

MATLAB

`>> roots([1 2 3])`
finds the roots of
 $x^2 + 2x + 3 = 0$

`>> roots([1 2 3 5])`

$$x^3 + 2x^2 + 3x + 5 = 0$$

`>> roots([1 0 0 0 2])`

$$x^4 + 2 = 0$$

`>> conv([1 2], [1 2 3])`

$(x+2)(x^2 + 2x + 3)$
multiplies two polynomials

`>> conv([1 3 0 2], [1 5 3])`

$$(x^3 + 3x^2 + 2)(x^2 + 5x + 3)$$

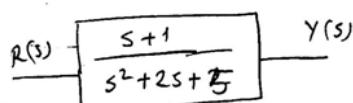
`>> P = [1 2 3]`

`>> Q = [2 3]`

`>> conv(P, Q)`

$$(x^2 + 2x + 3)(2x + 3)$$

Step response



$$\text{if } X(s) = \frac{1}{s} \quad Y(t) = ?$$

`s = tf('s')` write once

$$H = (s+1)/(s^2 + 2s + 5)$$

`step(H)`

`impulse(H)`

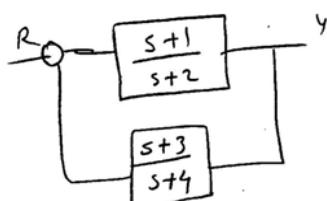


$$G_1 = (s+2)/(s+4)$$

$$G_2 = (s+3)/(s^2 + 2s + 3)$$

`G3 = series(G1, G2)`

$$G_3 = \frac{(s+2)(s+3)}{(s+4)(s^2 + 2s + 3)}$$



$$G_1 = (s+1)/(s+2)$$

$$G_2 = (s+3)/(s+4)$$

`G3 = feedback(G1, G2)`

$$G_3 = \frac{G_1}{1 + G_1 G_2}$$

`step(G3)`

Step response of closed loop

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Blok Diyagramlar ve İşaret Akış Diyagramları

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1

Karmaşık sistemler bir çok alt sistemin bir araya gelmesiyle oluşmuştur.

Eğer karmaşık bir sistemi tek bir transfer fonksiyonuna veya alt sisteme indirgelyebilirsek tüm sistemi analitik olarak daha kolay inceleyebiliriz.

Karmaşık sistemleri tek bir transfer fonksiyonuna iki yöntemle indirgelyebiliriz:

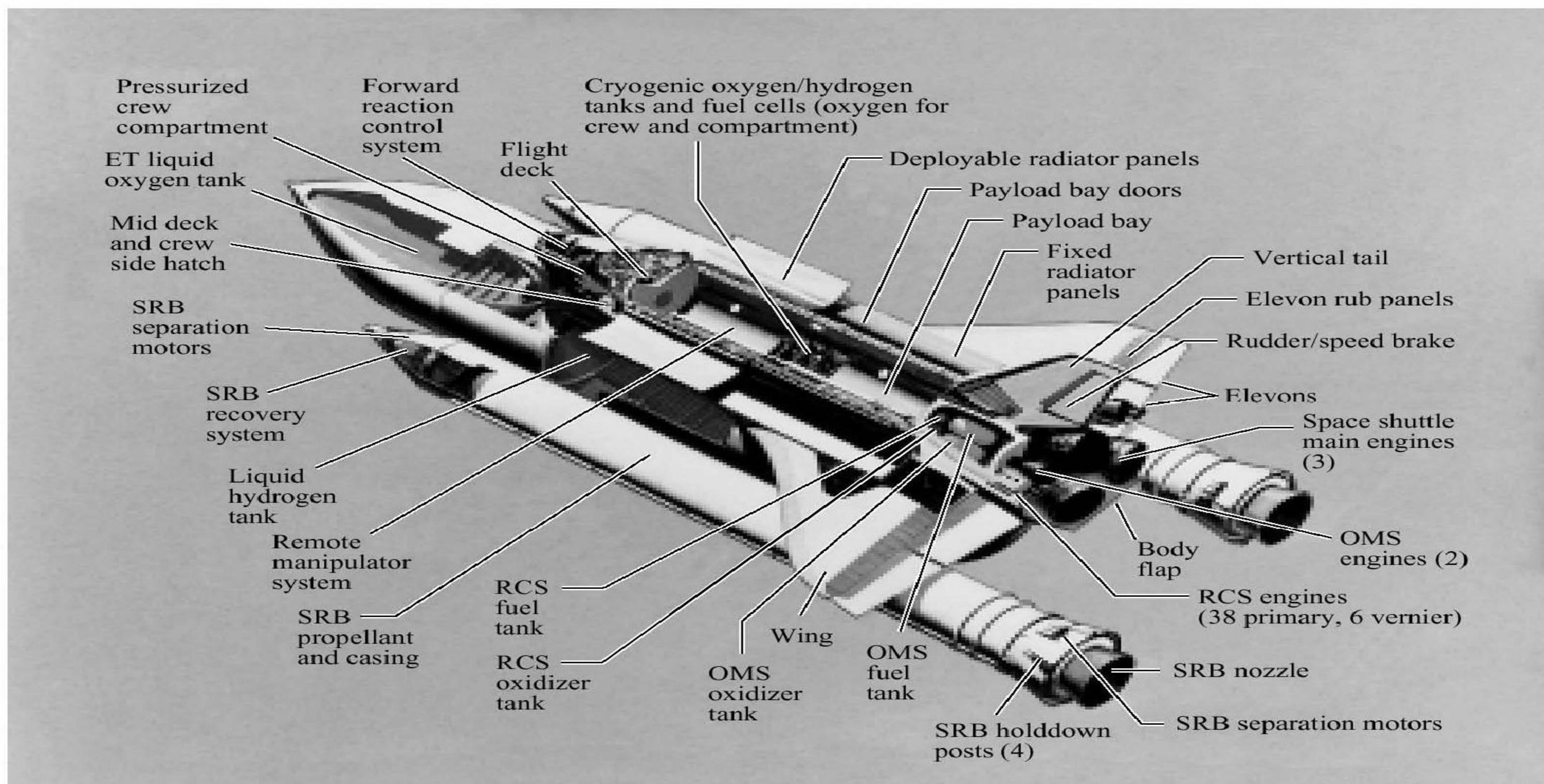
1. Blok Diyagramları
2. İşaret Akış Diyagramları

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2

BLOK DİYAGRAMLAR



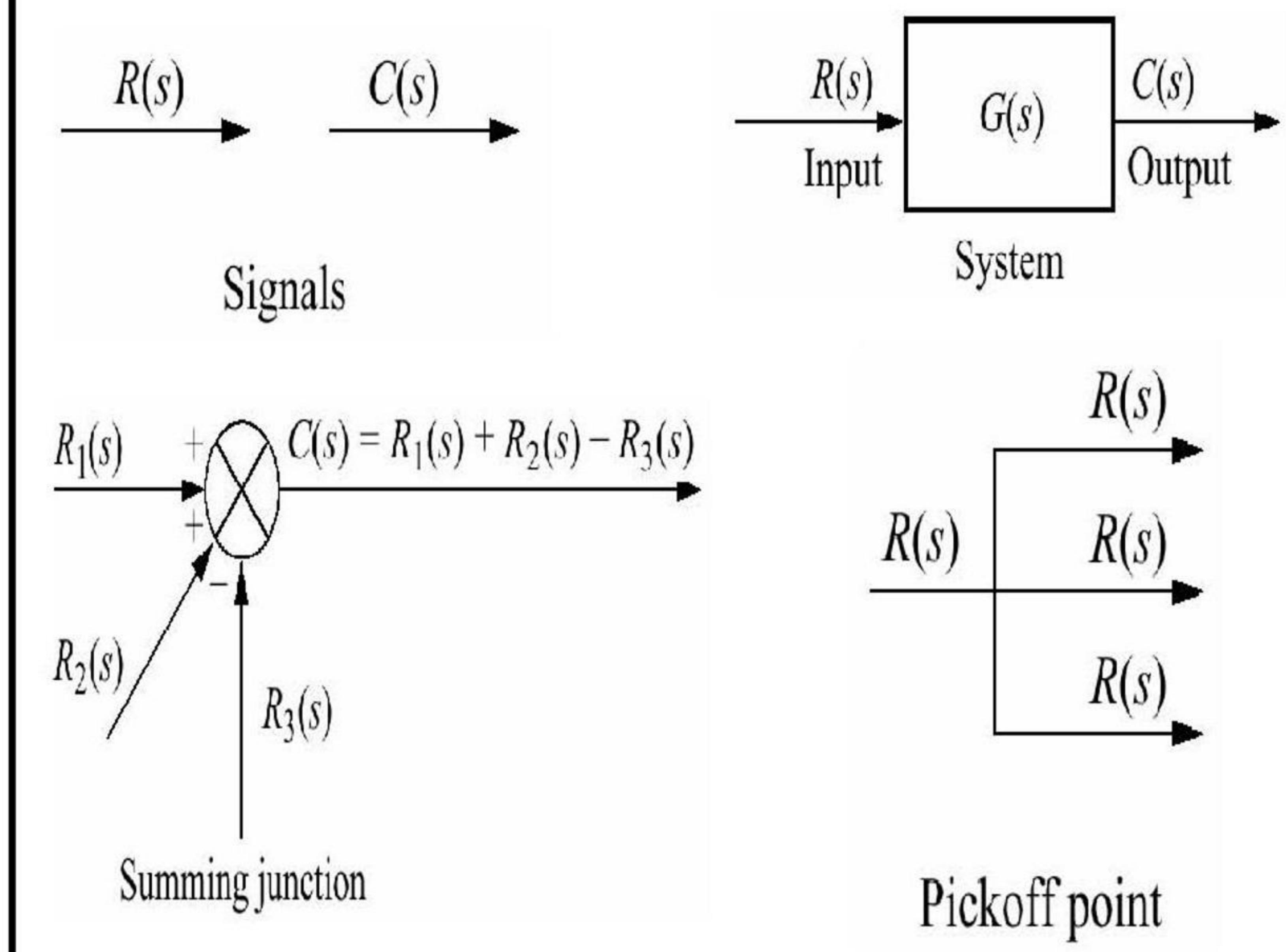
Birden fazla sistemden oluşan uzay aracı

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3

Bir önceki sayfada gördüğümüz gibi karmaşık bir sistem birden fazla alt sistemin biraraya gelmesi ile oluşmuştur. Bu alt sistemler arasında ilişkilenmeyi sağlayan basit operatörler vardır:



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4

Bir sistemin blok diyagramı sistem parçalarının işlevlerinin ve işaret akışının şéklini gösterimidir.

Tüm sistemimi oluşturan alt sistemleri işaret akışına göre tüm sistemin blok diyagramını oluşturmak üzere ilişkilendirmek zor değildir. Böylece tüm sistemin performansına her bir alt sistemin katkısını belirleyebiliriz.

Bir sistemin blok diyagramı sistemin dinamik davranışını temsil eder, sistemin fiziksel yapısı hakkında bilgi vermez.

Birbiriyle alakasız iki ayrı sistemin blok diyagramları aynı olabilir.

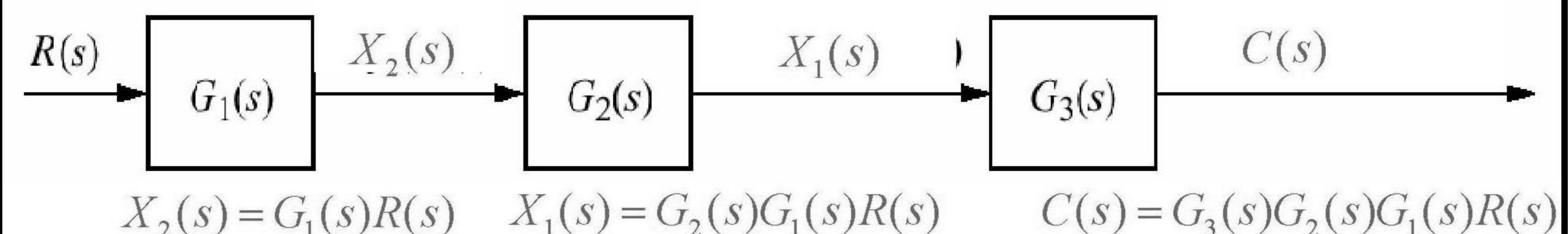
Bir sistemin blok diyagram gösterimi tek değildir. Yapılacak analize göre bir sistem farklı blok diyagramlar şeklinde gösterilebilir.

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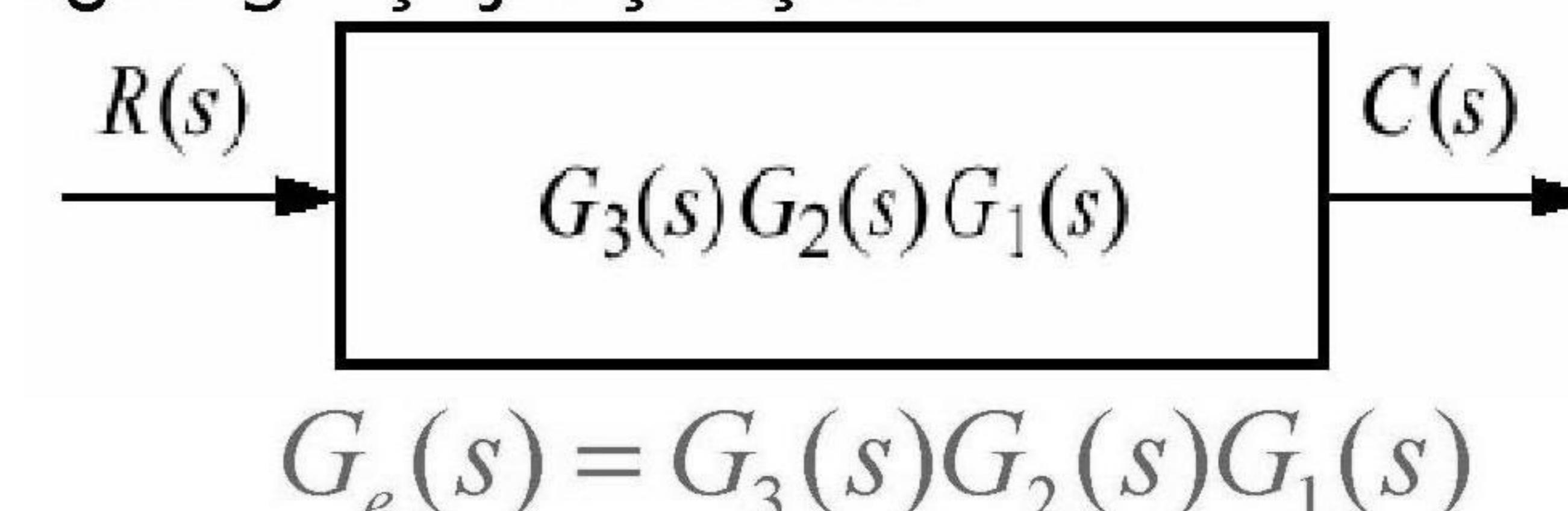
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5

Ardarda (Kaskat) Bağlantı:



Sistemin eşdeğer giriş-çıkış ilişkisi:



Bu eşdeğer giriş- çıkış ilişkisi alt sistemlerin birbirlerini yüklemeyenleri varsayımlı ile doğrudur. Eğer yüklenme söz konusu ise eşdeğer giriş çıkış ilişkisi oluşturulurken yüklenme etkisi göz önünde bulundurulmalıdır.

