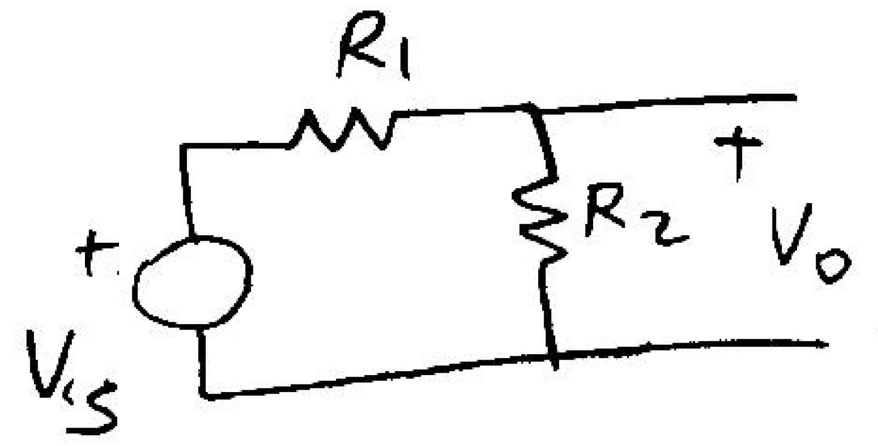
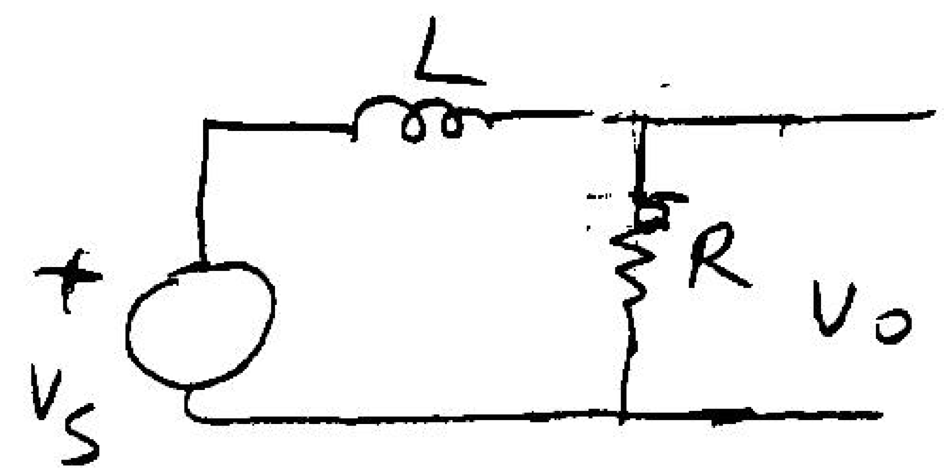
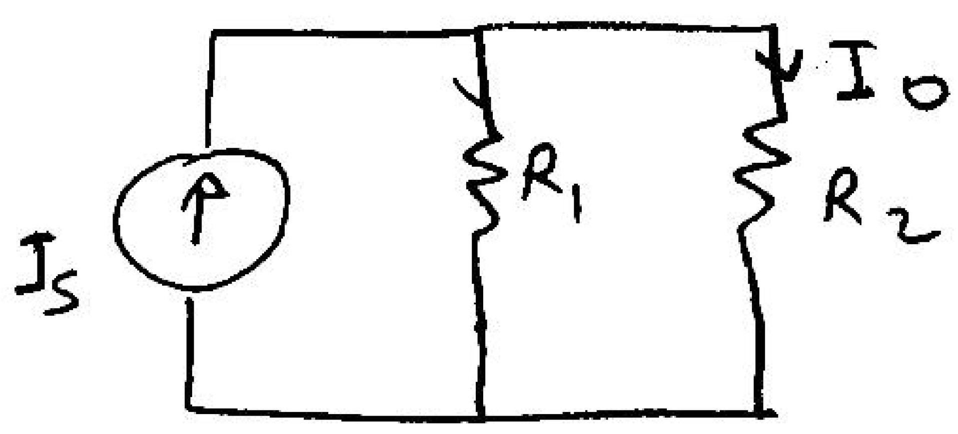
TRANSFER FUNCTION



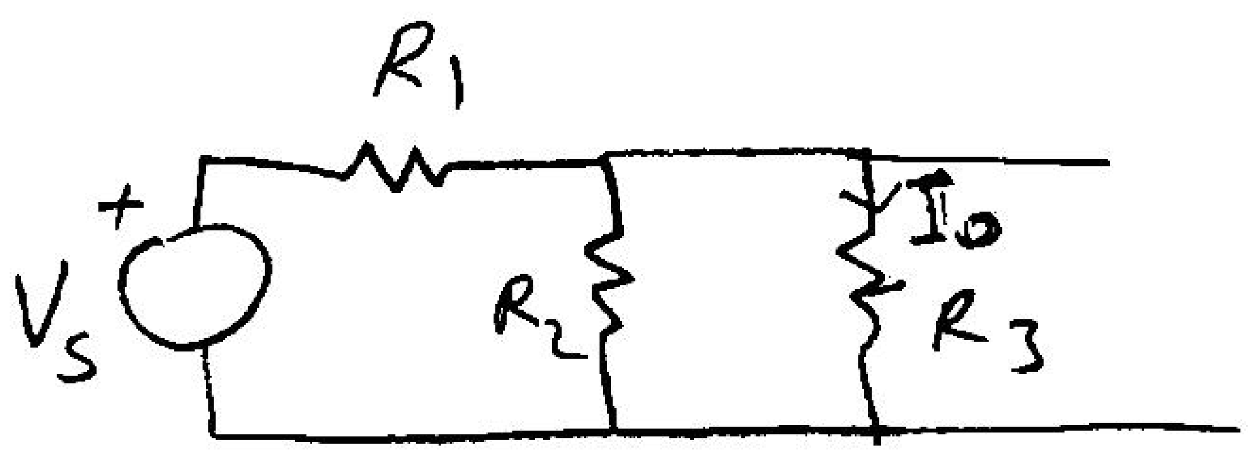
$$\frac{V_o}{V_s} = \frac{R_2}{R_2 + R_1}$$



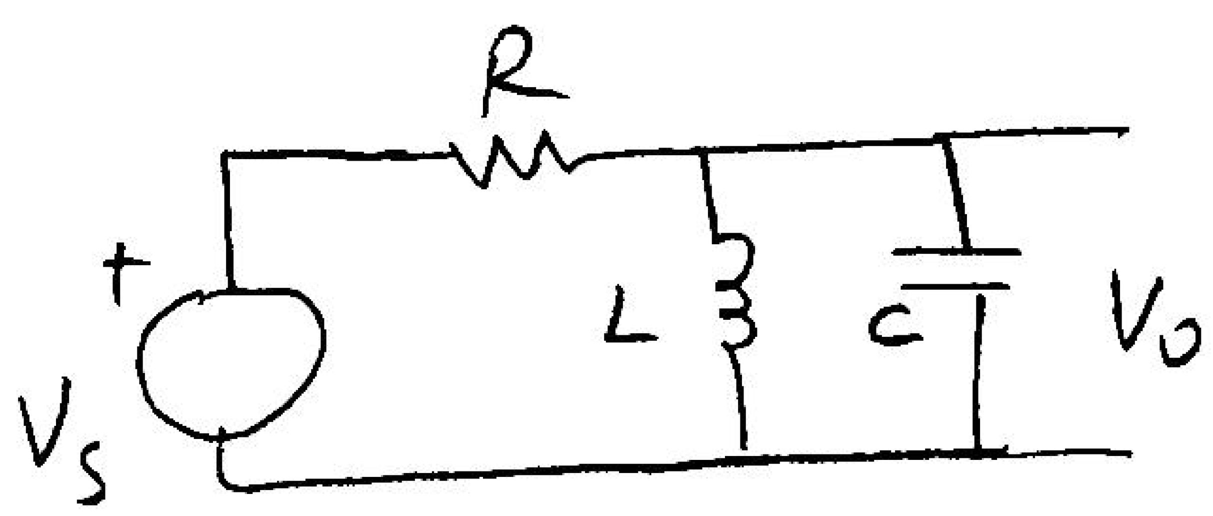
$$\frac{V_o}{V_s} = \frac{R}{R + jwL}$$



$$\frac{I_o}{I_s} = \frac{R_1}{R_1 + R_2}$$



$$\frac{I_o}{V_s} = \frac{R_{23}}{R_{23} + R_1} \cdot \frac{1}{R_3} \qquad \left(R_{23} = R_2 // R_3\right)$$

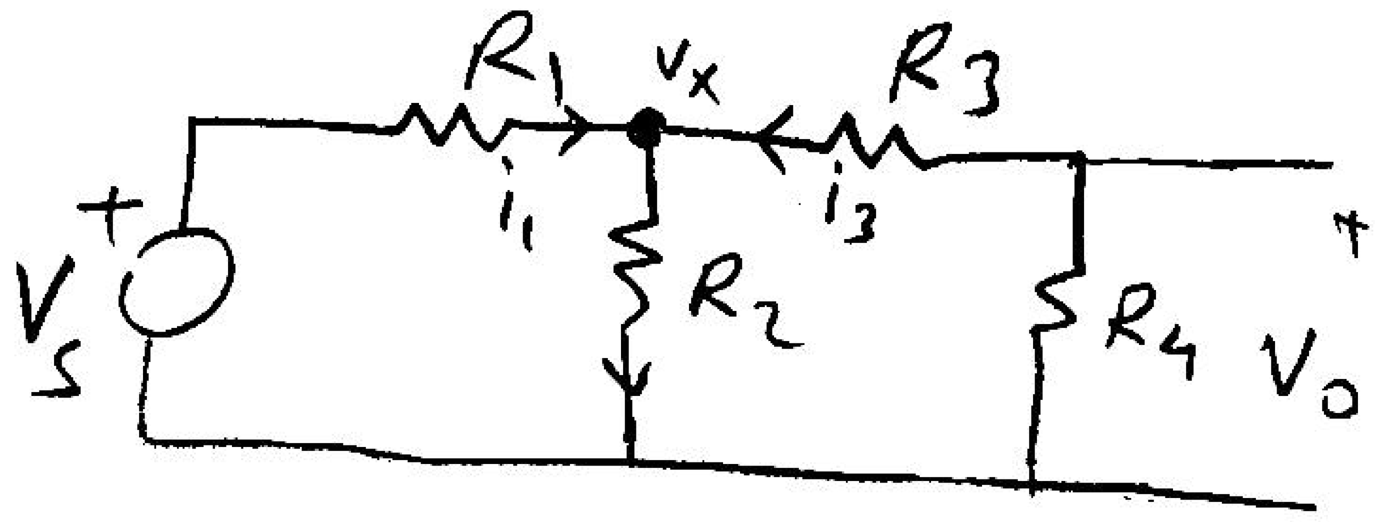


$$\frac{V_o}{V_s} = \frac{Z}{Z + R} \qquad Z = jwL // \frac{1}{jwC} = \frac{jwL}{1 - w^2LC}$$

$$\text{Transfer function} = \frac{\text{Ouput Voltage (or current)}}{\text{Input Voltage (or current)}}$$

Example: find the transfer function

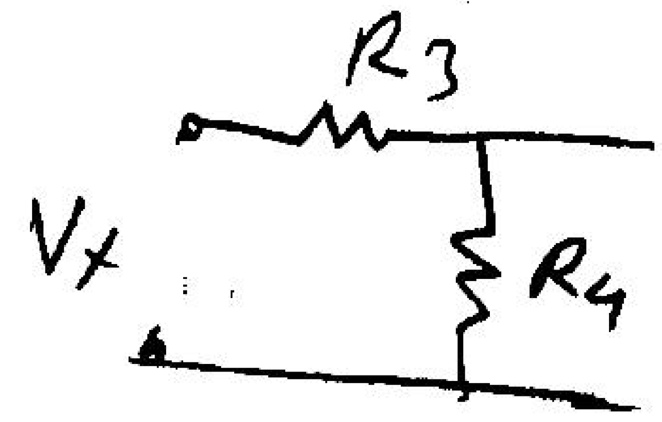
$\frac{V_o}{V_s}$ and fill the table. $R_1=10$ $R_2=20$ $R_3=15$ $R_4=25$



V_s	10	20	100	-20
V_o	?	?	?	?

Solution $I_2 = i_1 + i_3$

$$\frac{V_s - V_x}{R_1} + \frac{V_o - V_x}{R_3} = \frac{V_x}{R_2}$$



$$\frac{V_o}{V_x} = \frac{R_4}{R_3 + R_4}$$

$$V_x = \underbrace{\frac{R_3 + R_4}{R_4}}_{\alpha} V_o = \alpha V_o$$

$$\frac{V_s - \alpha V_o}{R_1} + \frac{V_o - \alpha V_o}{R_3} = \frac{\alpha V_o}{R_2}$$

$$\frac{V_s}{R_1} - \frac{\alpha V_o}{R_1} + \frac{(1-\alpha)}{R_3} V_o = \frac{\alpha V_o}{R_2}$$

$$\frac{V_s}{R_1} = V_o \overbrace{\left(\frac{\alpha}{R_1} + \frac{\alpha - 1}{R_3} + \frac{\alpha}{R_2} \right)}^{m} \Rightarrow V_o = \frac{1}{m R_1} V_s \Rightarrow \frac{V_o}{V_s} = \frac{1}{m R_1}$$

$$\alpha = \frac{R_3 + R_4}{R_4} = \frac{15 + 25}{25} = \frac{40}{25} = 1.6$$

$$m = \frac{1.6}{10} + \frac{1.6 - 1}{15} + \frac{1.6}{20} = 0.28$$

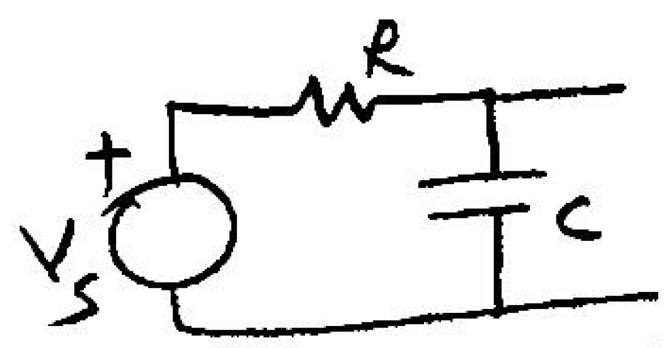
$$V_o = \frac{1}{0.28 \times 10} V_s = 0.357\, V_s$$

V_s	10	20	100	-20
V_o	3.57	7.14	35.7	-7.14

e221

Example Problem calculate output voltage

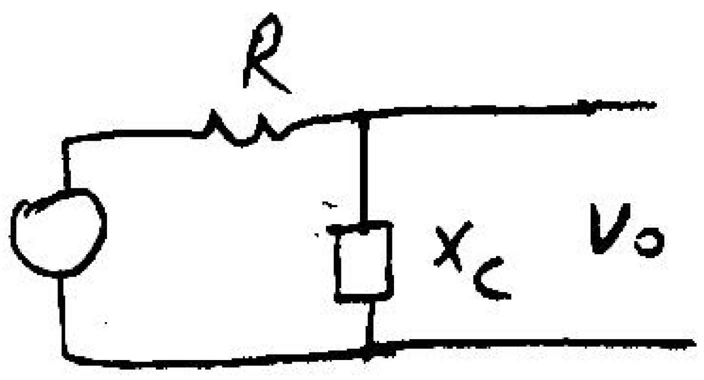
if a) $V_s = 10 \cos t$ b) $V_s = 10 \cos 2t$ c) $V_s = 10 \cos(2t + 30°)$



d) $V_s = 10 \sin 3t$

$R = 2\Omega \quad c = 0.1f$

solution



$\frac{V_o}{V_s} = \frac{X_c}{R + X_c}$ (Voltage divider)

$$V_o = \frac{X_c}{R + X_c} V_s = \frac{\frac{1}{jwc}}{R + \frac{1}{jwc}} V_s = \frac{1}{jwcR + 1} V_s = \frac{1}{jw\,0{,}1 \times 2 + 1}$$

a) $V_s = 10 \cos t \Rightarrow \underline{V_s} = 10 \qquad w = 1$

$$V_o = \frac{1}{jw\,0{,}2 + 1} V_s = \frac{1}{j \times 1 \times 0{,}2 + 1} 10 = \frac{1}{1 + 0{,}2j} 10 = \frac{10}{\sqrt{1^2 + 0{,}2^2}\; e^{j\theta}}$$

$\theta = \tan^{-1} \frac{0{,}2}{1} = 11{,}3°$

$$V_o = \frac{10}{\sqrt{1.04}\; e^{j11}} = \frac{10}{1{,}019} e^{-j\,11.3} = 9.80\, e^{-j11.3°}$$

$$\boxed{V_o(t) = 9.80 \cos(t - 11{,}3°)}$$

b) $V_s = 10 \cos 2t \qquad \underline{V_s} = 10 \qquad w = 2$

$$V_o = \frac{1}{jw\,0{,}2 + 1} V_s = \frac{1}{j \times 2 \times 0{,}2 + 1} \cdot 10 = \frac{10}{1 + 0{,}4j} = \frac{10}{\sqrt{1 + 0{,}4^2}\; e^{j\theta}} =$$

$\theta = \tan^{-1} \frac{0{,}4}{1} = 21{,}8°$

$$V_o = \frac{10}{\sqrt{1.16}\; e^{j\,21.8}} = 9{,}28\, e^{-j\,21.8}$$

$$\boxed{V_o(t) = 9{,}28 \cos(2t - 21{,}8)}$$

c) $V_s = 10 \cos(2t + 30)$ $\quad \underline{V_s} = 10 e^{j30}$ $\quad w = 2$

$$V_o = \frac{V_s}{jw\,0{,}2 + 1} = \frac{V_s}{j2\times 0{,}2 + 1} = \frac{10 e^{j30}}{1 + 0{,}4j} = \frac{10 e^{j30}}{\sqrt{1{,}16}\; e^{j21{,}8}}$$

$$= \frac{10}{\sqrt{1{,}16}} e^{j(30 - 21{,}8)} = 9{,}28\; e^{j8{,}2^\circ}$$

$V_o(t) = 9{,}28 \cos(2t + 8{,}2^\circ)$

d) $V_s = 10 \sin 3t = 10 \cos(3t - 90)$ $\quad \underline{V_s} = 10 e^{-j90}$ $\quad w = 3$

$$V_o = \frac{V_s}{jw\,0{,}2 + 1} = \frac{10 e^{-j90}}{j\times 3\times 0{,}2 + 1} = \frac{10 e^{-j90}}{1 + j0{,}6} = \frac{10 e^{-j90}}{\sqrt{1^2 + 0{,}6^2}\; e^{j\theta}}$$

$\theta = \tan^{-1}\frac{0{,}6}{1} = 30{,}9^\circ$

$$V_o = \frac{10}{\sqrt{1{,}36}} e^{j(-90 - 30{,}9)} = 8{,}57\; e^{-j120{,}9}$$

$V_o(t) = 8{,}57 \cos(3t - 120{,}9)$

or

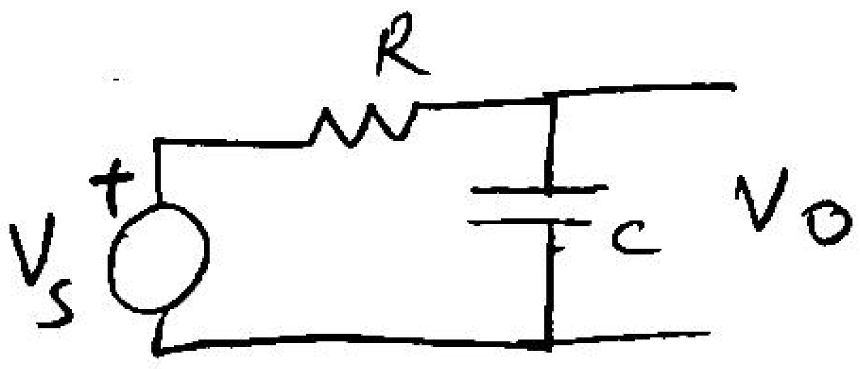
$V_o(t) = 8{,}57 \sin(3t - 120{,}9 + 90) = 8{,}57 \sin(3t - 30{,}9)$

e 223

Example Problem: Calculate output voltage

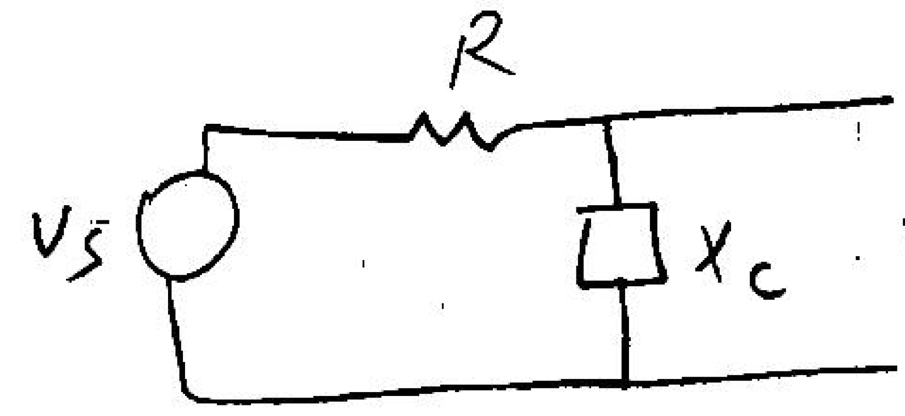
if a) $V_s = 10 \cos t$ b) $V_s = 10 \cos 2t$ c) $V_s = 10 \cos(2t+30)$

d) $V_s = 10 \sin 3t$ e) $V_s = 10 \cos 10t$



$R = 2\Omega$ $c = 0{,}1\ f$

Solution:



$$V_o = \frac{X_c}{R + X_c} V_s$$ (Voltage divid-

$$\frac{V_o}{V_s} = \frac{X_c}{R + X_c} = \frac{\frac{1}{jwc}}{R + \frac{1}{jwc}} = \frac{1}{jwcR + 1} = \frac{1}{jw.0{,}1 \times 2 + 1} = \frac{1}{jw0{,}2+1}$$

$V_s = 10 \cos t \Rightarrow \underline{V_s} = 10\, e^{j0} = 10$ $w = 1.$

$V_s = 10 \cos 2t \Rightarrow V_s = 10$ $w = 2$

$V_s = 10 \cos 3t \Rightarrow V_s = 10$ $w = 3$

$\vdots$

$V_s = 10 \cos 10t \Rightarrow V_s = 10$ $w = 10$

$$\frac{V_o}{V_s} = \frac{1}{jw0{,}2+1} \qquad \left|\frac{V_o}{V_s}\right| = \frac{|1|}{|jw0{,}2+1|} = \frac{1}{\sqrt{(0{,}2w)^2 + 1^2}}$$

$$\angle \frac{V_o}{V_s} = \angle 1 - \angle jw0{,}2+1$$

$$= 0 - \tan^{-1} \frac{0{,}2w}{1}$$

$$w = 1 \Rightarrow \left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{(0{,}2 \times 1)^2 + 1^2}} = \frac{1}{\sqrt{1.04}} = 0{,}98$$

$$\angle \frac{V_o}{V_s} = -\tan^{-1} 0{,}2w = -\tan 0{,}2 = -11.3^\circ$$

$$\left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{(0{,}2w)^2+1}}$$

$w=2$ $\quad \left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{(0{,}2\times 2)^2+1}} = \frac{1}{\sqrt{1.16}} = 0.928$

$$\angle\frac{V_o}{V_s} = -\tan^{-1} 0{,}2w = -\tan^{-1} 0{,}2\times 2 = -\tan^{-1} 0{,}4 = -21.8$$

$w=3$. $\quad \left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{(0{,}2\times 3)^2+1}} = \frac{1}{\sqrt{1.36}} = 0{,}857$

$$\angle\frac{V_o}{V_s} = -\tan^{-1} 0{,}2\times 3 = -30{,}9^{\circ}$$

$w=4 \Rightarrow \left|\frac{V_o}{V_s}\right| =$ $\qquad \angle\frac{V_o}{V_s} =$

$w=5 \Rightarrow \left|\frac{V_o}{V_s}\right| =$ $\qquad \angle\frac{V_o}{V_s} =$

⋮

$w=10 \Rightarrow \left|\frac{V_o}{V_s}\right| =$ $\qquad \angle\frac{V_o}{V_s} =$

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w	1	2	3	4
$\left\|\frac{V_o}{V_s}\right\|$	0,98	0,928	0.857	
$\angle\frac{V_o}{V_s}$	−11,3	−21,8	−30,9	

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a) $V_s = 10\cos t \Rightarrow V_o = 10\times 0{,}98\ \cos(t-11{,}3) = 9{,}8\ \cos(t-11{,}3)$

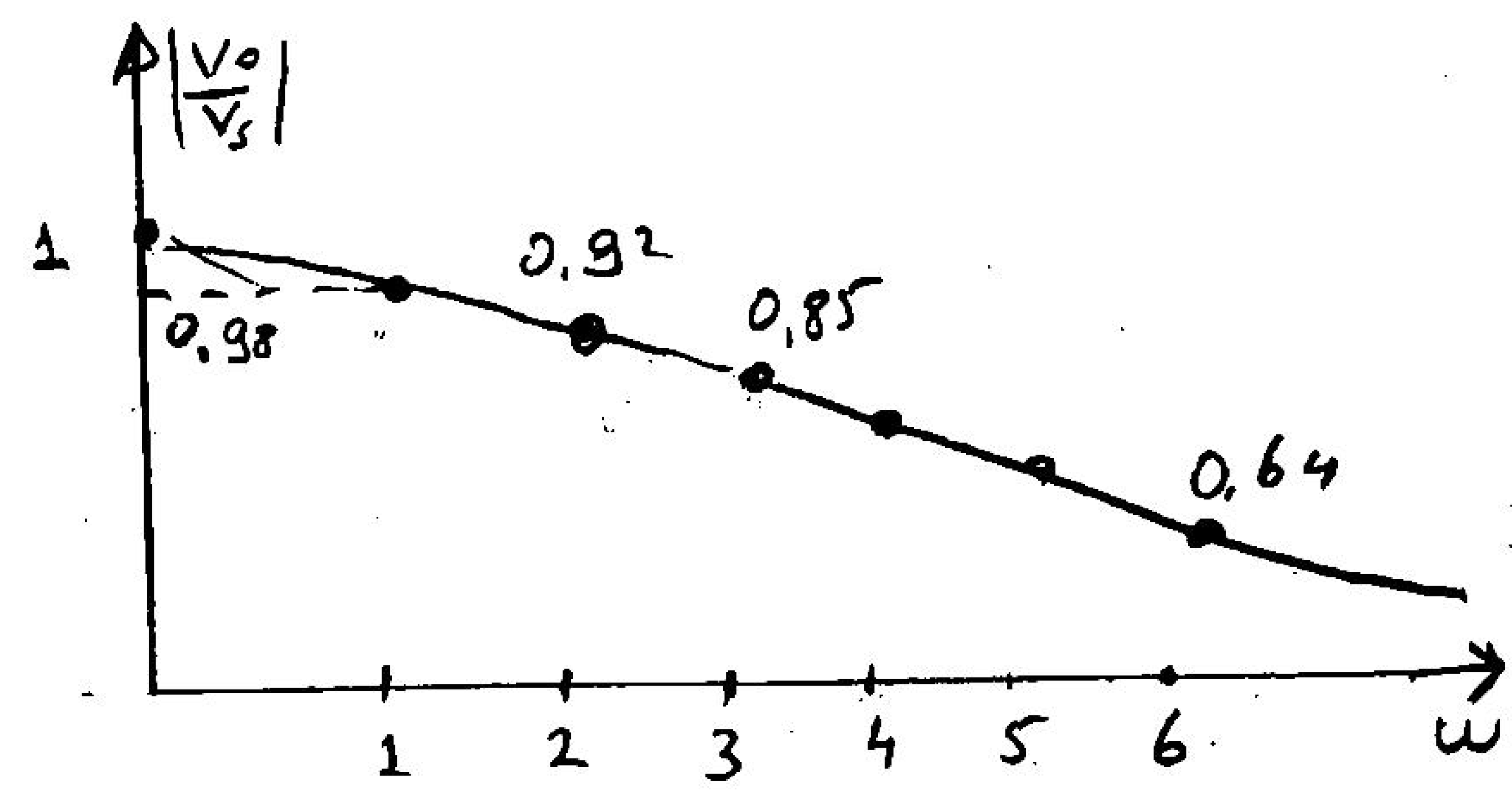
b) $V_s = 10\cos 2t \Rightarrow V_o = 10\times 0.928\ \cos(2t-21{,}8) = 9{,}28\ \cos(2t-21{,}8)$

c) $V_s = 10\cos(2t+30) \Rightarrow V_o = 10\times 0{,}928\ \cos(2t+30-21{,}8) = 9{,}28\ \cos(2t+8{,}2)$

