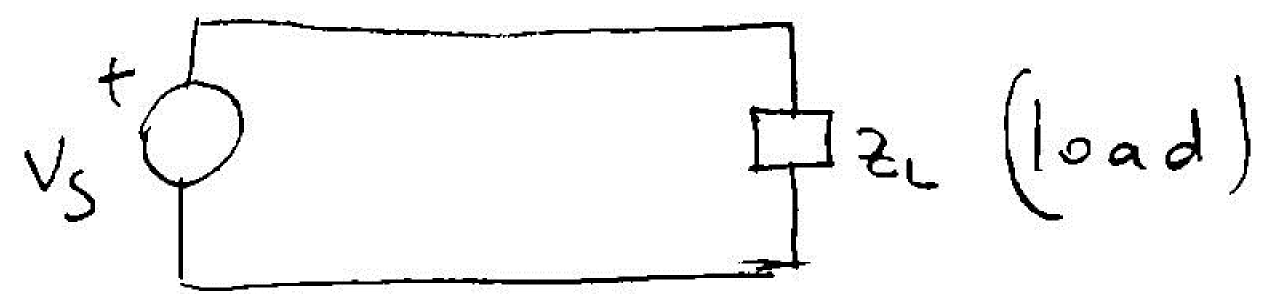
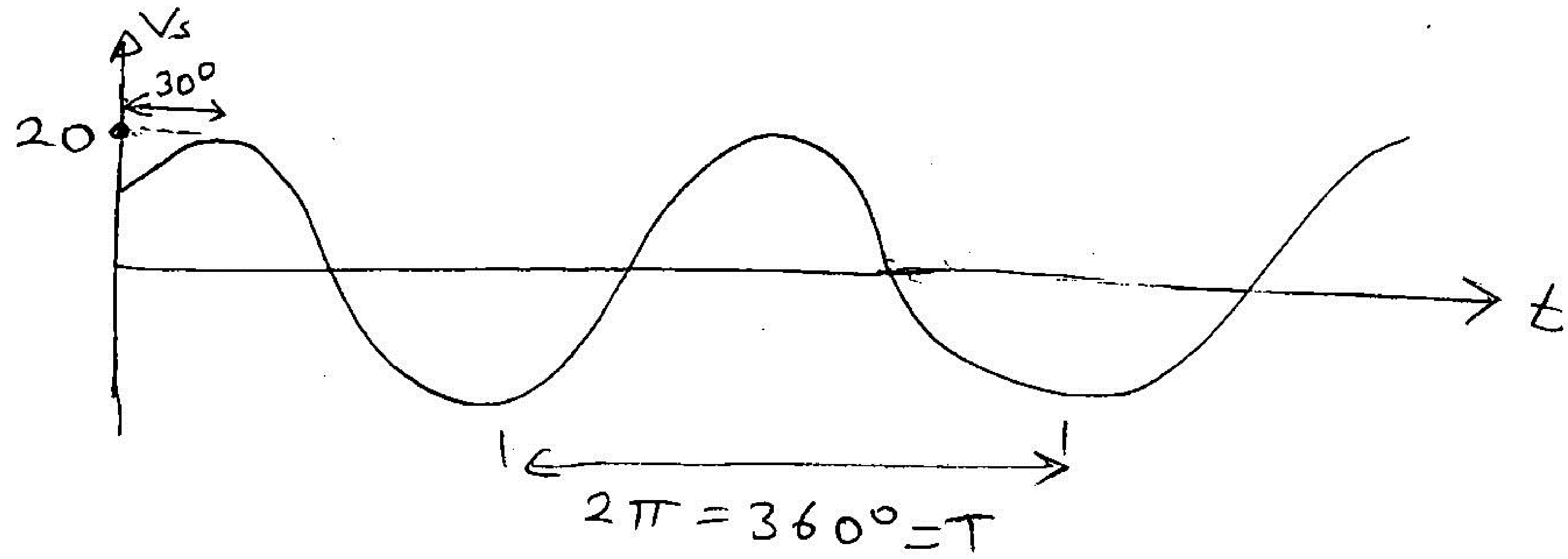
Balanced Three-Phase Voltages



$v_s = 20\cos(60t - 30)$

source



mono phase (single phase) system

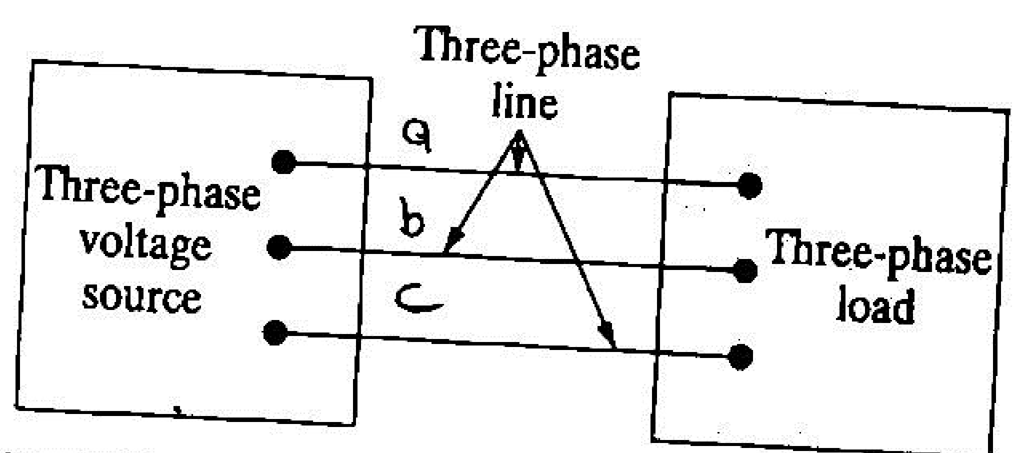
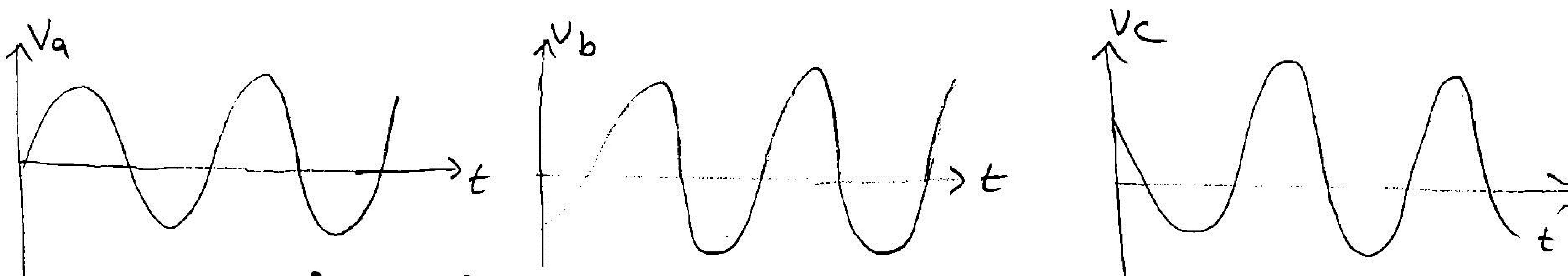
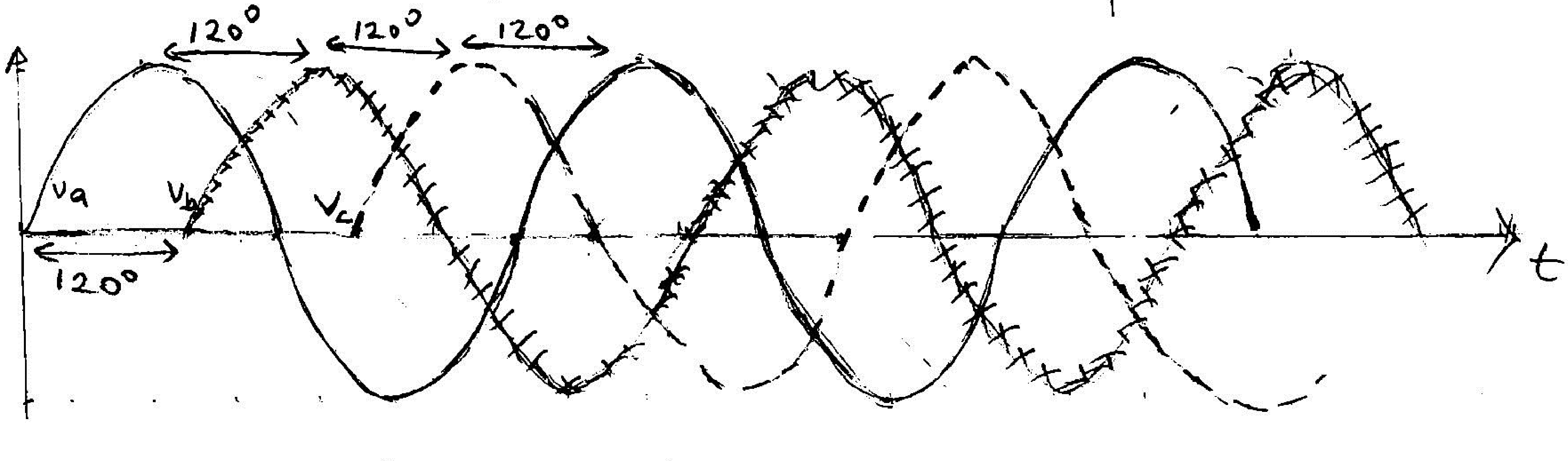


Figure 11.1 A basic three-phase circuit.





Three phase system

A set of balanced three-phase voltages consists of three sinusoidal voltages that have identical amplitudes and frequencies but are out of phase with each other by exactly 120°. Standard practice is to refer to the three phases as a, b, and c, and to use the a-phase as the reference phase. The three voltages are referred to as the **a-phase voltage**, the **b-phase voltage**, and the **c-phase voltage**.