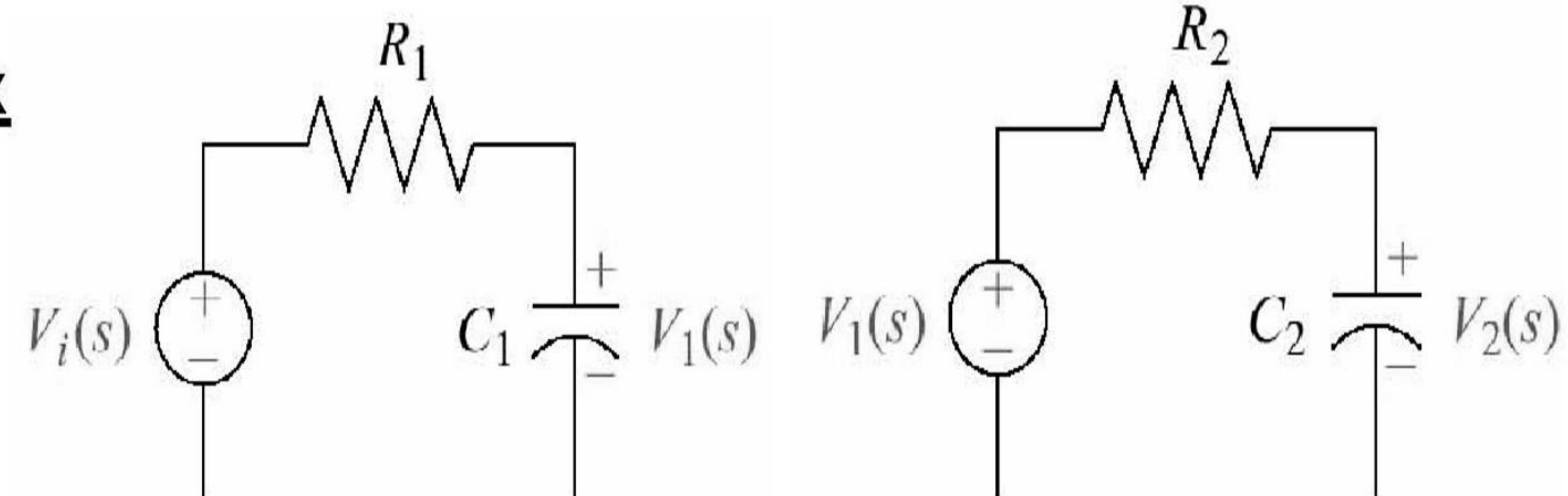
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6

Yüklenme etkisini basitçe şöyle ifade edebiliriz: Eğer bir alt sistemin çıkışına başka bir alt sistem ekleneşi ile değişmiyorsa sistem yüklenmiyor demektir, eğer değişiyorsa yüklenme etkisi vardır ve eş değer sistem oluşturulurken göz önüne alınması gereklidir.

Örnek



$$G_1(s) = \frac{V_1(s)}{V_i(s)}$$

$$G_1(s) = \frac{V_1(s)}{V_i(s)} = \frac{\frac{1}{R_1C_1}}{s + \frac{1}{R_1C_1}}$$

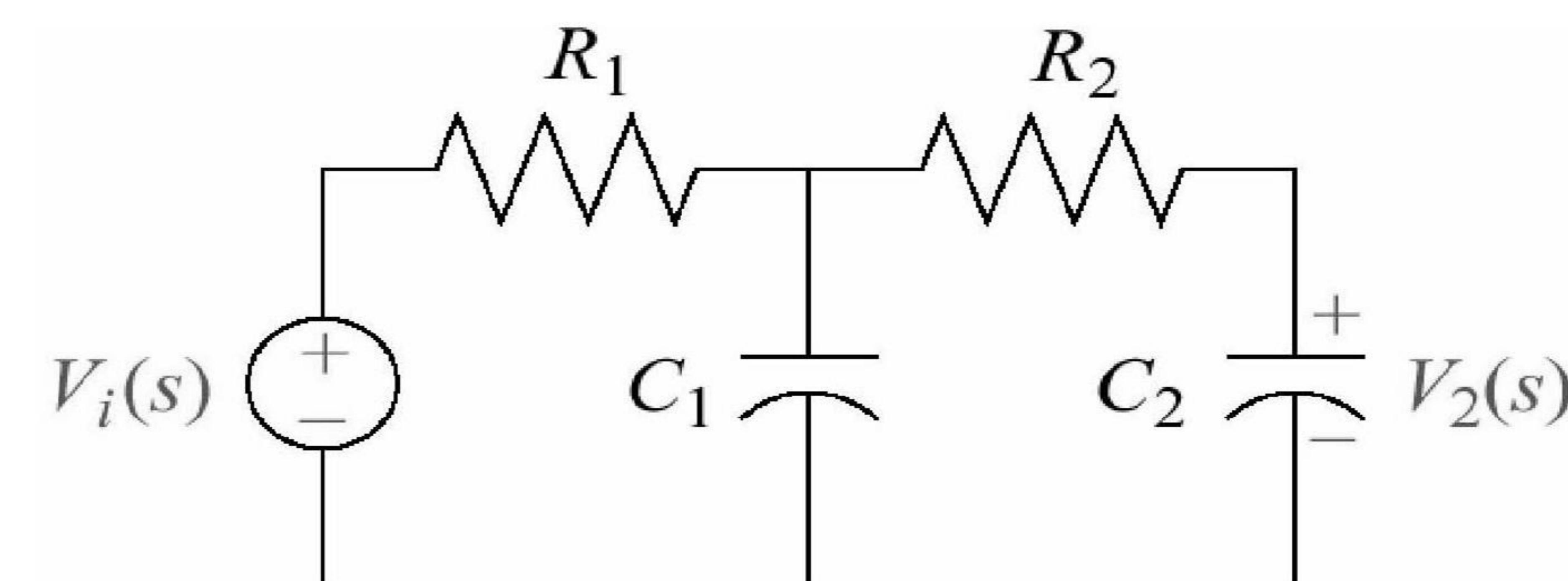
$$G_2(s) = \frac{V_2(s)}{V_1(s)}$$

$$G_2(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{R_2C_2}}{s + \frac{1}{R_2C_2}}$$

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7



$$G_T(s) = \frac{V_2(s)}{V_i(s)} \neq G_2(s)G_1(s)$$

Giriş-Cıkış ilişkisini kurduğumuzda:

$$G(s) = \frac{V_2(s)}{V_i(s)} = \frac{\frac{1}{R_1C_1R_2C_2}}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2} + \frac{1}{R_2C_1}\right)s + \frac{1}{R_1C_1R_2C_2}}$$

Olarak elde ederiz.

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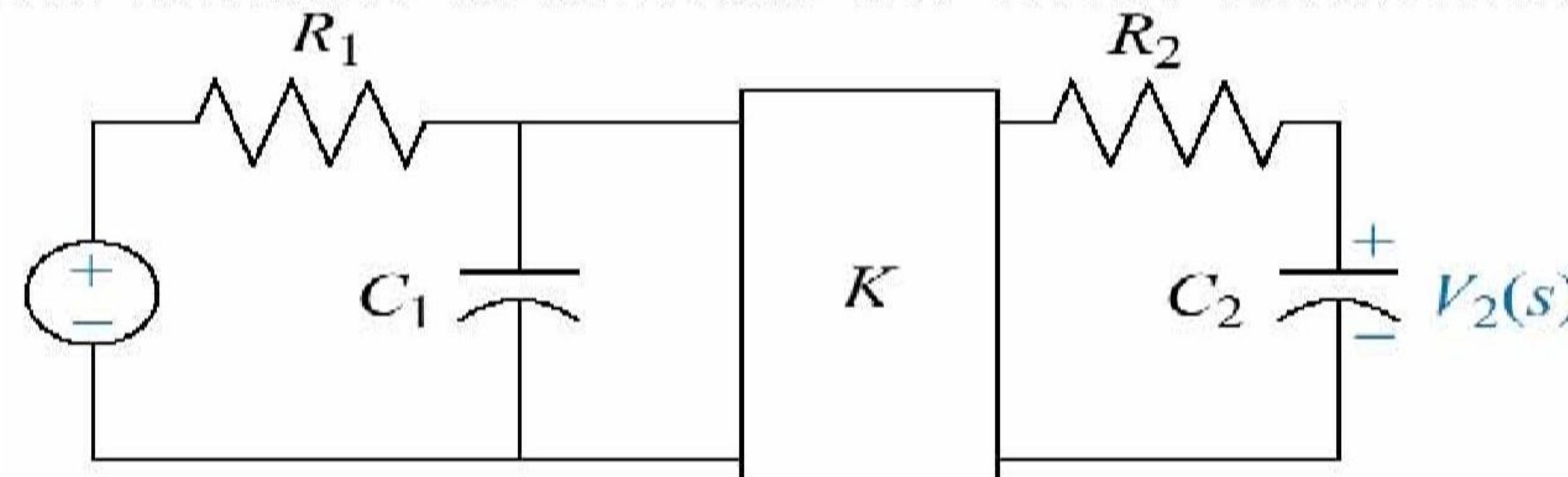
8

Yükleme etkisi göz önüne alınmazsa:

$$G(s) = \frac{V_2(s)}{V_i(s)} = \frac{\frac{1}{R_1 C_1 R_2 C_2}}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} \right) s + \frac{1}{R_1 C_1 R_2 C_2}}$$

Gördüğü gibi iki giriş-cıkış ilişkisi arasında fark var.

İki alt sistem arasında yüklenme etkisini ortadan kaldırmak için genellikle iki alt sistem arasında op-amp kullanılır.

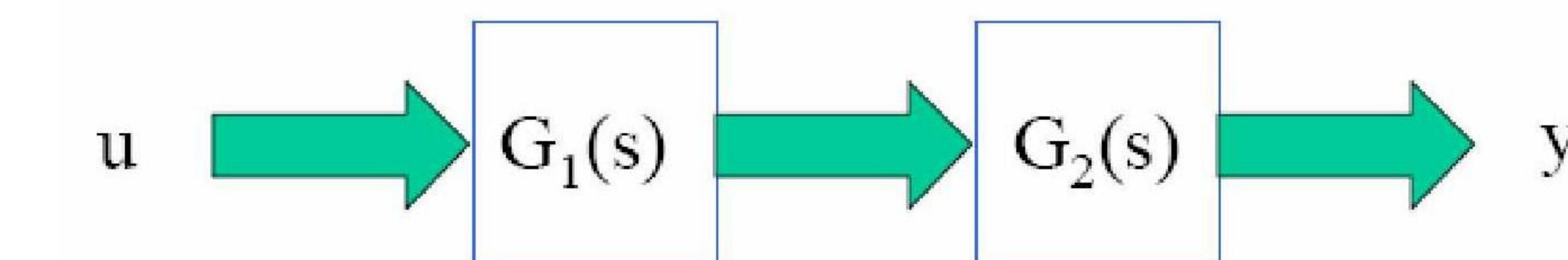


$$G_T(s) = \frac{V_2(s)}{V_i(s)} = K G_2(s) G_1(s)$$

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Genel olarak yüklenme etkisinin giriş-cıkış ilişkisindeki etkisi:



$$\frac{Y(s)}{U(s)} = \left[G_1(s) \frac{1}{1 + \frac{Z_{o1}}{Z_{i2}}} \right] G_2(s)$$

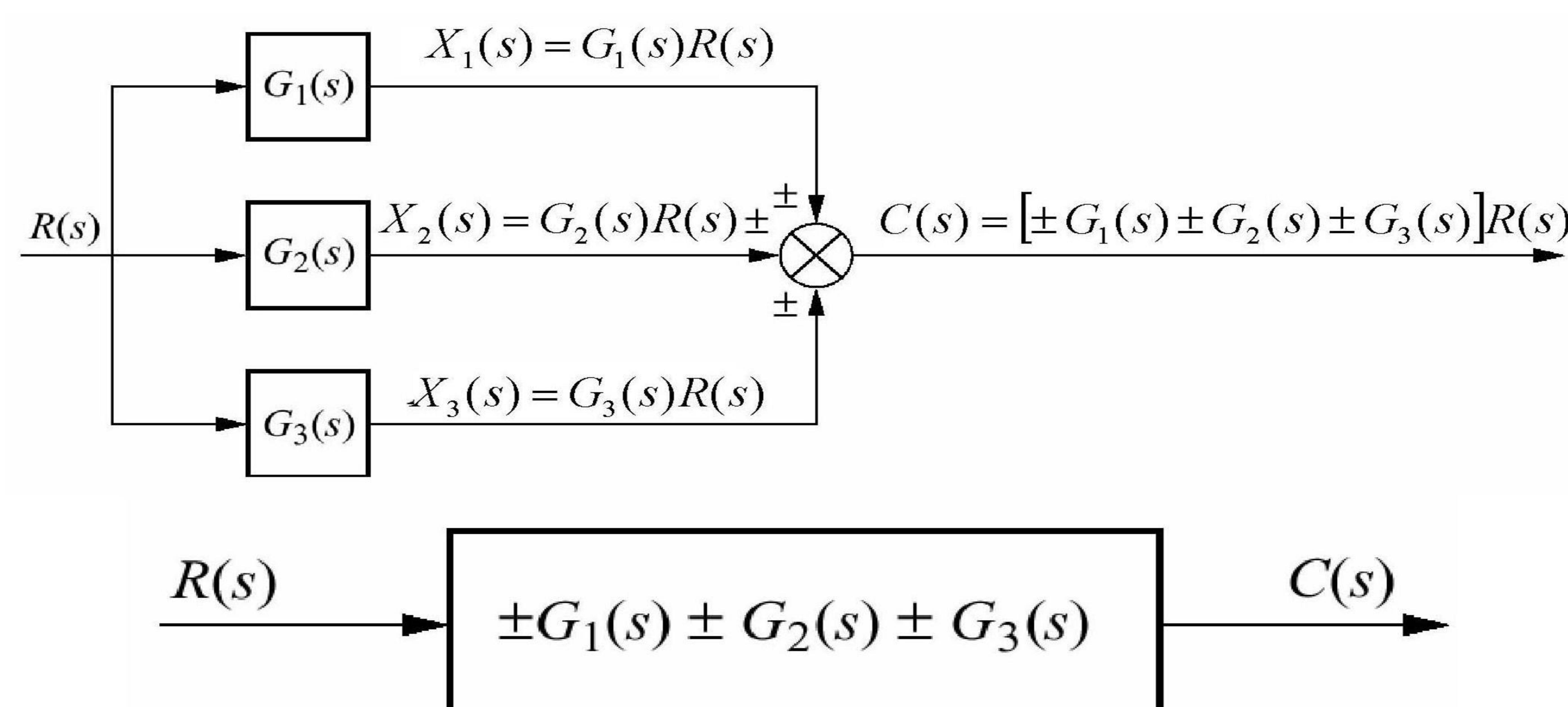
$\frac{Z_{o1}}{Z_{i2}} \ll 1$ ise yüklenme etkisi ihmal edilebilir.