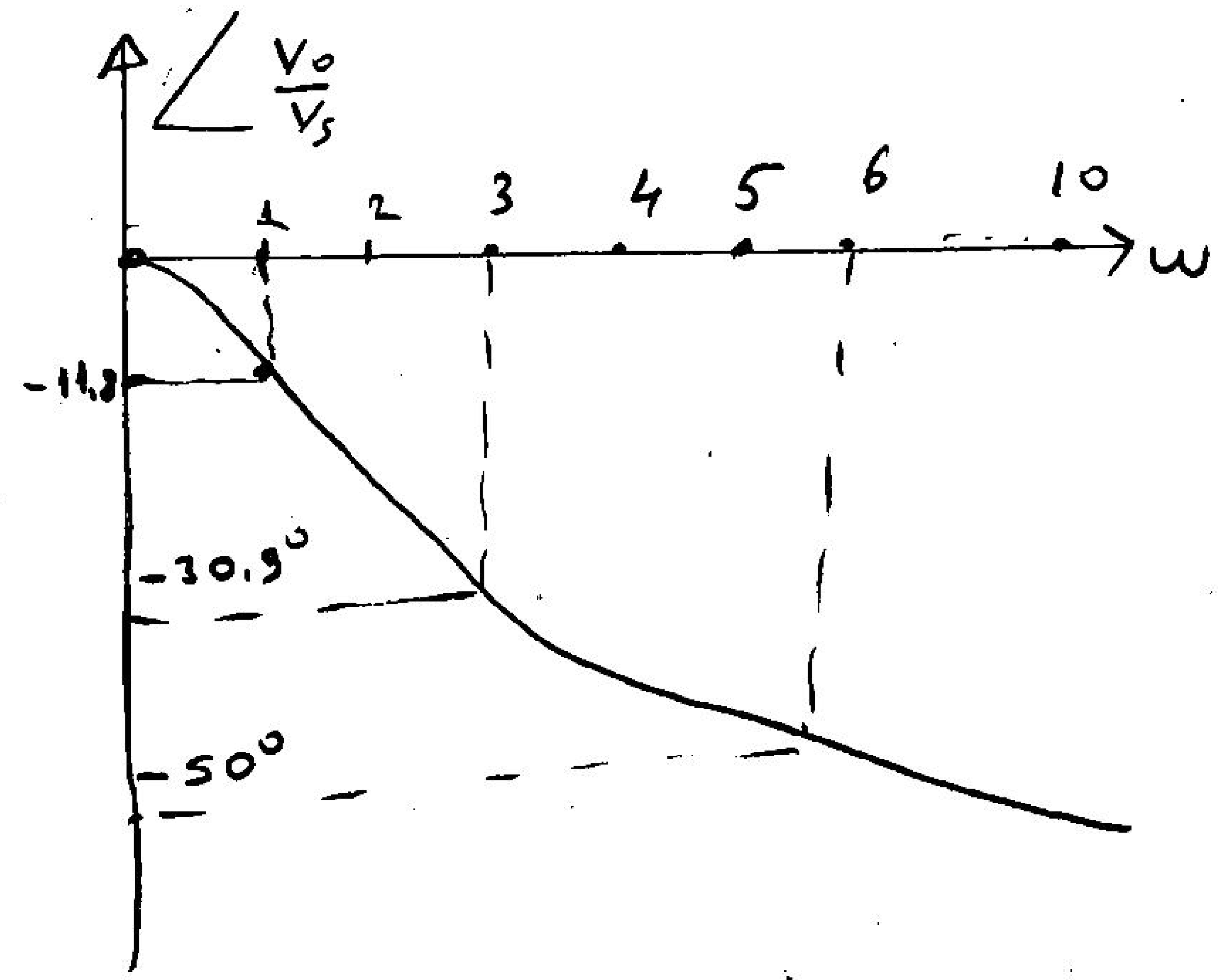
d) $V_s = 10\sin 3t \Rightarrow V_o = 10\times 0{,}857\ \sin(3t-30{,}9) = 8{,}57\ \sin(3t-30{,}9)$

SPECTRUM

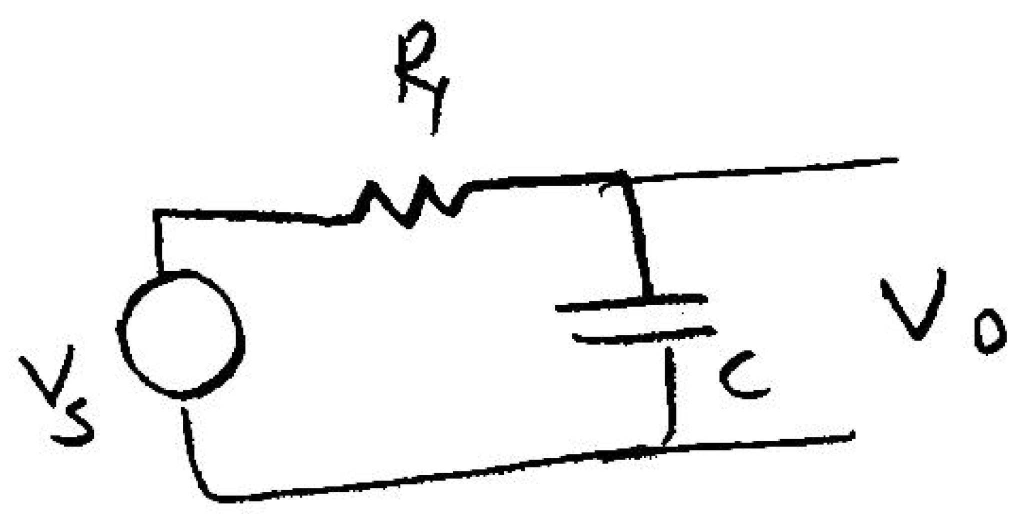
ω	0	1	2	3	4	5	6	8	10	100
$\left\lvert \frac{V_o}{V_s} \right\rvert$	1	0,98	0,92	0,85	0,78	0,707	0,64	0,53	0,44	0,05
$\angle \frac{V_o}{V_s}$	0	-11,9	-21,8	-30,9	-38	-45	-50	-54	-63	-87



Amplitude spectrum

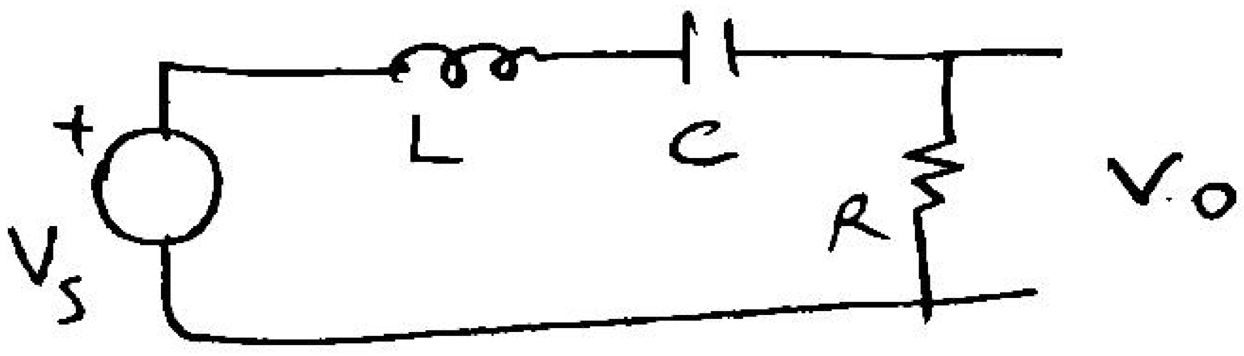


phase spectrum



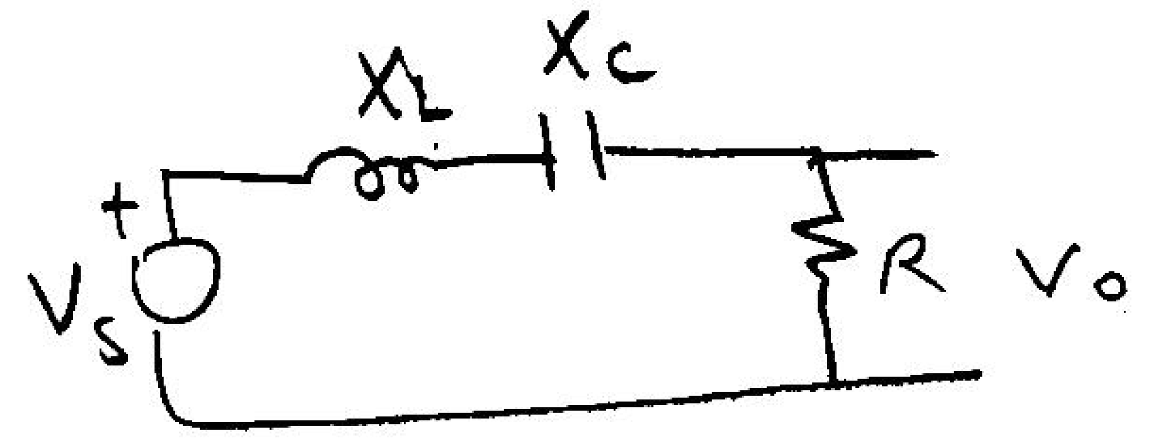
$$\frac{V_o}{V_s} = \frac{1}{j\omega CR + 1} = \frac{1}{0,2\omega j + 1}$$

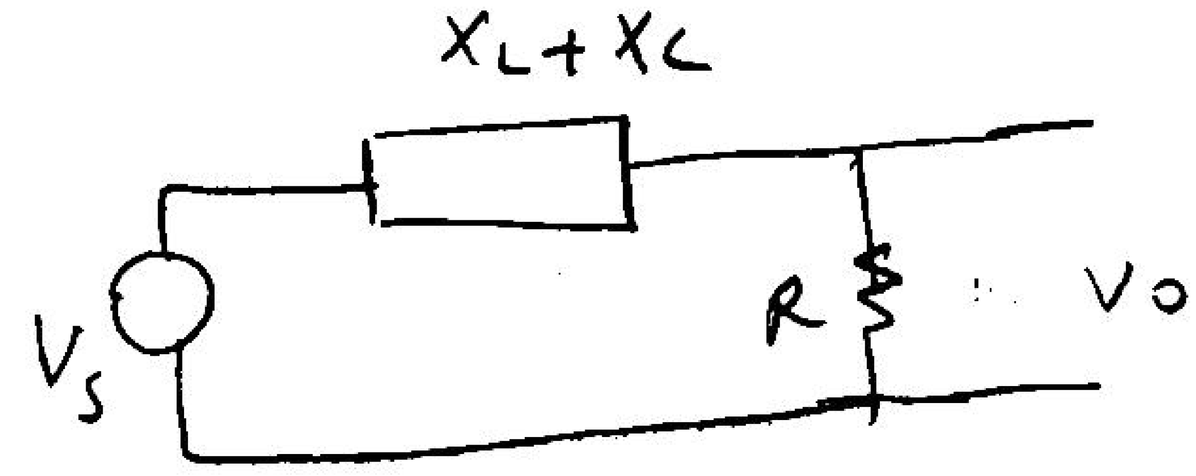
Example problem: calculate transfer function e226 and draw amplitude and phase spectrum



$L = 0{,}1 \quad C = 0{,}4 \quad R = 10$

Solution:





$$\frac{V_o}{V_s} = \frac{R}{R + X_L + X_C} = \frac{R}{R + jwL + \frac{1}{jwC}} = \frac{R}{R + j\left(wL - \frac{1}{wC}\right)}$$

$$= \frac{10}{10 + j\left(0{,}1w - \frac{1}{0{,}4w}\right)}$$

$$\left|\frac{V_o}{V_s}\right| = \frac{10}{\sqrt{10^2 + \left(0{,}1w - \frac{1}{0{,}4w}\right)^2}}$$

$$\angle\frac{V_o}{V_s} = \angle 10 - \angle 10 + j\left(0{,}1w - \frac{1}{0{,}4w}\right)$$

$$\downarrow$$

$$= 0 - \tan^{-1}\left(\frac{0{,}1w - \frac{1}{0{,}4w}}{10}\right)$$

$$w = 1 \Rightarrow \left|\frac{V_o}{V_s}\right| = \frac{10}{\sqrt{10^2 + \left(0{,}1\times1 - \frac{1}{0{,}4\times1}\right)^2}} =$$

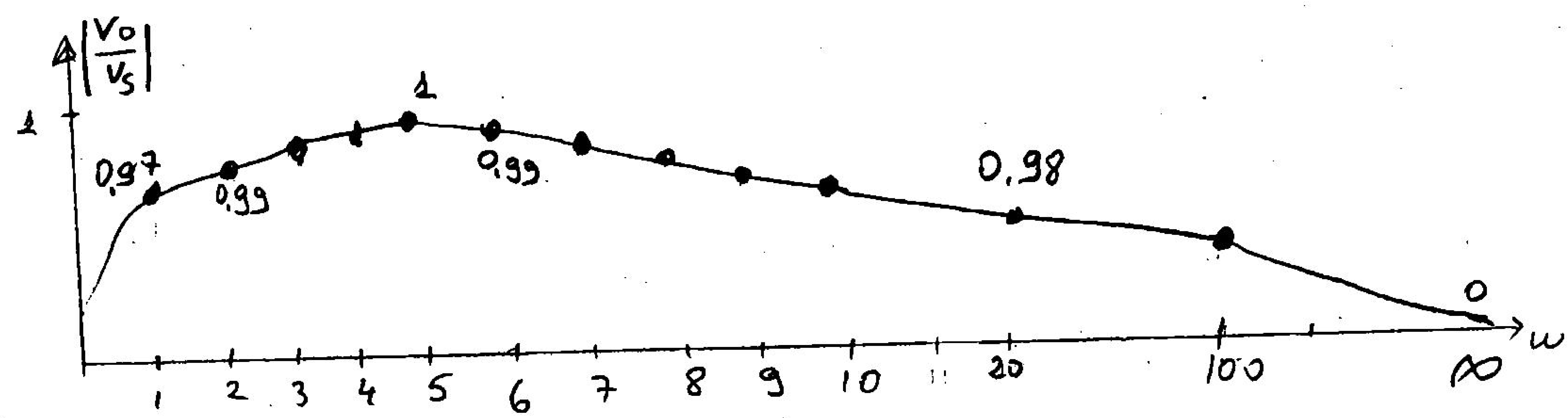
$$\angle\frac{V_o}{V_s} = -\tan^{-1}\left(\frac{0{,}1\times1 - \frac{1}{0{,}4\times1}}{10}\right) =$$

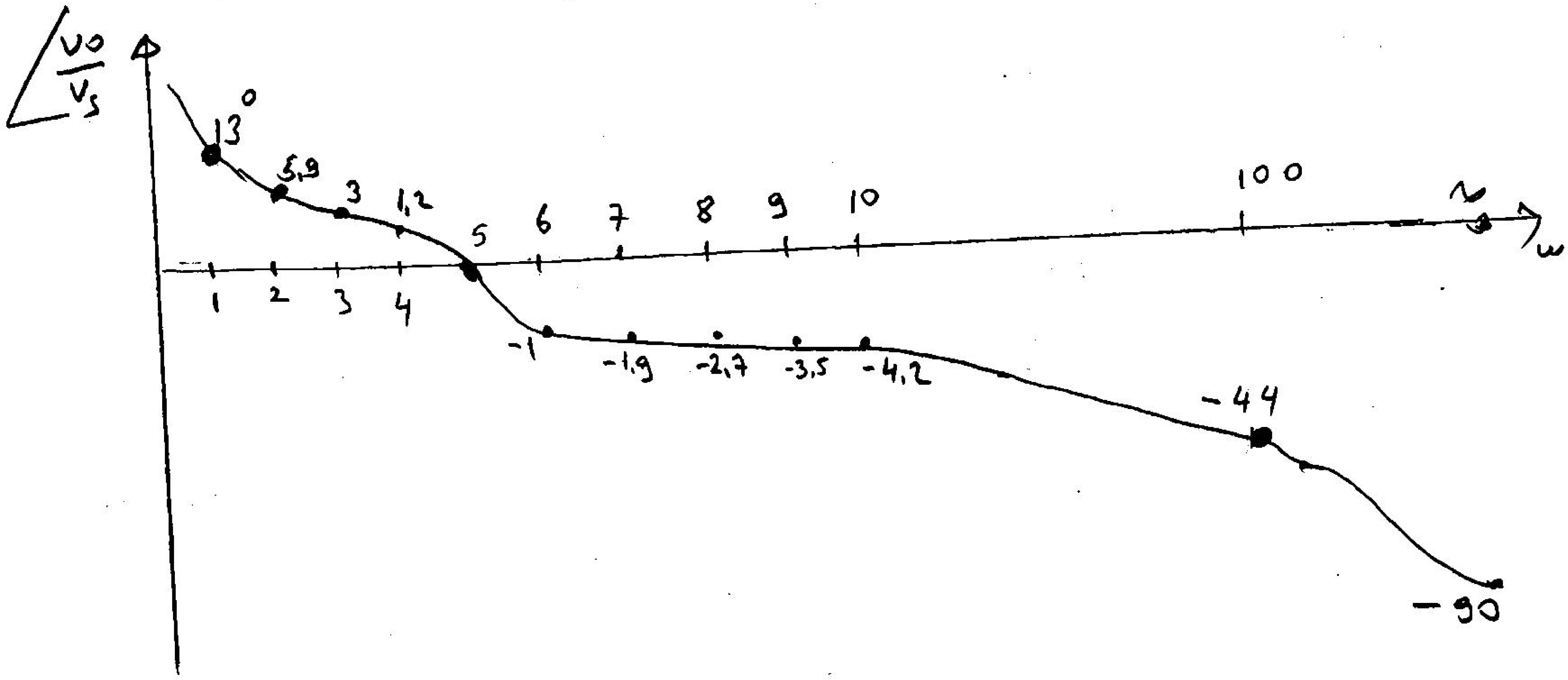
e227

$$\omega = 2 \Rightarrow \left|\frac{V_o}{V_s}\right| = \frac{10}{\sqrt{10^2+\left(0{,}1\omega - \frac{1}{0{,}4\omega}\right)^2}} = \frac{10}{\sqrt{10^2+\left(0{,}1\times 2 - \frac{1}{0{,}4\times 2}\right)}} =$$

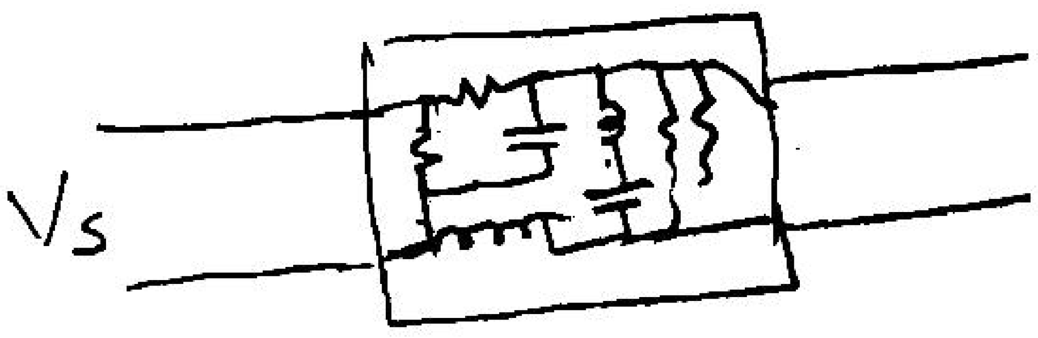
$$\angle \frac{V_o}{V_s} = -\tan^{-1}\left(0{,}1\times 2 - \frac{1}{0{,}4\times 2}\right) =$$

ω	1	2	3	4	5	6	7	8	9	10	20	30	40	50	100	∞
$\left\lvert\frac{V_o}{V_s}\right\rvert$	0,97	0,99	0,99	0,99	1	0,99	0,99	0,99	0,99	0,99	0,98	0,96	0,93	0,89	0,7	0
$\angle \frac{V_o}{V_s}$	13°	$5{,}9^\circ$	3°	$1{,}2^\circ$	0	-1°	$-1{,}9^\circ$	$-2{,}7^\circ$	$-3{,}5$	$-4{,}2^\circ$	-10°	-16°	-21°	-26°	-44	-90





Example Problem: Calculate the output

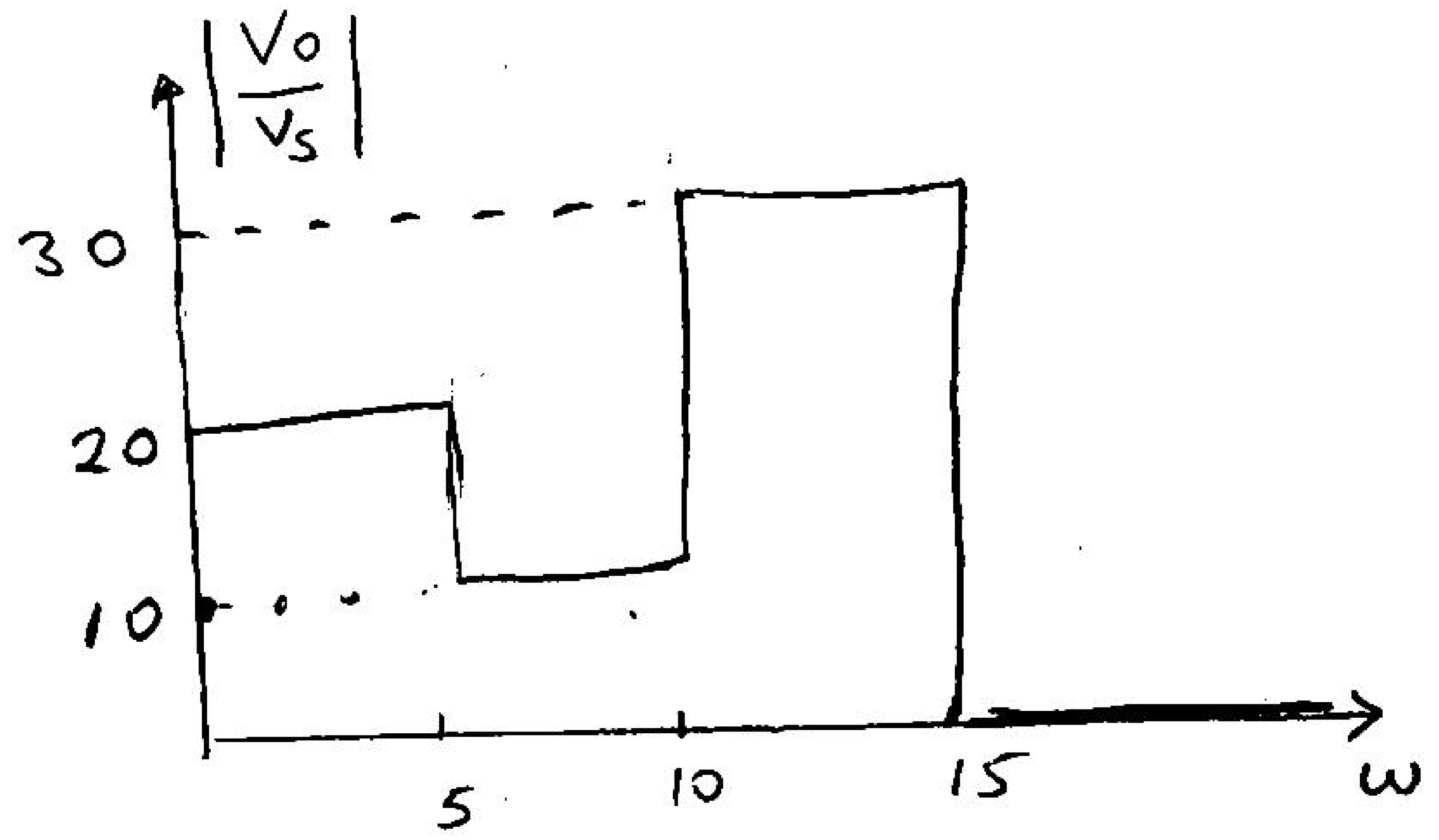


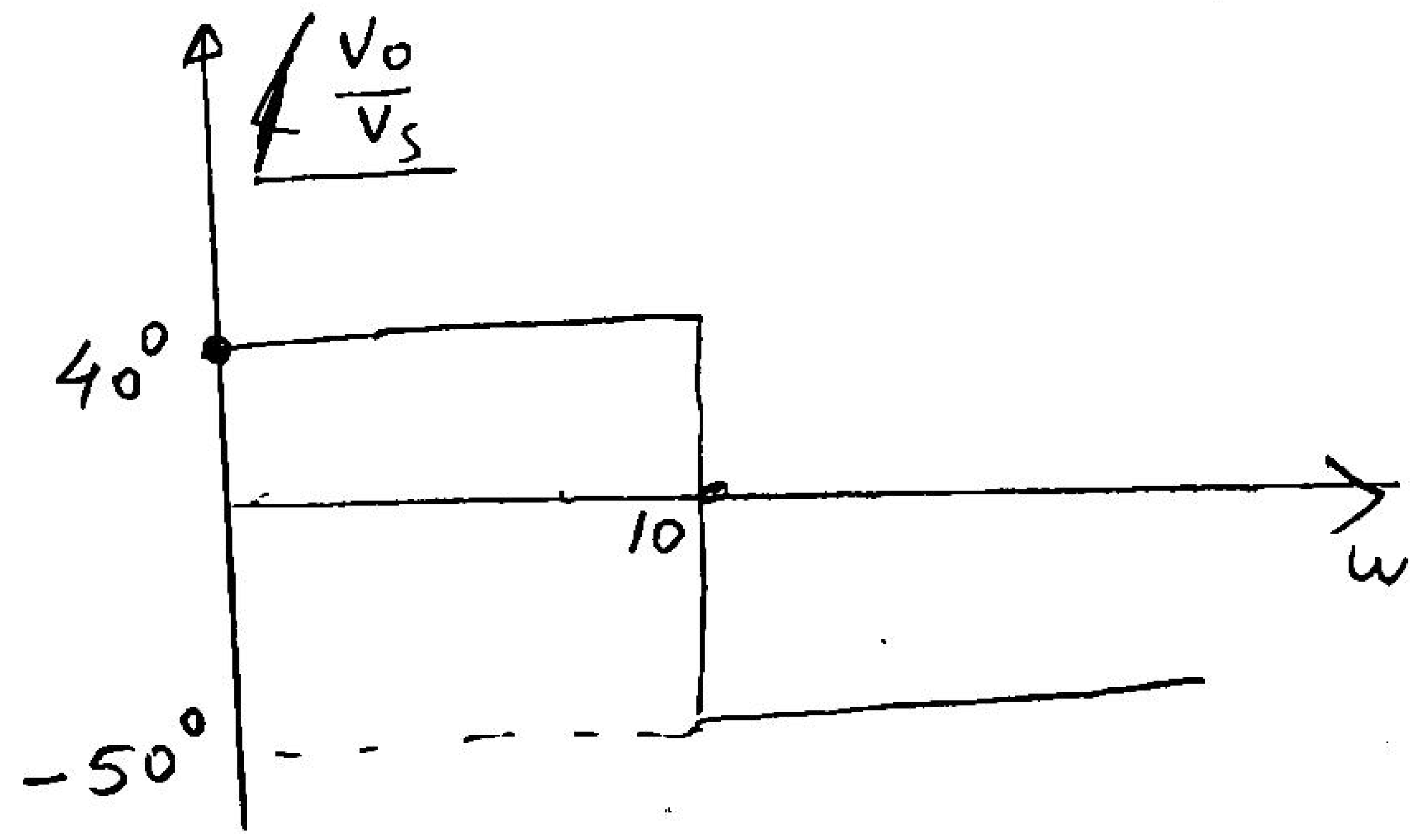
a) $V_s = 4\cos 2t$

b) $V_s = 5\cos(6t+20)$

c) $V_s = 2\cos(13t+60)$

d) $V_s = 10\cos 25t$





a) $V_s = 4\cos 2t$ $\quad \omega = 2 \quad \left|\frac{V_o}{V_s}\right| = 20 \quad \angle\frac{V_o}{V_s} = 40^\circ$

$$V_o = 4 \times 20\cos(2t + 40^\circ) = 80\cos(2t+40)$$

b) $V_s = 5\cos(6t+20)$ $\quad \omega = 6 \quad \left|\frac{V_o}{V_s}\right| = 10 \quad \angle\frac{V_o}{V_s} = 40^\circ$

$$V_o = 5 \times 10\cos(6t + 20 + 40) = 50\cos(6t+60)$$

c) $V_s = 2\cos(13t+60)$ $\quad \omega = 13 \quad \left|\frac{V_o}{V_s}\right| = 30 \quad \angle\frac{V_o}{V_s} = -50^\circ$

$$V_o = 2 \times 30\cos(13t + 60 - 50) = 60\cos(13t+10)$$

d) $V_s = 10\cos 25t$ $\quad \omega = 25 \quad \left|\frac{V_o}{V_s}\right| = 0 \quad \angle\frac{V_o}{V_s} = -50$