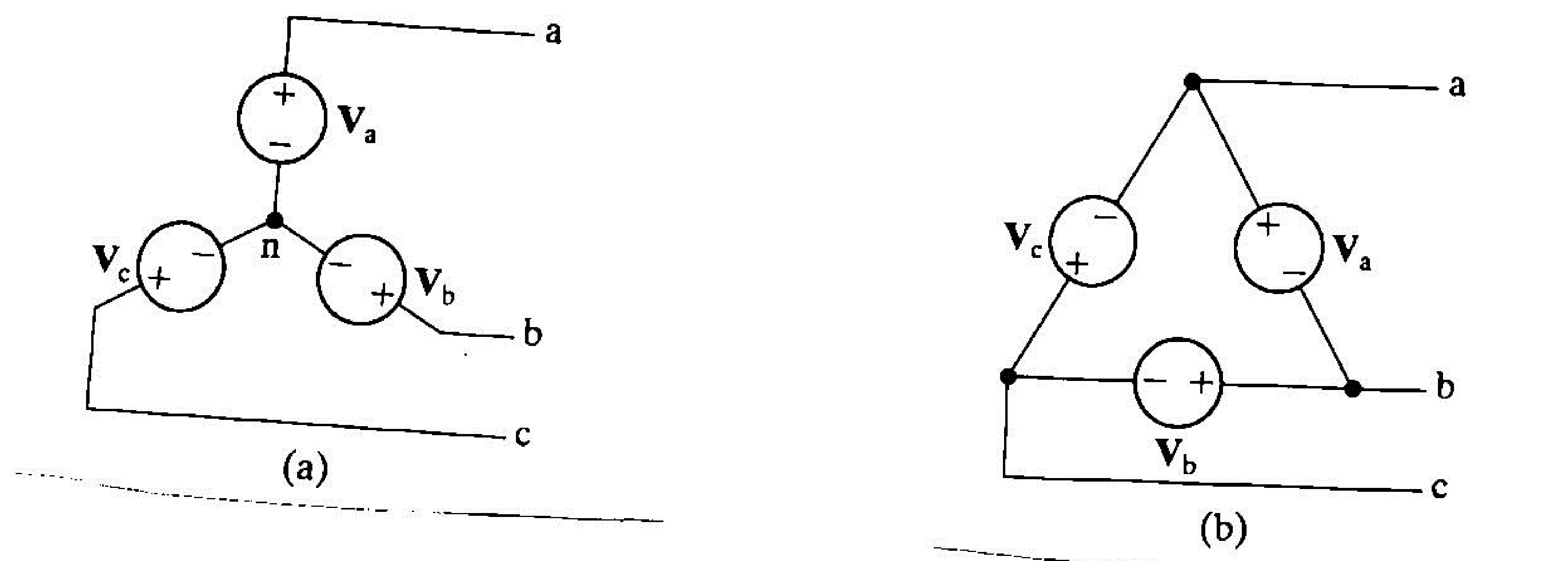
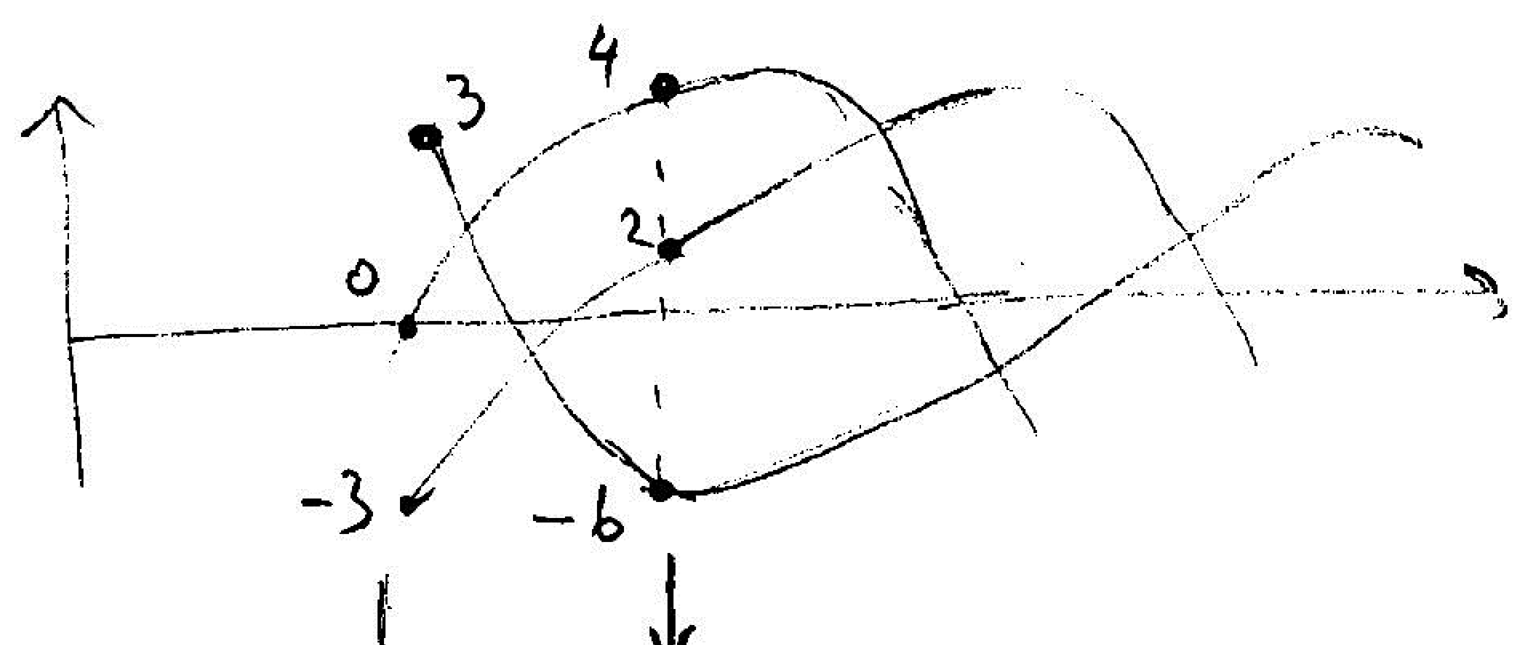


Figure 11.4 The two basic connections of an ideal three-phase source. (a) A Y-connected source. (b) A Δ-connected source.

$$\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c = 0. \tag{11.3}$$

Because the sum of the phasor voltages is zero, the sum of the instantaneous voltages also is zero; that is,

$$v_a + v_b + v_c = 0. \tag{11.4}$$



$-3+3+0=0$ $\quad 4+2-6=0$

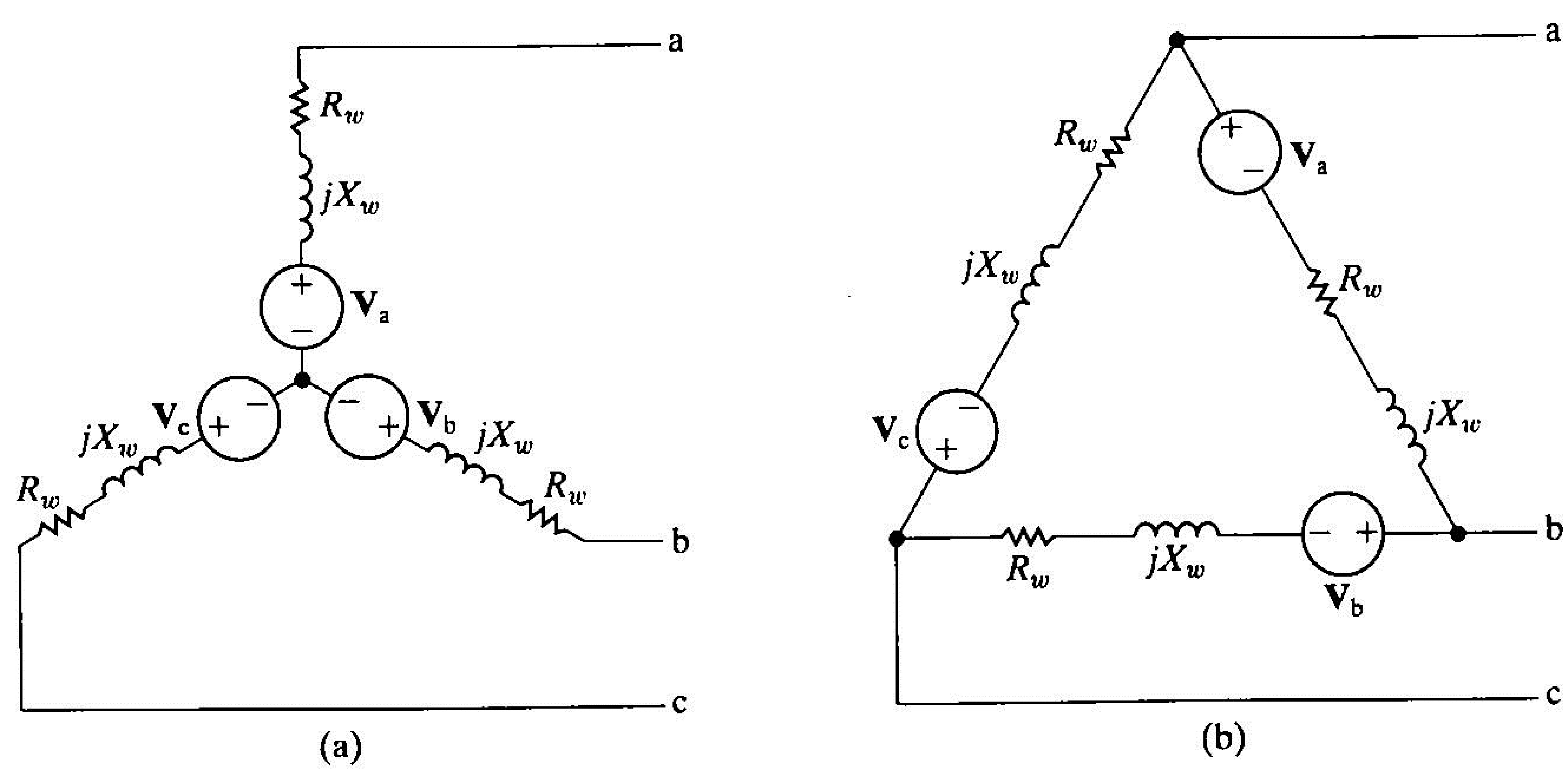
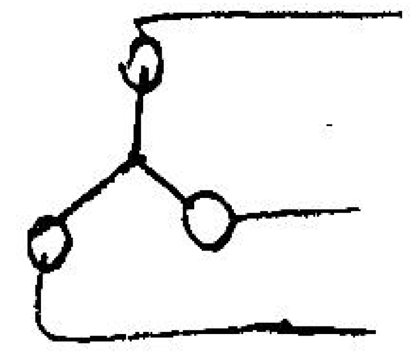
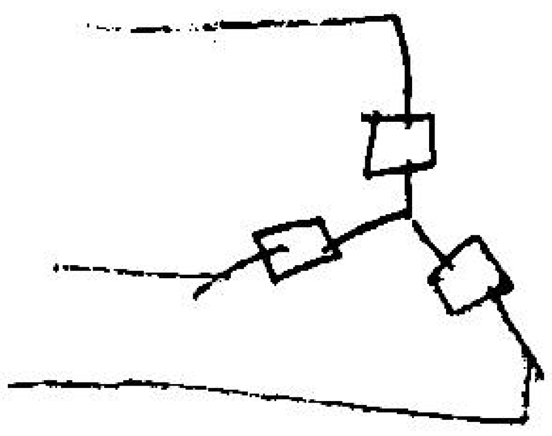


Figure 11.5 A model of a three-phase source with winding impedance: (a) a Y-connected source; and (b) a Δ-connected source.