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10

Paralel Bağlantı:



$$G_e = \pm G_1(s) \pm G_2(s) \pm G_3(s)$$

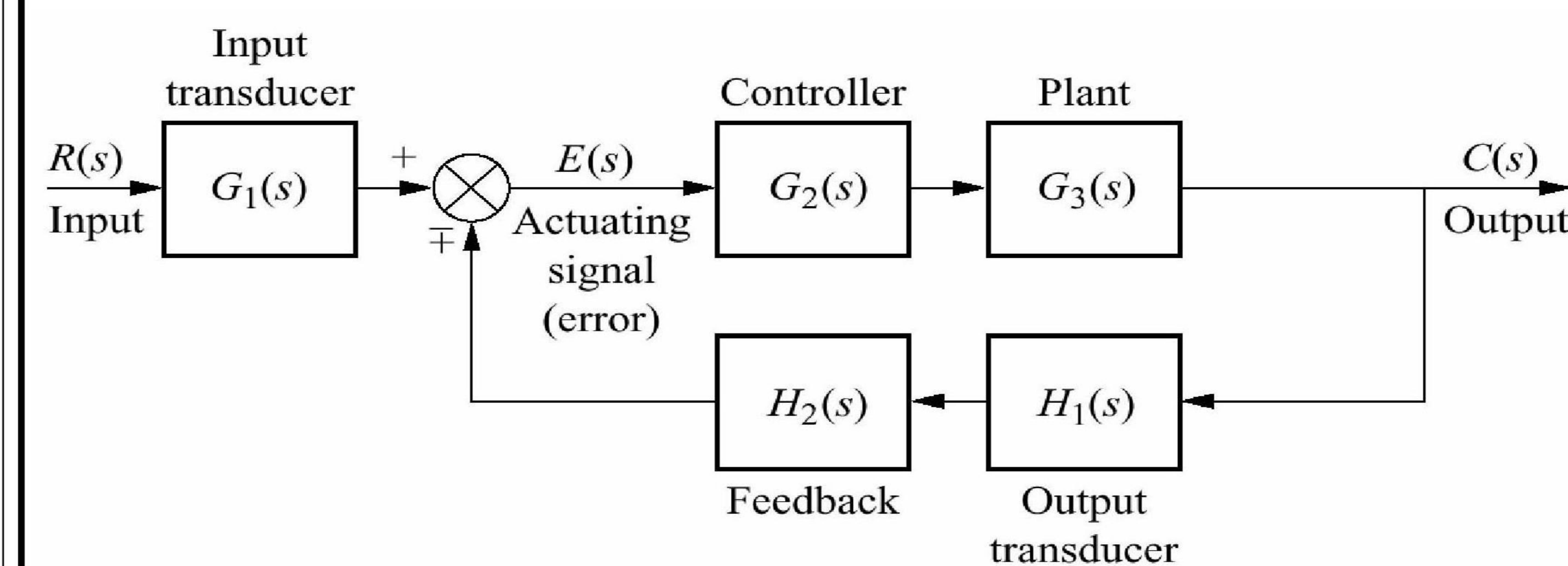
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11

Geri-besleme Bağlantısı:

Geri besleme bağlantısı kontrol tasarımları yapan mühendis için temel bir konudur.

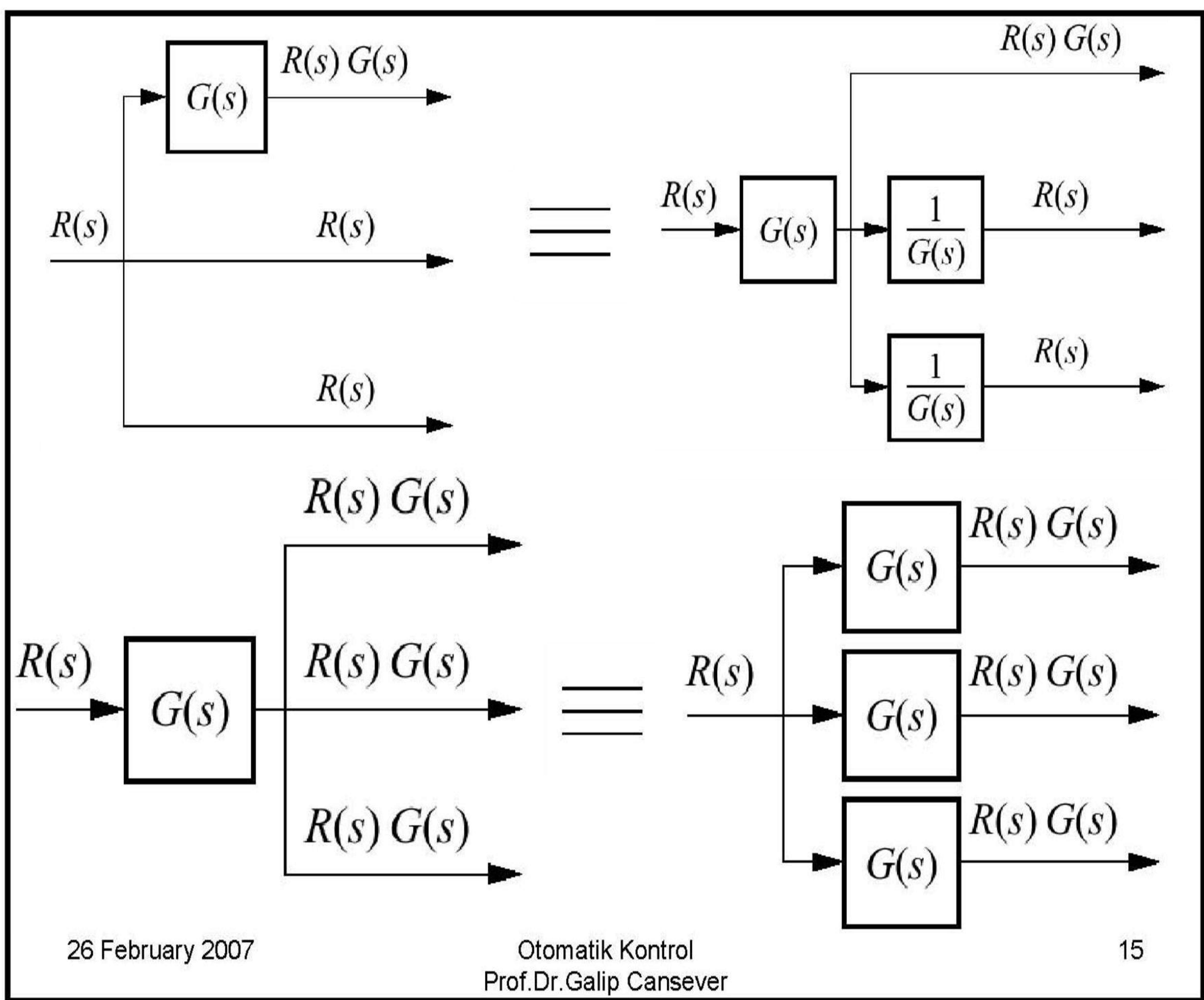
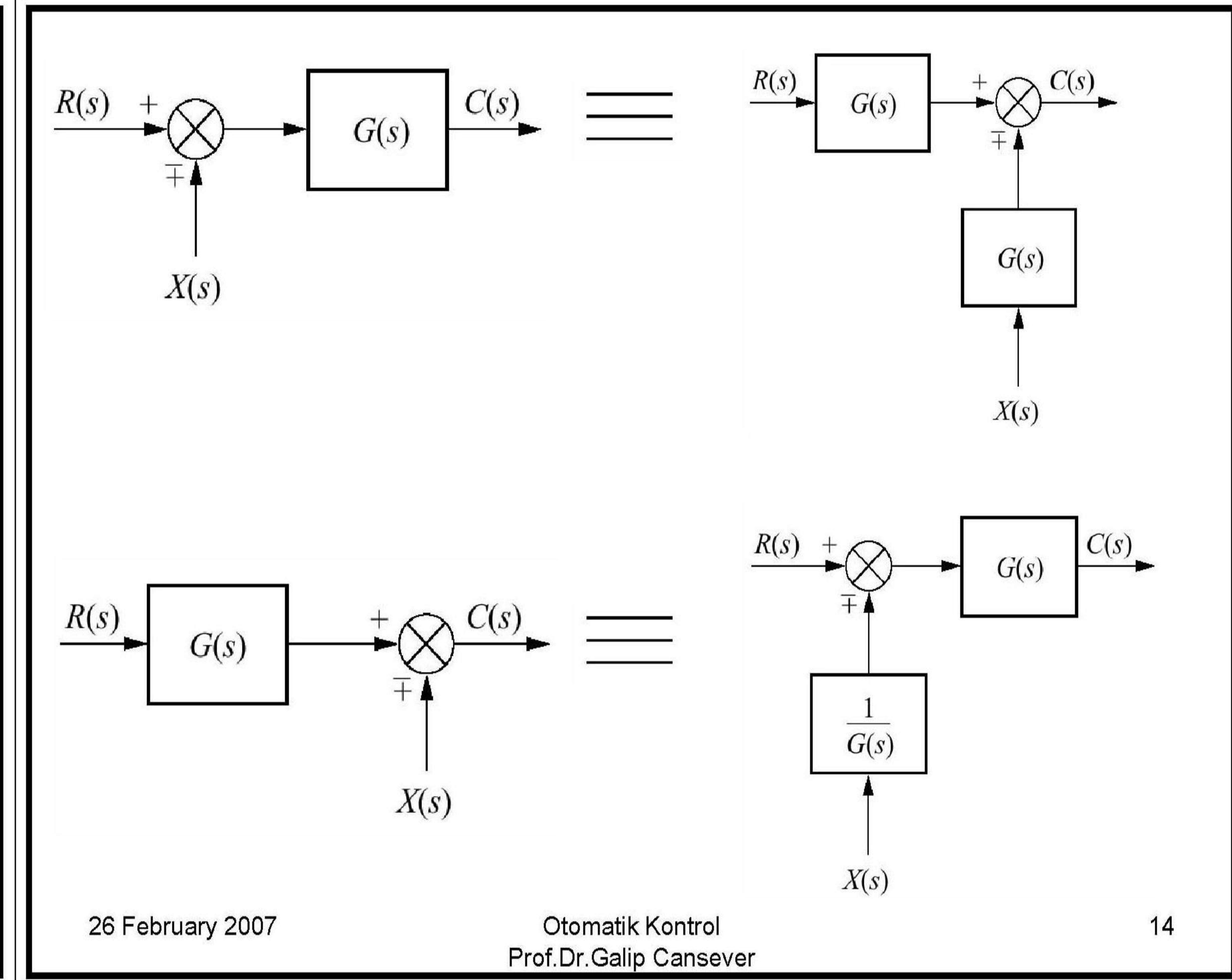
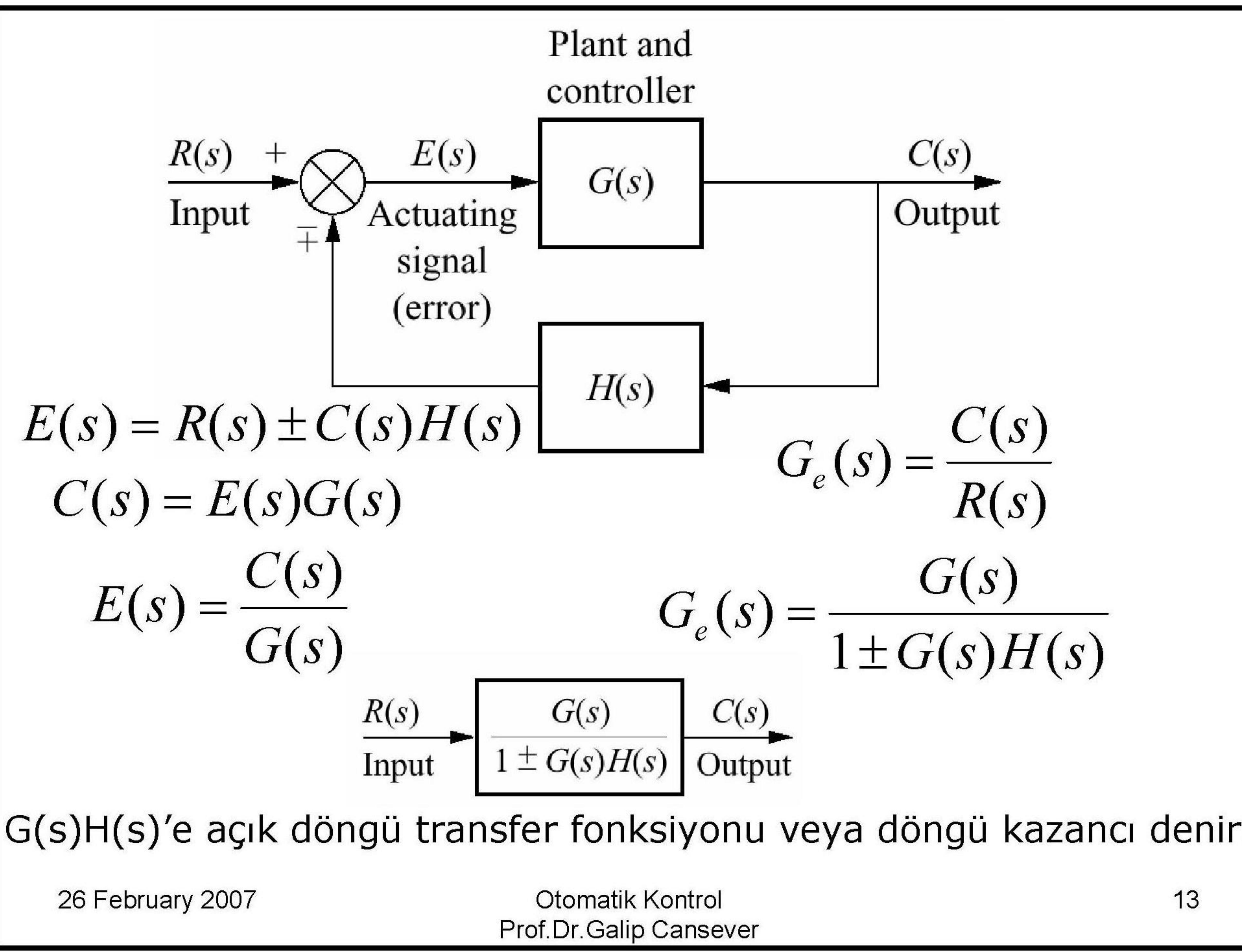


Blok diyagramını sadeleştirelim;