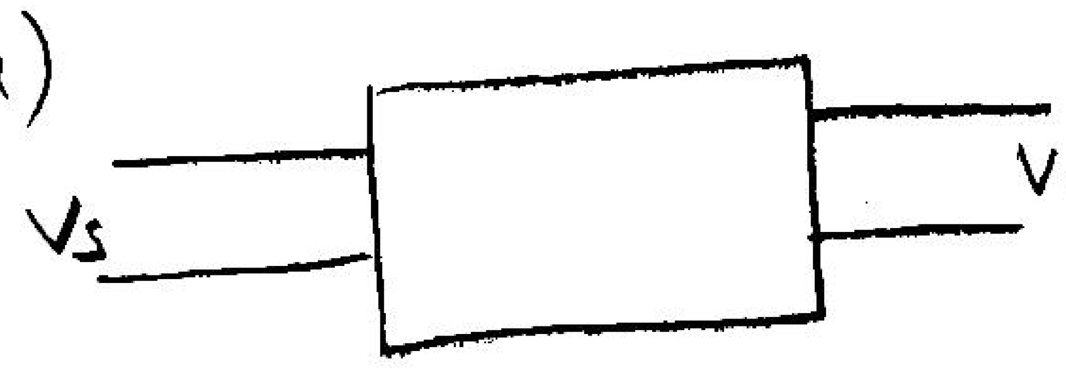
$$V_o = 10 \times 0\cos(25t - 50) = 0$$

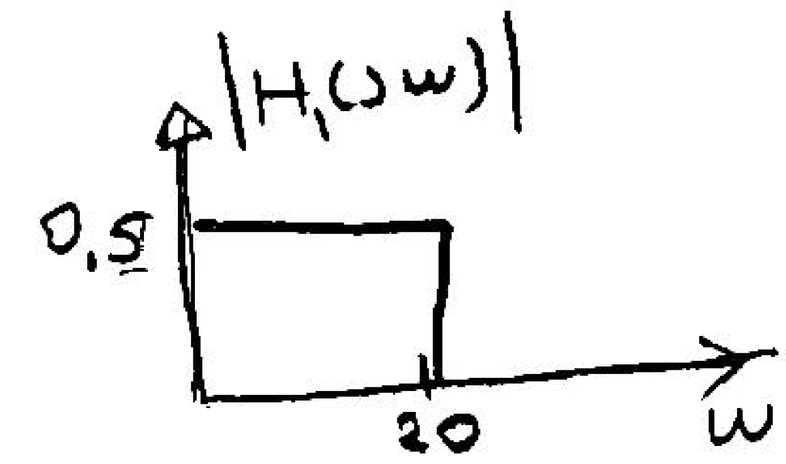
Filters

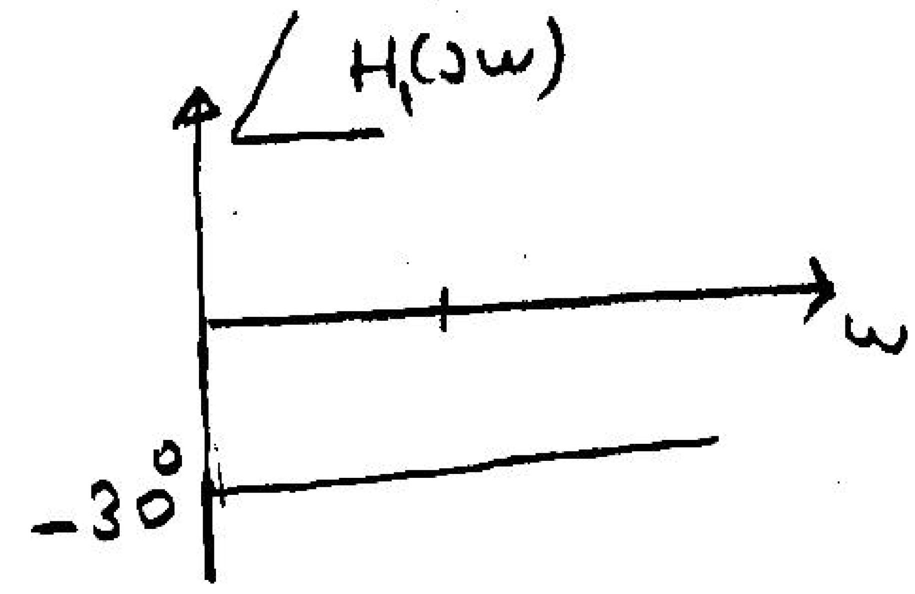
e229

$V_s = \cos 5t + \cos 50t + \cos 500t$

a)



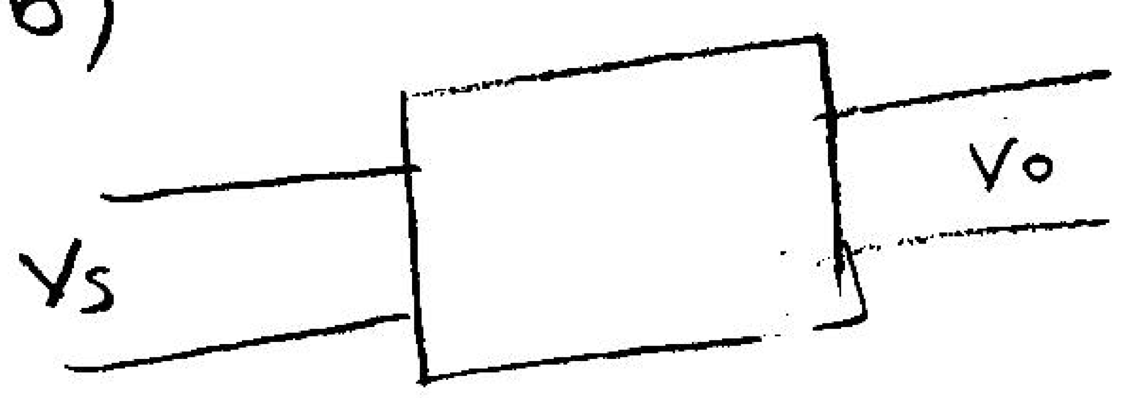


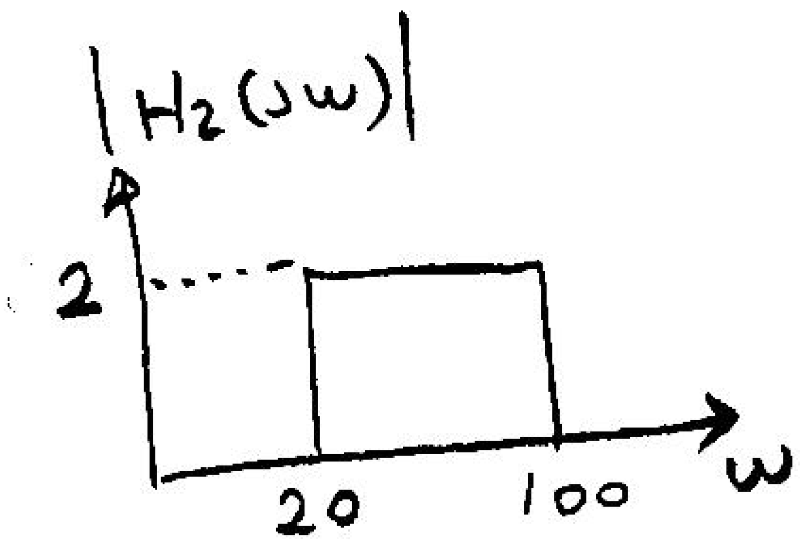


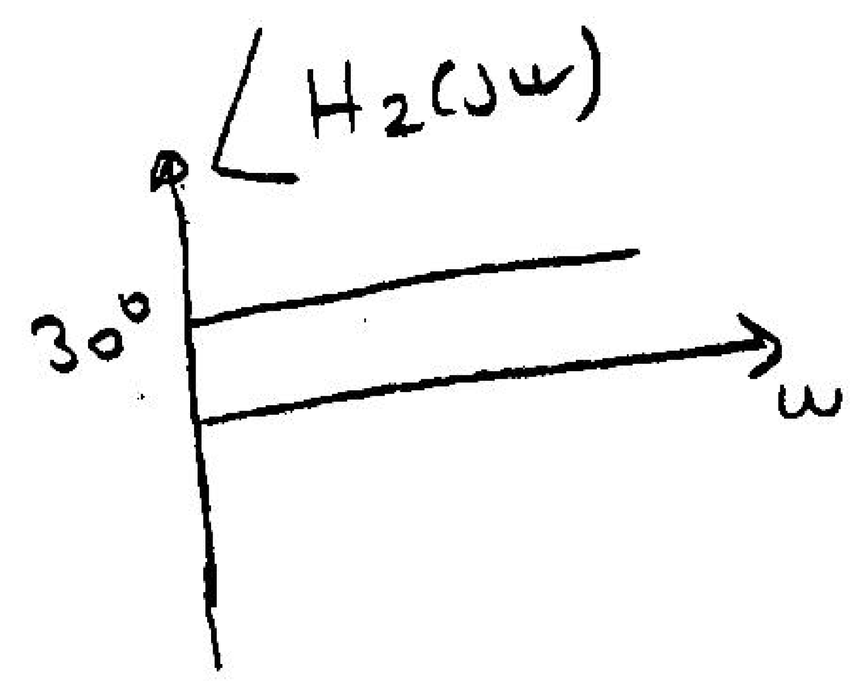
$H_1(jw) = \frac{V_o}{V_s}$

$V_o = 0.5 \cos(5t+30) + 0 + 0$

b)



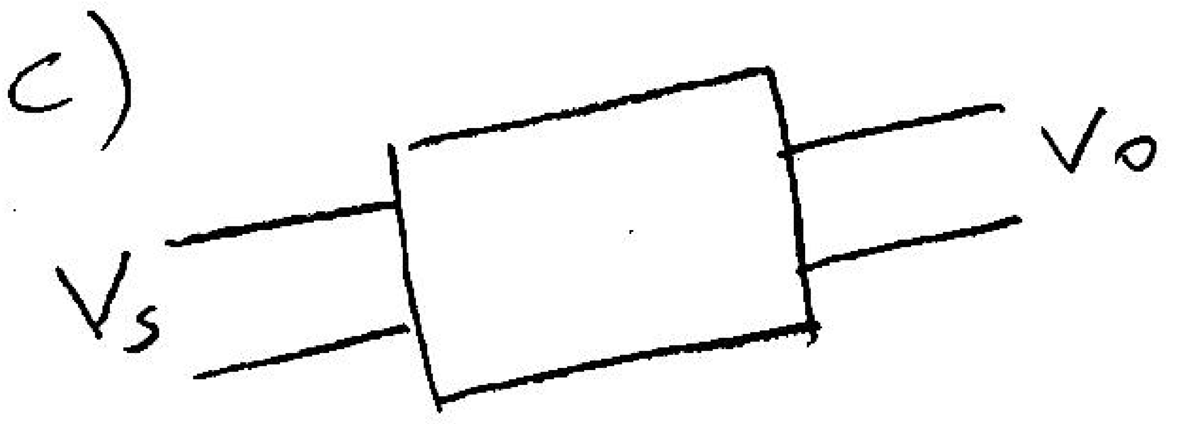


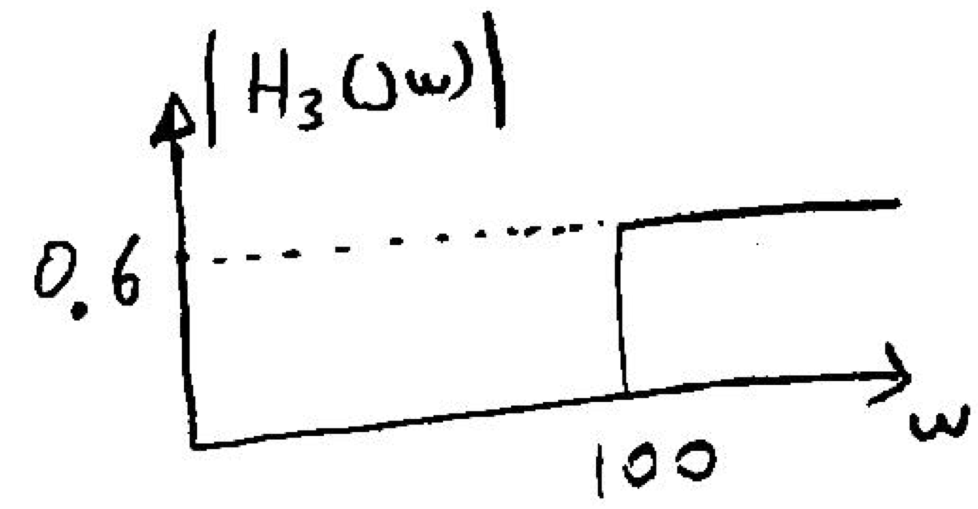


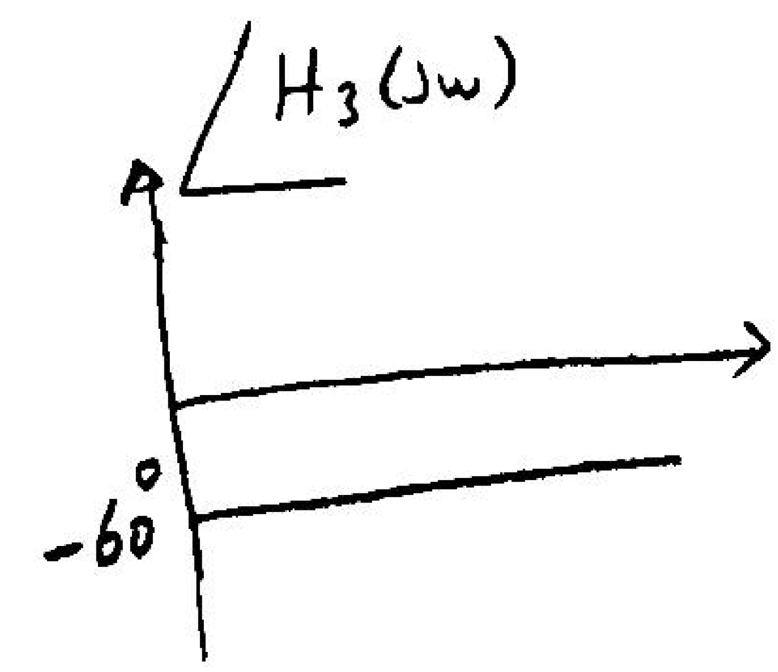
$H_2(jw) = \frac{V_o}{V_s}$

$V_o = 0 + 2 \cos(50t+30) + 0$

c)



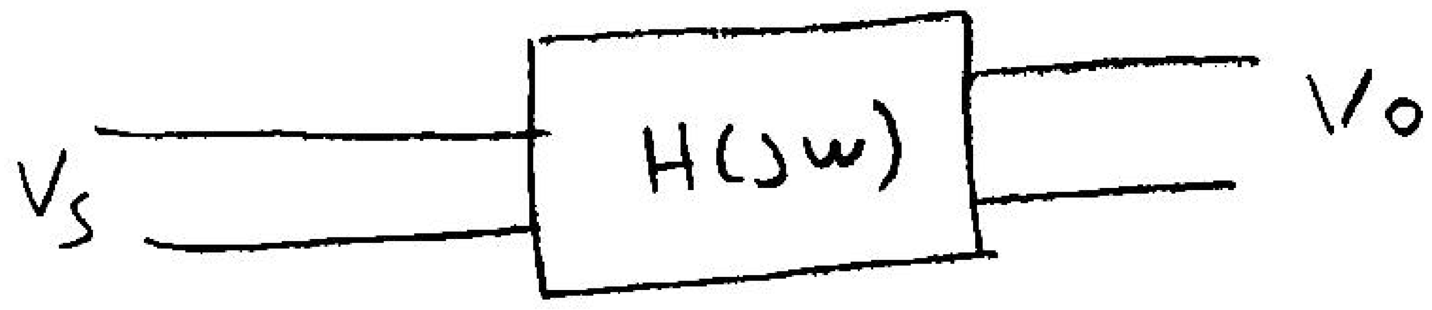




$H_3(jw) = \frac{V_o}{V_s}$

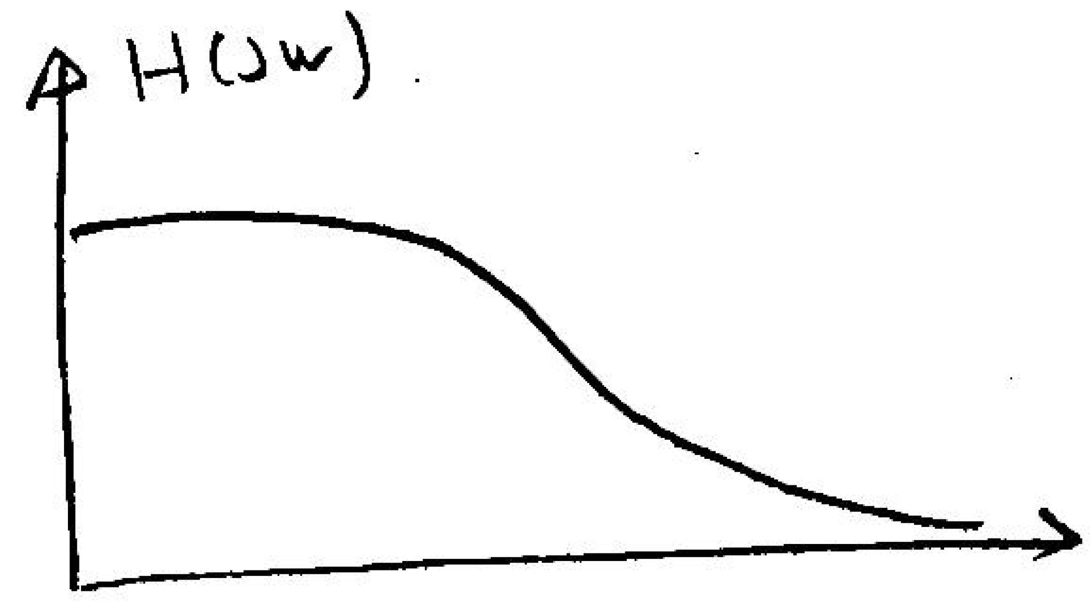
$V_o = 0 + 0 + 0.6 \cos(500t - 60)$

e230

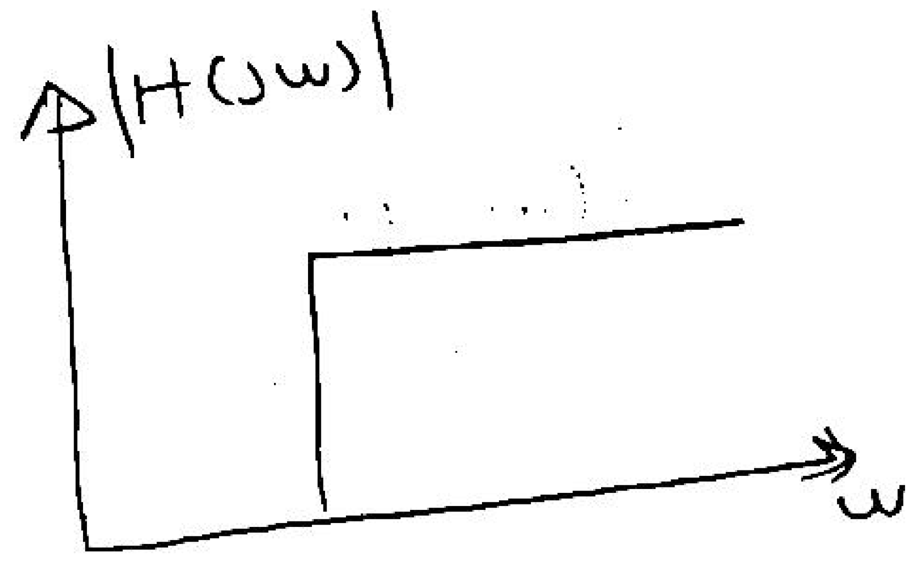


$$H(jw) = \frac{V_o}{V_s}$$

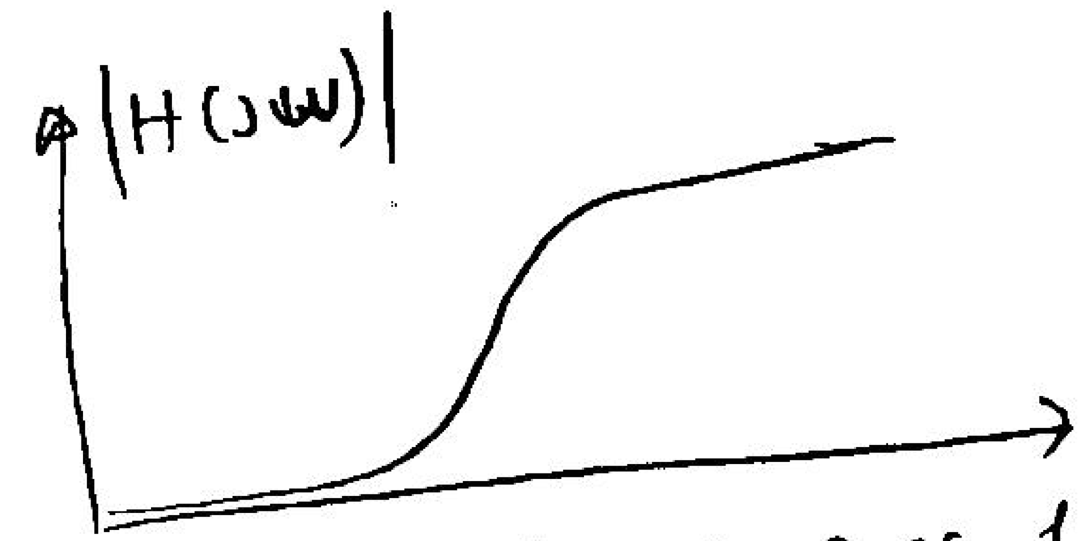
ideal low pass filter amplitude spectrum



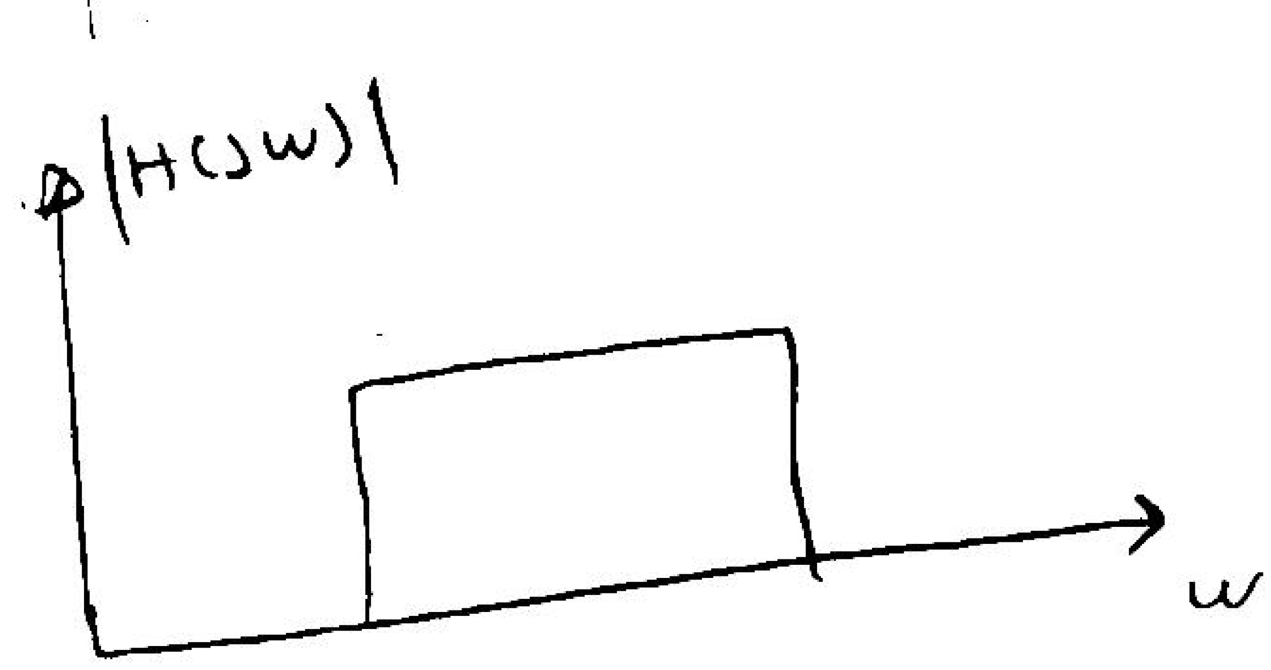
Practical low pass fi amplitude spectrum