Source Y   load Y

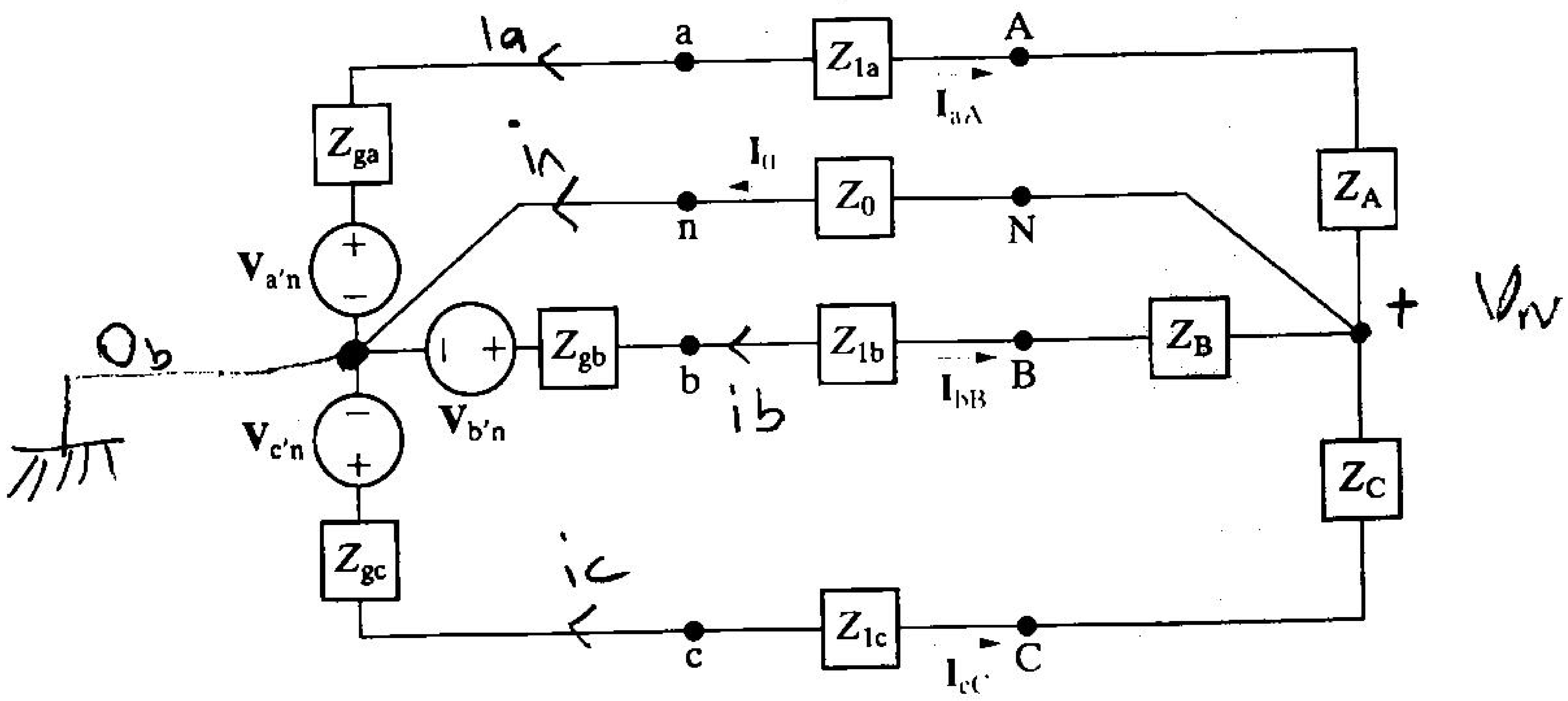


Figure 11.6 A three-phase Y-Y system.

Z_{ga}, Z_{gb}, and Z_{gc} represent the internal impedance of generator

Z_{1a}, Z_{1b}, and Z_{1c} represent the impedance of the lines

, Z_0 is the impedance of the neutral conductor

Z_A, Z_B, and Z_C represent the impedance of each phase of the load.

$$\dot{I}_a + \dot{I}_b + \dot{I}_c + \dot{I}_n = 0$$

loop equation from O_A to O_B

$$-V_N + (Z_A + Z_{1a} + Z_{ga})\,\dot{I}_a + V_{a'n} = 0 \Rightarrow \dot{I}_a = \frac{V_N - V_{a'n}}{Z_A + Z_{1a} + Z_{ga}}$$

$$-V_N + Z_0 \dot{I}_n = 0 \Rightarrow \dot{I}_n = \frac{V_N}{Z_0}$$

$$\Rightarrow \dot{I}_b = \frac{V_N - V_{b'n}}{Z_B + Z_{1b} + Z_{gb}}$$

$$\Rightarrow \dot{I}_c = \frac{V_N - V_{c'n}}{Z_C + Z_{1c} + Z_{gc}}$$

$I_n + I_a + I_b + I_c = 0$

$$\frac{\mathbf{V}_N}{Z_0} + \frac{\mathbf{V}_N - \mathbf{V}_{a'n}}{Z_A + Z_{1a} + Z_{ga}} + \frac{\mathbf{V}_N - \mathbf{V}_{b'n}}{Z_B + Z_{1b} + Z_{gb}} + \frac{\mathbf{V}_N - \mathbf{V}_{c'n}}{Z_C + Z_{1c} + Z_{gc}} = 0. \quad (11.5)$$

CONDITIONS FOR A BALANCED THREE-PHASE CIRCUIT

1. The voltage sources form a set of balanced three-phase voltages. In Fig. 11.6, this means that $\mathbf{V}_{a'n}$, $\mathbf{V}_{b'n}$, and $\mathbf{V}_{c'n}$ are a set of balanced

$|V_{a'n}| = |V_{b'n}| = |V_{c'n}|$

$\angle V_{b'n} = \angle V_{a'n} + 120^\circ$

$\angle V_{c'n} = \angle V_{b'n} + 120^\circ = \angle V_{a'n} + 240 = \angle V_{a'n} - 120$

2. The impedance of each phase of the voltage source is the same. In Fig. 11.6, this means that $Z_{ga} = Z_{gb} = Z_{gc}$.
3. The impedance of each line (or phase) conductor is the same. In Fig. 11.6, this means that $Z_{1a} = Z_{1b} = Z_{1c}$.
4. The impedance of each phase of the load is the same. In Fig. 11.6, this means that $Z_A = Z_B = Z_C$.

If the circuit in Fig. 11.6 is balanced, we may rewrite Eq. 11.5 as

$$\mathbf{V}_N\left(\frac{1}{Z_0} + \frac{3}{Z_\phi}\right) = \frac{\mathbf{V}_{a'n} + \mathbf{V}_{b'n} + \mathbf{V}_{c'n}}{Z_\phi}, \quad (11.6)$$

where

$$Z_\phi = Z_A + Z_{1a} + Z_{ga} = Z_B + Z_{1b} + Z_{gb} = Z_C + Z_{1c} + Z_{gc}.$$

$$\mathbf{V}_N = 0. \quad (11.7)$$

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{a'n} - \mathbf{V}_N}{Z_A + Z_{1a} + Z_{ga}} = \frac{\mathbf{V}_{a'n}}{Z_\phi}, \quad (11.8)$$

$$\mathbf{I}_{bB} = \frac{\mathbf{V}_{b'n} - \mathbf{V}_N}{Z_B + Z_{1b} + Z_{gb}} = \frac{\mathbf{V}_{b'n}}{Z_\phi}, \quad (11.9)$$

$$\mathbf{I}_{cC} = \frac{\mathbf{V}_{c'n} - \mathbf{V}_N}{Z_C + Z_{1c} + Z_{gc}} = \frac{\mathbf{V}_{c'n}}{Z_\phi}. \quad (11.10)$$

if the three phase is balanced we can analyse each phase seperately.

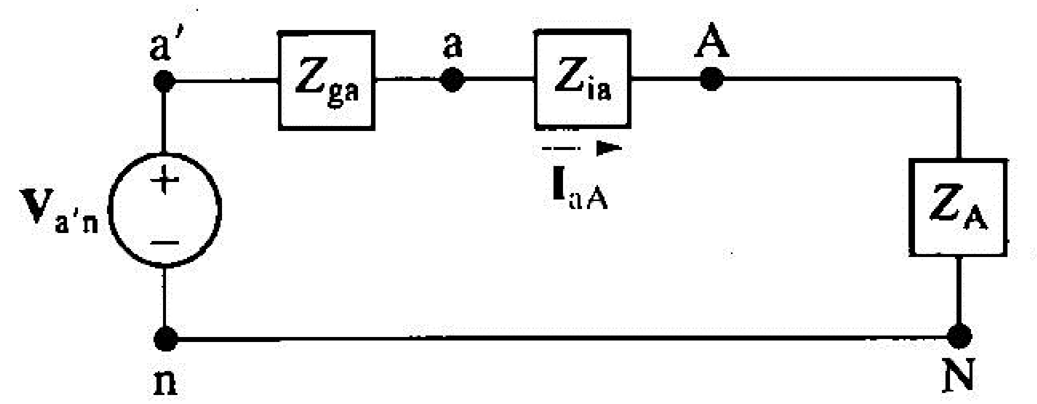


Figure 11.7 A single-phase equivalent circuit.