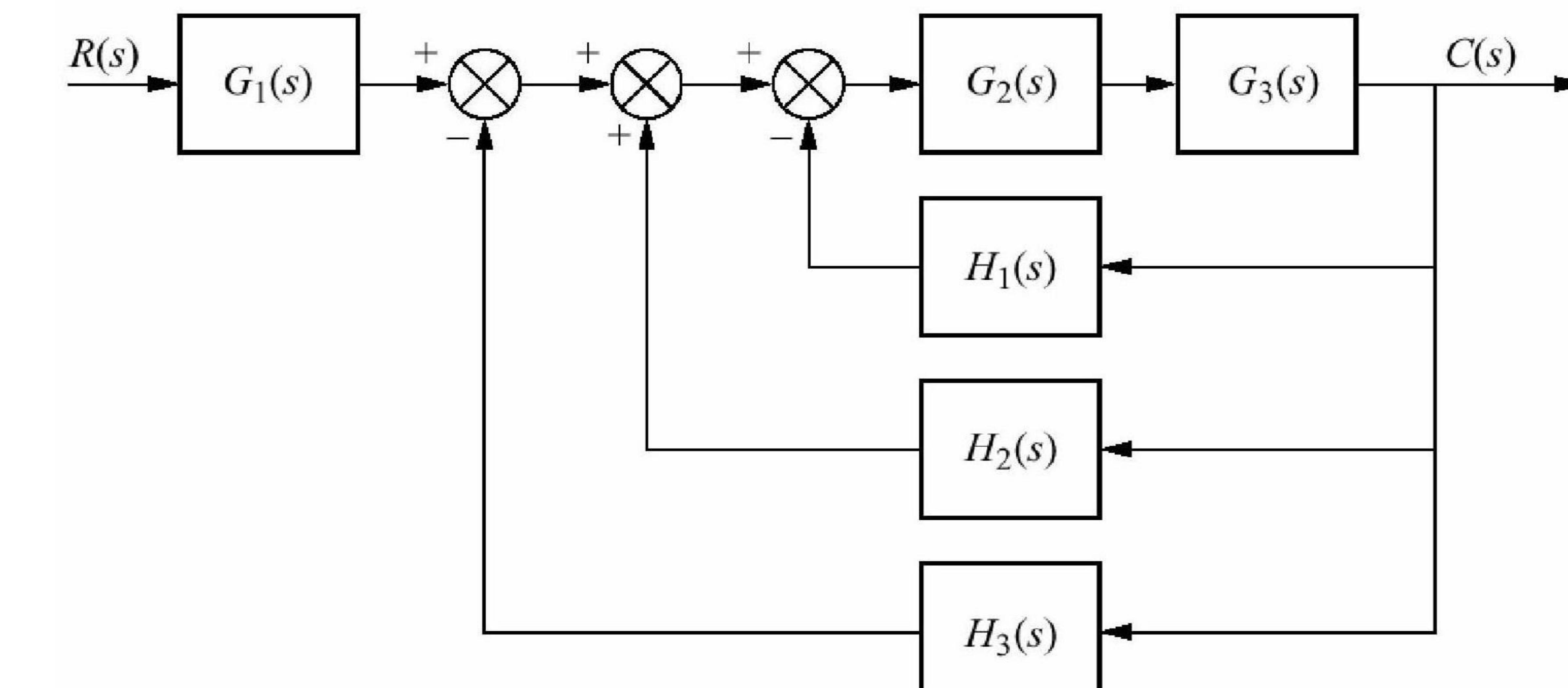
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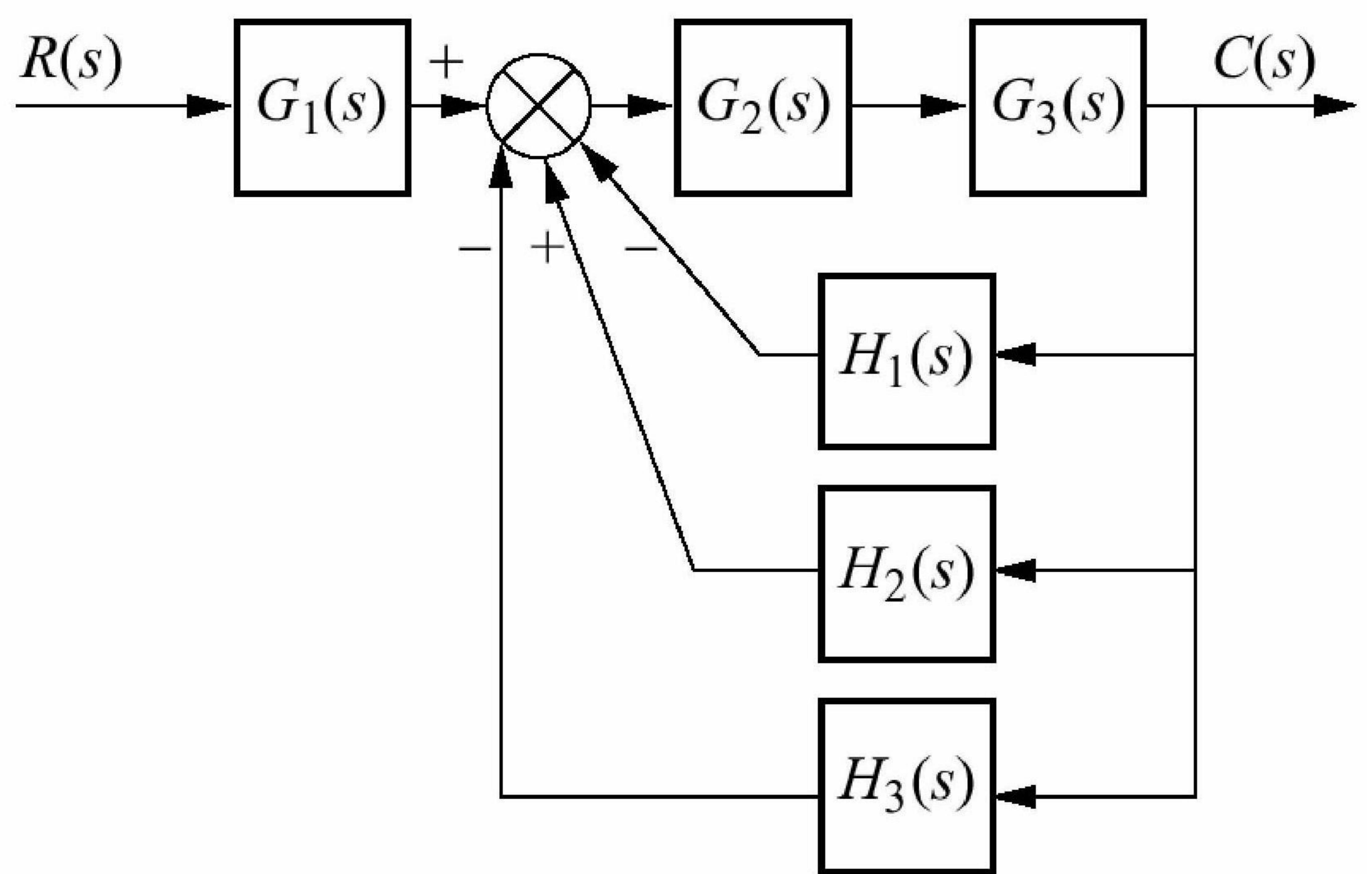
12



Örnek: Aşağıdaki blok diyagramı tek bir giriş-çıkış'a indirgeyin.



Önce tek bir toplayıcı da geri besleme sinyallerini toplayabiliyoruz

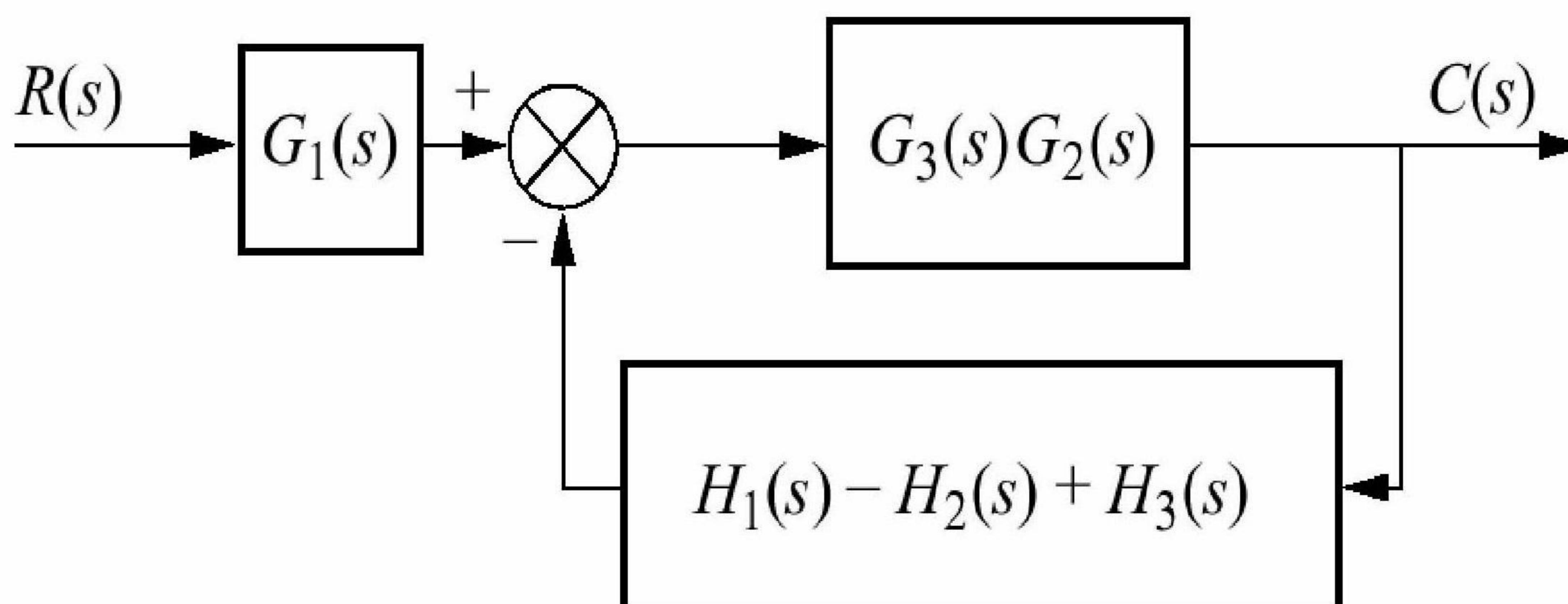


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17

H₁(s), **H₂(s)** ve **H₃(s)** lar aynı giriş işaretine sahipler çıkışları toplanmaktadır. Ayrıca **G₂(s)** ve **G₃(s)** ard ardadır. Bu durumda;



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18

Geri besleme bağlantısında dikkate alındığında sonuç olarak tek giriş ve çıkışlı blok diyagramı;

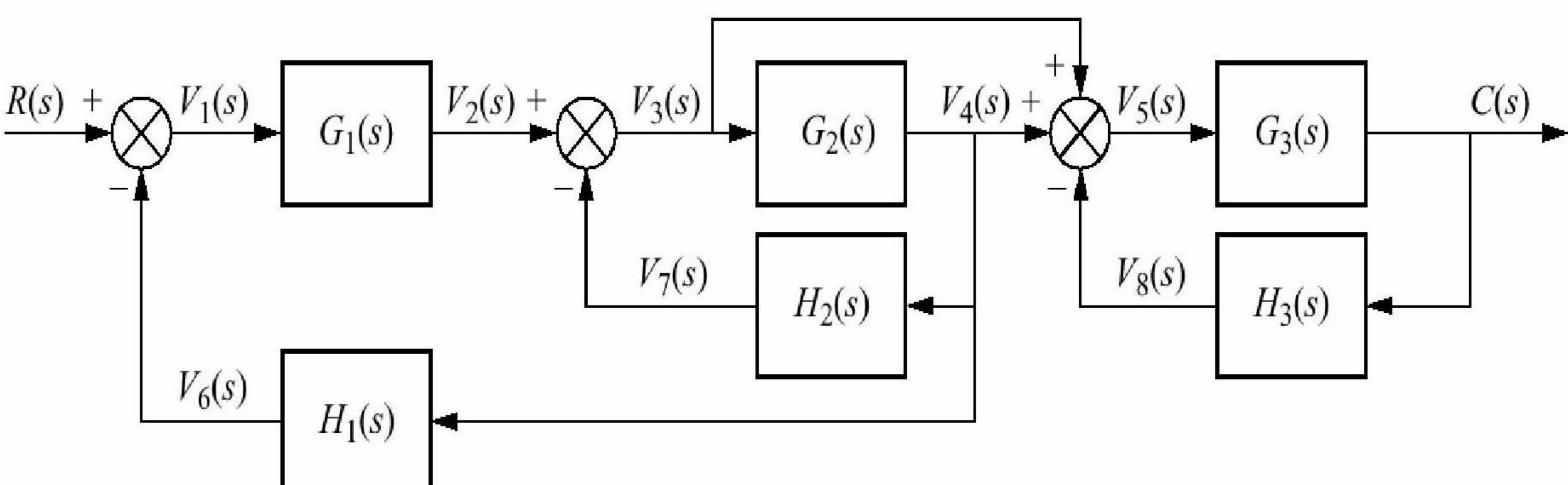
$$R(s) \rightarrow \frac{G_3(s)G_2(s)G_1(s)}{1 + G_3(s)G_2(s)[H_1(s) - H_2(s) + H_3(s)]} C(s)$$

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19

Örnek: Aşağıdaki blok diyagramı tek bir giriş-cıkış a indirgeyin.

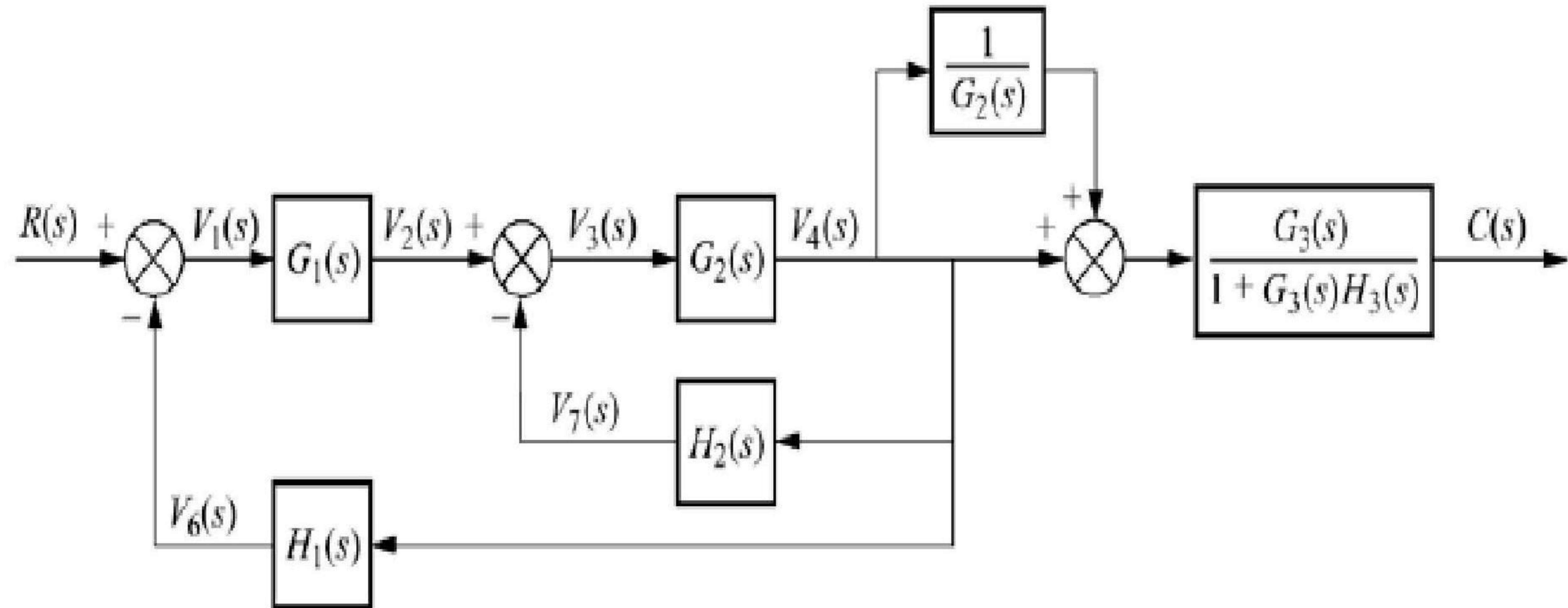


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20

Birim geribesleme ucunu $\mathbf{G}_2(s)$ 'in sağına alalım ve $\mathbf{G}_3(s)$ ve $\mathbf{H}_3(s)$ in geri beslemesini tek blok haline getirelim:

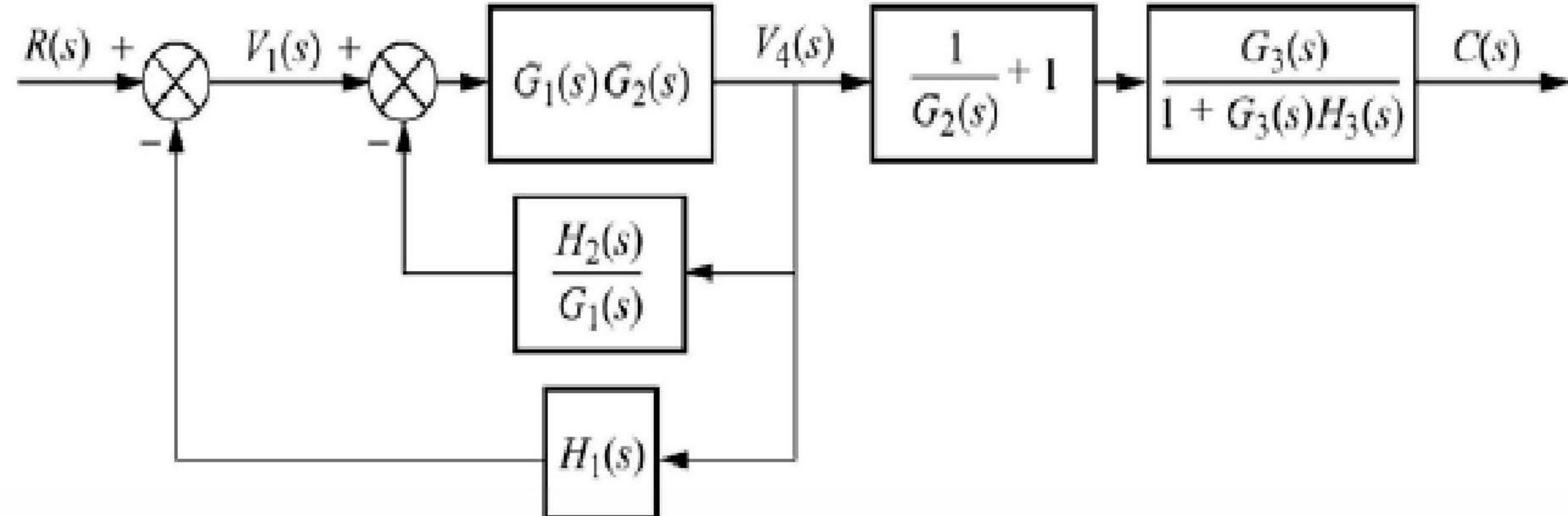


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21

$1/\mathbf{G}_2(s)$ ile birim işaretin birleştirilmesi ve $\mathbf{G}_1(s)$ 'i toplayıcının sağına alalım:

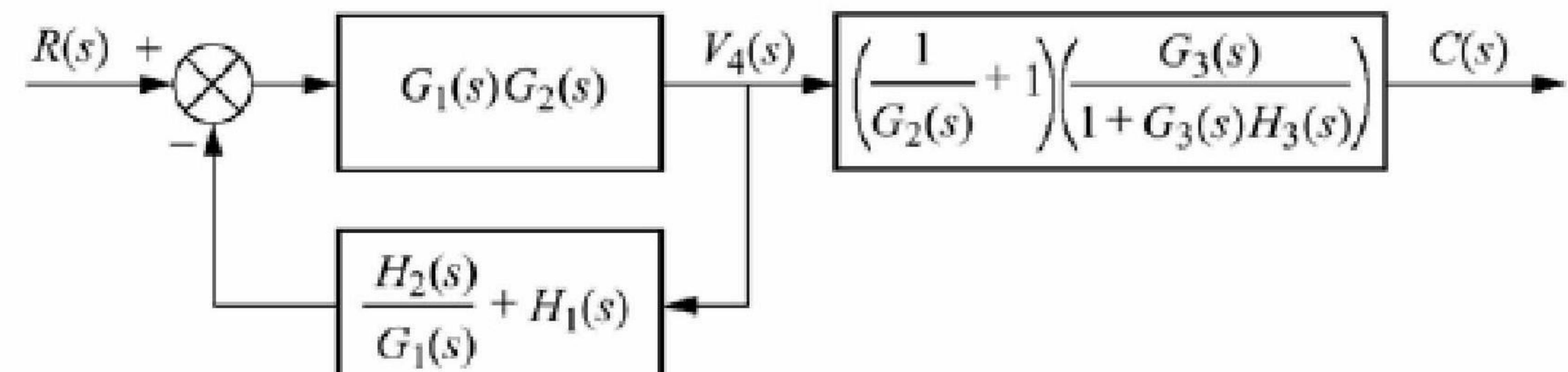


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22

Toplayıcıları birleştirelim ard arda bağlantıyı tek blok diyagrama dönüştürelim:

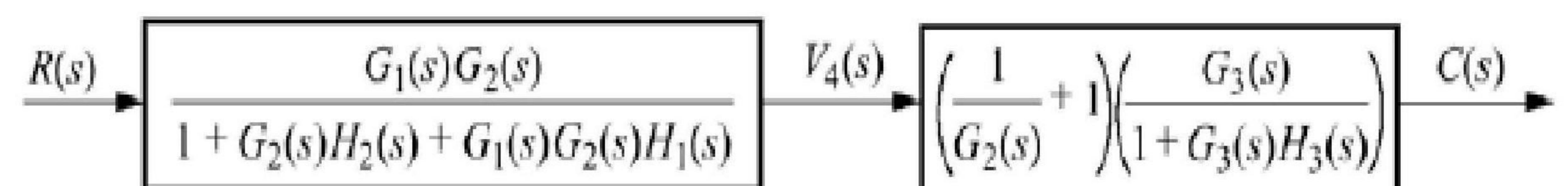


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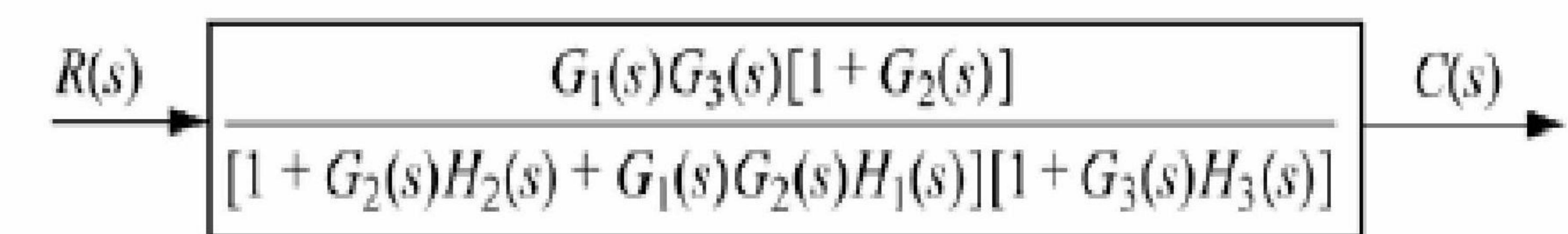
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23

Geri besleme bağlantısının blok işlemini yapalım:



Ard arda bağlantı işelmini gerçekleyelim:



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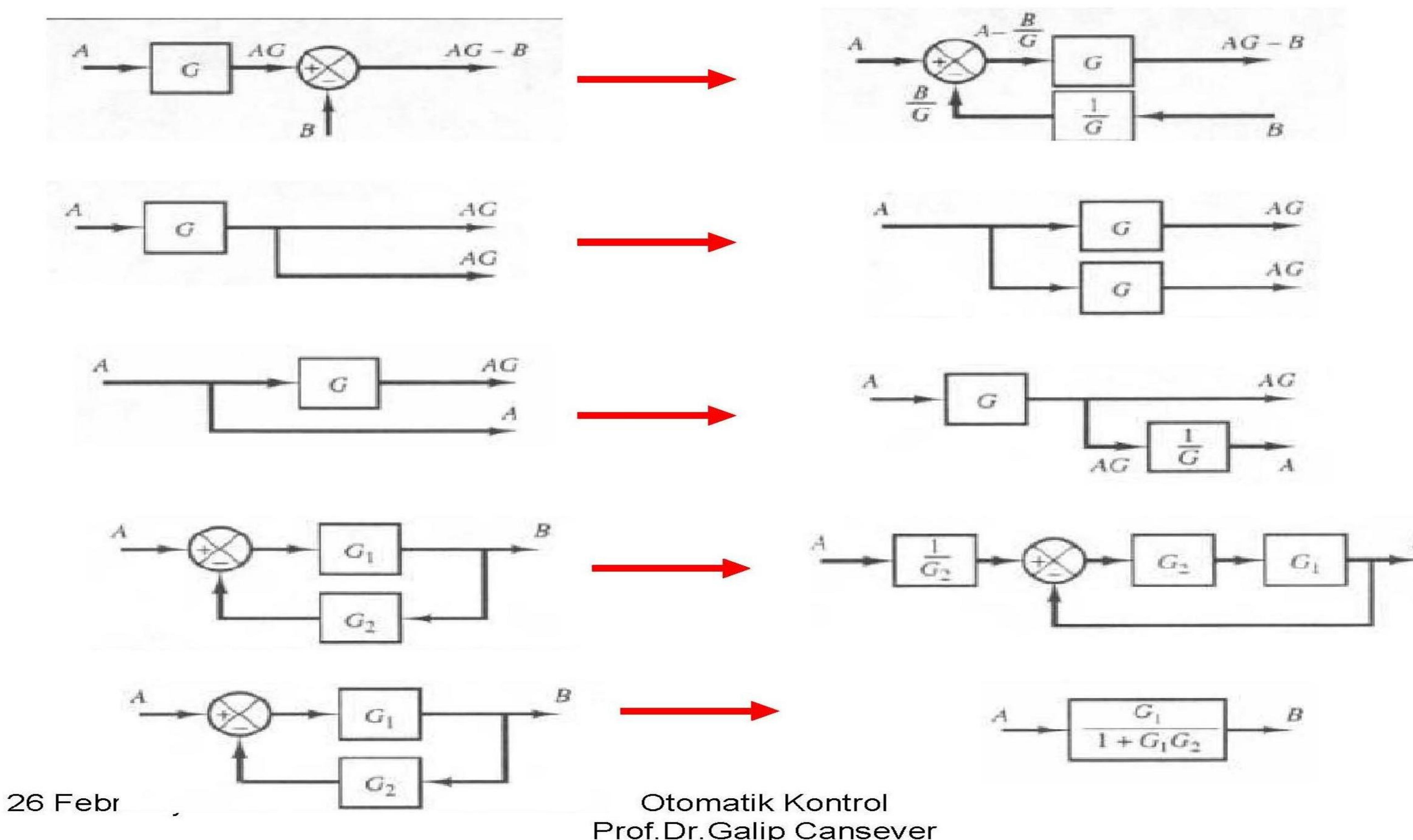
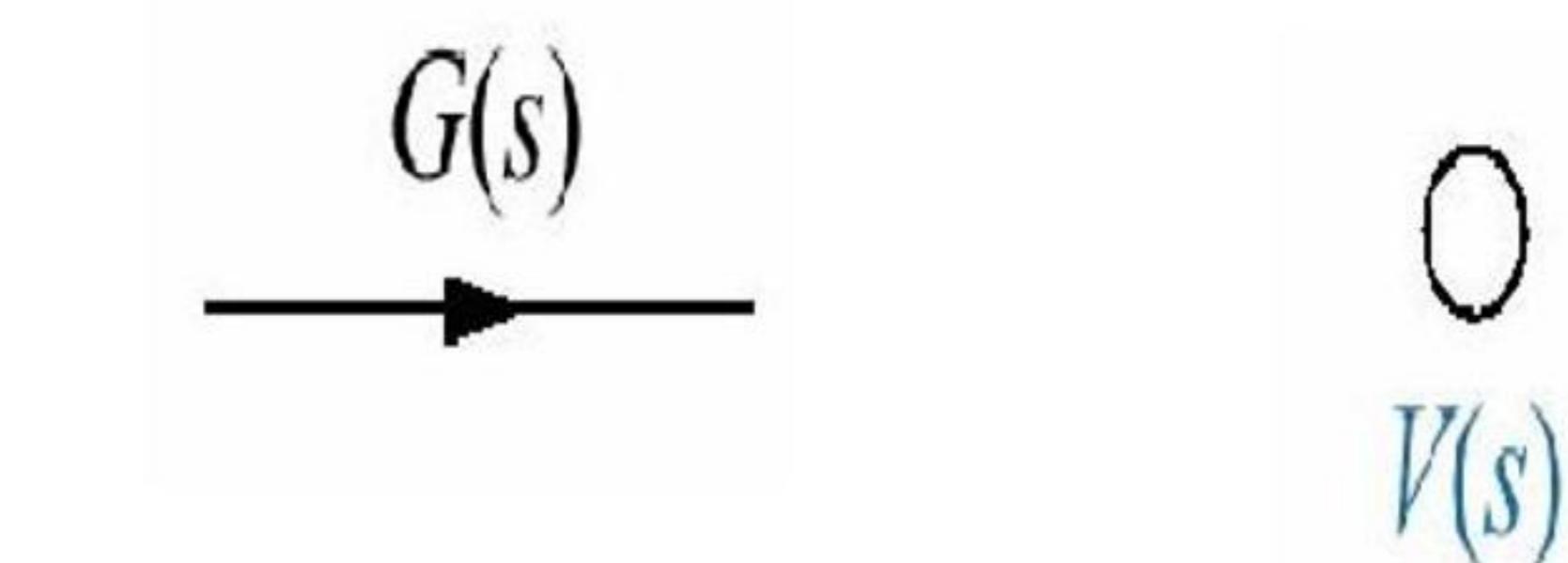
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24

İŞARET AKIŞ DİYAGRAMLARI

İşaret akış diyagramları bir diğer sistem temsilidir.

İşaret akış diyagramı, alt sistemi ifade eden dallardan ve işaretleri ifade eden nod'lardan oluşur.

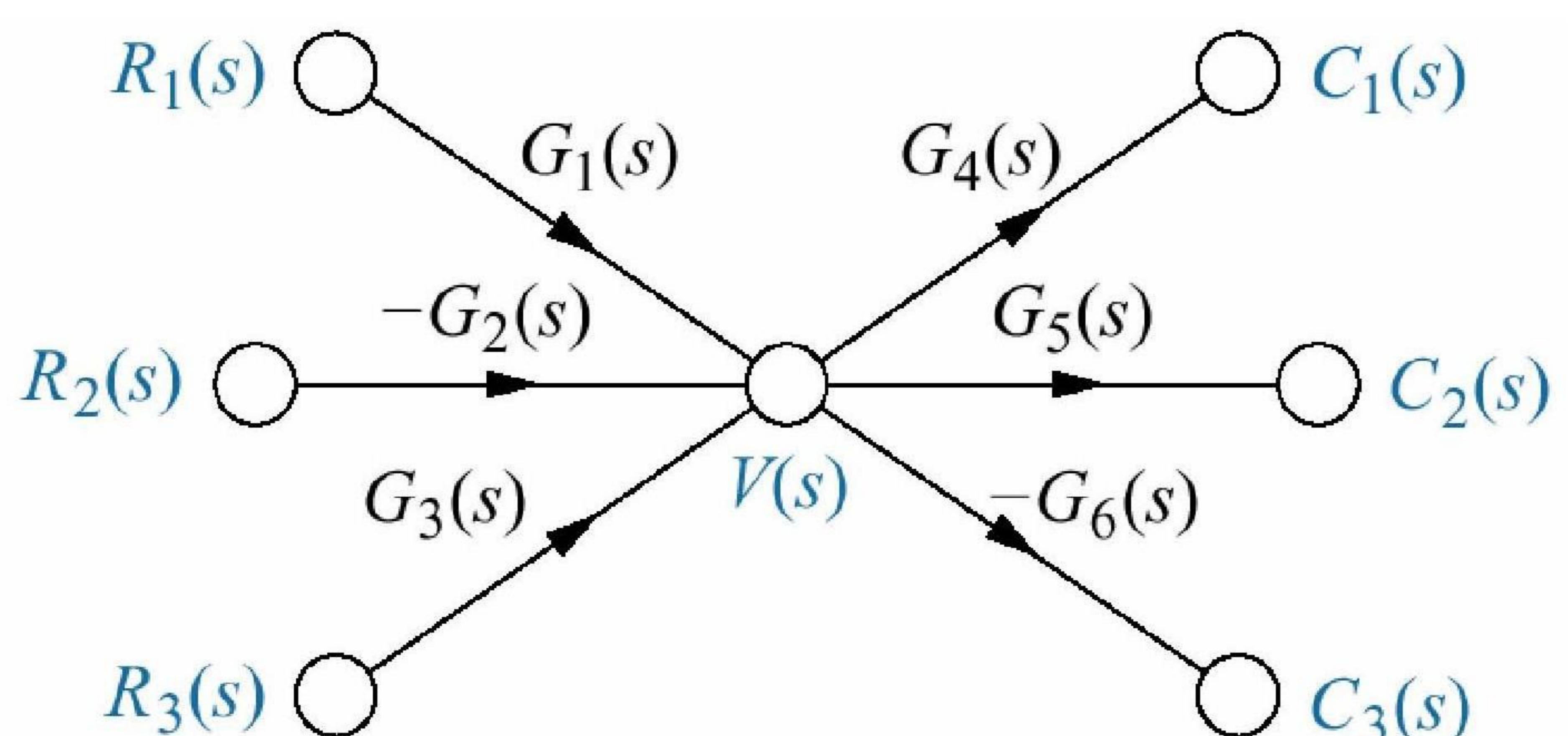


25

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26



Her bir işaret kendine doğru gelen işaretlerin toplamı ile ifade edilir.
Örneğin:

$$X(s) = R_1(s)G_1(s) - R_2(s)G_2(s) + R_3(s)G_3(s)$$

$$C_2(s) = X(s)G_5(s) = R_1(s)G_1(s)G_5(s) - R_2(s)G_2(s)G_5(s) + R_3(s)G_3(s)G_5(s)$$