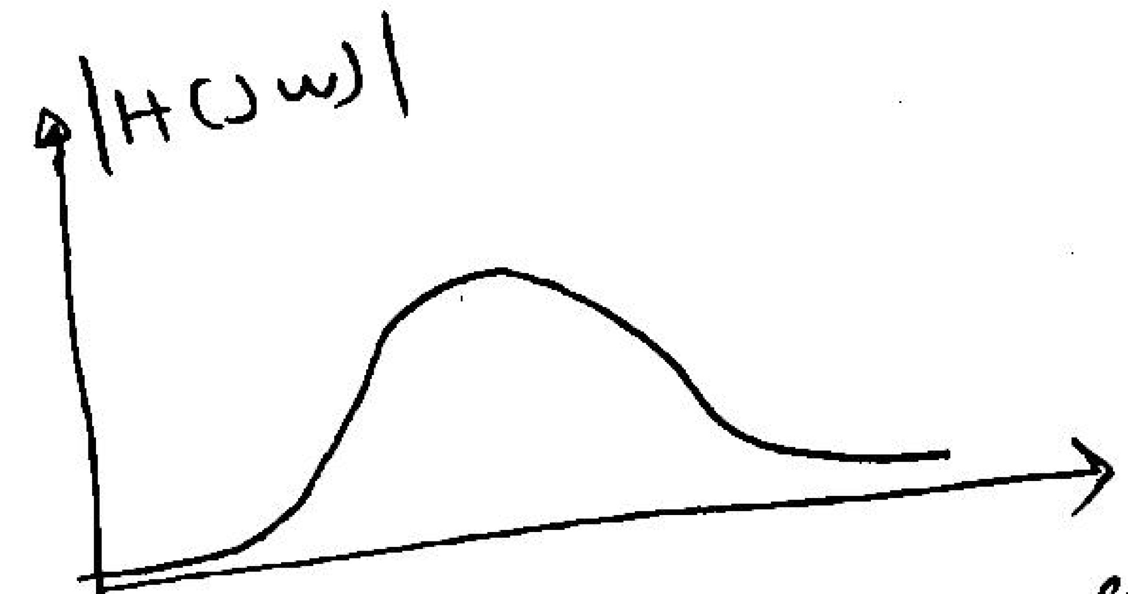
ideal high pass filter amplitude spectrum



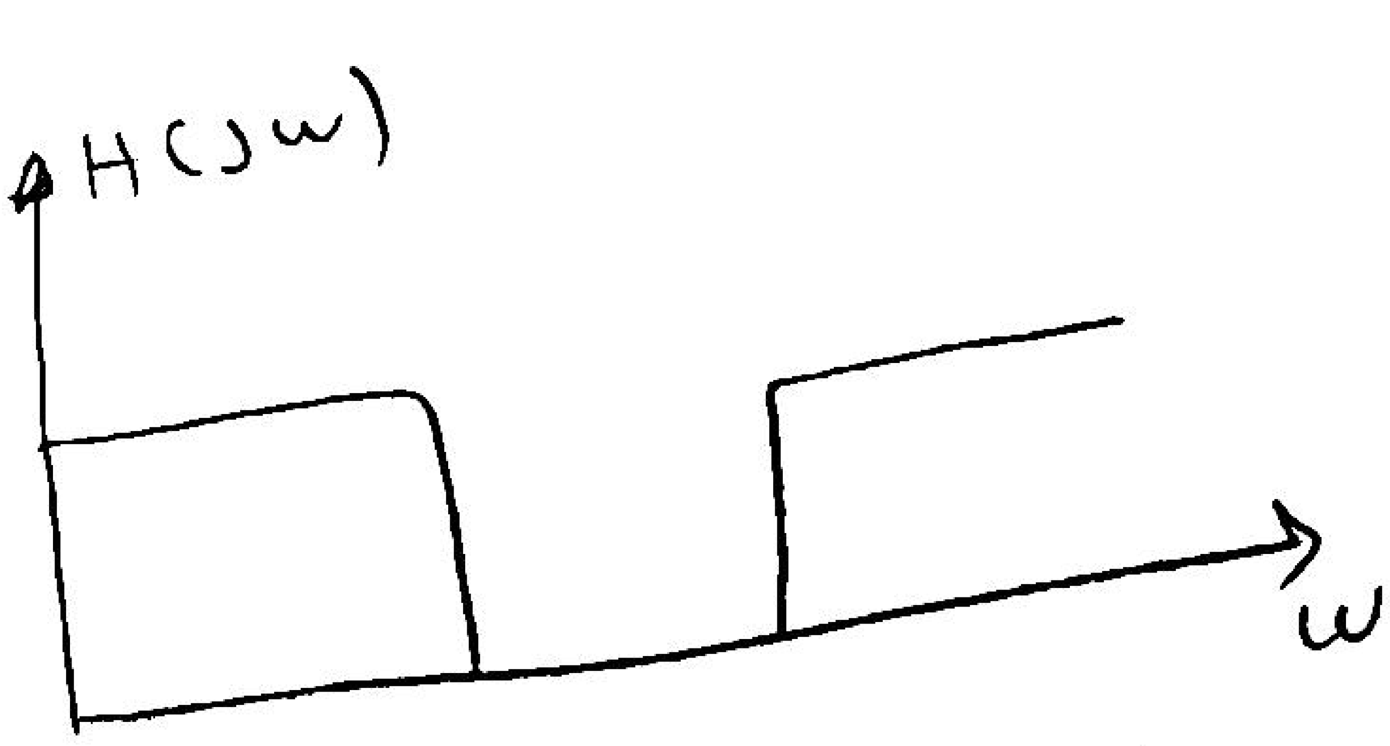
Practical high pass filter amplitude spectrum.



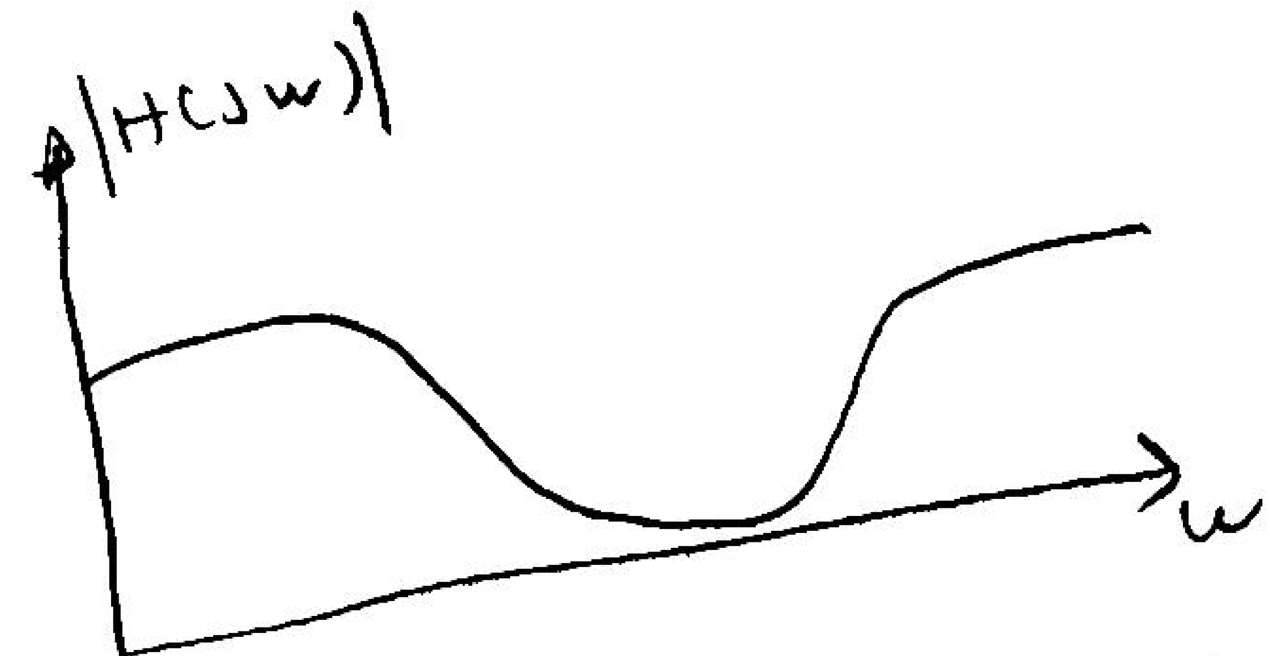
ideal band-pass filter amplitude spectrum



practical band pass filter amplitude spectrum



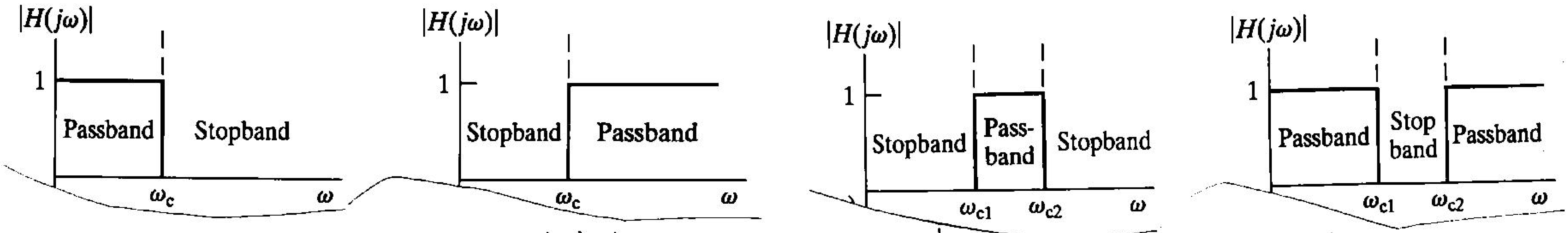
ideal band-stop filter amplitude spectrum



practical band stop filter amplitude spectrum.

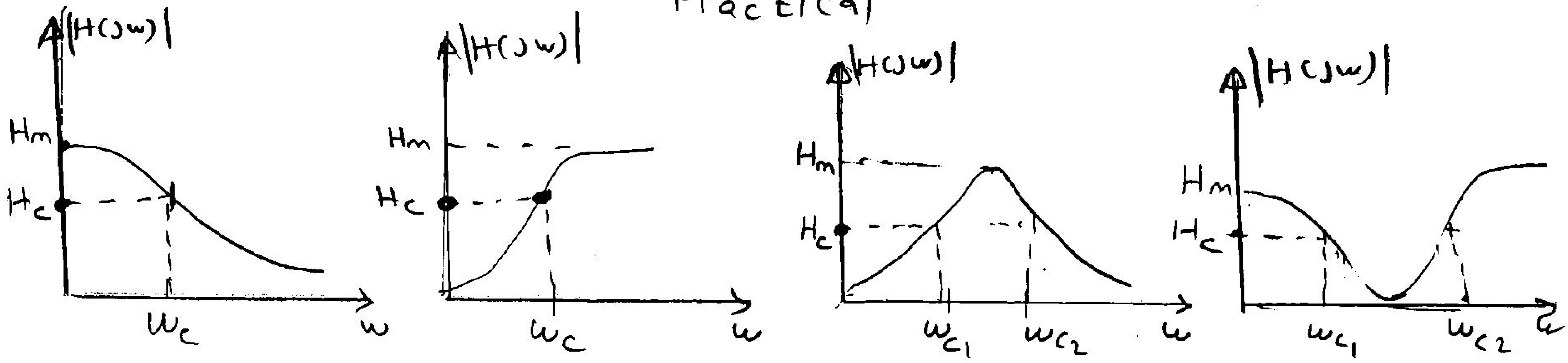
ideal

e231



Low pass | high pass | band pass | bandstop

Practical

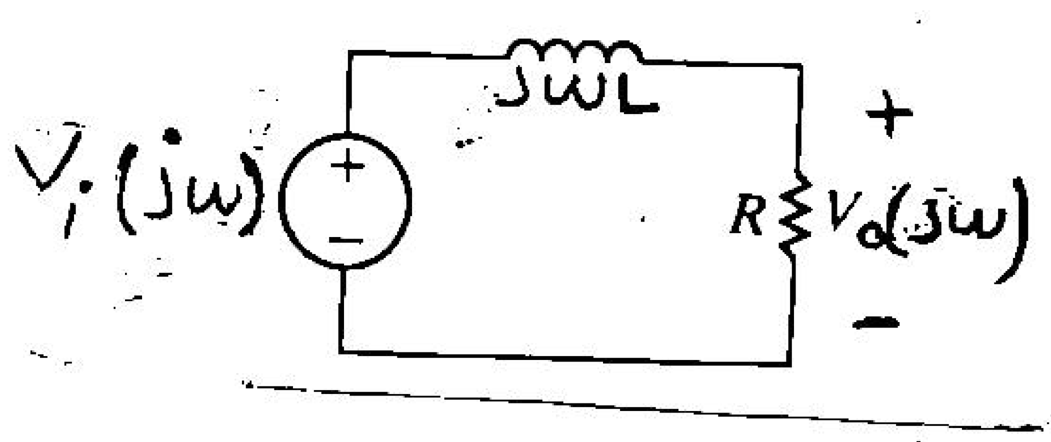


$$H_c = \frac{H_m}{\sqrt{2}} \qquad H(jw_c) = H_c \qquad H_m = |H(jw)|_{max}$$

W_c; Cutoff frequency

14.2 ◆ Low-Pass Filters

The Series *RL* Circuit



$$V_o(jw) = \frac{R}{jwL+R} V_i(jw)$$

$$H(jw) = \frac{V_o(jw)}{V_i(jw)} = \frac{R}{jwL+R}$$

$$|H(jw)| = \frac{R}{\sqrt{(wL)^2+R^2}}$$

$$\theta(jw) = -\tan^{-1}\frac{wL}{R}$$

at $w=0$ $|H(jw)|$ is max

$$H_m = |H(jw)|_{w=0} = \frac{R}{\sqrt{0^2+R^2}} = 1$$

at $w = w_c$ $|H(jw)| = \frac{H_m}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$$\frac{R}{\sqrt{(w_c L)^2+R^2}} = \frac{1}{\sqrt{2}}$$

$$\frac{R^2}{(w_c L)^2+R^2} = \frac{1}{2}$$

$$\frac{R^2}{(\omega_c L)^2 + R^2} = \frac{1}{2} \qquad 2R^2 = \omega_c^2 L^2 + R^2 \qquad 2R^2 - R^2 = \omega_c^2 L^2$$

e232

$$R^2 = \omega_c^2 L^2 \qquad \omega_c^2 = \frac{R^2}{L^2} \qquad \boxed{\omega_c = \frac{R}{L}}$$

EXAMPLE 14.1

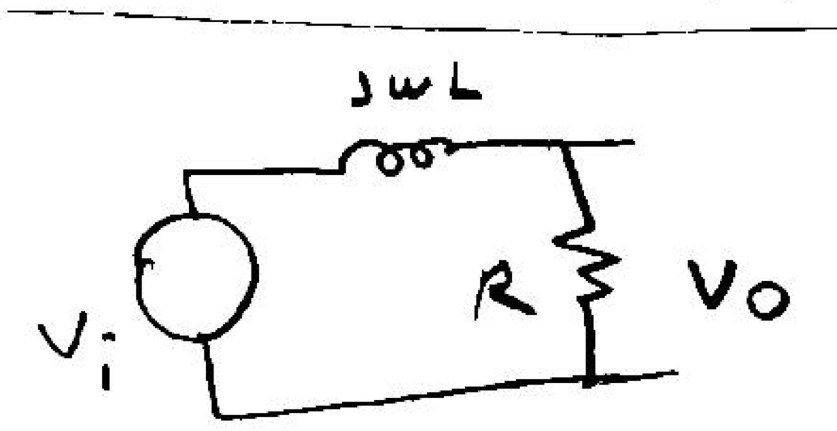
Electrocardiology is the study of the electric signals produced by the heart. These signals maintain the heart's rhythmic beat, and they are measured by an instrument called an electrocardiograph. This instrument must be capable of detecting periodic signals whose frequency is about 1 Hz (the normal heart rate is 72 beats per minute). The instrument must operate in the presence of sinusoidal noise consisting of signals from the surrounding electrical environment, whose fundamental frequency is 60 Hz—the frequency at which electric power is supplied.

Choose values for R and L in the circuit of Fig. 14.4(a) such that the resulting circuit could be used in an electrocardiograph to filter out any noise above 10 Hz and pass the electric signals from the heart at or near 1 Hz. Then compute the magnitude of V_o at 1 Hz, 10 Hz, and 60 Hz to see how well the filter performs.

SOLUTION

The problem is to select values for R and L that yield a low-pass filter with a cutoff frequency of 10 Hz. From Eq. 14.12, we see that R and L cannot be specified independently to generate a value for ω_c. Therefore, let's choose a commonly available value of L, 100 mH. Before we use Eq. 14.12 to compute the value of R needed to obtain the desired cutoff frequency, we need to convert the cutoff frequency from hertz to radians per second:

$$\omega_c = 2\pi(10) = 20\pi \text{ rad/s.}$$



$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R}{j\omega L + R}$$

$$\omega_c = \frac{R}{L} = 20$$

$L = 100\,mH = 0.1\,H$

$$\frac{R}{L} = 20\pi$$

$$\frac{R}{0.1} = 20\pi \Rightarrow R = 2\pi\,\Omega$$
$$= 6.28\,\Omega$$

$$|H(j\omega)| = \frac{R}{\sqrt{(\omega L)^2 + R^2}} = \frac{6.28}{\sqrt{(\omega \times 0.1)^2 + 6.28^2}}$$

at $f = 1$, $\omega = 2\pi$

$$|H(j2\pi)| = \frac{6.28}{\sqrt{(2\times\pi\times 0.1)^2 + 6.28^2}} = 0.995$$

at $f = 10$ hz $\quad \omega = 2\pi \times 10 = 20\pi$

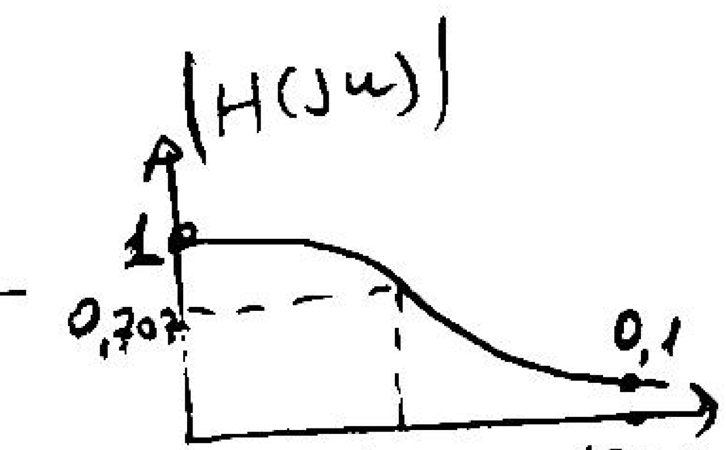
$$|H(j20\pi)| = \frac{6.28}{\sqrt{(20\pi \times 0.1)^2 + 6.28^2}} = 0.707$$

at $f = 60$ hz $\quad \omega = 2\pi \times 60 = 120\pi$

$$H(j120\pi) = \frac{6.28}{\sqrt{(120\pi\, 0.1)^2 + 6.28^2}} = 0.164$$

Assume $|V_i| = 1V$

f(Hz)	$\|V_i\|$(V)	$\|V_o\|$(V)
1	1.0	0.995
10	1.0	0.707
60	1.0	0.164



The Series RC Circuit

e233

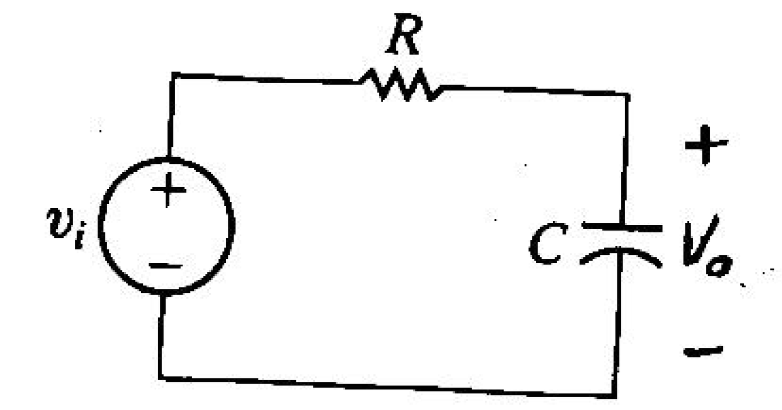
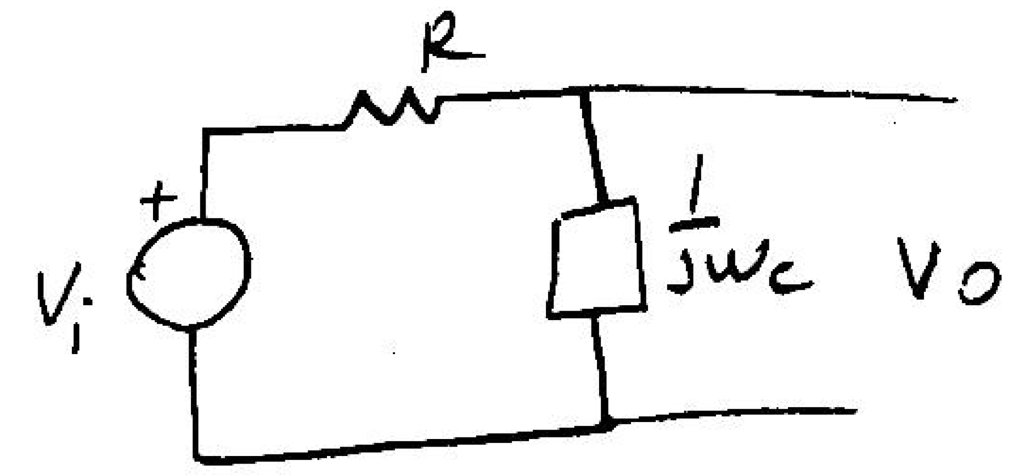


Figure 14.7 A series RC low-pass filter.



$$V_o(jw) = \frac{\frac{1}{jwc}}{R + \frac{1}{jwc}} V_i(jw) = \frac{1}{jwCR + 1} V_i(jw)$$

$$H(jw) = \frac{V_o(jw)}{V_i(jw)} = \frac{1}{jwCR + 1}$$

$$|H(jw)| = \frac{1}{\sqrt{(wCR)^2 + 1^2}} \qquad \theta(jw) = -\tan^{-1}\frac{wCR}{1}$$

at $w = 0$ $|H(jw)|$ is maximum

$$|H(jw)|_{max} = \frac{1}{\sqrt{0^2 + 1^2}} = 1$$

at $w = w_c$ $|H(jw_c)| = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{(w_c CR)^2 + 1^2}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{1}{(w_c CR)^2 + 1^2} = \frac{1}{2}$$

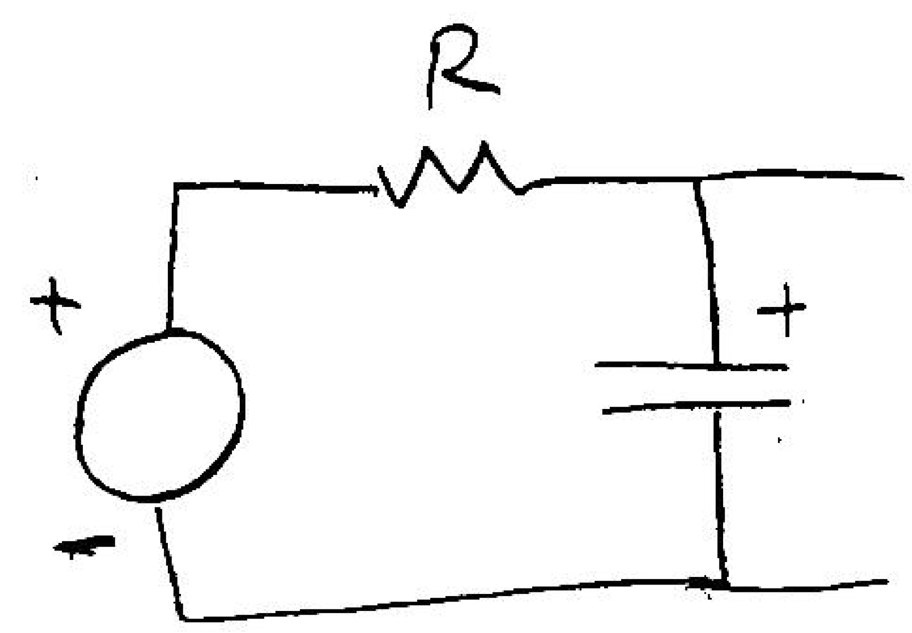
$$(w_c CR)^2 + 1^2 = 2 \qquad (w_c CR)^2 = 1 \qquad \boxed{w_c = \frac{1}{RC}}$$

EXAMPLE 14.2

e234

For the series RC circuit in Fig. 14.7:

a) Find the transfer function between the source voltage and the output voltage.
b) Determine an equation for the cutoff frequency in the series RC circuit.
c) Choose values for R and C that will yield a low-pass filter with a cutoff frequency of 3 kHz.