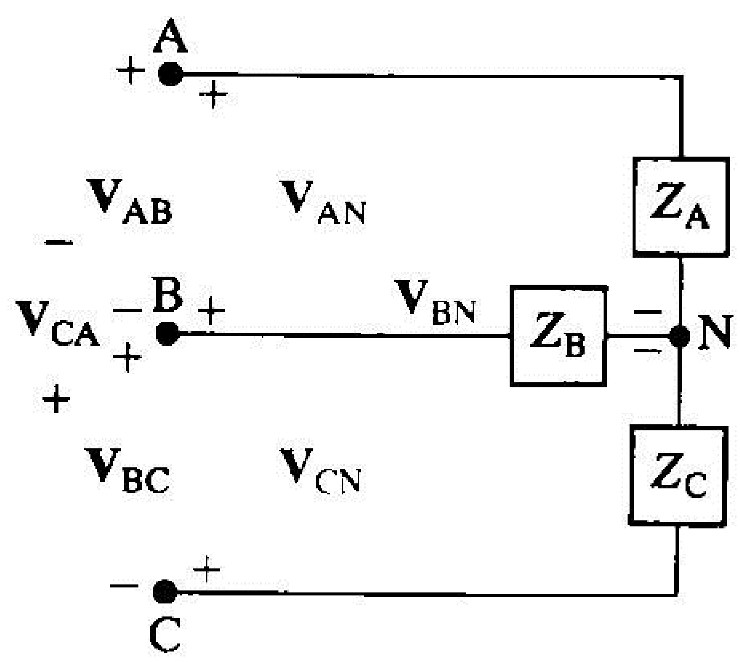


Figure 11.8 Line-to-line and line-to-neutral voltages.

$$V_{AB} = V_{AN} - V_{BN}, \tag{11.12}$$

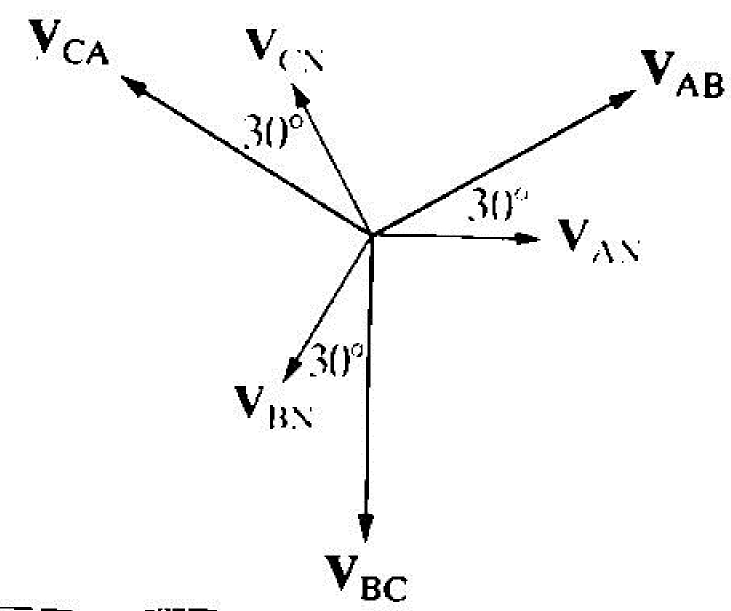
$$V_{BC} = V_{BN} - V_{CN}, \tag{11.13}$$

$$V_{CA} = V_{CN} - V_{AN}. \tag{11.14}$$

$$V_{AN} = V_\phi \angle 0^\circ, \tag{11.15}$$

$$V_{BN} = V_\phi \angle -120^\circ$$

$$V_{CN} = V_\phi \angle +120^\circ, \tag{11.17}$$



V_ϕ represents the magnitude of the line-to-neutral voltage.

$$V_{AB} = V_\phi \angle 0^\circ - V_\phi \angle -120^\circ = \sqrt{3} V_\phi \angle 30^\circ, \tag{11.18}$$

$$V_{BC} = V_\phi \angle -120^\circ - V_\phi \angle 120^\circ = \sqrt{3} V_\phi \angle -90^\circ, \tag{11.19}$$

$$V_{CA} = V_\phi \angle 120^\circ - V_\phi \angle 0^\circ = \sqrt{3} V_\phi \angle 150^\circ. \tag{11.20}$$

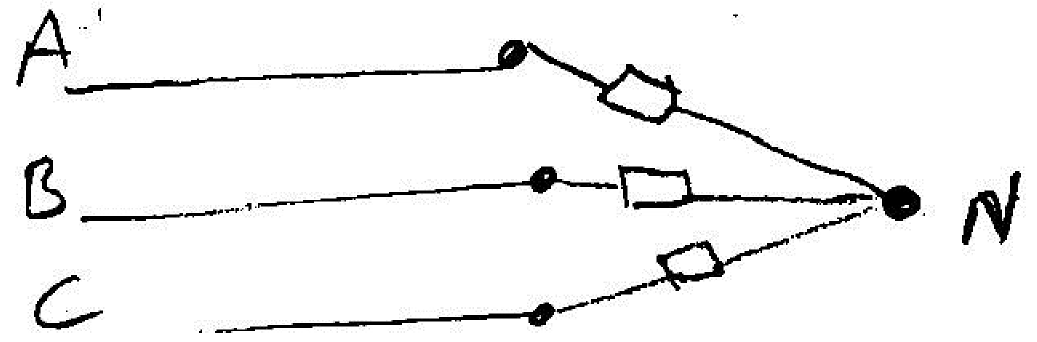
$$V_{AB} = V_\theta e^{j0} - V_\theta e^{-j120} = V_\theta \left(1 - e^{-j120}\right)$$

$$= V_\theta \left[1 - (\cos 120 - j \sin 120)\right]$$

$$= V_\theta \left[1 - \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)\right]$$

$$= V_\theta \left[\frac{3}{2} + j\frac{\sqrt{3}}{2}\right]$$

$$= V_\theta \sqrt{3} \left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right] = V_\theta \sqrt{3}\, e^{j30}$$



line voltage

phase //

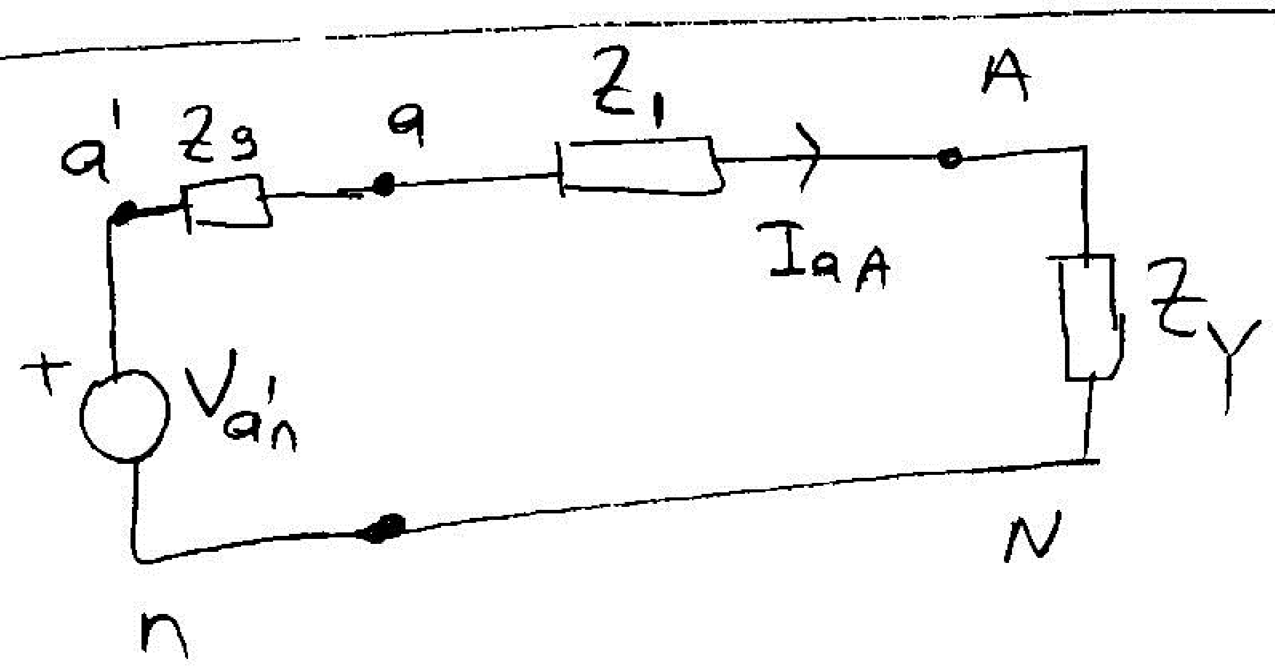
$$|V_{AB}| = \sqrt{3}\,|V_{AN}|$$

$$|V_{AB}| = |V_{BC}| = |V_{AC}|$$

$\angle V_{AB} = 0°$ $\qquad \angle V_{AN} = -30°$

$\angle V_{BC} = -120°$ $\qquad \angle V_{BN} = -150$

$\angle V_{AC} = +120°$ $\qquad \angle V_{CN} = 90$



Z_Y = load impedance wye connection

Z_1 = line impedence

Z_g = Source impedence

I_{aA} = line current

V_{AN} = load voltage (phase voltage)

$V_{a'n}$ = generator (source) voltage

V_{an} = generator output voltage

Example: line voltage is $190.52\angle 20°$ calculate phase voltage.

