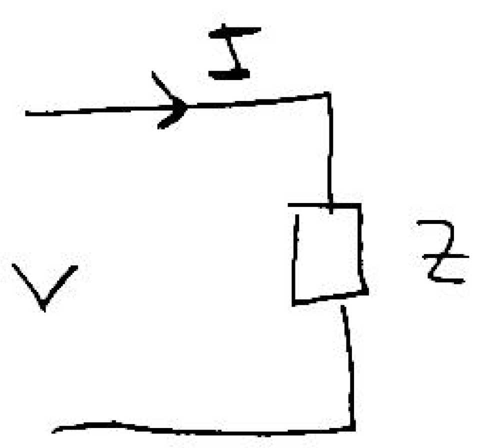
$$(a+bj)(a-bj) = a^2 - abj + abj + b^2$$
$$= a^2 + b^2$$

$$|a+bj| = \sqrt{a^2+b^2}$$

$$(a+bj)(a-bj) = |a+bj|^2$$

$$\varphi \, \varphi^* = |\varphi|^2$$

Example: calculate P, φ, S



$V_{eff} = 3+4j$

$Z = 8+6j$

Solution: $S = V_{eff} I^*_{eff}$

$$V = Z \cdot I$$

$$S = V I^* = V\left(\frac{V}{Z}\right)^* = \frac{V V^*}{Z^*} = \frac{|V|^2}{Z^*}$$

$$= \frac{|3+4j|^2}{(8+6j)^*} = \frac{3^2+4^2}{8-6j} = \frac{25}{8-6j}$$

$$= \frac{25(8+6j)}{(8-6j)(8+6j)} = \frac{200+150j}{8^2+6^2}$$

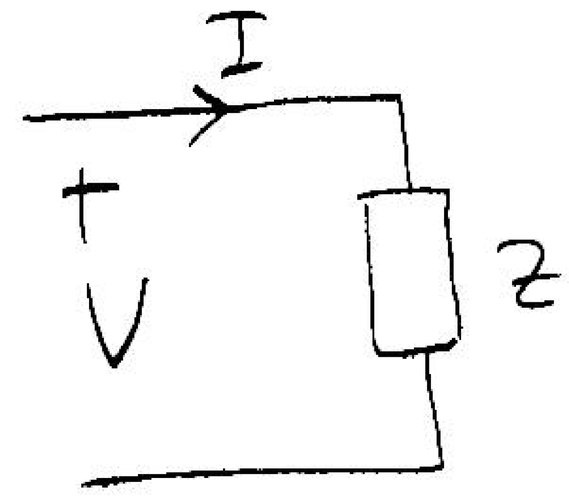
$$= \underbrace{2}_{P} + \underbrace{1.5}_{\varphi} j$$

$P = 2 \qquad \varphi = 1.5$

Appearent power $|S|$

$$|S| = |2+1.5j| = \sqrt{2^2+1.5^2} = 2.5$$

Example: Calculate P, φ, S



$I = 4+3j$

$Z = 8+6j$

solution $\quad S = \frac{1}{2} V I^*$

$$V = ZI$$

$$S = \frac{1}{2}(ZI) I^* = \frac{1}{2} Z |I|^2$$

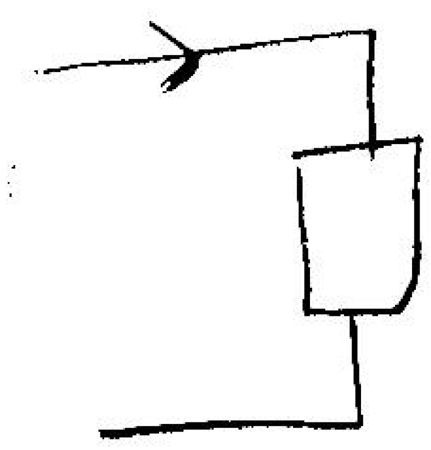
$$= \frac{1}{2}(8+6j)|4+3j|^2$$

$$= \frac{1}{2}(8+6j)\, 25 = 100 + 75j$$

$P = 100 \text{ W} \qquad \varphi = 75 \text{ VAR}$

$$|S| = \sqrt{100^2+75^2} = 125$$

Example: Calculate power factor



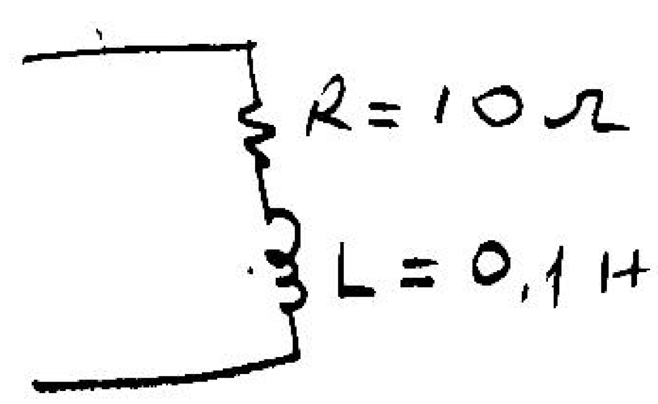
$Z = 10+15j$

$$\tan\theta = \frac{\varphi}{P} = \frac{Im\{Z\}}{Re\{Z\}} = \frac{15}{10} = 1.5$$

$$\theta = 56.3°$$

$\cos\theta = 0.55$ = Power factor

Example: Calculate power factor



$\omega = 10000$

$$Z = R + j\omega L = 10 + 0.1\, 10^{-3}\, 10^{4} j$$
$$= 10 + 10j$$

$$\theta = \tan^{-1}\frac{10}{10} = 45° \qquad \underline{\cos\theta = 0.707}$$