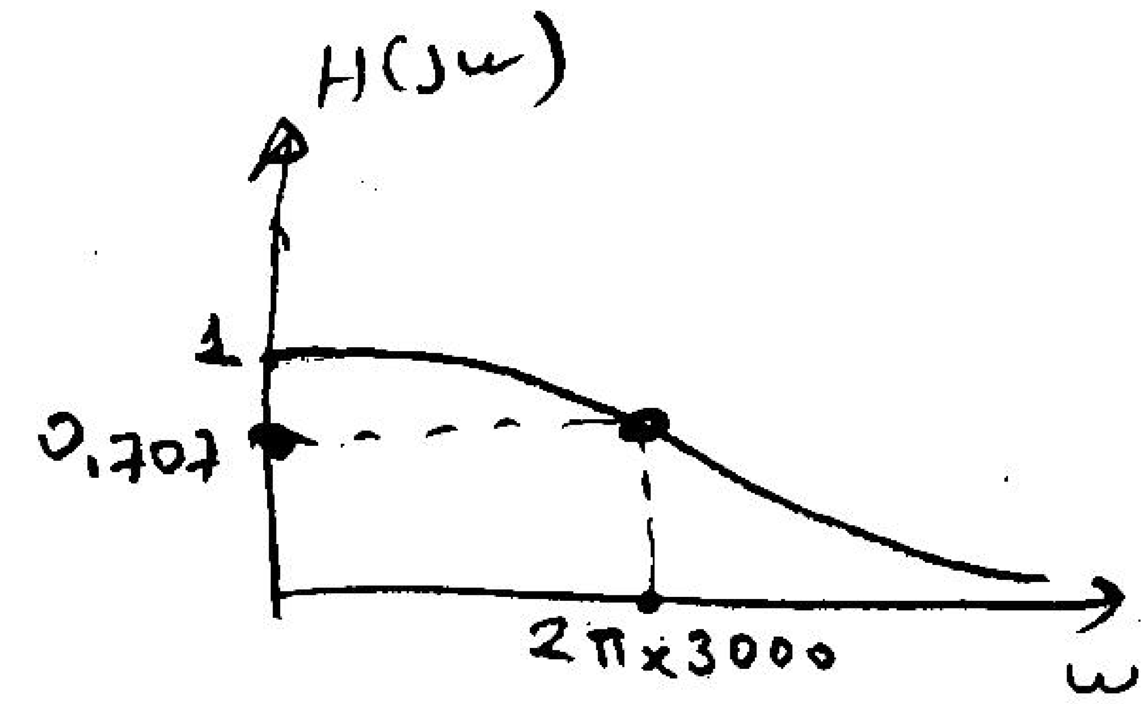


a, b, c in previous page

Because R and C cannot be computed independently, let's choose $C = 1\ \mu F$.

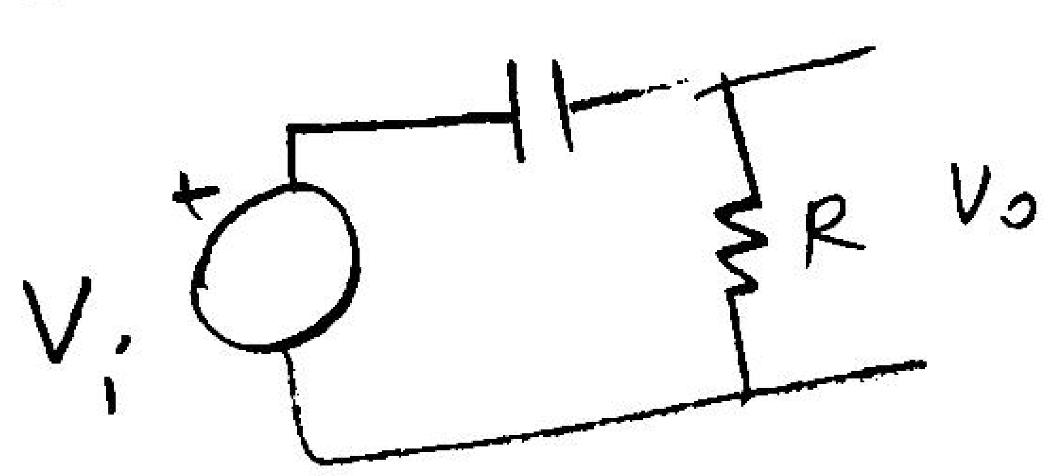
$$w_c = \frac{1}{RC}$$

$$R = \frac{1}{\omega_c C} = \frac{1}{(2\pi)(3 \times 10^3)(1 \times 10^{-6})} = 53.05\ \Omega.$$



HIGH PASS FILTERS

The Series *RC* Circuit—



$$V_o = \frac{R}{R + \frac{1}{jwc}} V_i(jw) = \frac{jwcR}{jwcR + 1} V_i(jw)$$

$$H(jw) = \frac{V_o(jw)}{V_i(jw)} = \frac{jwcR}{jwcR+1}$$

$$|H(jw)| = \frac{wcR}{\sqrt{(wcR)^2 + 1^2}} \qquad \theta(jw) = 90° - \tan^{-1}\frac{wcR}{1}$$

at $w = \infty$ $|H(jw)|$ is maximum

$$H(j\infty) = \lim_{w \to \infty} \frac{wcR}{\sqrt{(wcR)^2 + 1}} = 1$$

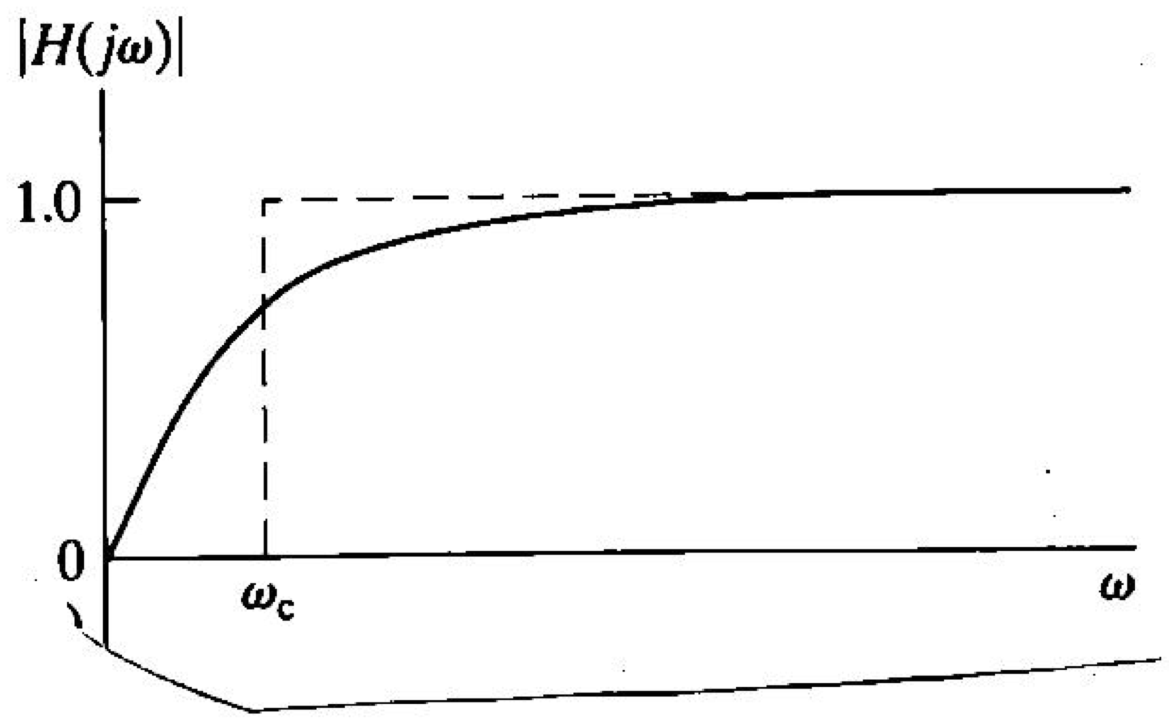
e235

at $\omega = \omega_c$ $\quad |H(j\omega_c)| = \dfrac{1}{\sqrt{2}}$

$$\frac{\omega_c CR}{\sqrt{(\omega_c CR)^2 + 1}} = \frac{1}{\sqrt{2}} \qquad \frac{(\omega_c CR)^2}{(\omega_c CR)^2 + 1} = \frac{1}{2}$$

$$2(\omega_c CR)^2 = (\omega_c CR)^2 + 1 \qquad (\omega_c CR)^2 = 1 \qquad \omega CR = 1$$

$$\boxed{\omega_c = \frac{1}{RC}}$$



$RC = 1$

ω	$\lvert H(j\omega)\rvert$
0	0
0.5	0.44
1	0.707
10	0.995
100	0.999995
∞	1

EXAMPLE 14.3

Show that the series *RL* circuit in Fig. 14.13 also acts like a high-pass filter:

a) Derive an expression for the circuit's transfer function.

b) Use the result from (a) to determine an equation for the cutoff frequency in the series RL circuit.

c) Choose values for *R* and *L* that will yield a high-pass filter with a cutoff frequency of 15 kHz.

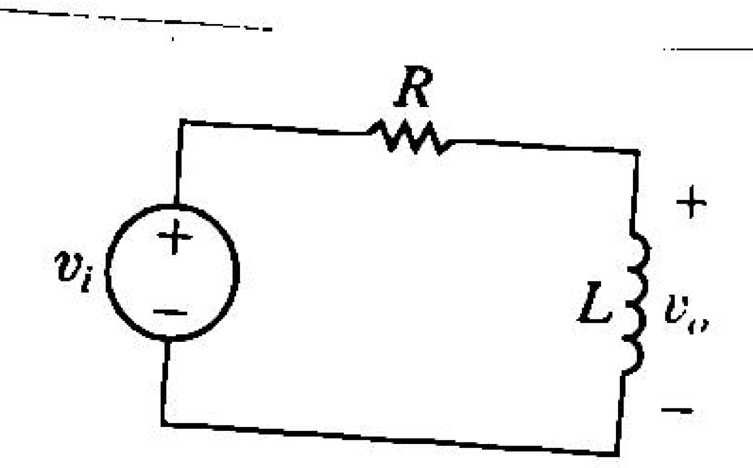
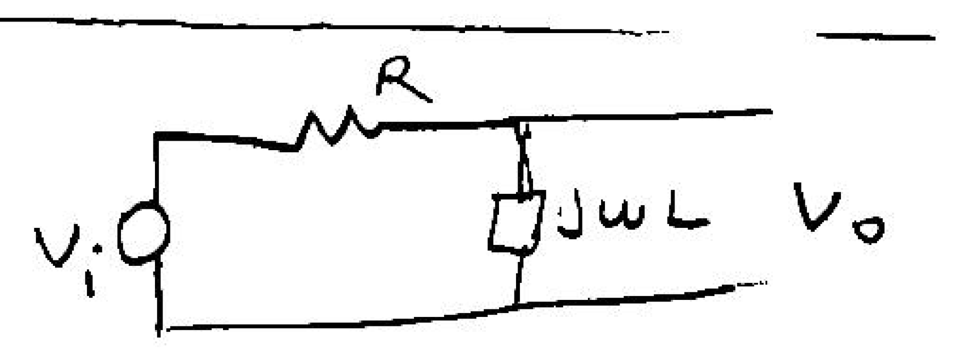


Figure 14.13 The circuit for Example 14.3.

Solution

a)



$$V_o = \frac{j\omega L}{j\omega L + R} V_i$$

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{j\omega L}{j\omega L + R}$$

b)

$$|H(j\omega)| = \frac{\omega L}{\sqrt{(\omega L)^2 + R^2}}$$

at $\omega = 0$ $\quad |H(j0)| = \dfrac{0 \times L}{\quad} = 0$

at $\omega = \infty$ $\quad |H(j\infty)| = 1$

at $\omega = \omega_c$ $\quad |H(j\omega_c)| = \dfrac{1}{\sqrt{2}}$

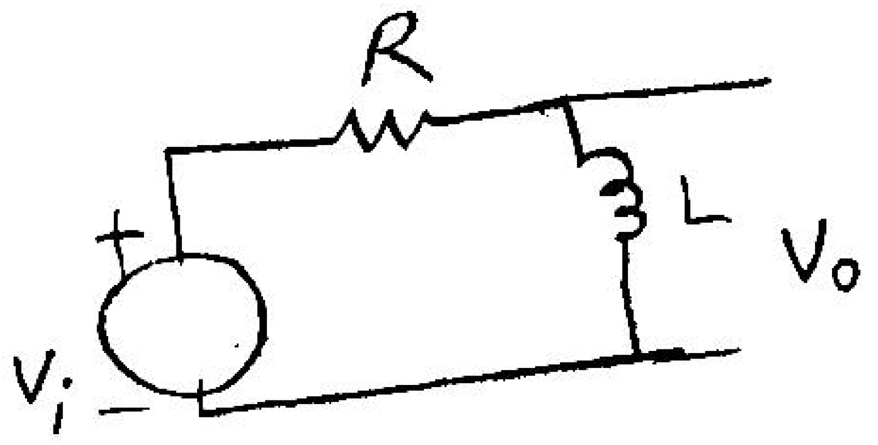
$$\frac{1}{\sqrt{2}} = \frac{\omega_c L}{\sqrt{(\omega_c L)^2 + R^2}} \Rightarrow \frac{1}{2} = \frac{(\omega_c L)^2}{(\omega_c L)^2 + R^2}$$

$$(\omega_c L)^2 + R^2 = 2(\omega_c L)^2$$

$$R^2 = (\omega_c L)^2 \Rightarrow \boxed{\omega_c = \frac{R}{L}}$$

c) $L = 0.1\,H$ (arbitrary اختياريا)

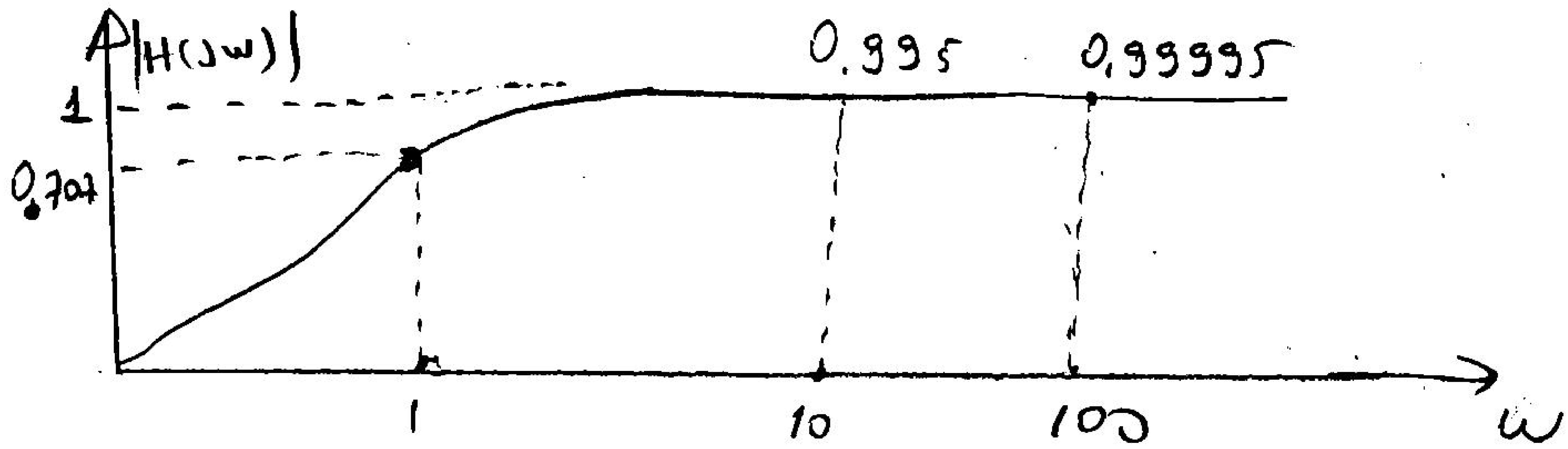
$R = \omega_c L = 2\pi\, 15000\ \ 0.1 = 9424\,\Omega$



$$|H(j\omega)| = \frac{\omega L}{\sqrt{(\omega L)^2 + R^2}} = \frac{\omega}{\sqrt{\omega^2 + 1^2}}$$

R=1
L=1

ω	0	0.1	0.5	1	10	100	∞
$\lvert H(j\omega)\rvert$	0	0,099	0,44	0,707	0,99	0,99995	1



14.4 ◆ Bandpass Filters

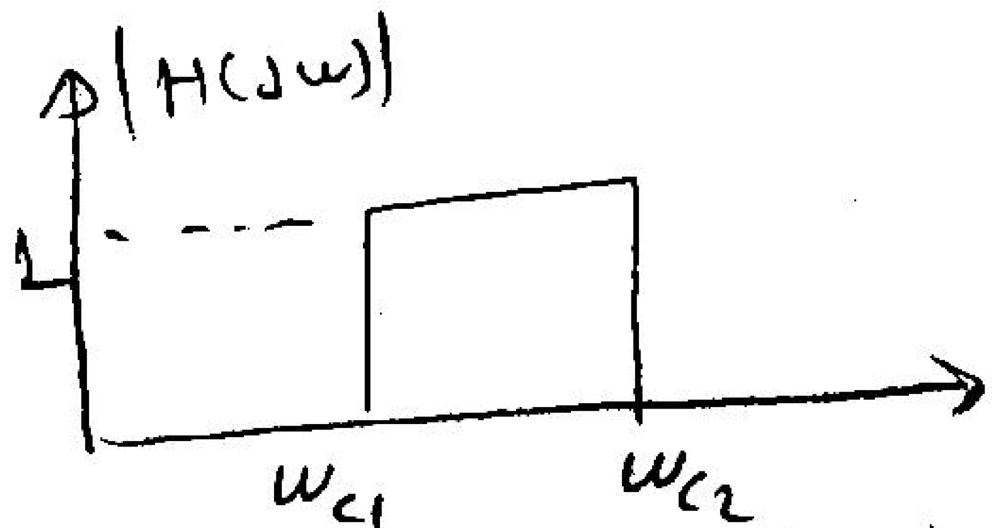
Bandwidth $\beta = \omega_{c2} - \omega_{c1}$

Center frequency

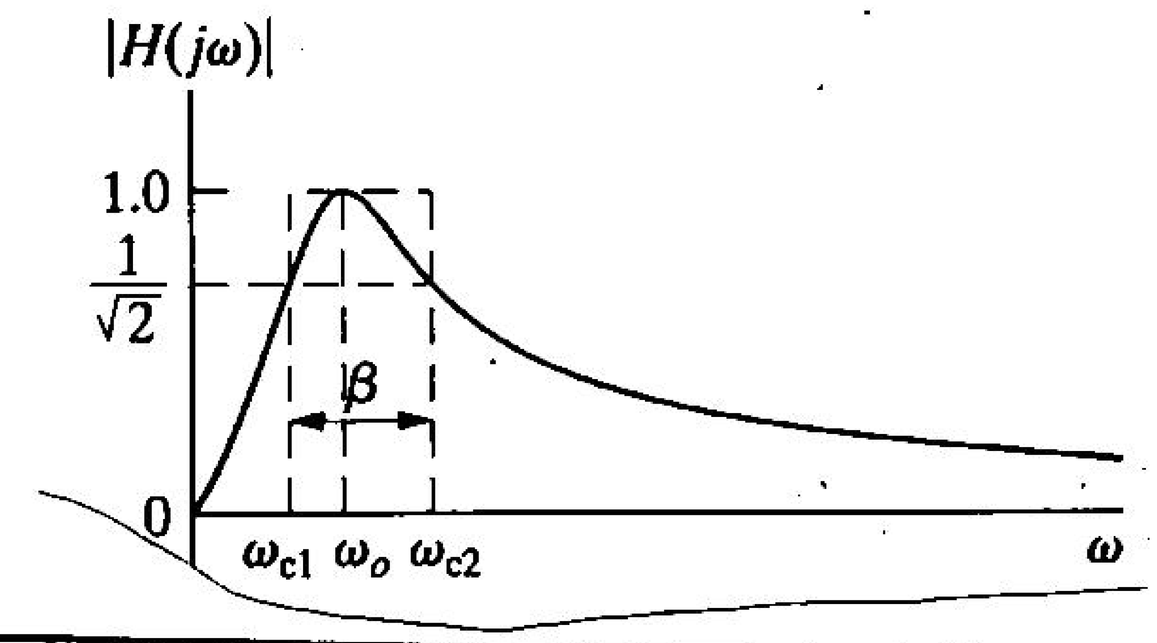
$$\omega_0 = \sqrt{\omega_{c1}\,\omega_{c2}}$$

Quality factor

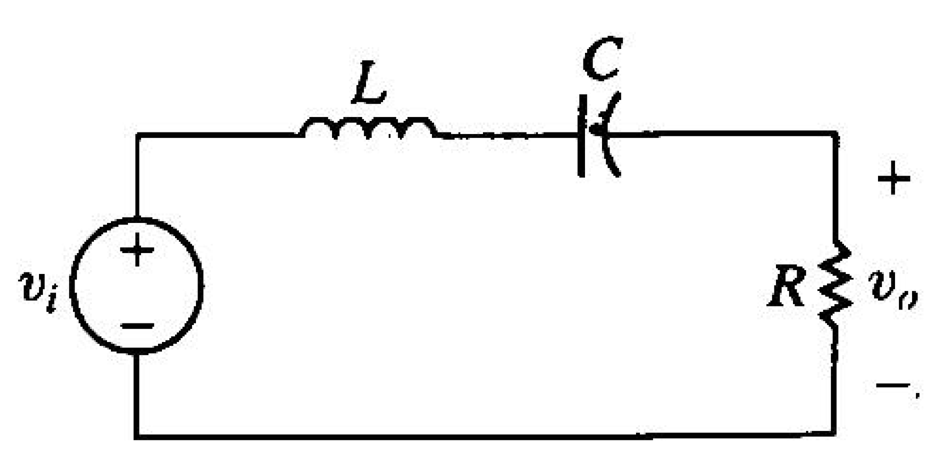
$$Q = \frac{\omega_0}{\beta}$$

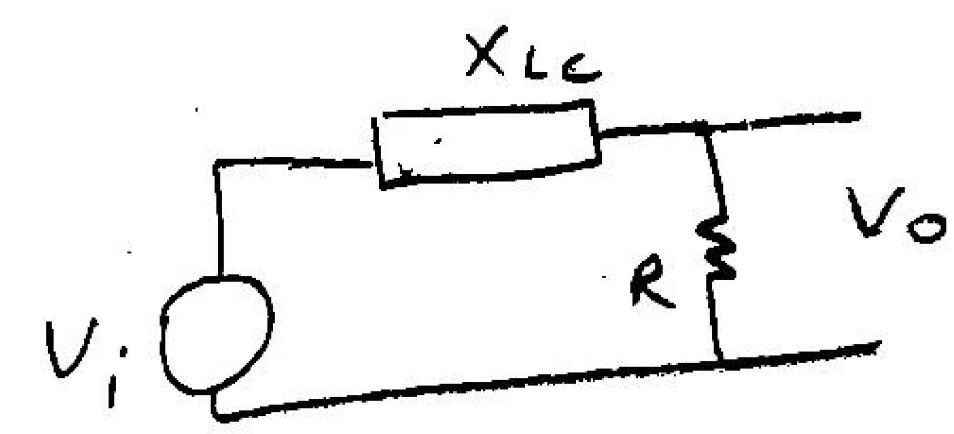


ideal



The Series *RLC* Circuit





$$V_o = \frac{R}{R + X_{LC}} V_i$$

$$X_{LC} = j\omega L + \frac{1}{j\omega C}$$

$$H(j\omega) = \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{R}{R + X_{LC}} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega CR}{j\omega CR + j^2\omega^2 LC + 1}$$

$$= \frac{j\omega CR}{1 - \omega^2 LC + j\omega CR}$$

$$|H(j\omega)| = \frac{\omega CR}{\sqrt{(1 - \omega^2 LC)^2 + (\omega CR)^2}}$$

$$\theta(j\omega) = 90 - \tan^{-1}\frac{\omega CR}{1 - \omega^2 LC}$$

it is clear that denominator (المقام) is minimum if $1-w^2LC=0$.

Thus $w^2LC=1$ $\quad w=\sqrt{\frac{1}{LC}}$ is the maximum of $|H(jw)|$

$$w=\sqrt{\frac{1}{LC}} \Rightarrow |H(jw)| = \frac{wCR}{\sqrt{0^2+(wCR)^2}} = 1$$

at $w=w_c$ $\quad |H(jw_c)| = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \frac{w_c RC}{\sqrt{(1-w_c^2LC)^2+(w_cCR)^2}} \Rightarrow \frac{1}{2} = \frac{(w_cRC)^2}{(1-w_c^2LC)^2+(w_cRC)^2}$$

$$(1-w_c^2LC)^2+(w_cRC)^2 = 2(w_cRC)^2$$

$$1-2w_c^2LC+w_c^4(LC)^2+(w_cRC)^2 = 2(w_cRC)^2$$

$$1-2w_c^2LC+w_c^4(LC)^2+(w_cRC)^2-2(w_cRC)^2 = 0$$

$$w_c^4(LC)^2+w_c^2\left[-2LC-(RC)^2\right]+1=0$$

$w_c^2=x \qquad w_c^4=x^2$

$$\omega_{c1} = -\frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\left(\frac{1}{LC}\right)}, \qquad (14.29)$$

$$\omega_{c2} = \frac{R}{2L}+\sqrt{\left(\frac{R}{2L}\right)^2+\left(\frac{1}{LC}\right)}. \qquad (14.30)$$

$\beta = w_{c2}-w_{c1}$

$Q=\frac{w_o}{\beta}=\sqrt{\frac{L}{CR^2}}$

$$\omega_{c1} = -\frac{\beta}{2}+\sqrt{\left(\frac{\beta}{2}\right)^2+\omega_o^2}, \qquad (14.34)$$

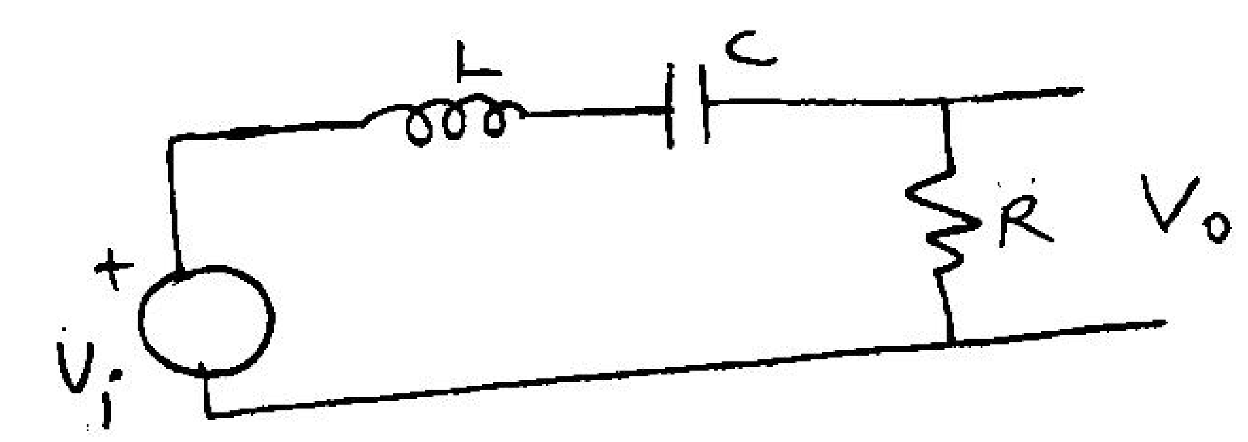
$$\omega_{c2} = \frac{\beta}{2}+\sqrt{\left(\frac{\beta}{2}\right)^2+\omega_o^2}. \qquad (14.35)$$

$$\omega_{c1} = \omega_o \cdot \left[-\frac{1}{2Q}+\sqrt{1+\left(\frac{1}{2Q}\right)^2}\right], \qquad (14.36)$$

$$\omega_{c2} = \omega_o \cdot \left[\frac{1}{2Q}+\sqrt{1+\left(\frac{1}{2Q}\right)^2}\right]. \qquad (14.37)$$

EXAMPLE 14.5

A graphic equalizer is an audio amplifier that allows you to select different levels of amplification within different frequency regions. Using the series *RLC* circuit in Fig. 14.19(a), choose values for *R*, *L*, and *C* that yield a bandpass circuit able to select inputs within the 1–10 kHz frequency band. Such a circuit might be used in a graphic equalizer to select this frequency band from the larger audio band (generally 0–20 kHz) prior to amplification.