Solution:

line voltage = line to line voltage $|V_{AB}| = |V_{BC}| = |V_{AC}|$

$$V_{AB} = 190.52\angle 20°$$

$$V_{AN} = ?$$

$$|V_{AN}| = \frac{|V_{AB}|}{\sqrt{3}} = \frac{190.52}{\sqrt{3}} = 110$$

$$\angle V_{AN} = V_{AB} - 30 = 20 - 30 = -10°$$

$$V_{AN} = 110\angle -10°$$

phase voltage $= 110\angle -10°$

Example: $V_{AN} = 10\cos(314t + 30)$ $Z_Y = 30 + 40j$. calculate line current

$$Z_Y = 30 + 40j = 50e^{j53.1}$$

$$I_{aA} = \frac{V_{AN}}{Z_Y} \qquad V_{AN} = 10e^{j30}$$

Here we always use effective value $V_{AN} = \frac{10}{\sqrt{2}} e^{j30}$

$$I_{aA} = \frac{\frac{10}{\sqrt{2}} e^{j30}}{50e^{j53.1}} = 0.14e^{-j23.1°}$$

EXAMPLE 11.1

A balanced three-phase Y-connected generator with positive sequence has an impedance of $0.2 + j0.5\ \Omega/\phi$ and an internal voltage of $120\ \text{V}/\phi$. The generator feeds a balanced three-phase Y-connected load having an impedance of $39 + j28\ \Omega/\phi$. The impedance of the line connecting the generator to the load is $0.8 + j1.5\ \Omega/\phi$. The a-phase internal voltage of the generator is specified as the reference phasor.

a) Construct the a-phase equivalent circuit of the system.

b) Calculate the three line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.

c) Calculate the three phase voltages at the load, $\mathbf{V}_{AN}$, $\mathbf{V}_{BN}$, and $\mathbf{V}_{CN}$.

d) Calculate the line voltages $\mathbf{V}_{AB}$, $\mathbf{V}_{BC}$, and $\mathbf{V}_{CA}$ at the terminals of the load.

e) Calculate the phase voltages at the terminals of the generator, $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, and $\mathbf{V}_{cn}$.

f) Calculate the line voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ at the terminals of the generator.

solution

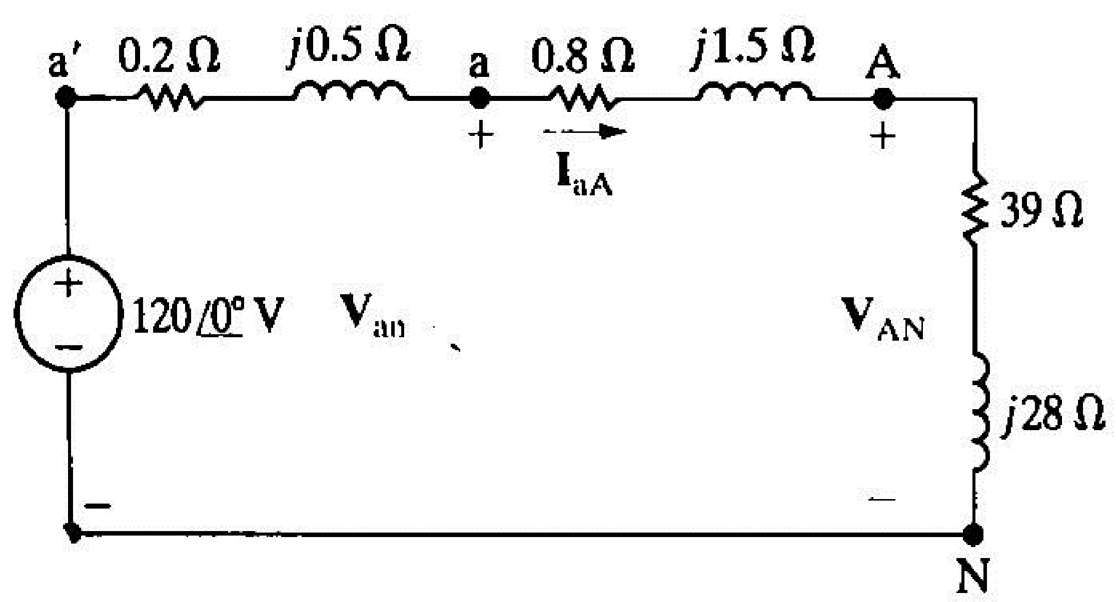
$0.2 + j0.5 = Z_{ga} = Z_{gb} = Z_{gc}$

$120^{V} = |V_{a'n}| = |V_{b'n}| = |V_{c'n}|$

$39 + j28 = Z_A = Z_B = Z_C$

$0.8 + j1.5 = Z_{1a} = Z_{1b} = Z_{1c}$

a) Figure 11.10 shows the single-phase equivalent circuit.



11.10 The single-phase equivalent circuit for Example 11.1.

$10\,e^{j30}$

effective

se

from the figure

b) The a-phase line current is

$$\mathbf{I}_{aA} = \frac{120\,\angle 0^\circ}{(0.2 + 0.8 + 39) + j(0.5 + 1.5 + 28)}$$

$$= \frac{120\,\angle 0^\circ}{40 + j30}$$

$$= 2.4\,\angle -36.87^\circ \text{ A}.$$

$|I_{bB}| = |I_{bC}| = |I_{aA}|$

$\angle I_{bB} = \angle I_{bA} - 120$

$\angle I_{cC} = \angle I_{bA} + 120 = \angle I_{bB} - 240$

$$\mathbf{I}_{bB} = 2.4\,\angle -156.87^\circ \text{ A},$$

$$\mathbf{I}_{cC} = 2.4\,\angle 83.13^\circ \text{ A}.$$

c) $V_{AN} = I\,Z =$

$= (2.4\,\angle -36.87)(39 + j28)$

$$\mathbf{V}_{AN} = (39 + j28)(2.4\,\angle -36.87^\circ)$$

$$= 115.22\,\angle -1.19^\circ \text{ V}.$$

$|V_{BN}| = |V_{AN}| = |V_{CN}|$

$\angle V_{BN} = \angle V_{AN} - 120$

$\angle V_{CN} = \angle V_{AN} + 120$

$$\mathbf{V}_{BN} = 115.22\,\angle -121.19^\circ \text{ V},$$

$$\mathbf{V}_{CN} = 115.22\,\angle 118.81^\circ \text{ V}.$$

d) For a positive phase sequence, the line voltages lead the phase voltages by 30°; thus

$$\mathbf{V}_{AB} = (\sqrt{3}\,\angle 30^\circ)\mathbf{V}_{AN}$$

$$= 199.58\,\angle 28.81^\circ \text{ V},$$

$$\mathbf{V}_{BC} = 199.58\,\angle -91.19^\circ \text{ V},$$

$$\mathbf{V}_{CA} = 199.58\,\angle 148.81^\circ \text{ V}.$$

e) The phase voltage at the a terminal of the source is (from the figure)

$$\mathbf{V}_{an} = 120 - (0.2 + j0.5)(2.4 \angle -36.87^\circ)$$
$$= 120 - 1.29 \angle 31.33^\circ$$
$$= 118.90 - j0.67$$
$$= 118.90 \angle -0.32^\circ \text{ V.}$$

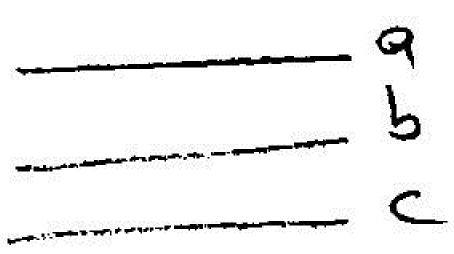
For a positive phase sequence,

$$\mathbf{V}_{bn} = 118.90 \angle -120.32^\circ \text{ V,}$$
$$\mathbf{V}_{cn} = 118.90 \angle 119.68^\circ \text{ V.}$$

f) The line voltages at the source terminals are

$$\mathbf{V}_{ab} = (\sqrt{3} \angle 30^\circ)\mathbf{V}_{an}$$
$$= 205.94 \angle 29.68^\circ \text{ V,}$$
$$\mathbf{V}_{bc} = 205.94 \angle -90.32^\circ \text{ V,}$$
$$\mathbf{V}_{ca} = 205.94 \angle 149.68^\circ \text{ V.}$$

Sumary



Line to line Voltage V_{ab}, V_{bc}, V_{ac}

line to neutral Voltage V_{an}, V_{bn}, V_{cn}
(phase voltage)

$|V_{ab}| = |V_{bc}| = |V_{ac}|$

$|V_{an}| = |V_{bn}| = |V_{cn}|$

$|V_{ab}| = \sqrt{3}\,|V_{an}|$

$\angle V_{bc} = \angle V_{ab} - 120$

$\angle V_{ac} = \angle V_{ab} + 120$

$\angle V_{an} + 30$

1. The magnitude of the line-to-line voltage is $\sqrt{3}$ times the magnitude of the line-to-neutral voltage.
2. The line-to-line voltages form a balanced three-phase set of voltages.
3. The set of line-to-line voltages leads the set of line-to-neutral voltages by 30°.

11.1 The voltage from A to N in a balanced three-phase circuit is $240 \angle -30^\circ$ V. If the phase sequence is positive, what is the value of $\mathbf{V}_{BC}$?

ANSWER: $415.69 \angle -120^\circ$ V.

$V_{AN} = 240\, e^{-j30}$

$|V_{AB}| = |V_{AN}|\sqrt{3} = 240\sqrt{3} = 415.69$

$\angle V_{AB} = \angle V_{AN} + 30 = -30 + 30 = 0$

$V_{AB} = 415.69\, e^{j0}$

$\angle V_{BC} = \angle V_{AB} - 120$

$= 0 - 120 = -120$

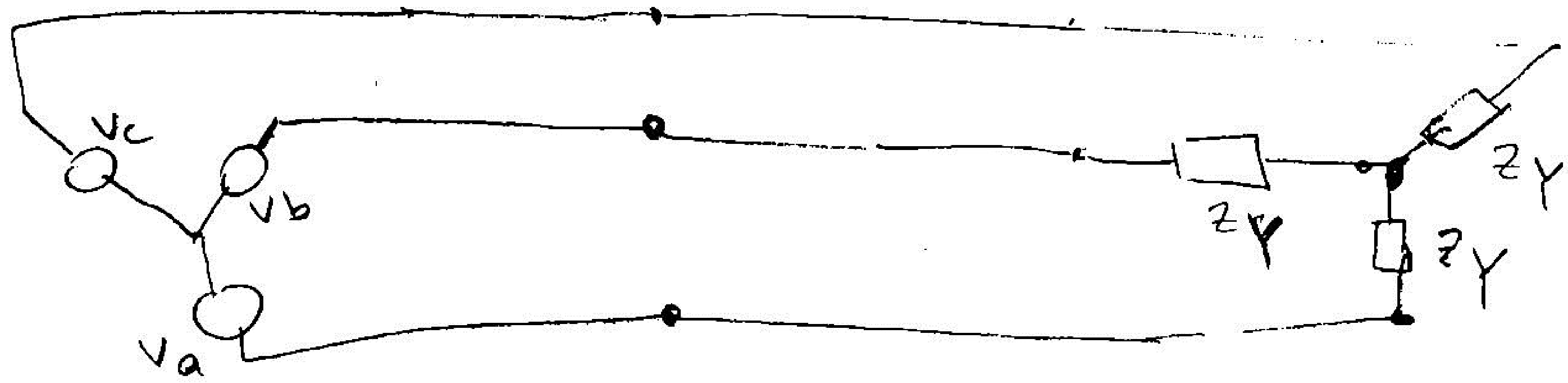
$|V_{AB}| = |V_{BC}| = |V_{AC}|$

$V_{BC} = 415.63\, e^{-120}$

429

11.3 The phase voltage at the terminals of a balanced three-phase Y-connected load is 2400 V. The load has an impedance of $16 + j12\ \Omega/\phi$ and is fed from a line having an impedance of $0.10 + j0.80\ \Omega/\phi$. The Y-connected source at the sending end of the line has a phase sequence of acb and an internal impedance of $0.02 + j0.16\ \Omega/\phi$. Use the a-phase voltage at the load as the reference and calculate (a) the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$; (b) the line voltages at the source, $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$; and (c) the internal phase-to-neutral voltages at the source, $\mathbf{V}_{a'n}$, $\mathbf{V}_{b'n}$, and $\mathbf{V}_{c'n}$.