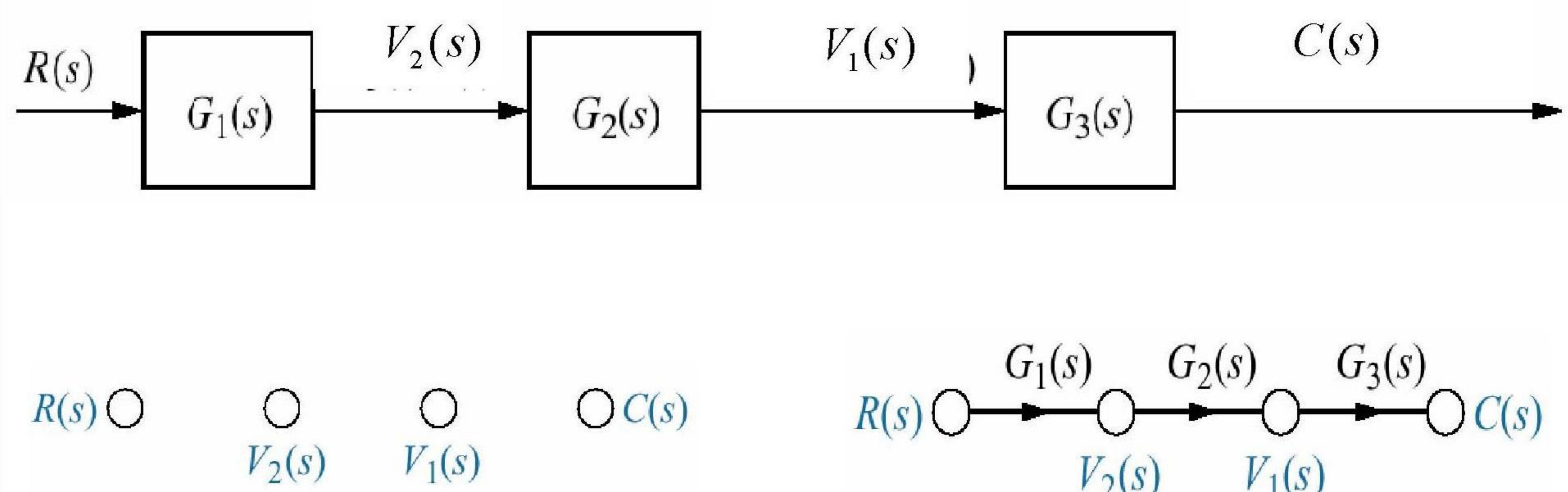
$$C_3(s) = -X(s)G_6(s) = -R_1(s)G_1(s)G_6(s) + R_2(s)G_2(s)G_6(s) - R_3(s)G_3(s)G_6(s)$$

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27

Örnek: Aşağıdaki sistemin işaret akış diyagramını oluşturun.

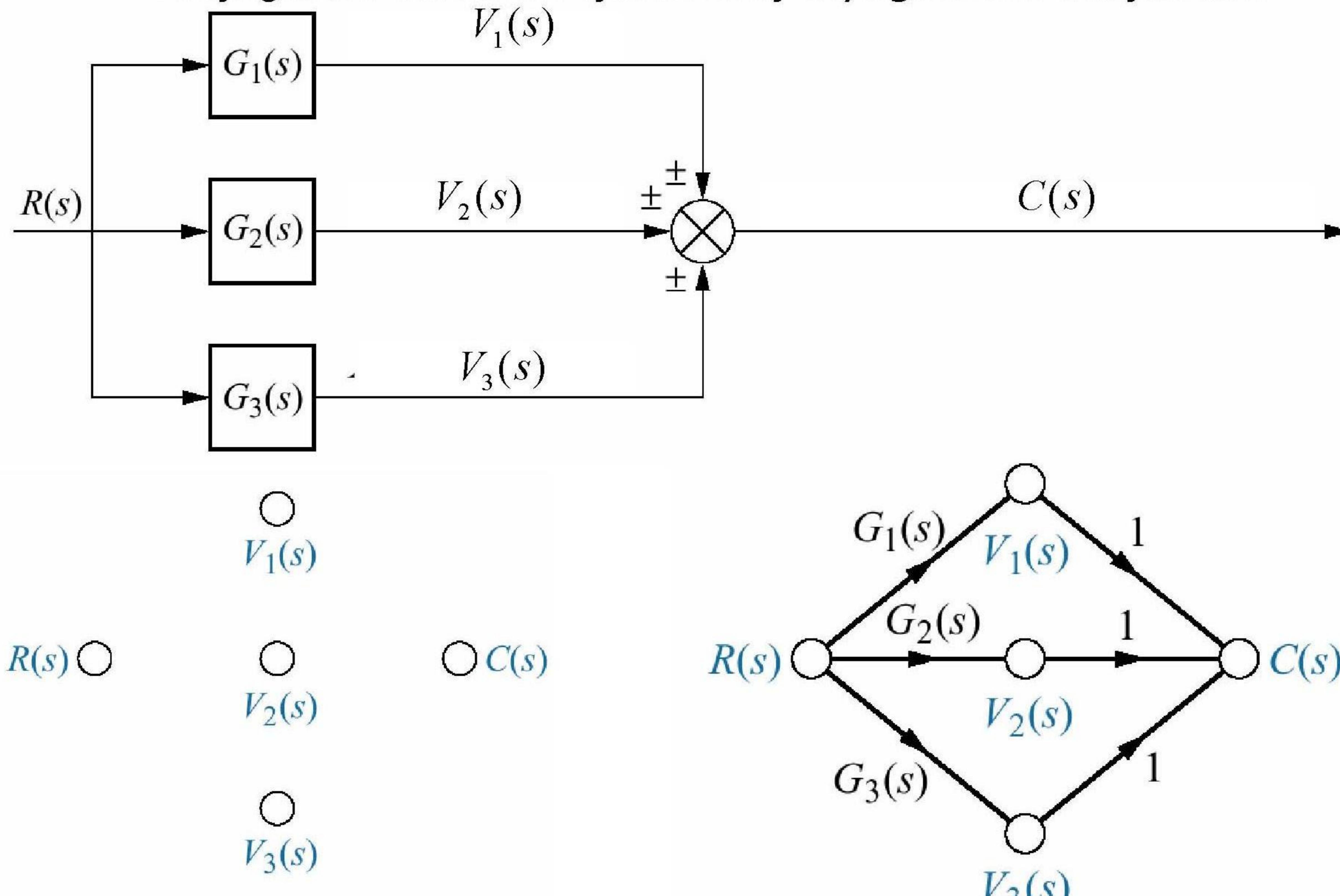


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28

Örnek: Aşağıdaki sistemin işaret akış diyagramını oluşturun.

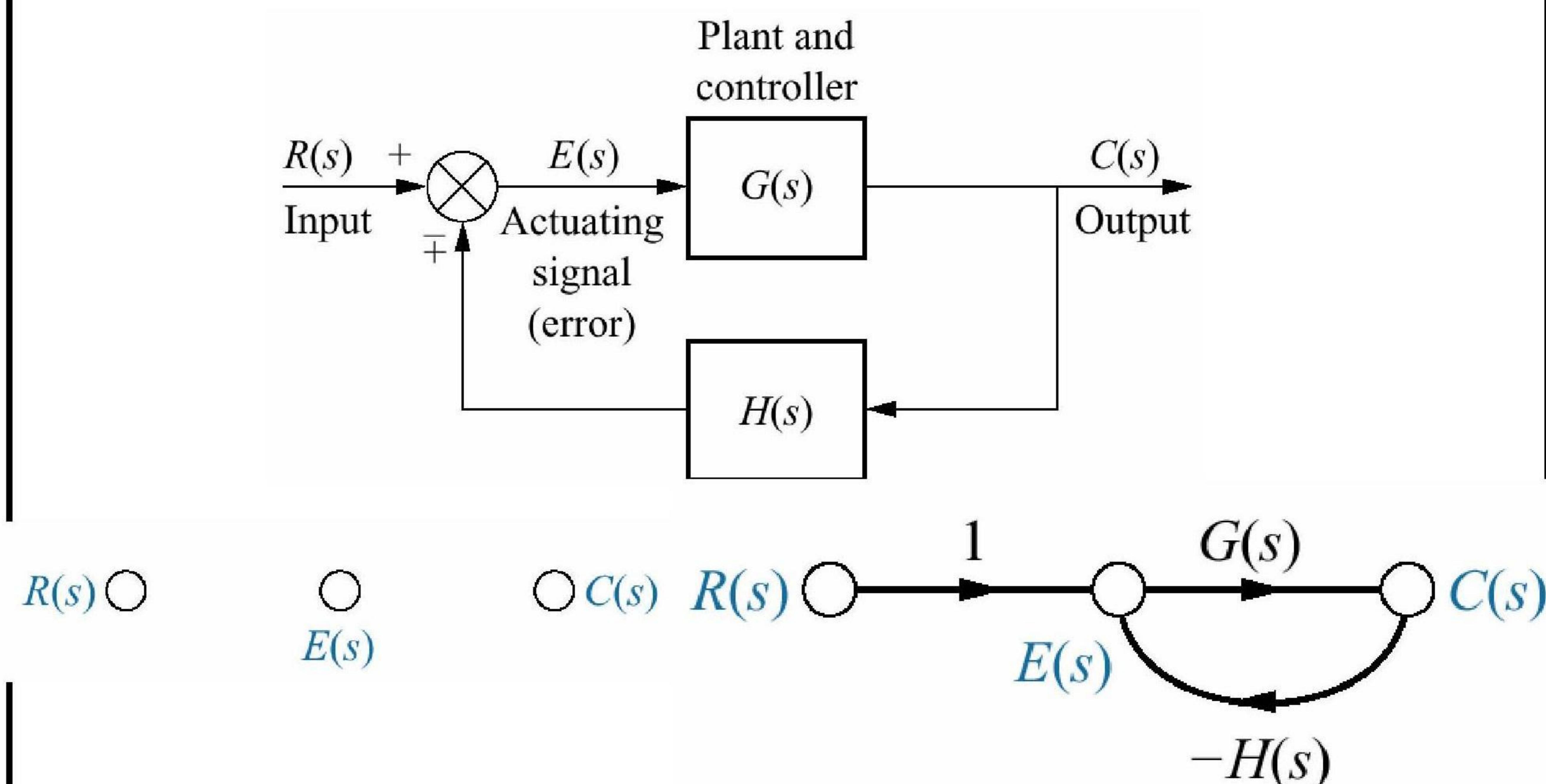


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29

Örnek: Aşağıdaki sistemin işaret akış diyagramını oluşturun.

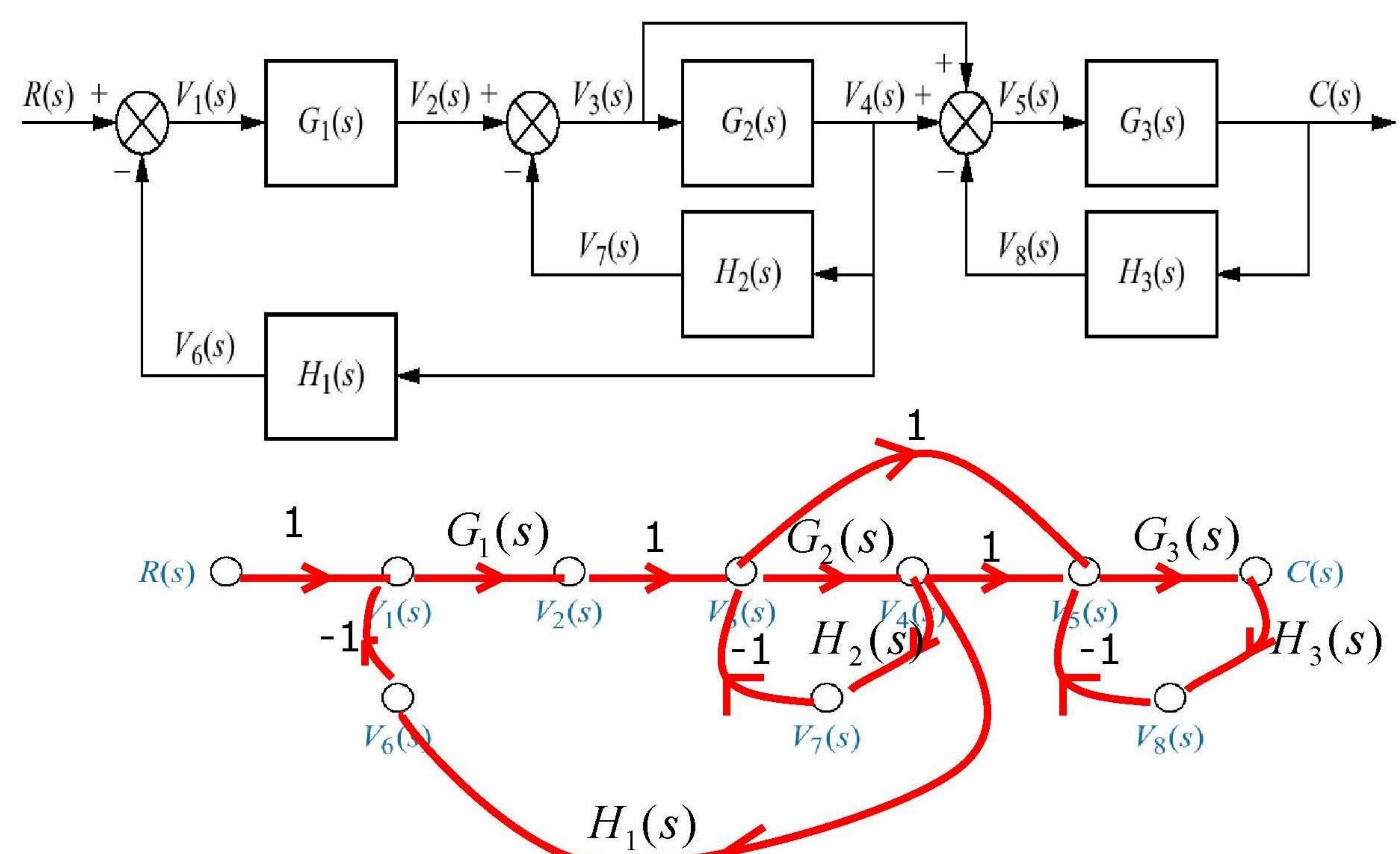


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30

Örnek: Aşağıdaki sistemin işaret akış diyagramını oluşturun.