Solution:

1 – 10 KHz

$f_{c1} = 1$ KHz $\quad f_{c2} = 10$ KHz

$w_{c1} = 2\pi \times 1000 \qquad w_{c2} = 2\pi\ 10\,000$

$$w_o = \sqrt{w_{c1}\, w_{c2}} = \sqrt{2\pi \times 1000\ \ 2\pi\ \ 10\,000}$$

$$w_o = 2\pi \times 3162.2$$

$$w_o = \sqrt{\frac{1}{LC}} \Rightarrow w_o^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{w_o^2 C}$$

$C = 1\,\mu f = 10^{-6} f$ (arbitrary)

$$L = \frac{1}{w_o^2 C} = \frac{1}{(2\pi\ 3162)^2 \cdot 10^{-6}} = 2.5\ 10^{-3}$$

$L = 2.5$ mH

$$\varphi = \frac{w_o}{\beta} = \frac{w_o}{w_{c2} - w_{c1}} = \frac{2\pi \times 3\,162}{2\pi\,(10\,000 - 1000)}$$

$$= 0.3514$$

$$\varphi = \sqrt{\frac{L}{CR^2}} \Rightarrow \varphi^2 = \frac{L}{CR^2}$$

$$R = \sqrt{\frac{L}{\varphi^2 C}} = \sqrt{\frac{2.5\ 10^{-3}}{0.35^2\ 10^{-6}}} =$$

$$= 142.8\,\Omega$$

Draw the amplitude spectrum for $C = 10^{-6}$

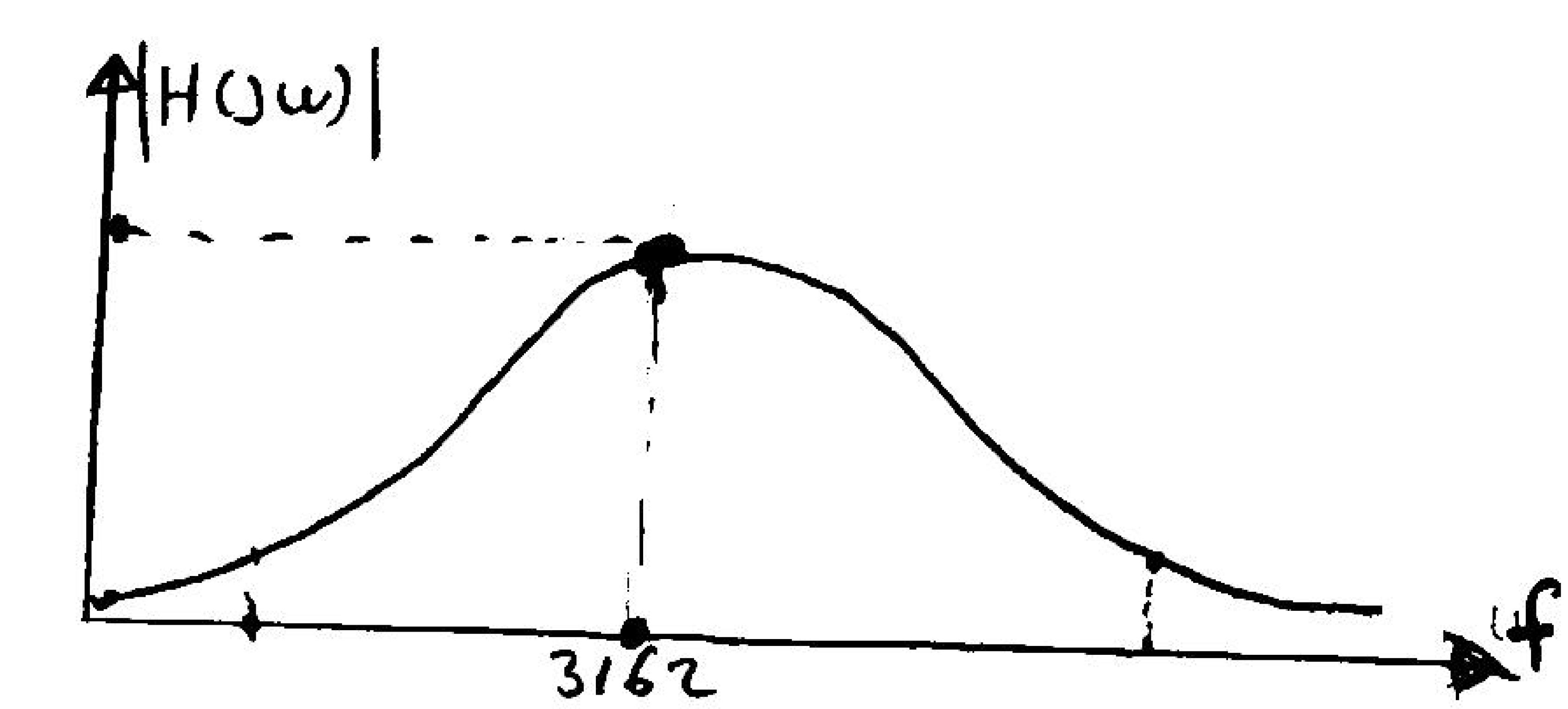
$R = 142.8 \qquad L = 2.5 \times 10^{-3}$

$$|H(jw)| = \frac{wCR}{\sqrt{(1 - w^2 LC)^2 + (wCR)^2}}$$

$$= \frac{w\ 10^{-6}\ 142}{\sqrt{(1 - w^2\ 2.5 \times 10^{-3}\ 10^{-6})^2 + (w\ 10^{-6}\ 142)^2}}$$

for $w = 0 \Rightarrow H(jw) = 0$

f	w	$\lvert H(jw)\rvert$
0	0	0
1	6.28	0.0009
10	62.8	0.0090
1000	6283	0.7055
3160	19854	0.999986
3162	19867	0.999989
3170	19917	0.999988
5000	31415	0.9504
10 000	62831	0.7112
100000	628318	0.0906
∞	∞	0



EXAMPLE 14.6

a) Show that the *RLC* circuit in Fig. 14.22 is also a bandpass filter by deriving an expression for the transfer function $H(s)$.

b) Compute the center frequency, ω_o.

c) Calculate the cutoff frequencies, ω_{c1} and ω_{c2}, the bandwidth, β, and the quality factor, Q.

d) Compute values for R and L to yield a bandpass filter with a center frequency of 5 kHz and a bandwidth of 200 Hz, using a 5 μF capacitor.

$C = 5\mu F$

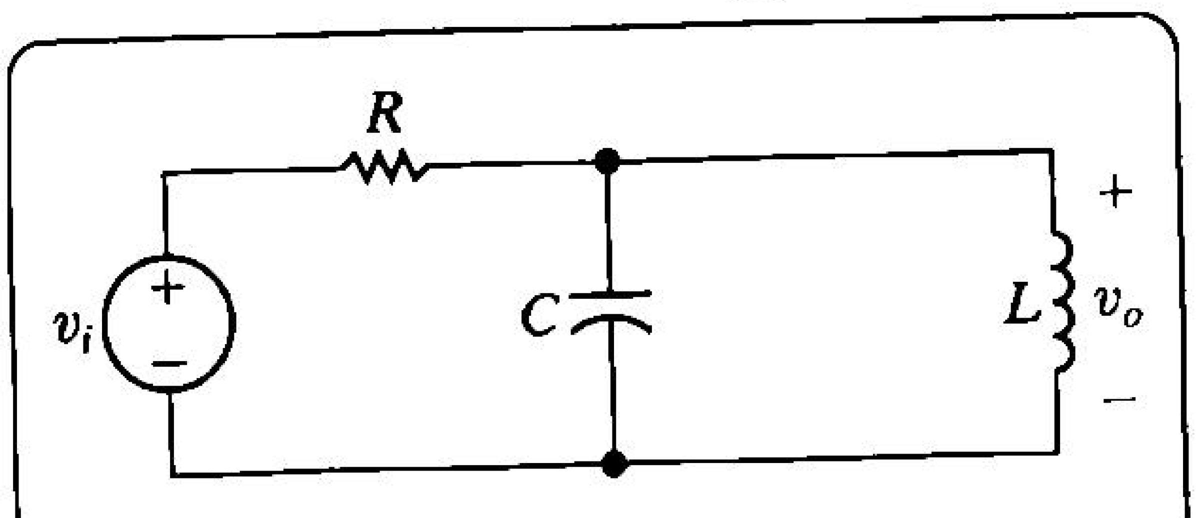
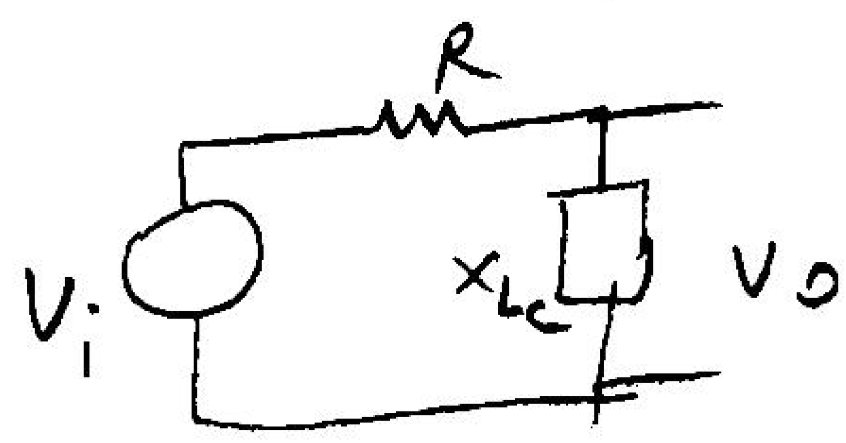


Figure 14.22 The circuit for Example 14.6.

solution



$X_{LC} = JwL // \frac{1}{Jwc}$

$$X_{LC} = \frac{JwL \cdot \frac{1}{Jwc}}{JwL + \frac{1}{Jwc}} = \frac{JwL}{1 - w^2LC}$$

$$H(Jw) = \frac{V_o(Jw)}{V_i(Jw)} = \frac{X_{LC}}{X_{LC} + R} = \frac{\frac{JwL}{1-w^2LC}}{\frac{JwL}{1-w^2LC} + R}$$

$$= \frac{JwL}{JwL + R(1 - w^2LC)}$$

$$|H(Jw)| = \frac{wL}{\sqrt{[R(1-w^2LC)]^2 + (wL)^2}}$$

$$\theta(Jw) = 90^\circ - \tan^{-1}\frac{wL}{R(1-w^2LC)}$$

if $1 - w^2LC = 0 \Rightarrow |H(Jw)| = 1$

maximum value of $|H(Jw)|$ is 1

$$1 - w_o^2LC = 0 \Rightarrow w_o = \sqrt{\frac{1}{LC}}$$

at $w = w_c$ $|H(Jw_c)| = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \frac{w_cL}{\sqrt{[R(1-w_c^2LC)]^2 + (w_cL)^2}}$$

$$\frac{1}{2} = \frac{(w_cL)^2}{R^2(1-w_c^2LC)^2 + (w_cL)^2}$$

$$R^2(1-w_c^2LC)^2 + (w_cL)^2 = 2(w_cL)^2$$

$$R^2[1 - 2w_c^2LC + w_c^4(LC)^2] - (w_cL)^2 = 0$$

$$w_c^4(RLC)^2 + w_c^2(-2R^2LC - L^2) + R^2 = 0$$

$w_c^2 = x$ $\quad w_c^4 = x^2$

$$\omega_{c1} = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}},$$

$$\omega_{c2} = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}.$$

$$\beta = \omega_{c2} - \omega_{c1} = \frac{1}{RC}.$$

$$Q = \omega_o/\beta = \sqrt{\frac{R^2C}{L}}.$$

d) $w_o = 2\pi \times 5000$ $\quad C = 5 \times 10^{-6}$ $\quad \beta = 200$

$$R = \frac{1}{\beta C} = 159.15\Omega$$

$$L = \frac{1}{w_o^2C} = 202.6\ 10^{-6}$$

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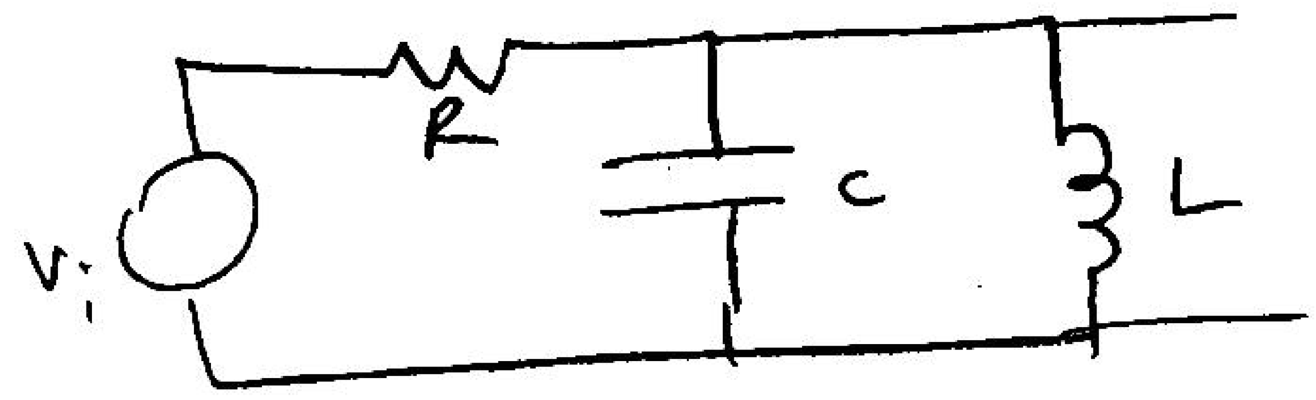
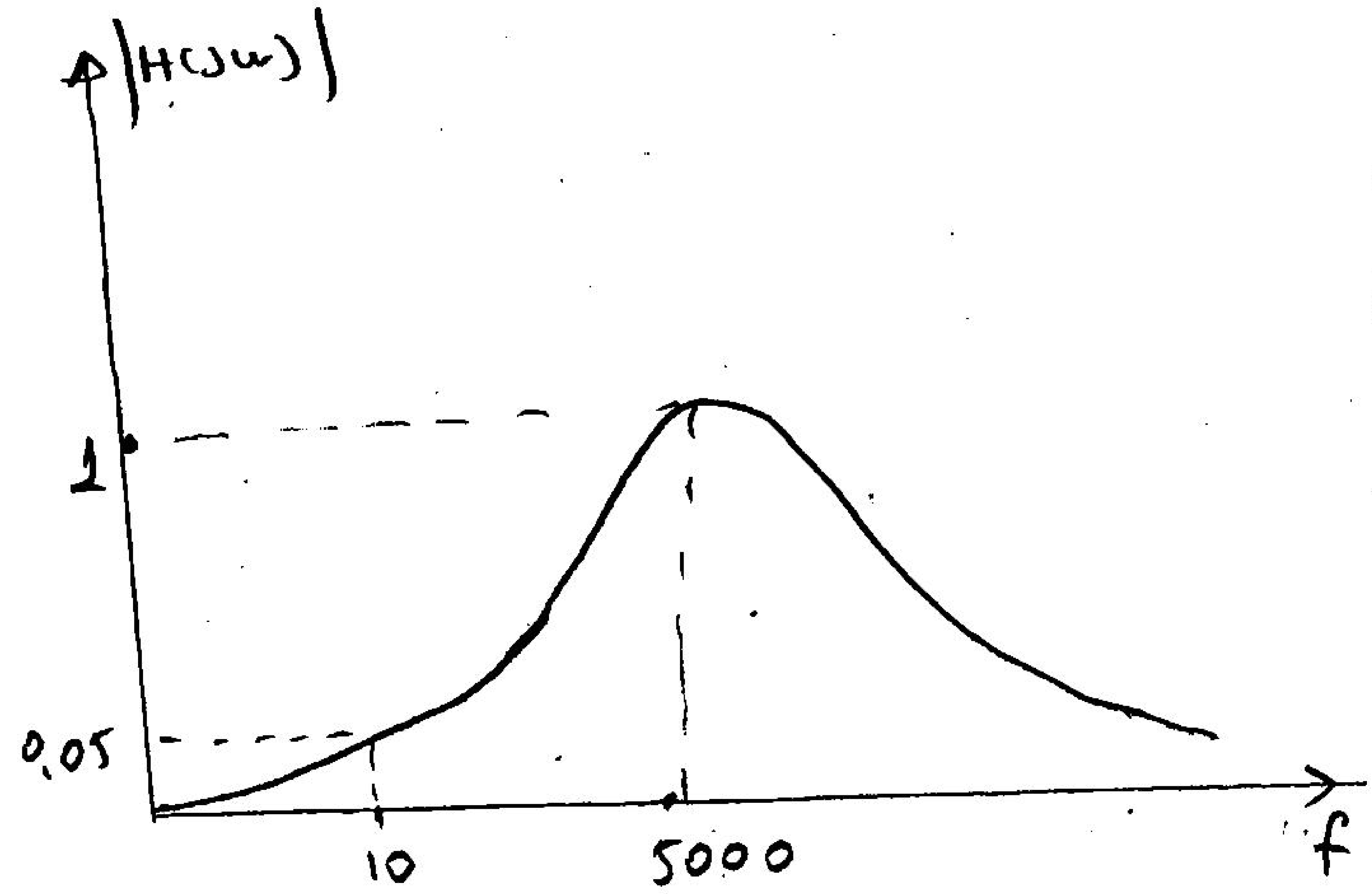
Draw the amplitude spectrum for $C = 5\mu F$ $R = 160\Omega$ $L = 2 \times 10^{-4}$



$$|H(j\omega)| = \frac{\omega L}{\sqrt{[R(1-\omega^2 LC)]^2 + (\omega L)^2}}$$

$$|H(j\omega)| = \frac{\omega \, 2\times10^{-4}}{\sqrt{[160(1-\omega^2 \, 2\times10^{-4} \times 5\times10^{-6})]^2 + (\omega \times 2\times10^{-4})^2}}$$

f	ω	|H(ω)|
0	0	0
1	6.28	0.0050
10	62.8	0.0502
100	628	0.449
1000	6283	0.9822
4990	31353	0.99999
5000	31416	1.000
5010	31479	0.99999
5100	32044	0.9999
10000	62832	0.9828
100000	628319	0.787
∞	0	0



Example

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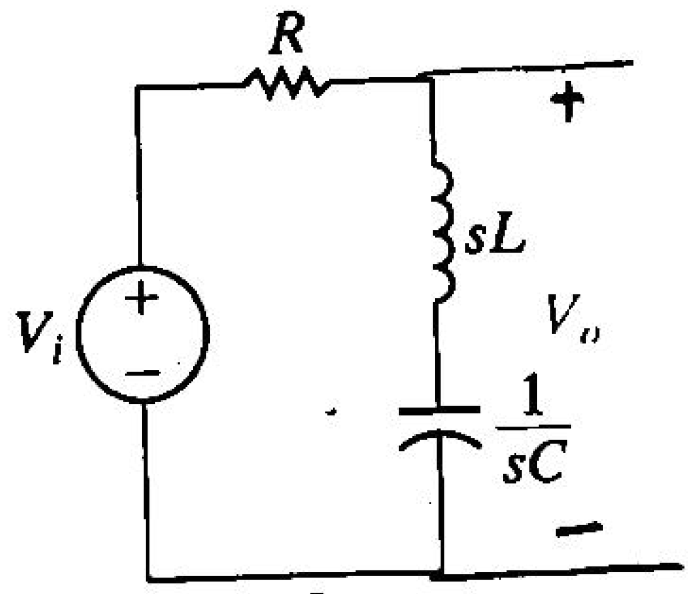
Example



$S = jw$

$$H(s) = \frac{s^2 + 1/LC}{s^2 + (R/L)s + 1/LC}$$

$\omega_o = \sqrt{1/LC}$ $\beta = R/L$

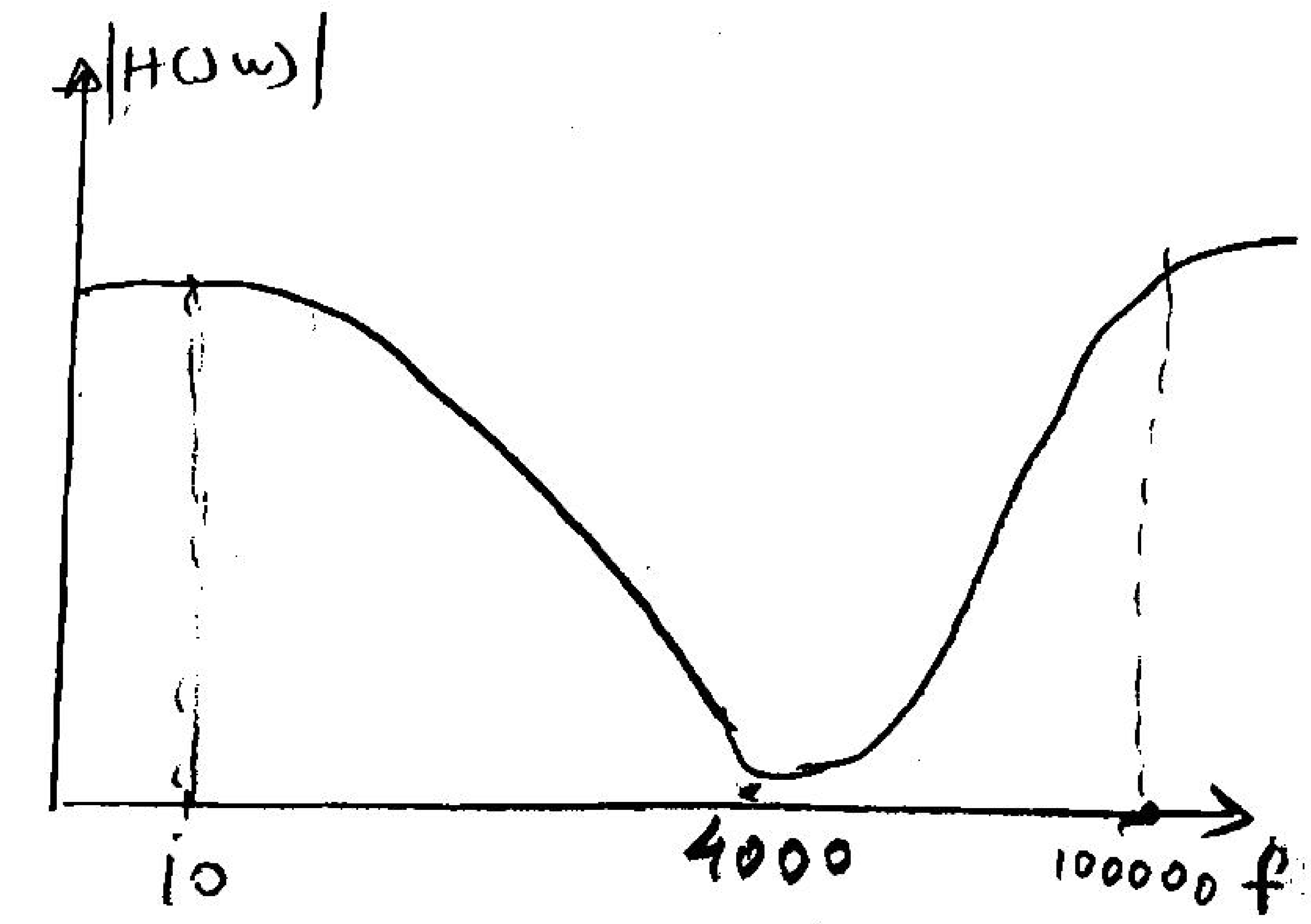
Draw amplitude spectrum for

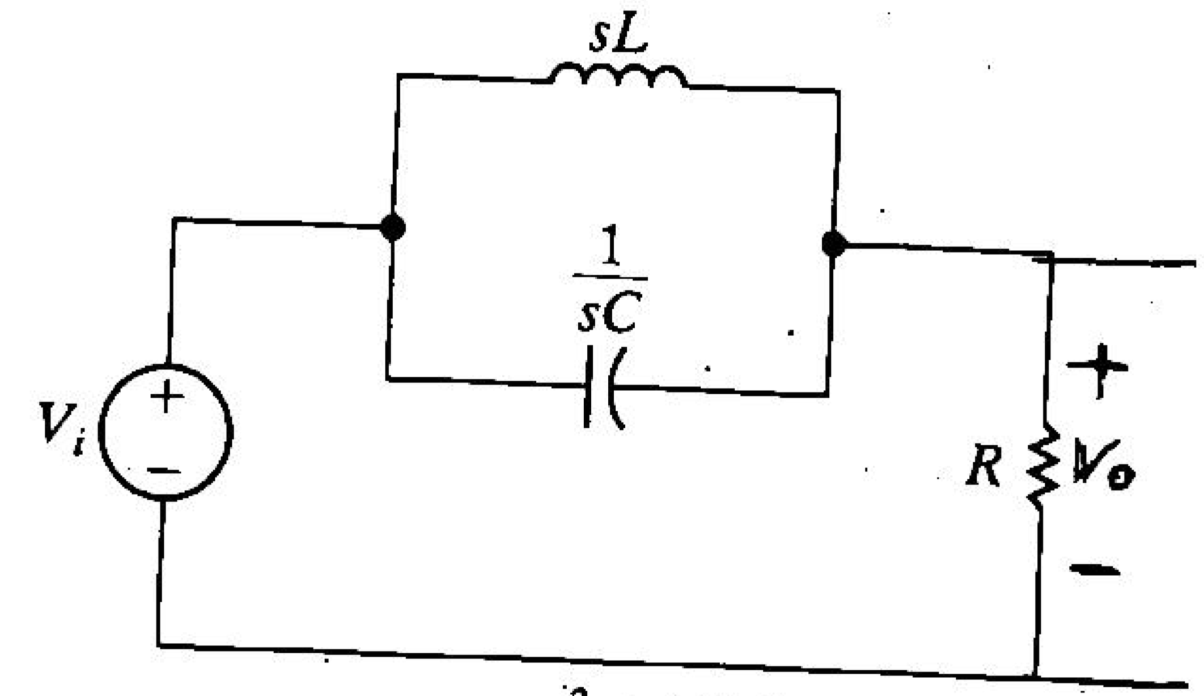
$C = 0.5275\ 10^{-6}\ F$ $L = 3\times10^{-3}$ $R = 15\,\Omega$

$$H(jw) = \frac{(jw)^2 + 1/LC}{(jw)^2 + \left(\frac{R}{L}\right) jw + \frac{1}{LC}}$$

$$|H(jw)| = \frac{\left|-w^2 + 1/LC\right|}{\sqrt{\left(\frac{1}{LC} - w^2\right)^2 + \left(\frac{Rw}{L}\right)^2}}$$

f	w	|H(jw)|
0	0	1
1	6.28	1
10	62.8	1
100	628	0.9999
1000	6283	0.9986
3990	25069	0.272
4000	25132	0.02
4010	25195	0.231
5000	31415	0.914
10.000	62831	0.9955
∞	∞	1