ANSWER: (a) $\mathbf{I}_{aA} = 120\ \angle{-36.87^\circ}$ A, $\mathbf{I}_{bB} = 120\ \angle{83.13^\circ}$ A, and $\mathbf{I}_{cC} = 120\ \angle{-156.87^\circ}$ A; (b) $\mathbf{V}_{ab} = 4275.02\ \angle{-28.38^\circ}$ V, $\mathbf{V}_{bc} = 4275.02\ \angle{91.62^\circ}$ V, and $\mathbf{V}_{ca} = 4275.02\ \angle{-148.38^\circ}$ V; (c) $\mathbf{V}_{a'n} = 2482.05\ \angle{1.93^\circ}$ V, $\mathbf{V}_{b'n} = 2482.05\ \angle{121.93^\circ}$ V, and $\mathbf{V}_{c'n} = 2482.05\ \angle{-118.07^\circ}$ V.



(studied)

Source wye

load wye

load delta

Source wye

If the load in a three-phase circuit is connected in a delta, it can be transformed into a wye by using the delta-to-wye transformation discussed in Section 9.6. When the load is balanced, the impedance of each leg of the wye is one third the impedance of each leg of the delta, or

$$Z_Y = \frac{Z_\Delta}{3}, \tag{11.21}$$

which follows directly from Eqs. 9.51–9.53. After the Δ load has been replaced by its Y equivalent, the a-phase can be modeled by the single-phase equivalent circuit shown in Fig. 11.11.

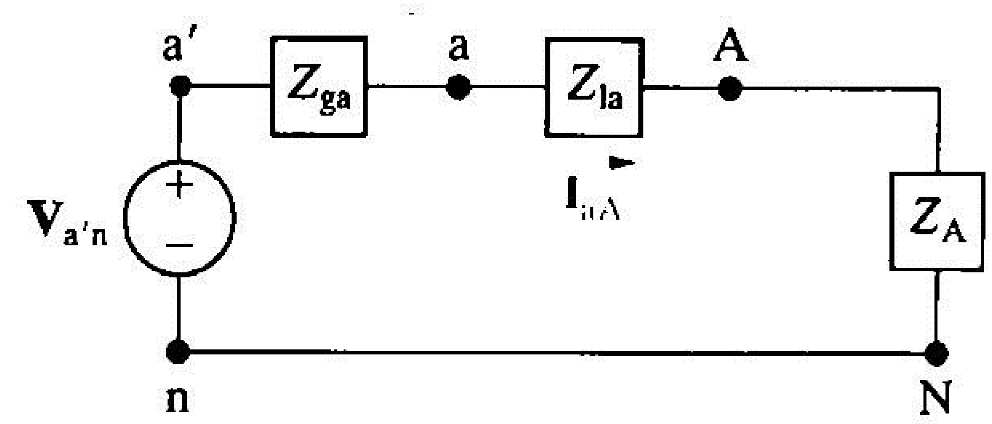
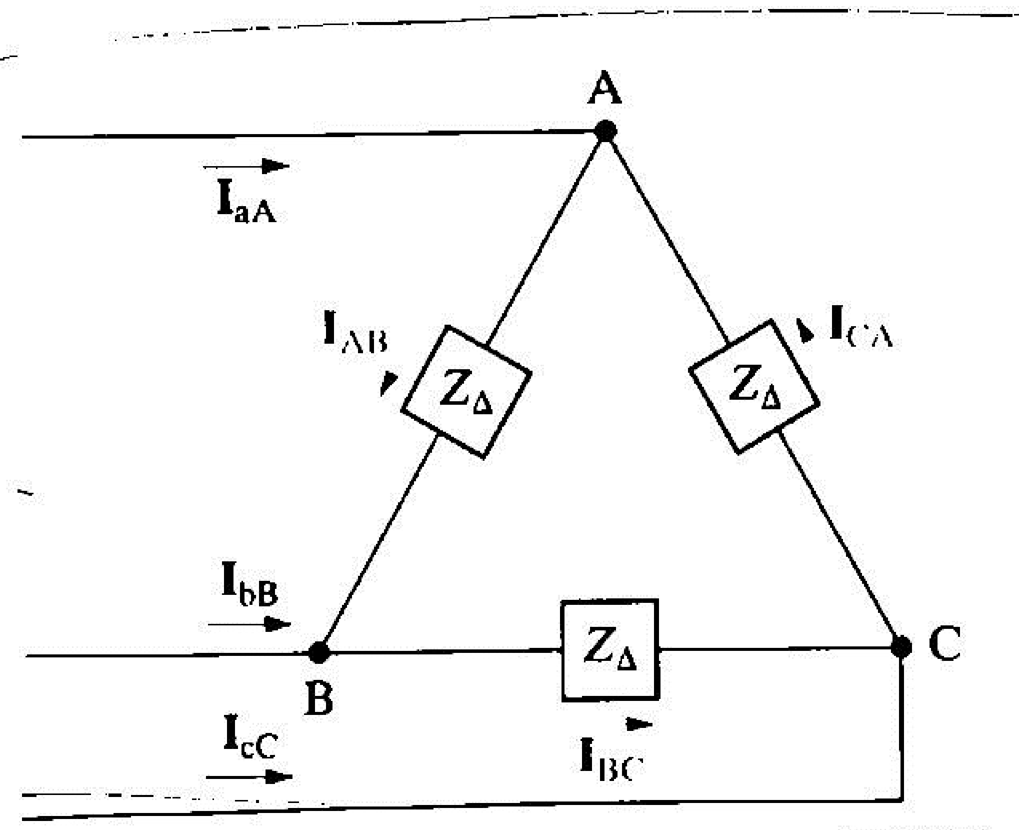


Figure 11.11 A single-phase equivalent circuit.

RELATIONSHIP BETWEEN THREE-PHASE DELTA-CONNECTED AND WYE-CONNECTED IMPEDANCE



$$\mathbf{I}_{AB} = I_\phi \angle 0^\circ, \tag{11.22}$$

$$\mathbf{I}_{BC} = I_\phi \angle -120^\circ, \tag{11.23}$$

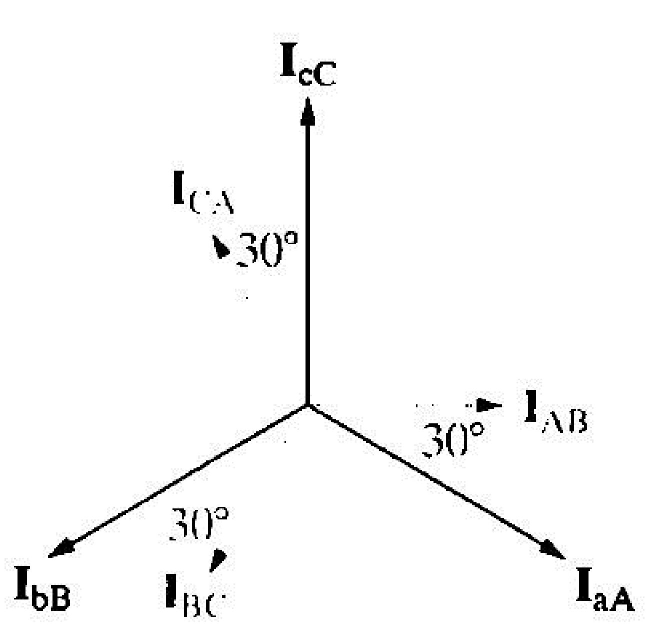
$$\mathbf{I}_{CA} = I_\phi \angle 120^\circ. \tag{11.24}$$

Balanced Case

$$\begin{aligned}\mathbf{I}_{aA} &= \mathbf{I}_{AB} - \mathbf{I}_{CA} \\ &= I_\phi \angle 0^\circ - I_\phi \angle 120^\circ \\ &= \sqrt{3} I_\phi \angle -30^\circ,\end{aligned} \tag{11.25}$$

$$\begin{aligned}\mathbf{I}_{bB} &= \mathbf{I}_{BC} - \mathbf{I}_{AB} \\ &= I_\phi \angle -120^\circ - I_\phi \angle 0^\circ \\ &= \sqrt{3} I_\phi \angle -150^\circ,\end{aligned} \tag{11.26}$$

$$\begin{aligned}\mathbf{I}_{cC} &= \mathbf{I}_{CA} - \mathbf{I}_{BC} \\ &= I_\phi \angle 120^\circ - I_\phi \angle -120^\circ \\ &= \sqrt{3} I_\phi \angle 90^\circ.\end{aligned} \tag{11.27}$$



EXAMPLE 11.2

The Y-connected source in Example 11.1 feeds a Δ-connected load through a distribution line having an impedance of $0.3 + j0.9\ \Omega/\phi$. The load impedance is $118.5 + j85.8\ \Omega/\phi$. Use the a-phase internal voltage of the generator as the reference.

a) Construct a single-phase equivalent circuit of the three-phase system.
b) Calculate the line currents $\mathbf{I}_{aA}$, $\mathbf{I}_{bB}$, and $\mathbf{I}_{cC}$.
c) Calculate the phase voltages at the load terminals.
d) Calculate the phase currents of the load.
e) Calculate the line voltages at the source terminals.