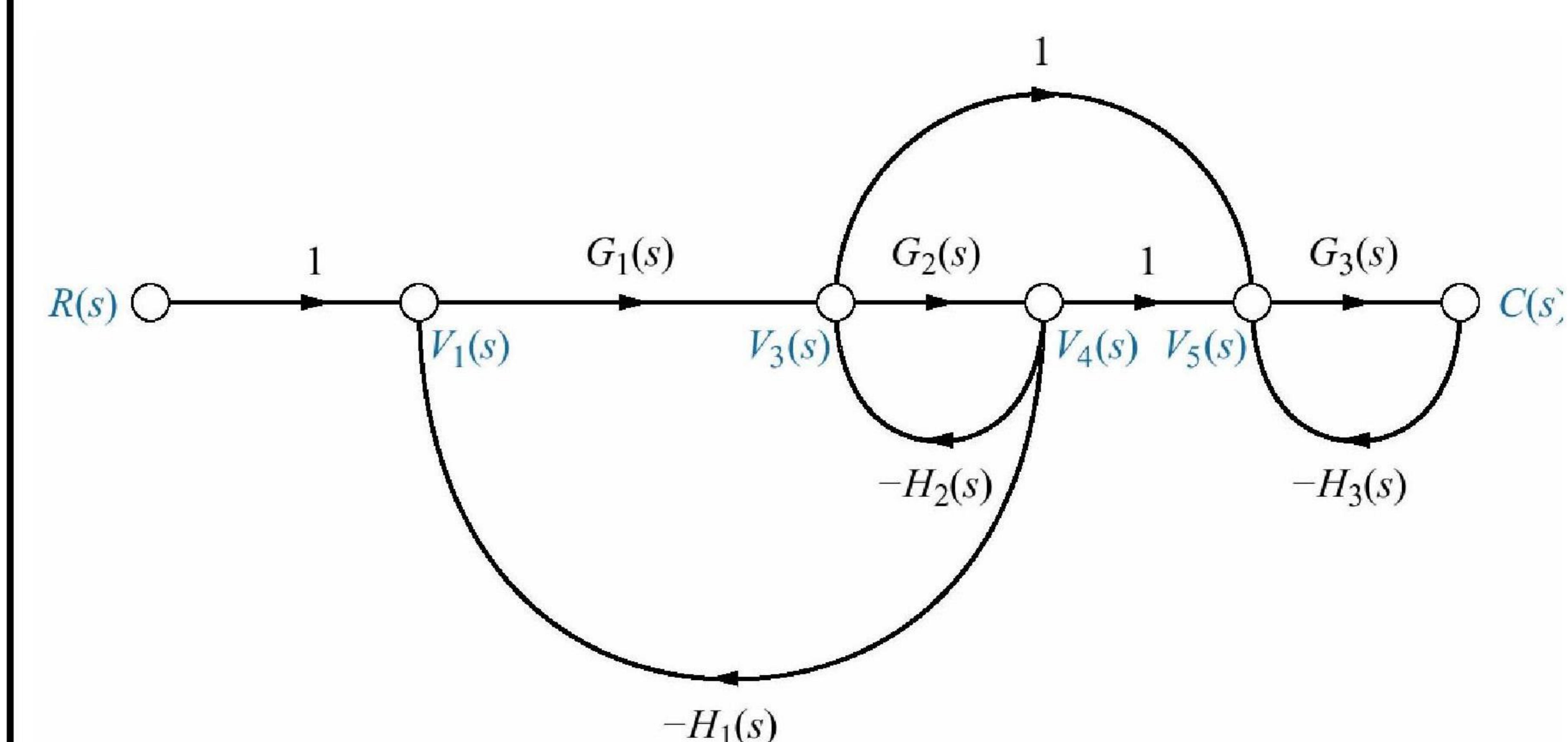


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31

V₂, V₆, V₇ ve V₈ gibi tek bir girişi ve tek bir çıkışı olan nod'ları sadeleştirilebiliriz, bu durumda:



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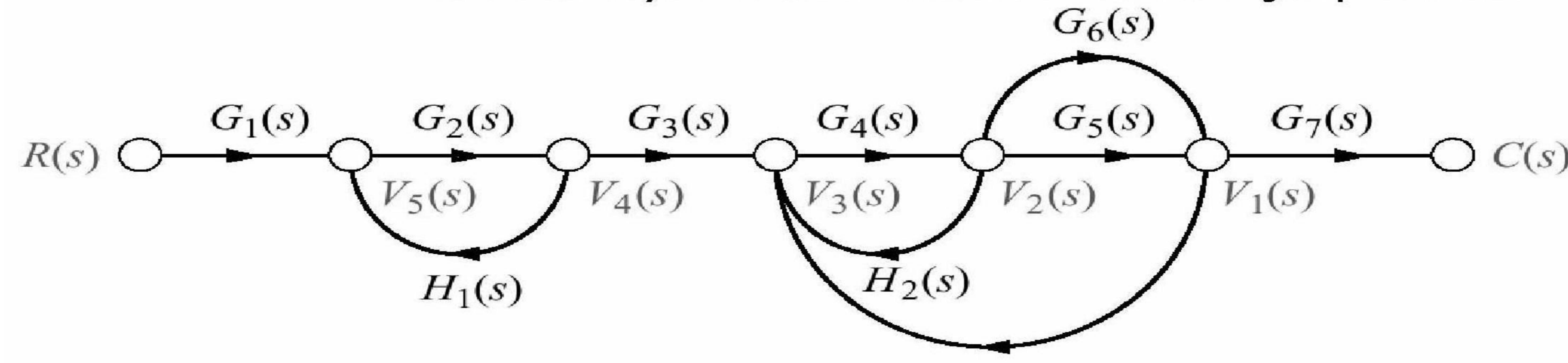
32

İ işaret Akış Diyagramının Sadeleştirilmesi, S.J.Mason

Yasası:

Blok diyagramları gibi işaret akış diyagramlarını sadeleştirebiliriz. S.J. Mason 1953 de işaret akış diyagramlarını tek bir giriş çıkış ilişkisine çeviren formülüzasyon geliştirmiştir.

Döngü Kazancı: Bir nod'da başlayıp başka bir nod'dan geçmeden tekrar aynı nod'a dönen dal'ları çarpımıdır.

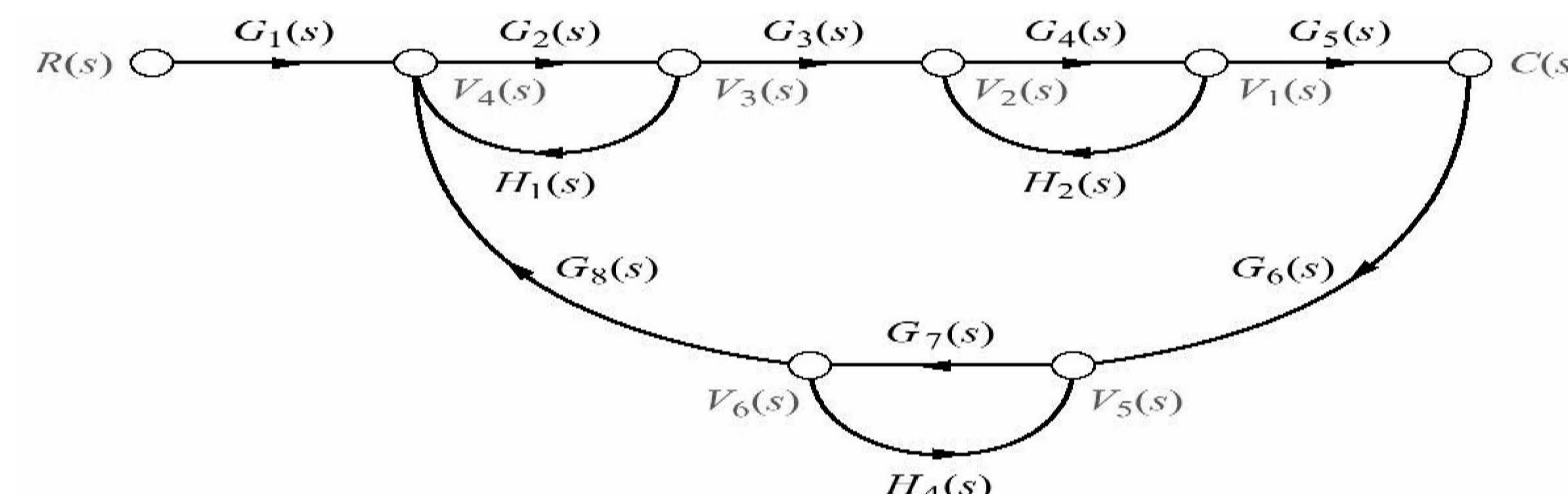


1. $G_2(s)H_1(s)$
2. $G_4(s)H_2(s)$
3. $G_4(s)G_5(s)H_3(s)$
4. $G_4(s)G_6(s)H_3(s)$

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Örnek: Aşağıdaki işaret akış diyagramı verilen sistemin $C(s)/R(s)$ taransfer fonksiyonunu bulunuz.



Önce ileri yol kazancını belirleyelim: $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)$

Kapalı döngü kazançları:

- | | |
|-------------------|---|
| 1. $G_2(s)H_1(s)$ | 3. $G_7H_4(s)$ |
| 2. $G_4(s)H_2(s)$ | 4. $G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)$ |

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İleri Yol Kazancı: Giriş nod'undan başlayıp çıkış nod'una kadar olan kazançların çarpımıdır.

1. $G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)G_7(s)$
2. $G_1(s)G_2(s)G_3(s)G_4(s)G_6(s)G_7(s)$

Temassız Döngü: Ortak nod'u olmayan döngülerdir.

$G_2(s)H_1(s)$ döngüsü $G_4(s)H_2(s)$, $G_4(s)G_5(s)H_3(s)$, ve $G_4(s)G_6(s)H_3(s)$ döngüleri ile temassızdır.

Temassız Döngü Kazancı: Temassız döngü kazançlarının iki, üçlü, etc çarpımıdır.

- | | |
|---------------------------------------|---------------------------------------|
| 1. $[G_2(s)H_1(s)]G_4(s)H_2(s)$ | 3. $[G_2(s)H_1(s)]G_4(s)G_6(s)H_3(s)$ |
| 2. $[G_2(s)H_1(s)]G_4(s)G_5(s)H_3(s)$ | |

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34

Bu örnekte, aynı anda üç temassız döngü olmadığından üçlü çarpım ile temassız döngü kazancımız yok.

S.J.Mason Yasası:

$$G(s) = \frac{C(s)}{R(S)} = \frac{\sum_k T_k \Delta_k}{\Delta}$$

k = iler yol sayısı

T_k = k. ileri yol kazancı

$\Delta = 1 - \Sigma(\text{döngü kazançları}) + \Sigma(\text{ikili çarpım temassız döngü kazançları})$
 $- \Sigma(\text{üçlü çarpım temassız döngü kazançları})$
 $+ \Sigma(\text{dörtlü çarpım temassız döngü kazançları})$

$\Delta_k = \Delta - \Sigma(k. \text{ Yola temas eden döngü kazançları})$. Bir başka deyişle

Δ 'ya temas etmeyen döngü kazançları

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35

Dikkat edilecek olursa 1. döngü 2. ve 3. döngüler ile temas etmez. 2. döngü de 3. döngü ile temas etmez. 1., 2. ve 3. döngüler 4. döngü ile temas etmektedir. Bu durumda;

İkili çarpım temassız döngü kazançları:

$$1. G_2(s)H_1(s)G_4(s)H_2(s) \quad 2. G_2(s)H_1(s)G_7(s)H_4(s)$$

$$3. G_4(s)H_2(s)G_7(s)H_4(s)$$

Üçlü çarpım temassız döngü kazancı:

$$G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)$$

Δ_k 'yı oluşturalım:

$$\Delta = 1 - [G_2(s)H_1(s) + G_4(s)H_2(s) + G_7(s)H_4(s)]$$

$$+ [G_2(s)G_3(s)G_4(s)G_5(s)G_6(s)G_7(s)G_8(s)]$$

$$+ [G_2(s)H_1(s)G_4(s)H_2(s) + G_2(s)H_1(s)G_7(s)H_4(s)]$$

$$+ G_4(s)H_2(s)G_7(s)H_4(s)]$$

$$- [G_2(s)H_1(s)G_4(s)H_2(s)G_7(s)H_4(s)]$$

26 February 2007

Otomatik Kontrol
Prof.Dr.Galip Cansever

37

Δ_k 'yı oluşturalım: İleri yola temas etmeyen Δ 'nın parçası

$$\Delta_1 = 1 - G_7(s)H_4(s)$$

Sadeleşmiş işaret akış diyagramı J.S.Mason formülüne göre,

$$G(s) = \frac{C(s)}{R(S)} = \frac{\sum_k T_k \Delta_k}{\Delta} = \frac{T_1 \Delta_1}{\Delta}$$

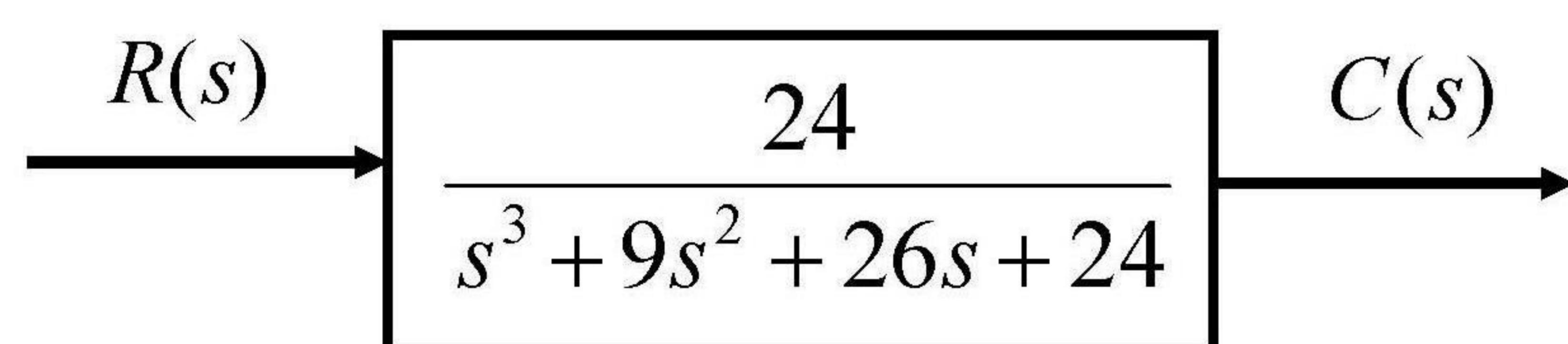
$$= \frac{[G_1(s)G_2(s)G_3(s)G_4(s)G_5(s)][1 - G_7(s)H_4(s)]}{\Delta}$$

26 February 2007

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38

Durum Denklemlerinin İşaret Akış Diyagramları



Sisteminin durum uzayı gösterimi:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