$S = jw$

$$H(s) = \frac{s^2 + 1/LC}{s^2 + s/RC + 1/LC}$$

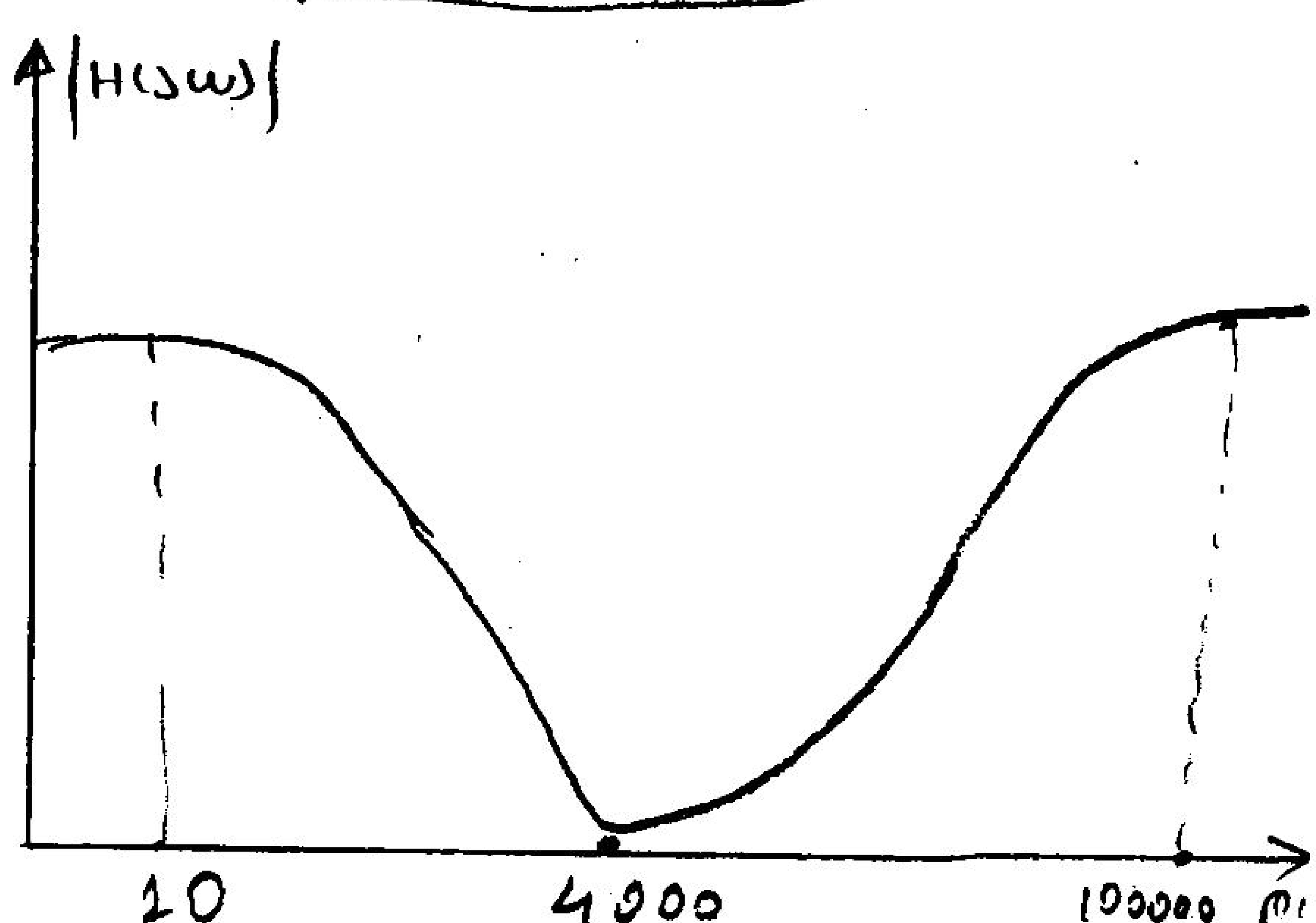
$\omega_o = \sqrt{1/LC}$ $\beta = 1/RC$

Draw amplitude spectrum for $C = 0.5\ 10^{-6}$ $L = 3\times10^{-3}$ $R = 15$

$$H(jw) = \frac{(jw)^2 + 1/LC}{(jw)^2 + \frac{1}{RC} jw + \frac{1}{LC}}$$

$$|H(jw)| = \frac{\left|\frac{1}{LC} - w^2\right|}{\sqrt{\left(\frac{1}{LC} - w^2\right)^2 + \left(\frac{w}{RC}\right)^2}}$$

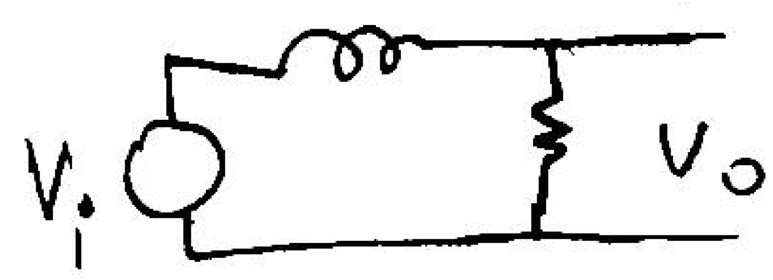
f	w	H(jw)
0	0	1
1	6.28	1
10	62.8	0.9999
100	628.3	0.9922
1000	6283	0.538
3990	25069	0.001
4000	25132	0.0001
4010	25195	0.001
5000	31415	0.0891
10000	62831	0.3853
100000	628318	0.9803
∞	∞	1



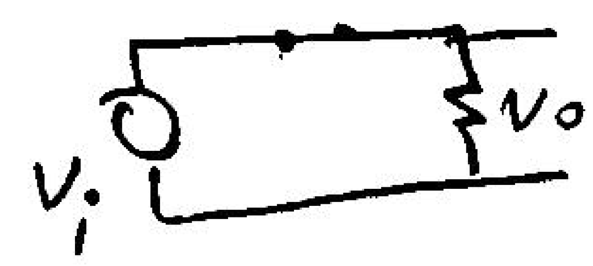
Frequency Response by Inspection

$Z_L = j\omega L$ $\quad \omega = 0 \Rightarrow Z_L = 0 \Rightarrow$ Short circuit

$\omega = \infty \Rightarrow Z_L = \infty \Rightarrow$ Open circuit

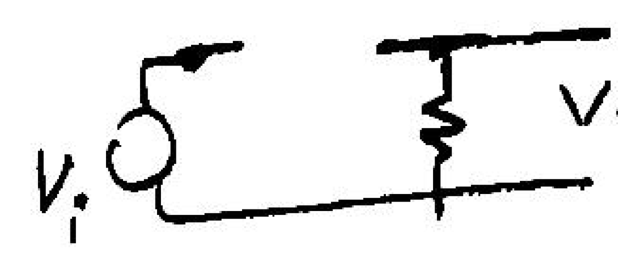


$\omega = 0 \Rightarrow$

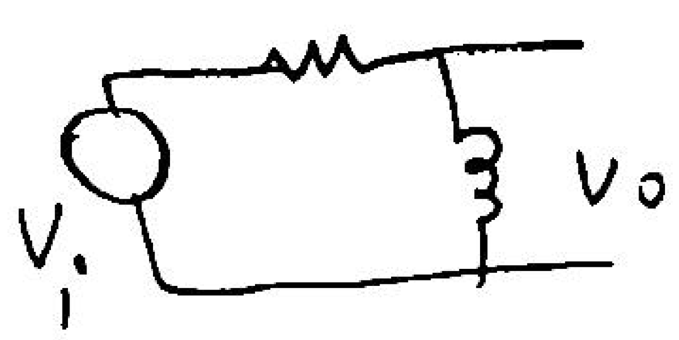


$V_o = V_i \quad |H| = \left|\frac{V_o}{V_i}\right| = 1$

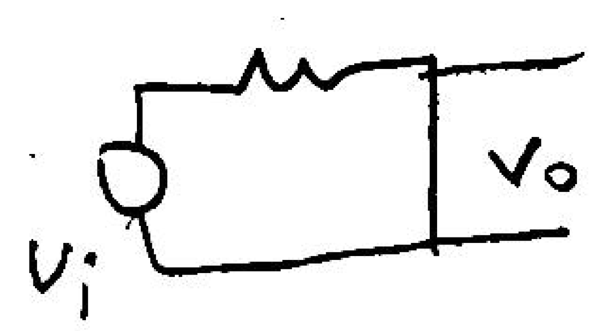
$\omega = \infty \Rightarrow$



$V_o = 0 \quad |H| = \left|\frac{V_o}{V_i}\right| = 0$

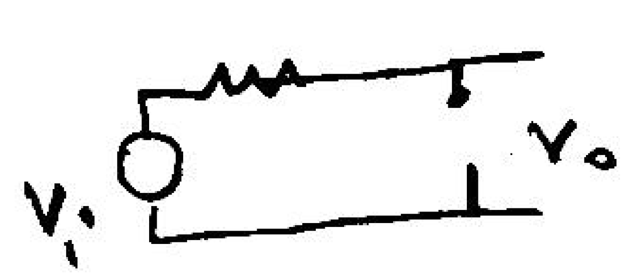


$\omega = 0 \Rightarrow$

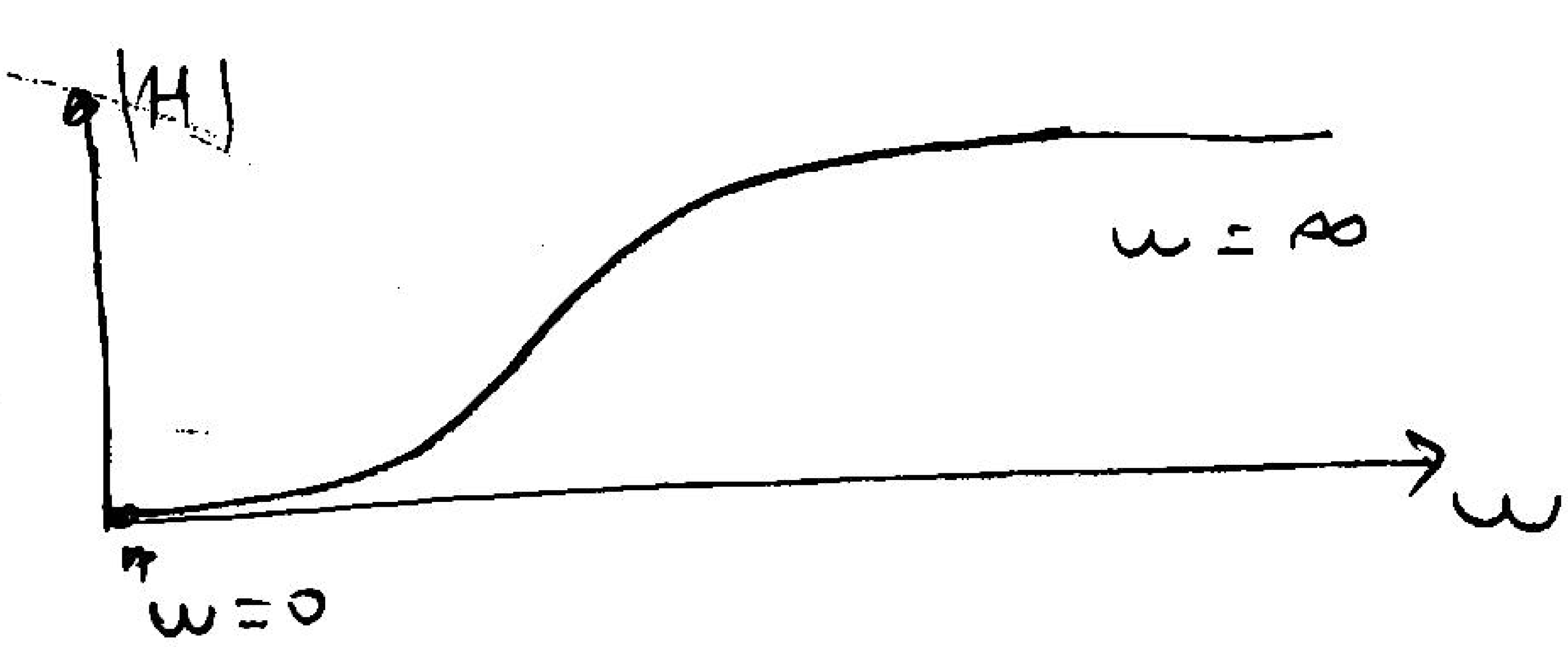


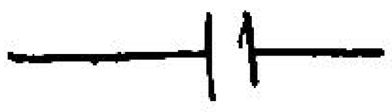
$V_o = 0 \quad |H| = \left|\frac{V_o}{V_i}\right| = 0$

$\omega = \infty \Rightarrow$

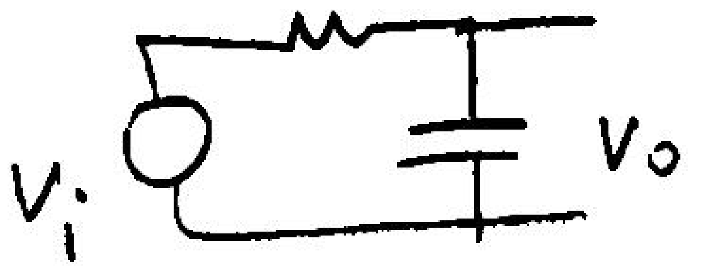


$V_o = V_i \quad |H| = \left|\frac{V_o}{V_i}\right| = 1$

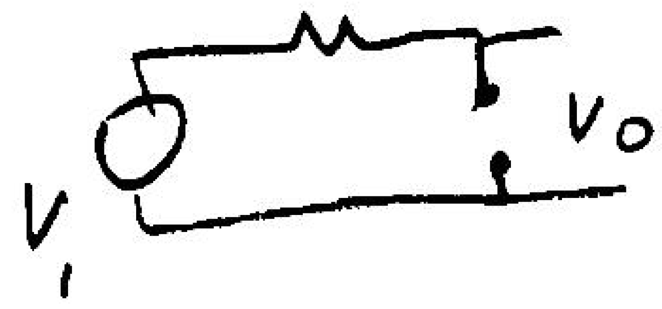


 $Z_c = \frac{1}{j\omega C}$ $\quad \omega = 0 \quad Z_c = \frac{1}{0} = \infty \Rightarrow$ Open circuit

$\omega = \infty \quad Z_c = \frac{1}{\infty} = 0 \Rightarrow$ short circuit

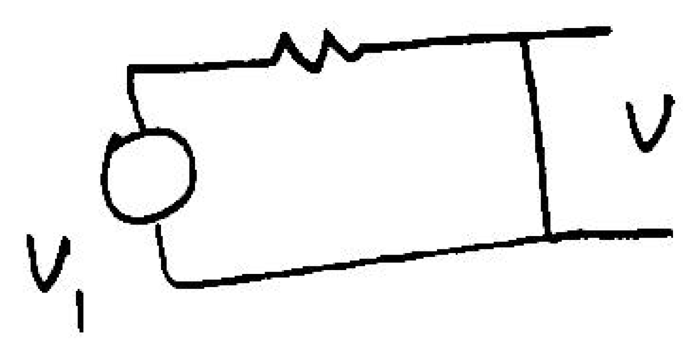


$\omega = 0$

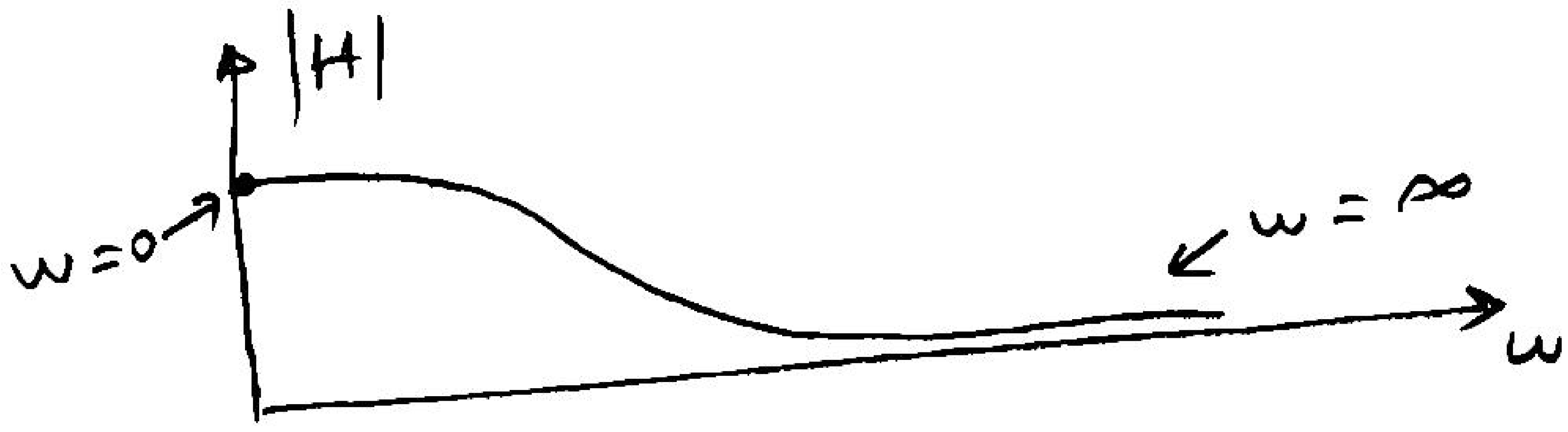


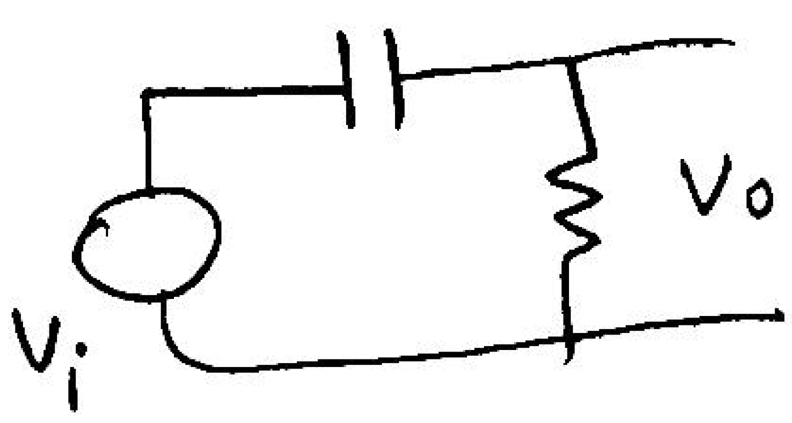
$V_o = V_i \quad |H| = \left|\frac{V_o}{V_i}\right| = 1$

$\omega = \infty$



$V_o = 0 \quad |H| = \left|\frac{V_o}{V_i}\right| = 0$

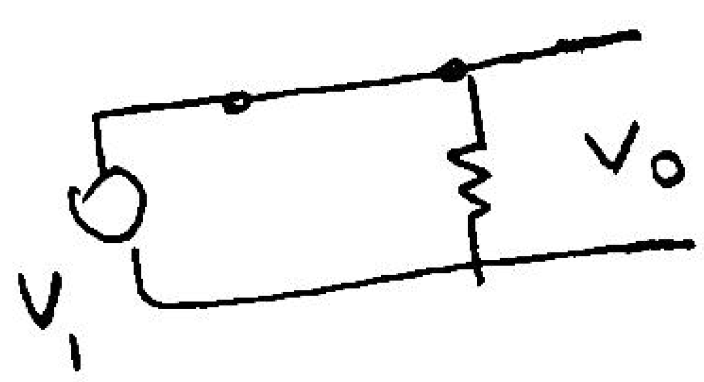




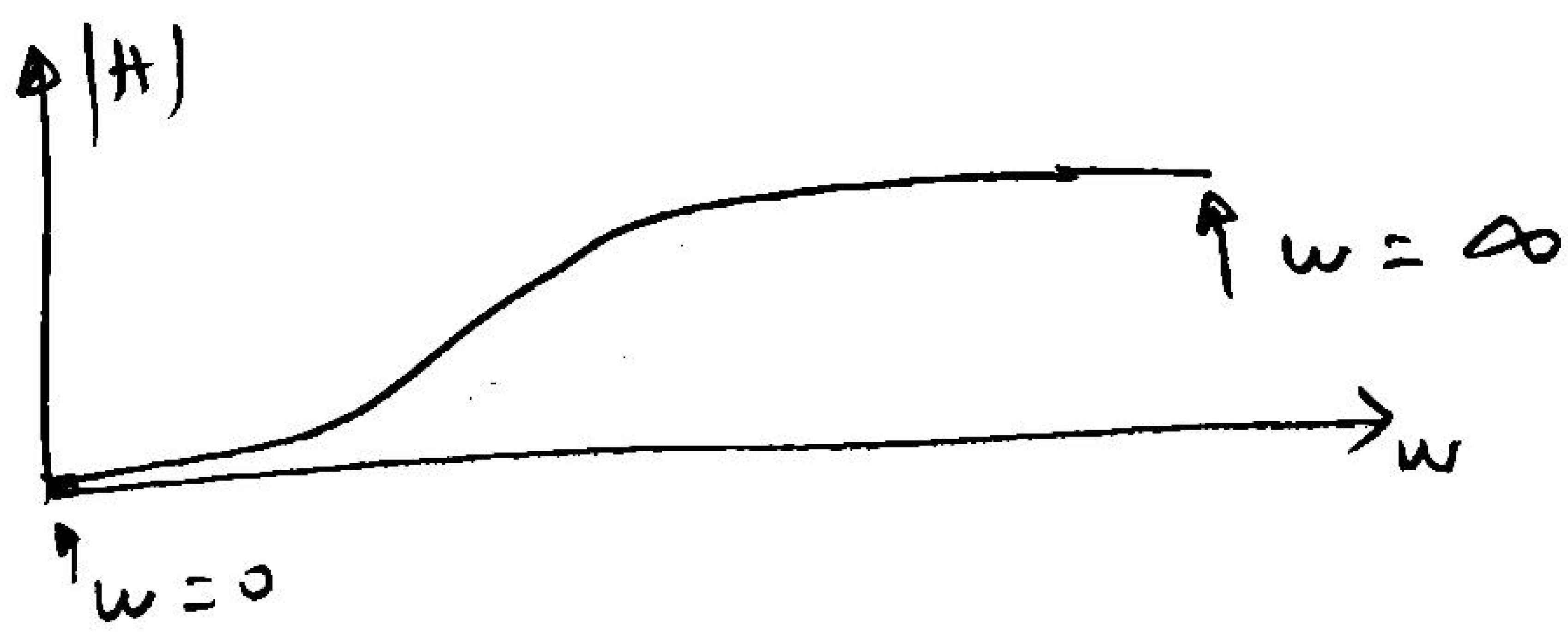
$\omega = 0$

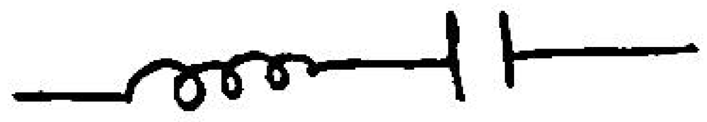
$V_o = 0 \quad |H| = \left|\frac{V_o}{V_i}\right| = 0$

$\omega = \infty$



$V_o = V_i \quad |H| = \left|\frac{V_o}{V_i}\right| = 1$

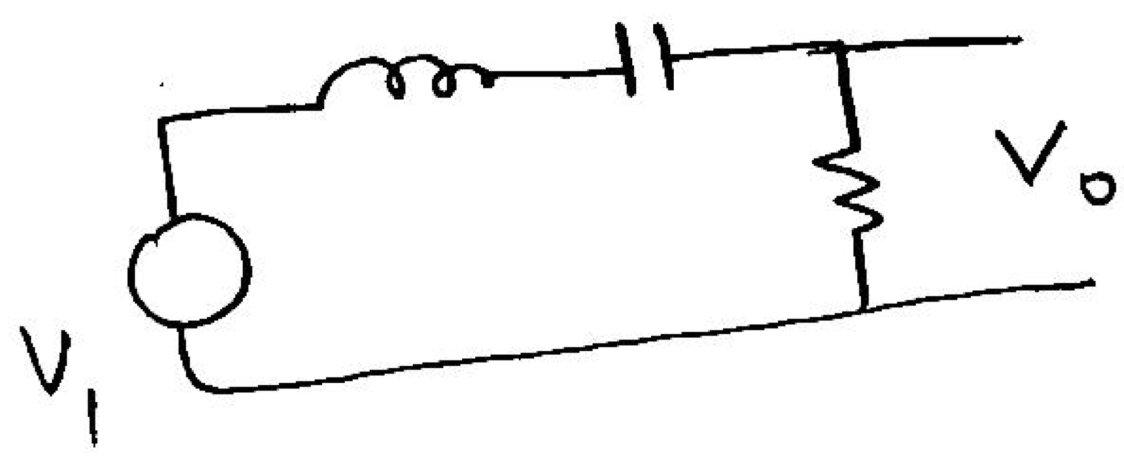



$$Z = jwL + \frac{1}{jwC} = jwL - \frac{j}{wC} = j\left[wL - \frac{1}{wC}\right]$$

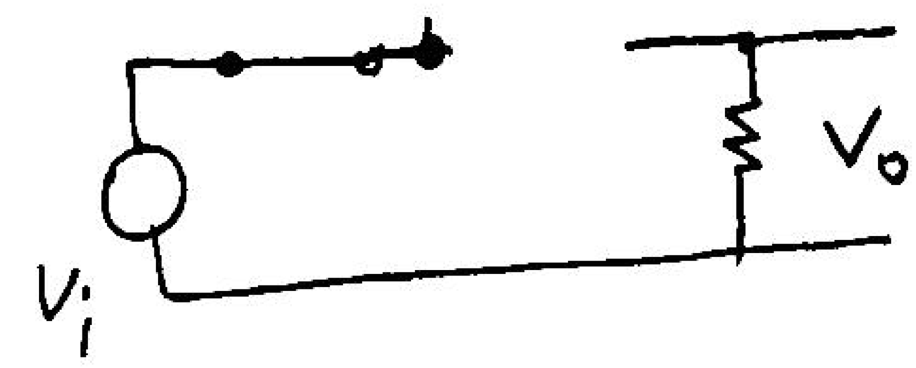
if $wL - \frac{1}{wC} = 0$ then $Z = 0$ short circuit

$$wL - \frac{1}{wC} = 0 \Rightarrow wL = \frac{1}{wC} \Rightarrow w^2 = \frac{1}{LC} \Rightarrow w = \frac{1}{\sqrt{LC}}$$

if $w = \frac{1}{\sqrt{LC}}$  $\Rightarrow$ short circuit
(serial resonance)