SOLUTION

from example 11.1

$Z_{sa} = Z_{sb} = Z_{sc} = 0.2 + j0.5$

line impedance

$Z_{la} = Z_{lb} = Z_{lc} = 0.3 + j0.9$

$Z_\Delta = 118.5 + j85.8$

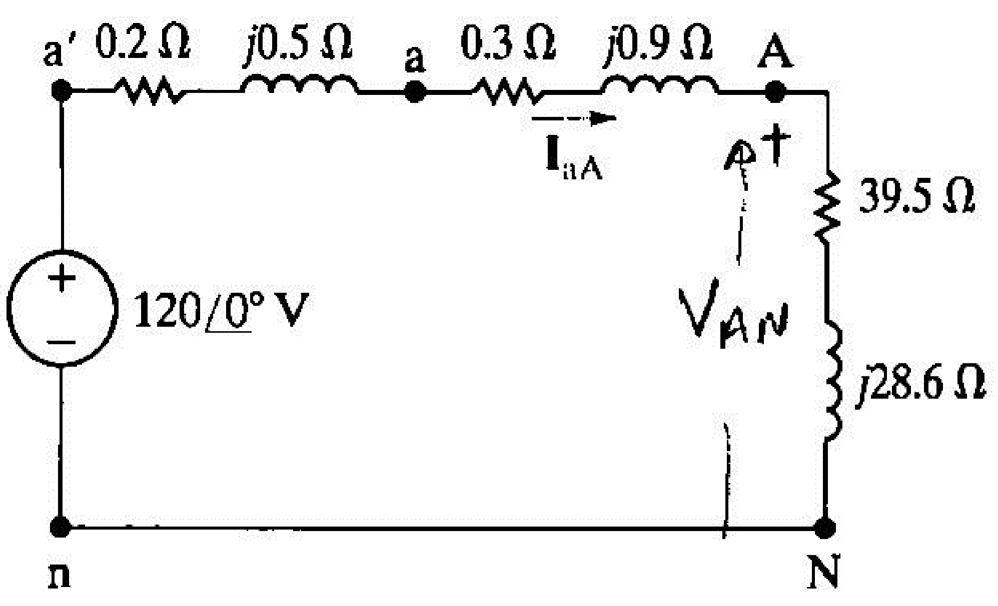


Figure 11.14 The single-phase equivalent circuit for Example 11.2.

a) Figure 11.14 shows the single-phase equivalent circuit. The load impedance of the Y equivalent is

$$\frac{118.5 + j85.8}{3} = 39.5 + j28.6\ \Omega/\phi.$$

$$\frac{Z_\Delta}{3} = Z_Y$$

b) The a-phase line current is (example 11.1)

$$\mathbf{I}_{aA} = \frac{120\angle 0^\circ}{(0.2 + 0.3 + 39.5) + j(0.5 + 0.9 + 28.6)}$$

$$= \frac{120\angle 0^\circ}{40 + j30} = 2.4\angle -36.87^\circ \text{ A.}$$

Hence

$$\mathbf{I}_{bB} = 2.4\angle -156.87^\circ \text{ A,} \rightarrow (I_{aA} - 120^\circ)$$

$$\mathbf{I}_{cC} = 2.4\angle 83.13^\circ \text{ A.} \quad (I_{aA} + 120^\circ)$$

c) Because the load is Δ connected, the phase voltages are the same as the line voltages. To calculate the line voltages, we first calculate $\mathbf{V}_{AN}$:

$$\mathbf{V}_{AN} = (39.5 + j28.6)(2.4\angle -36.87^\circ)$$

$$= 117.04\angle -0.96^\circ \text{ V.}$$

Because the phase sequence is positive, the line voltage $\mathbf{V}_{AB}$ is

$$\mathbf{V}_{AB} = (\sqrt{3}\angle 30^\circ)\,\mathbf{V}_{AN}$$

$$= 202.72\angle 29.04^\circ \text{ V.}$$

Therefore

$$\mathbf{V}_{BC} = 202.72\angle -90.96^\circ \text{ V,} \quad (V_{AB} - 120)$$

$$\mathbf{V}_{CA} = 202.72\angle 149.04^\circ \text{ V.} \quad (\angle V_{AB} + 120)$$

d) The phase currents of the load may be calculated directly from the line currents:

$$\mathbf{I}_{AB} = \left(\frac{1}{\sqrt{3}}\angle 30^\circ\right)\mathbf{I}_{aA}$$

$$= 1.39\angle -6.87^\circ \text{ A.}$$

Once we know $\mathbf{I}_{AB}$, we also know the other load phase currents:

$$\mathbf{I}_{BC} = 1.39\angle -126.87^\circ \text{ A,} \quad (\angle I_{AB} - 120)$$

$$\mathbf{I}_{CA} = 1.39\angle 113.13^\circ \text{ A.} \quad (\angle I_{AB} + 120)$$

Note that we can check the calculation of $\mathbf{I}_{AB}$ by using the previously calculated $\mathbf{V}_{AB}$ and the impedance of the Δ-connected load; that is,

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\phi} = \frac{202.72\angle 29.04^\circ}{118.5 + j85.8}$$

$$= 1.39\angle -6.87^\circ \text{ A.}$$

e) To calculate the line voltage at the terminals of the source, we first calculate $\mathbf{V}_{an}$. Figure 11.14 shows that $\mathbf{V}_{an}$ is the voltage drop across the line impedance plus the load impedance, so

$$\mathbf{V}_{an} = (39.8 + j29.5)(2.4\angle -36.87^\circ)$$

$$= 118.90\angle -0.32^\circ \text{ V.}$$

The line voltage $\mathbf{V}_{ab}$ is

$$\mathbf{V}_{ab} = (\sqrt{3}\,\angle 30^\circ)\,\mathbf{V}_{an},$$

or

$$\mathbf{V}_{ab} = 205.94\,\angle 29.68^\circ \text{ V}.$$

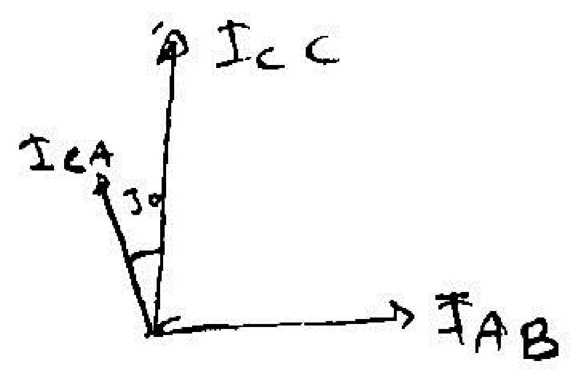
Therefore

$$\mathbf{V}_{bc} = 205.94\,\angle -90.32^\circ \text{ V},$$

$$\mathbf{V}_{ca} = 205.94\,\angle 149.68^\circ \text{ V}.$$

11.4 The current $\mathbf{I}_{CA}$ in a balanced three-phase Δ-connected load is $8\,\angle -15^\circ$ A. If the phase sequence is positive, what is the value of $\mathbf{I}_{cC}$?

ANSWER: $13.86\,\angle -45^\circ$ A.



$$\angle I_{cc} + 30 = \angle I_{CA}$$

$$|I_{cc}| = \sqrt{3}\, I_{CA}$$

$$I_{CA} = 8\, e^{-j15}$$

$$I_{cc} = 8\sqrt{3}\, e^{j(-15-30)}$$

$$= 13.86\, e^{-j45}$$

11.6 The line voltage $\mathbf{V}_{AB}$ at the terminals of a balanced three-phase Δ-connected load is $4160\,\angle 0^\circ$ V. The line current $\mathbf{I}_{aA}$ is $69.28\,\angle -10^\circ$ A.

a) Calculate the per-phase impedance of the load if the phase sequence is positive.

b) Repeat (a) for a negative phase sequence.

ANSWER: (a) $104\,\angle -20^\circ$ Ω; (b) $104\,\angle +40^\circ$ Ω.

$$|V_{AN}| = \frac{|V_{AB}|}{\sqrt{3}} = \frac{4160}{\sqrt{3}} = 2400$$

$$\angle V_{AN} = \angle V_{AB} - 30 = 0 - 30 = -30$$

$$Z_Y = \frac{V_{AN}}{I_{aA}} = \frac{2400\, e^{-j30}}{69.28\, e^{-j10}} =$$

$$= 34.64\, e^{-j20}$$

$$Z_\Delta = 3\, Z_Y = 3 \times 34.64\, e^{-j20}$$

$$= 104\, e^{-j20}$$

11.7 The line voltage at the terminals of a balanced Δ-connected load is 110 V. Each phase of the load consists of a 3.667 Ω resistor in parallel with a 2.75 Ω inductive impedance. What is the magnitude of the current in the line feeding the load?

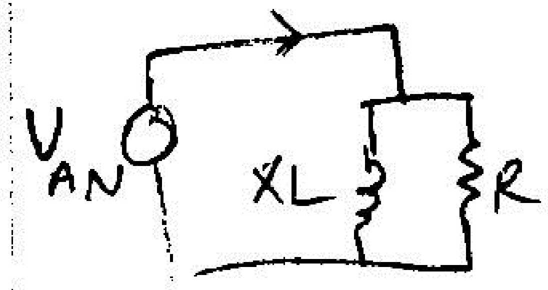
ANSWER: 86.60 A.

$$|V_{AN}| = \frac{|V_{AB}|}{\sqrt{3}}$$

$$\text{phase voltose} = \frac{\text{line voltose}}{\sqrt{3}}$$

$$|V_{AN}| = \frac{110}{\sqrt{3}} = 63.5008$$

$$R = 3.667 \quad X_L = 2.75$$



$$Z_\Delta = \frac{R X_L}{R + X_L} = \frac{3.66 \cdot 2.75j}{3.66 + 2.75j}$$

$$|Z_\Delta| = \frac{3.66 \times 2.75}{\sqrt{3.66^2 + 2.75^2}} = 2.20$$

$$|Z_Y| = \frac{|Z_\Delta|}{3} = \frac{2.20}{3} = 0.7333$$

$$|I_{aA}| = \frac{V_{AN}}{Z_Y} = \frac{63.50}{0.733} = 86.6\,A$$

Average Power in a Balanced Wye Load

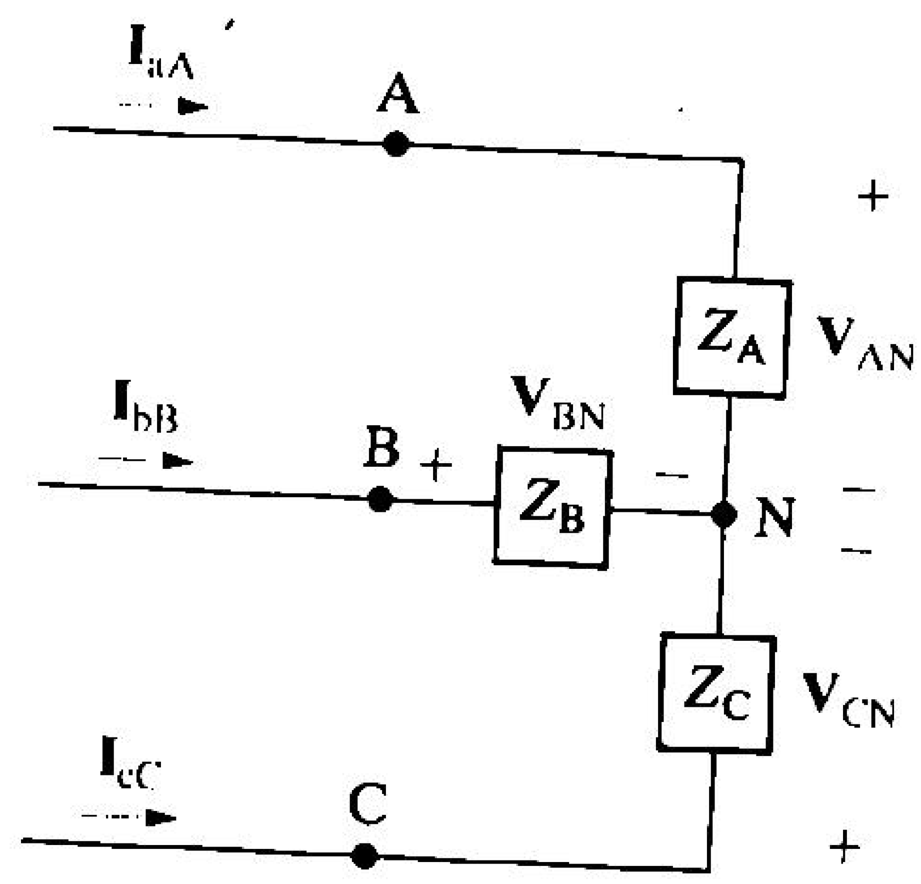


Figure 11.15 A balanced Y load used to introduce average power calculations in three-phase circuits.

$$V_\phi = |\mathbf{V}_{AN}| = |\mathbf{V}_{BN}| = |\mathbf{V}_{CN}|, \quad (11.31)$$

$$I_\phi = |\mathbf{I}_{aA}| = |\mathbf{I}_{bB}| = |\mathbf{I}_{cC}|, \quad (11.32)$$

and

$$\theta_\phi = \theta_{vA} - \theta_{iA} = \theta_{vB} - \theta_{iB} = \theta_{vC} - \theta_{iC}. \quad (11.33)$$

Moreover, for a balanced system, the power delivered to each phase of the load is the same, so

$$P_A = P_B = P_C = P_\phi = V_\phi I_\phi \cos\theta_\phi, \quad (11.34)$$

($V\phi$, $I\phi \Rightarrow$ rms values)
($P = \frac{1}{2} V_P I_P \cos \phi$)
(V_P, I_P peak values)

where P_ϕ represents the average power per phase.

The total average power delivered to the balanced Y-connected load is simply three times the power per phase, or

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi. \quad (11.35)$$

V_L and I_L represent the rms magnitudes of the line voltage and current, respectively,

Line voltage = $\sqrt{3}$ phase voltage

$V_L = \sqrt{3}\, V\phi \Rightarrow V\phi = \frac{V_L}{\sqrt{3}}$

$$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_\phi = \sqrt{3} V_L I_L \cos\theta_\phi. \quad (11.36)$$

Complex Power in a Balanced Wye Load

$$Q_\phi = V_\phi I_\phi \sin\theta_\phi, \quad (11.37)$$

$$Q_T = 3Q_\phi = \sqrt{3} V_L I_L \sin\theta_\phi. \quad (11.38)$$

$$S_\phi = P_\phi + jQ_\phi = \mathbf{V}_\phi \mathbf{I}_\phi^*, \quad (11.40)$$

$$S_T = 3S_\phi = \sqrt{3} V_L I_L \angle\theta_\phi^\circ. \quad (11.41)$$

EXAMPLE 11.3

a) Calculate the average power per phase delivered to the Y-connected load of Example 11.1.

b) Calculate the total average power delivered to the load.

c) Calculate the total average power lost in the line.

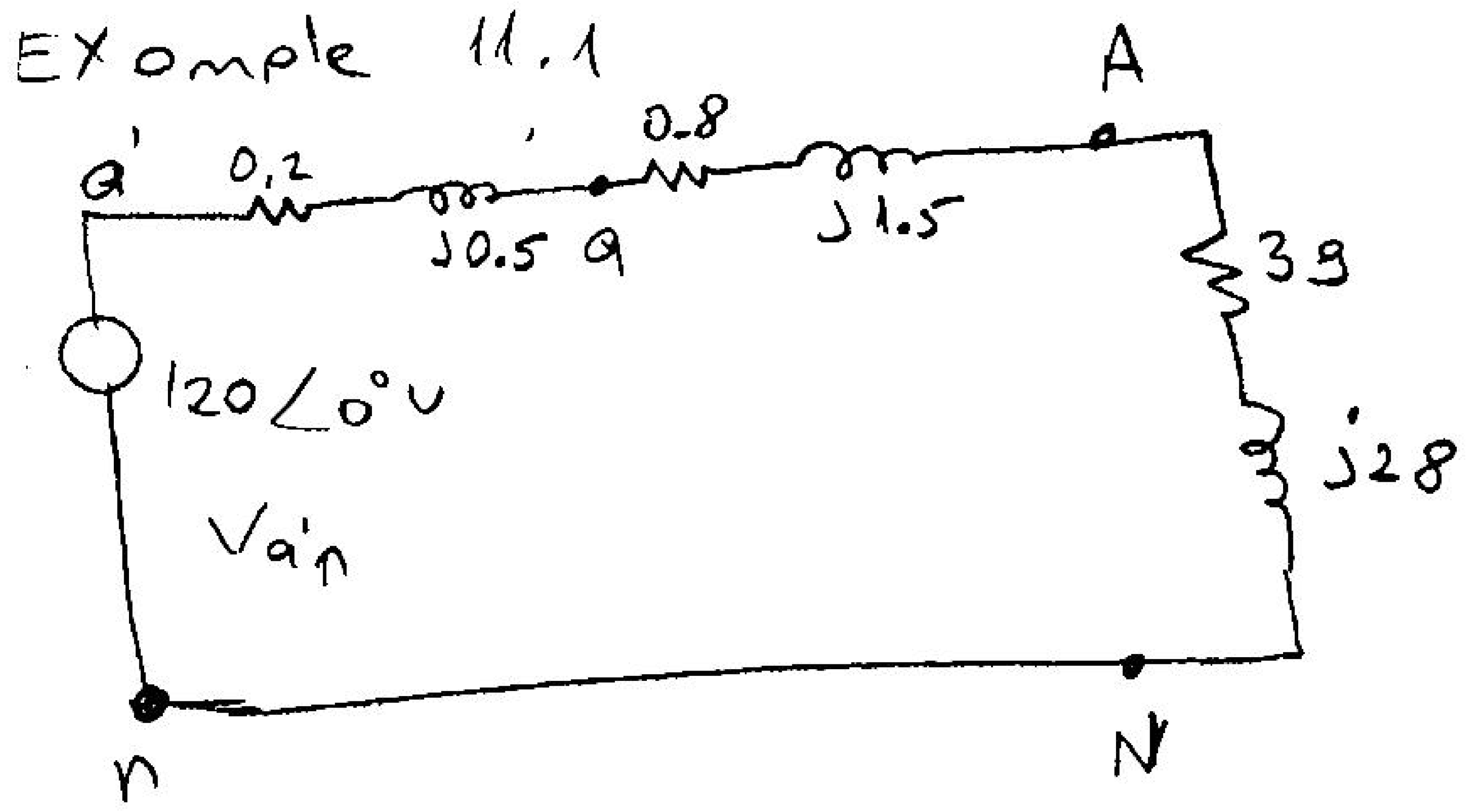
d) Calculate the total average power lost in the generator.

e) Calculate the total number of magnetizing vars absorbed by the load.

f) Calculate the total complex power delivered by the source.

SOLUTION

Example 11.1



$$I_{aA} = \frac{120}{0.2 + j0.5 + 0.8 + j0.5 + 39 + j28}$$

$$= 2.4 \angle -36.87$$

$$V_{AN} = I_{aA} * (39 + j28)$$

$$= 115.22 \angle -1.19$$

a) From Example 11.1, $V_\phi = 115.22$ V, $I_\phi = 2.4$ A, and $\theta_\phi = -1.19 - (-36.87) = 35.68°$. Therefore

$$P_\phi = (115.22)(2.4)\cos 35.68°$$

$$= 224.64 \text{ W}.$$

The power per phase may also be calculated from $I_\phi^2 R_\phi$, or

$$P_\phi = (2.4)^2(39) = 224.64 \text{ W}.$$

b) The total average power delivered to the load is $P_T = 3P_\phi = 673.92$ W. We calculated the line voltage in Example 11.1, so we may also use Eq. 11.36:

$$|V_L| = |V_\phi| \sqrt{3} = 115.22 \sqrt{3} =$$

$$= 199.56$$

$$P_T = \sqrt{3}\, V_L I_L \cos\phi$$

$$P_T = \sqrt{3}(199.58)(2.4)\cos 35.68°$$

$$= 673.92 \text{ W}.$$

c) The total power lost in the line is

$$3 \times I^2 R_L$$

$$P_{\text{line}} = 3(2.4)^2(0.8) = 13.824 \text{ W}.$$

d) The total internal power lost in the generator is

$$3 \times I^2 R_g$$

$$P_{\text{gen}} = 3(2.4)^2(0.2) = 3.456 \text{ W}.$$

e) The total number of magnetizing vars absorbed by the load is

$$\sqrt{3}\, V_L I_L \sin\phi$$

$$Q_T = \sqrt{3}(199.58)(2.4)\sin 35.68°$$

$$= 483.84 \text{ VAR}.$$

f) The total complex power associated with the source is

$$= -3 \times V_L \times I_L$$

$$S_T = 3S_\phi = -3(120)(2.4) \underline{/36.87°}$$

$$= -691.20 - j518.40 \text{ VA}.$$

The minus sign indicates that the internal power and magnetizing reactive power are being delivered to the circuit. We check this result by calculating the total and reactive power absorbed by the circuit:

$$P = 673.92 + 13.824 + 3.456$$

$$= 691.20 \text{ W (check)},$$

$$Q = 483.84 + 3(2.4)^2(1.5) + 3(2.4)^2(0.5)$$

$$= 483.84 + 25.92 + 8.64$$

$$= 518.40 \text{ VAR (check)}.$$