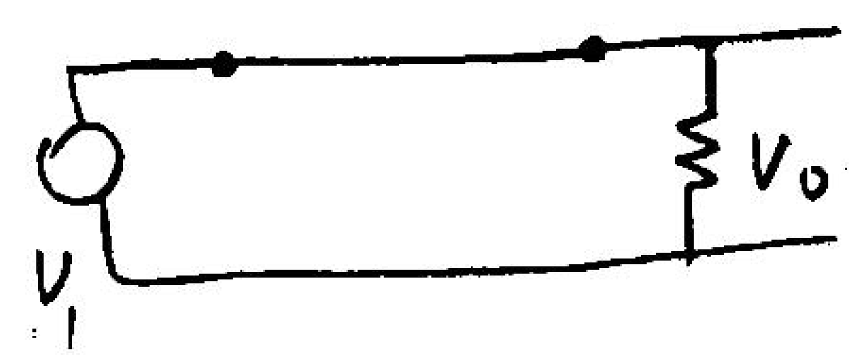
$w = 0$



$V_o = 0$

$|H| = \left|\frac{V_o}{V_1}\right| = 0$

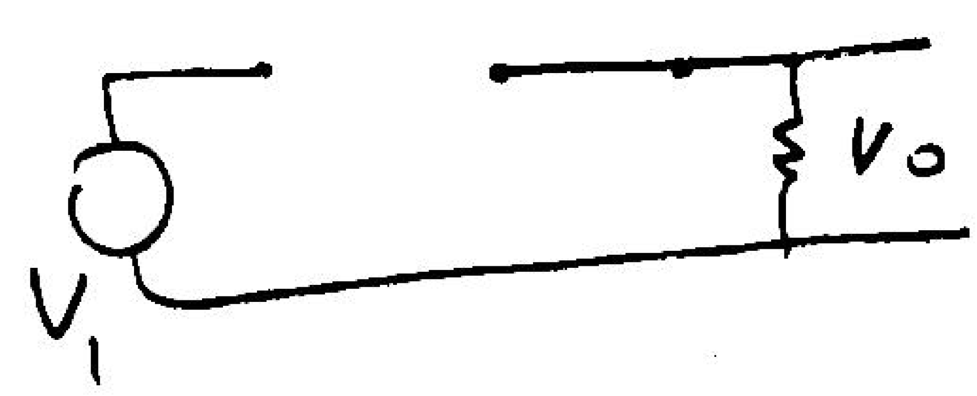
$w = \frac{1}{\sqrt{LC}}$



$V_o = V_1$

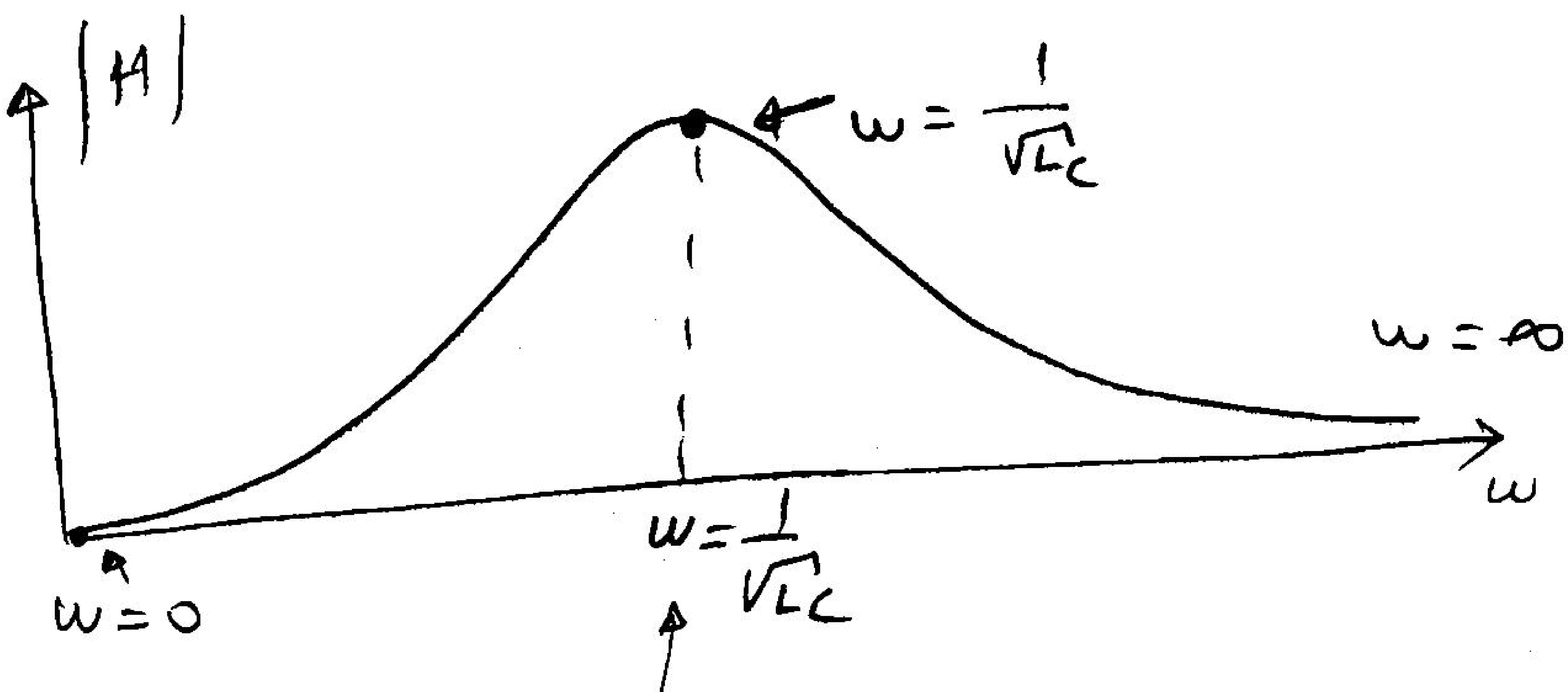
$|H| = \left|\frac{V_o}{V_1}\right| = 1$

$w = \infty$

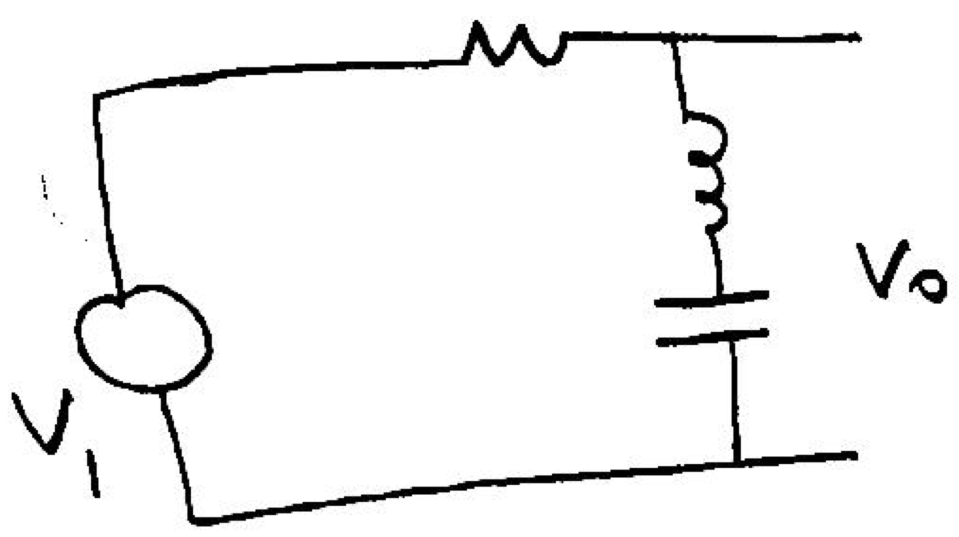


$V_o = 0$

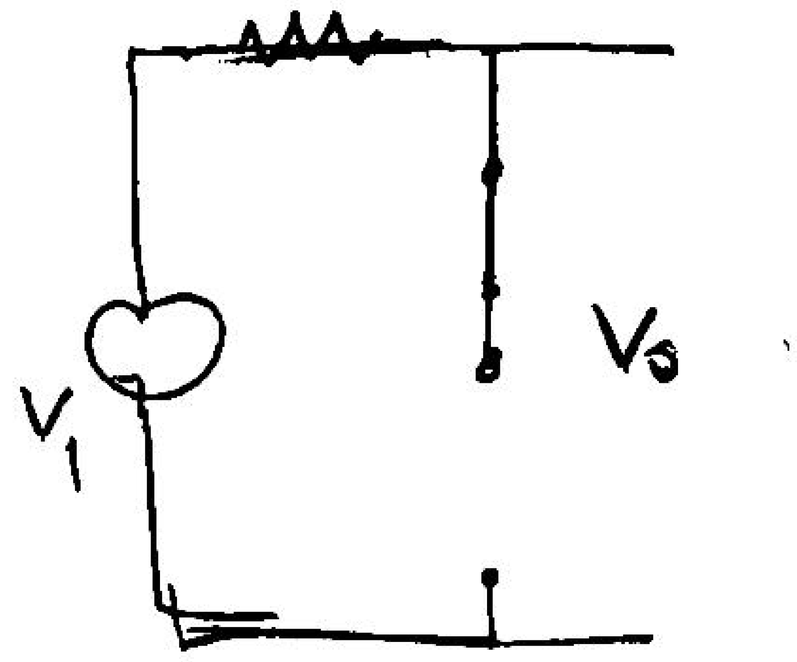
$|H| = \left|\frac{V_o}{V_1}\right| = 0$



resonance frequency

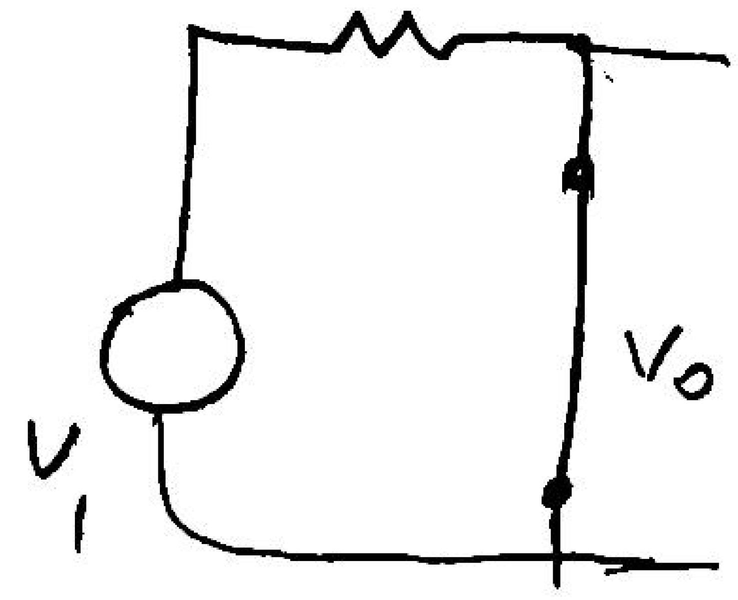


$\omega = 0$



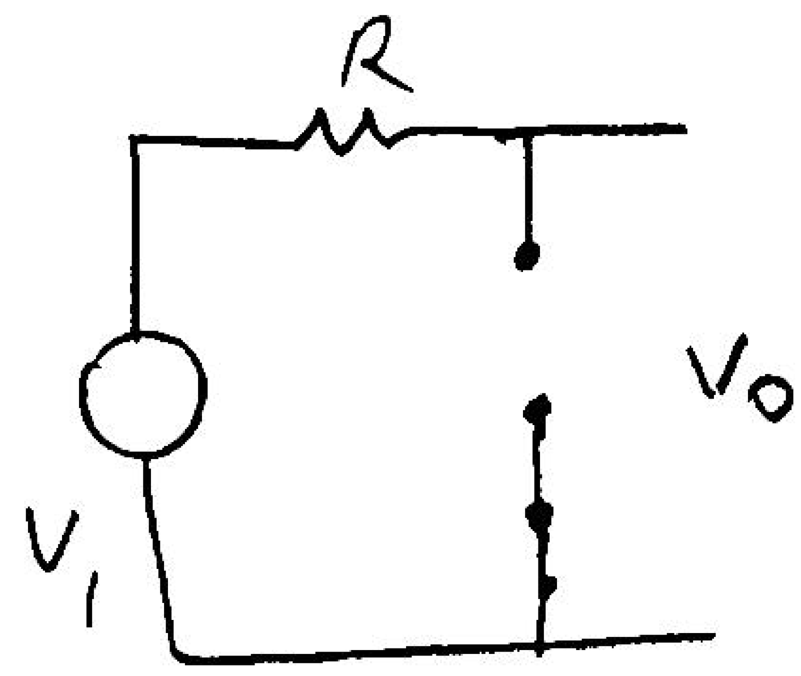
$V_o = V_1 \qquad |H| = \left|\frac{V_o}{V_1}\right| = 1$

$\omega = \frac{1}{\sqrt{LC}}$

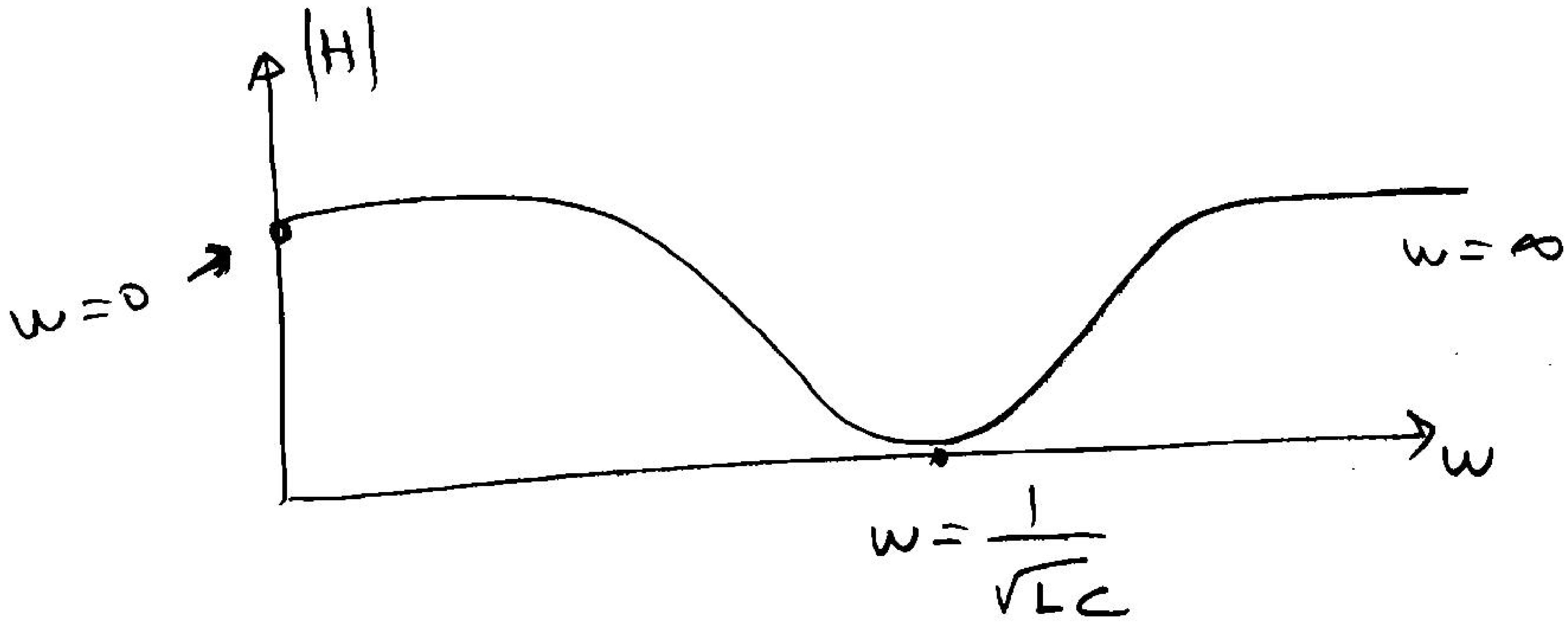


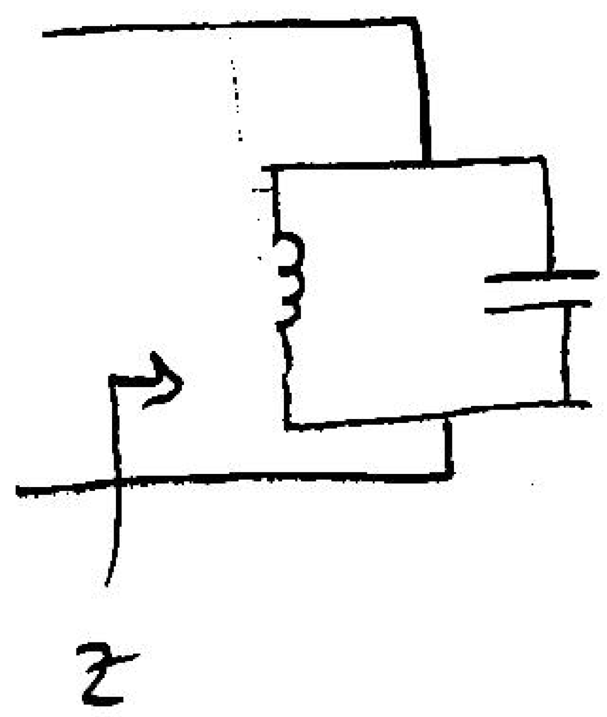
$V_o = 0 \qquad |H| = \left|\frac{V_o}{V_1}\right| = 0$

$\omega = \infty$



$V_o = V_1 \qquad |H| = \left|\frac{V_o}{V_1}\right| = 1$





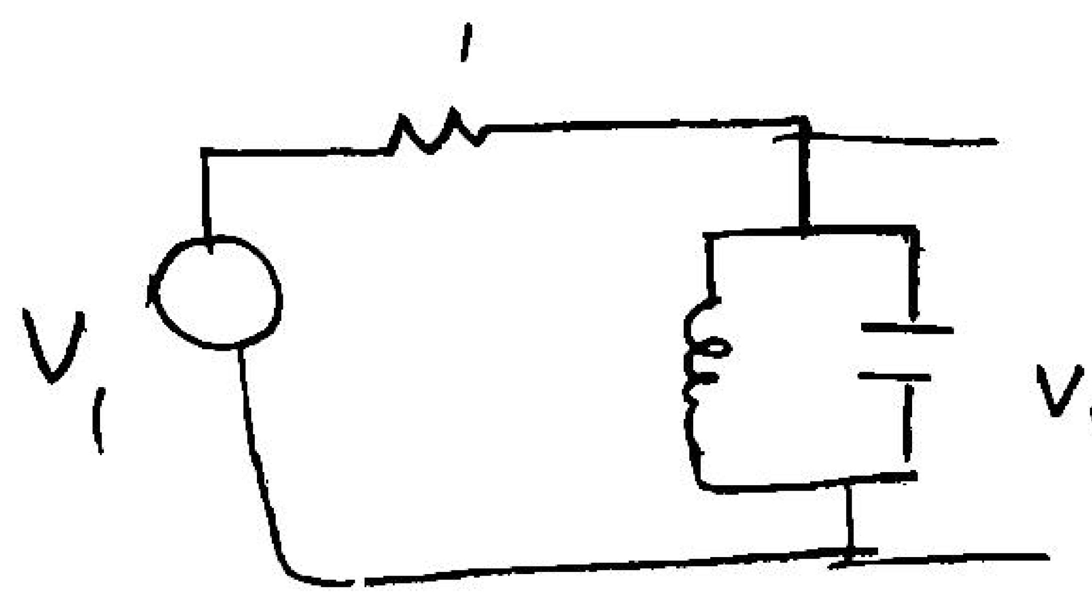
$$\frac{1}{Z} = \frac{1}{jwL} + \frac{1}{\frac{1}{jwC}} = \frac{1}{jwL} + jwC = -\frac{j}{wL} + jwC$$

$$\frac{1}{Z} = j\left(wC - \frac{1}{wL}\right) \Rightarrow Z = \frac{1}{j\left(wC - \frac{1}{wL}\right)}$$

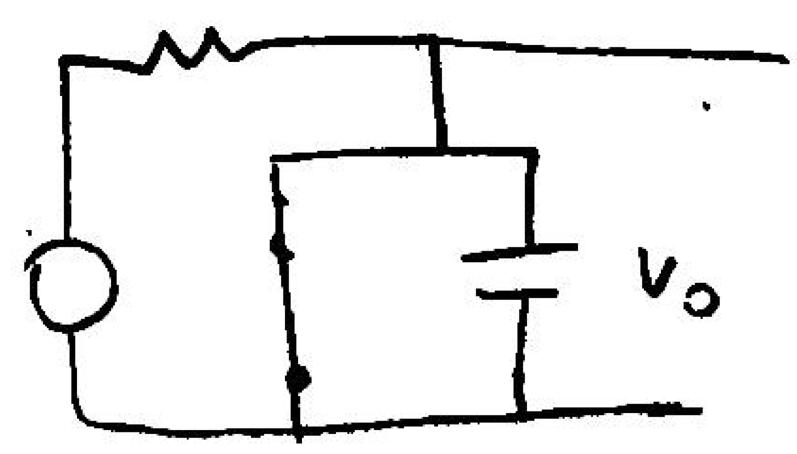
if $wC - \frac{1}{wL} = 0 \Rightarrow Z = \frac{1}{0} = \infty$ (open circuit)

$$wC - \frac{1}{wL} = 0 \Rightarrow wC = \frac{1}{wL} \Rightarrow w^2 = \frac{1}{LC} \Rightarrow w = \frac{1}{\sqrt{LC}}$$

Parallel resonance



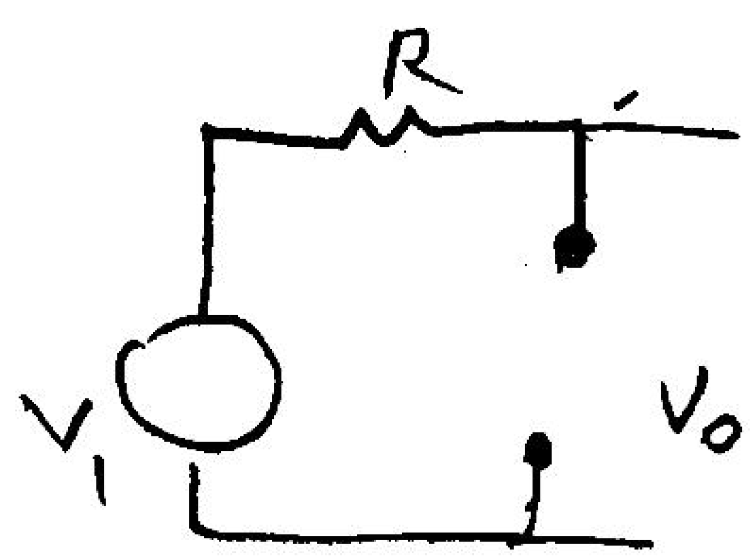
$w = 0 \Rightarrow$



$V_o = 0$

$|H| = \left|\frac{V_o}{V_1}\right| = 0$

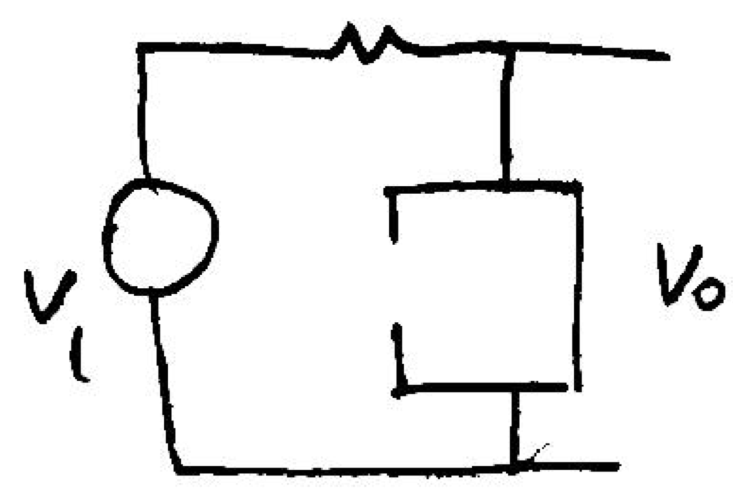
$w = \frac{1}{\sqrt{LC}}$



$V_o = V_1$

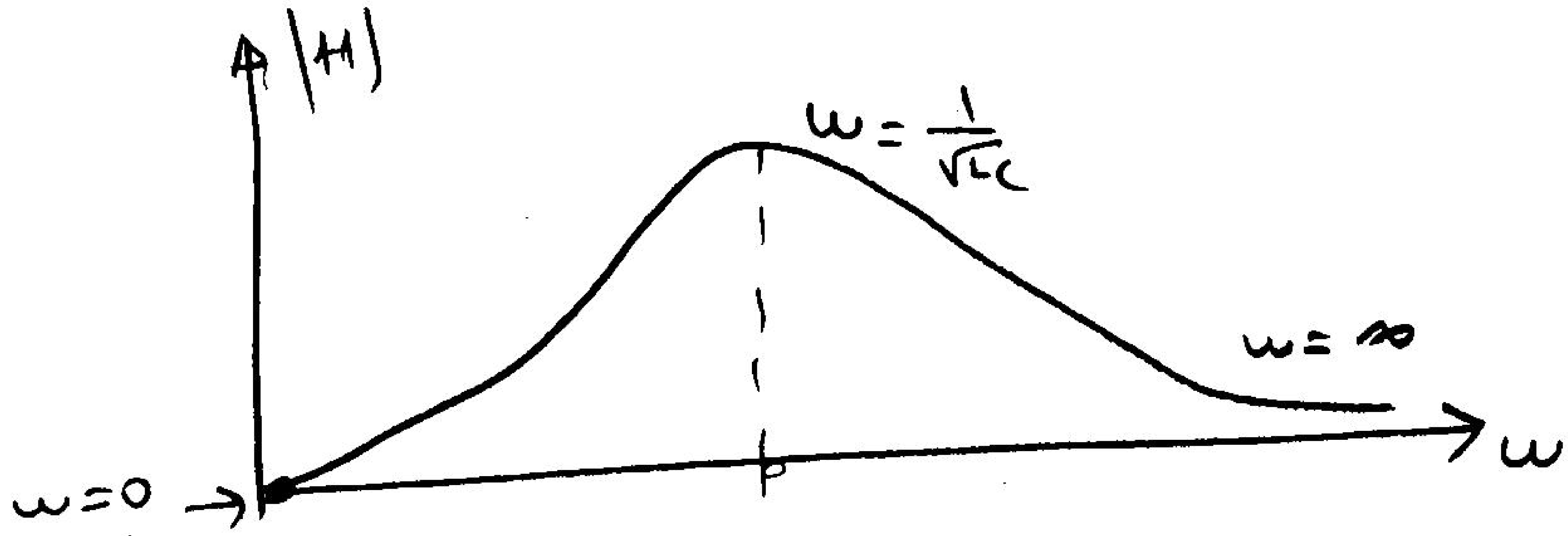
$|H| = \left|\frac{V_o}{V_1}\right| = 1$

$w = \infty$



$V_o = 0$

$|H| = \left|\frac{V_o}{V_1}\right| = 0$



Parallel resonance circuit response