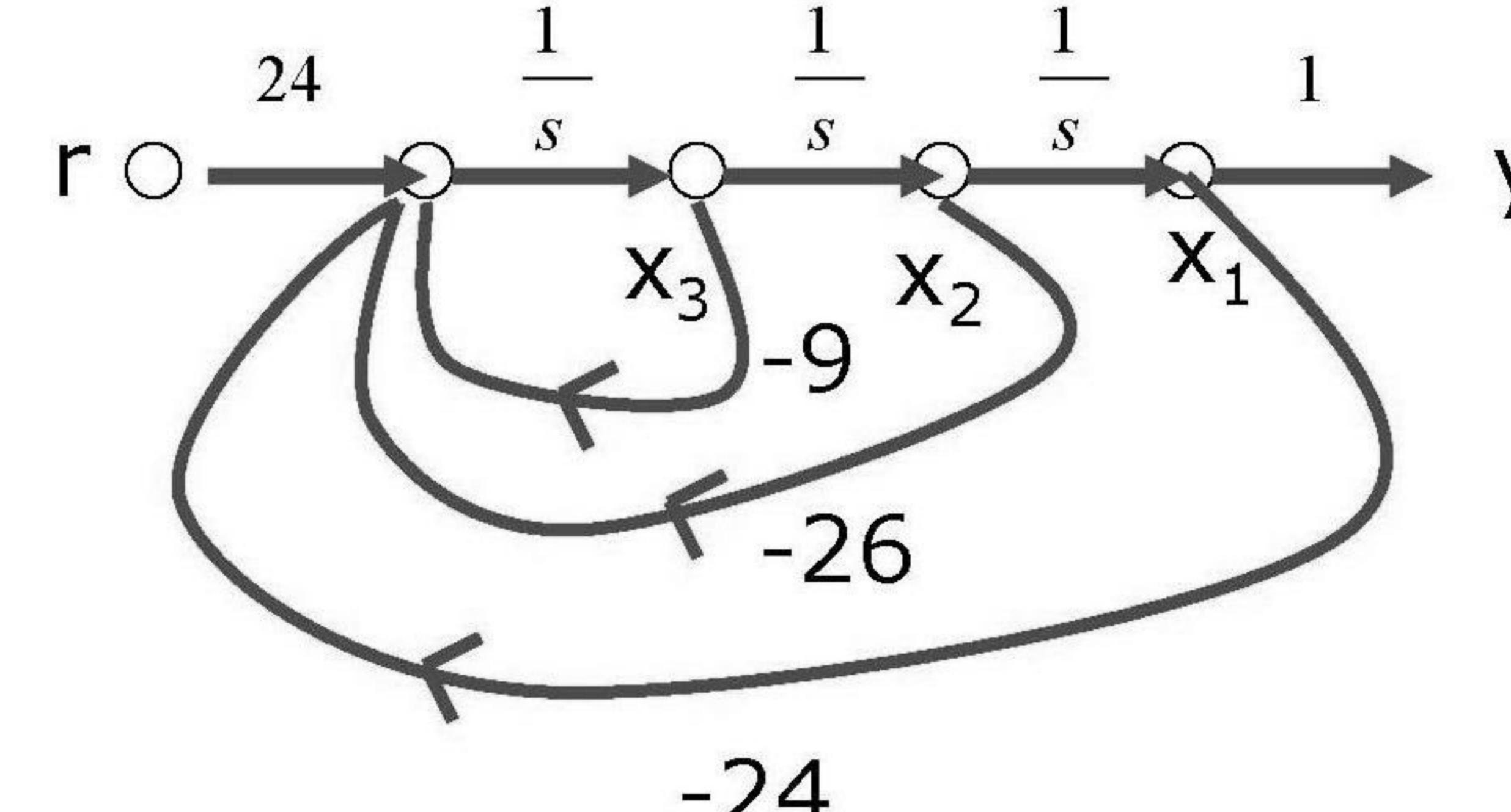
$$\dot{x}_3 = -24x_1 - 26x_2 - 9x_3 + 24r$$

$$y = x_1$$

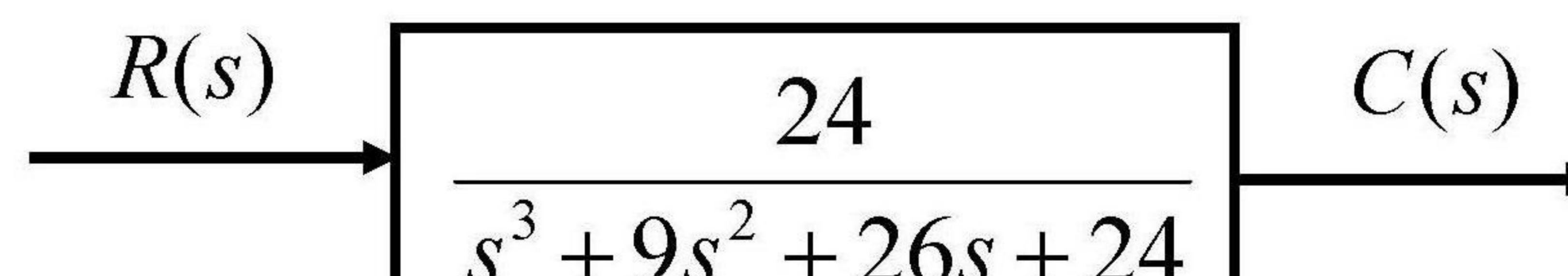
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39



Ard arda (Kaskat) Gösterim:



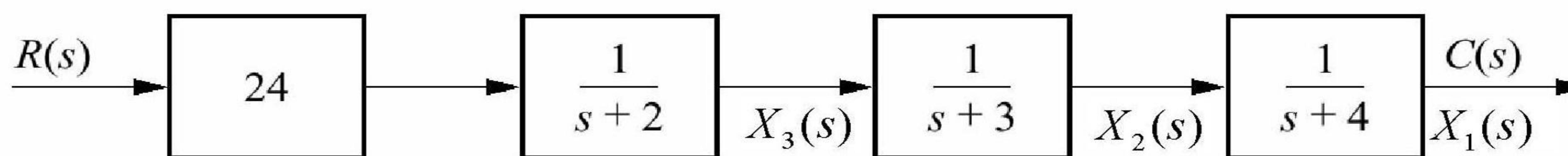
$$\frac{C(s)}{R(s)} = \frac{24}{(s+2)(s+3)(s+4)}$$

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40

Sistemimiz alternatif olarak;



Şeklinde gösterebiliriz

Her bir birinci derece blok'un çıkışını durum değişkeni olarak tanımlayalım.

Her bir blok'un giriş-cıkış ilişkisi: $\frac{C_i(s)}{R_i(s)} = \frac{1}{(s + a_i)}$

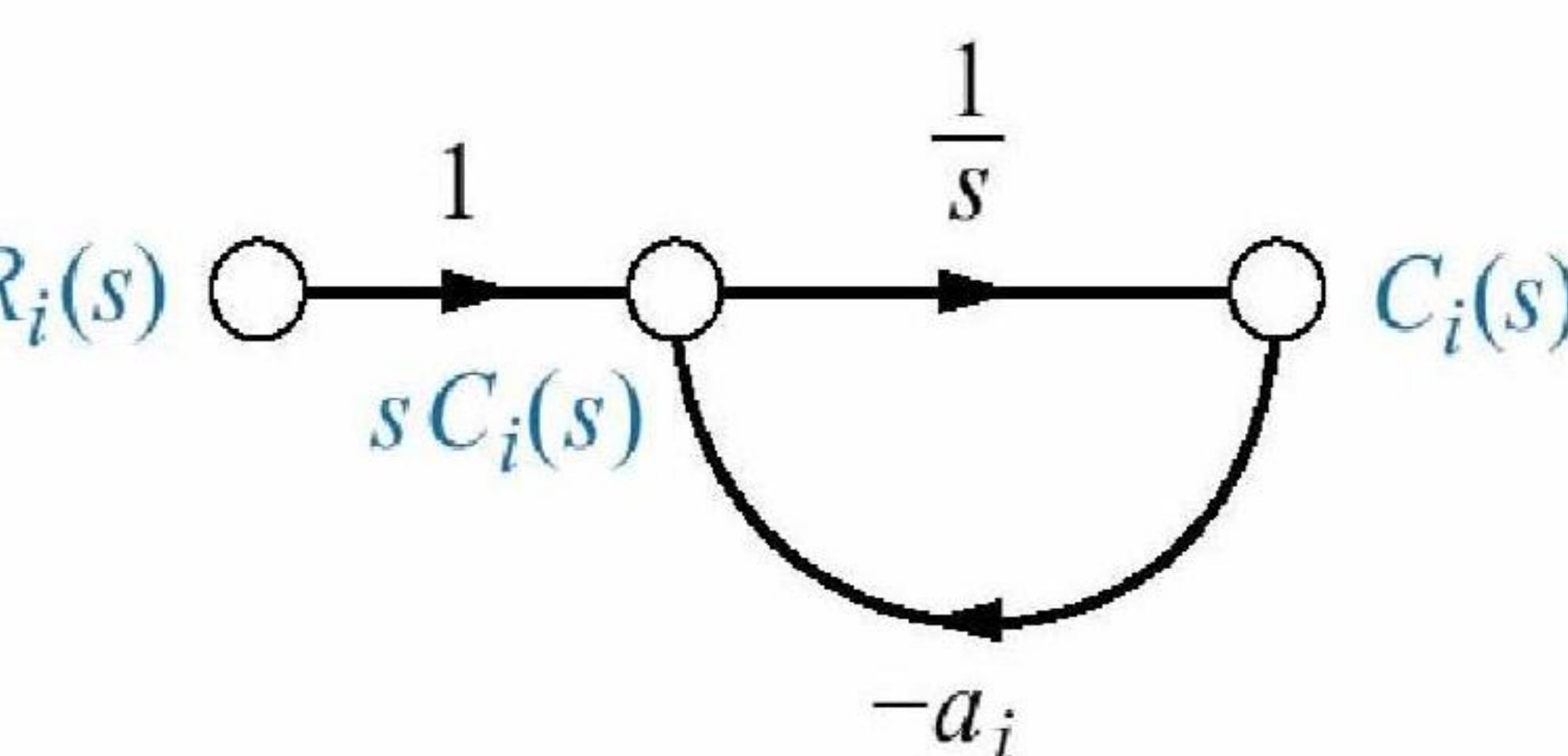
$C_i(s)(s + a_i) = R_i(s)$ Ters Laplas alalım,

$$\frac{dc_i}{dt} + a_i c_i = r_i(t)$$

$$\frac{dc_i}{dt} = -a_i c_i + r_i(t)$$

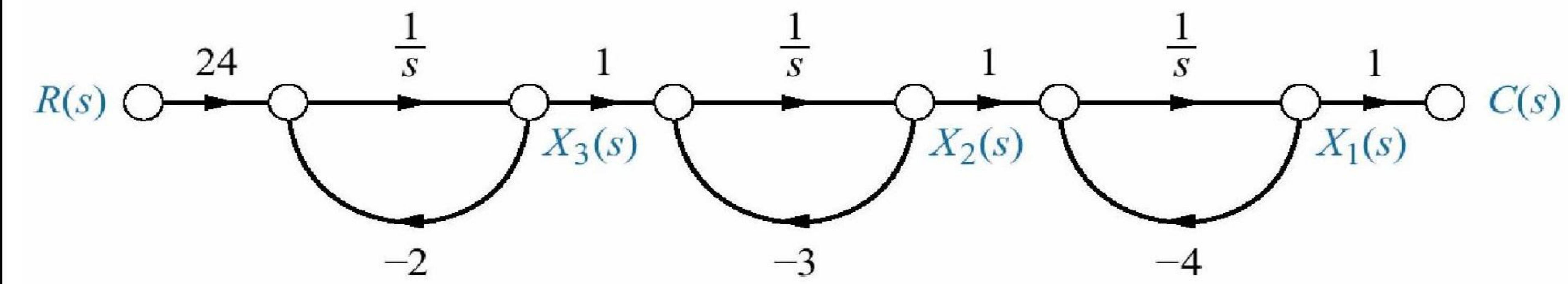
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41

Örneğimizdeki transfer fonksiyonlarını ard arda eklersek işaret akış diyagramımız:



Durum değişkeninin türevi her bir integratörün girişi olacağını hatırlayacak olursak durum dinamikleri;

$$\dot{x}_1 = -4x_1 + x_2$$

$$\dot{x}_2 = -3x_2 + x_3$$

$$\dot{x}_3 = -2x_3 + 24r$$

$$y(t) = c(t) = x_1$$

26 February 2007

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Prof.Dr.Galip Cansever

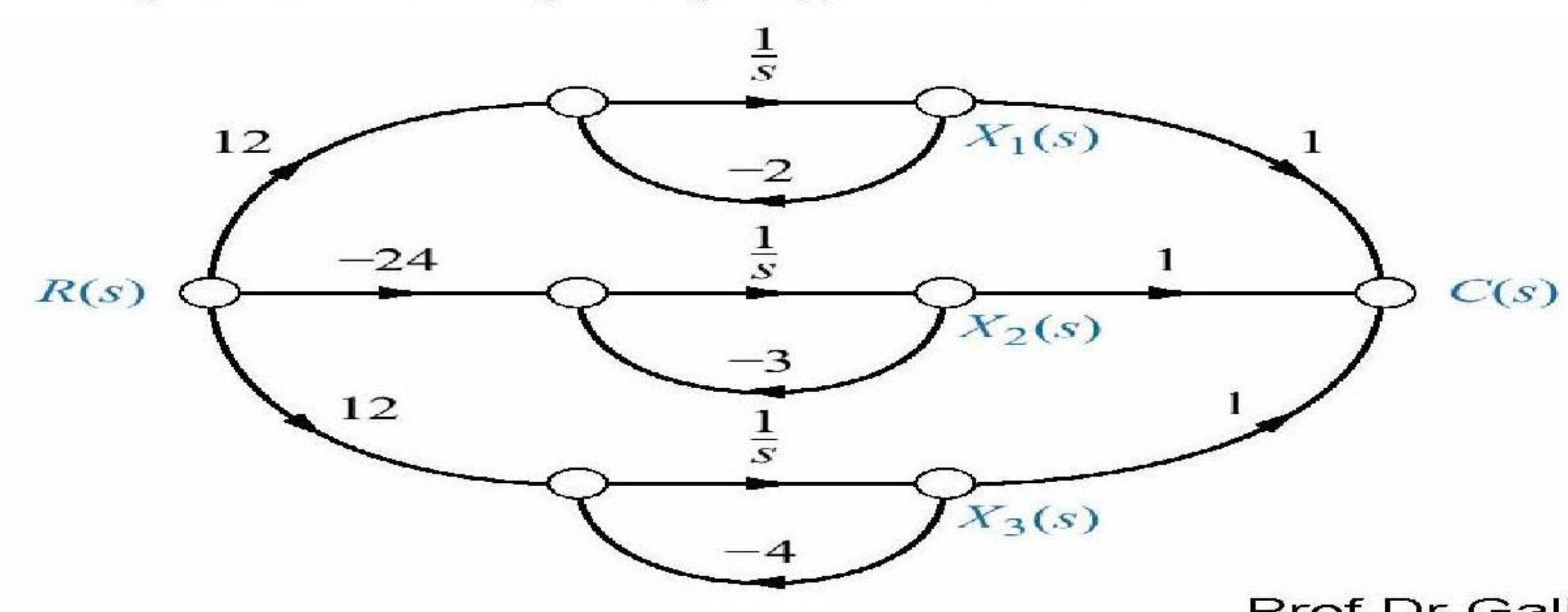
42

Paralel Gösterim: Sistemi temsil eden bir diğer gösterimdir.

$$\frac{C(s)}{R(s)} = \frac{24}{(s+2)(s+3)(s+4)} = \frac{12}{(s+2)} - \frac{24}{(s+3)} + \frac{12}{(s+4)}$$

$$C(s) = \frac{12}{(s+2)} R(s) - \frac{24}{(s+3)} R(s) + \frac{12}{(s+4)} R(s)$$

İşaret akış diyagramı:



$$\dot{x}_1 = -2x_1 + 12r(t)$$

$$\dot{x}_2 = -3x_2 - 24r(t)$$

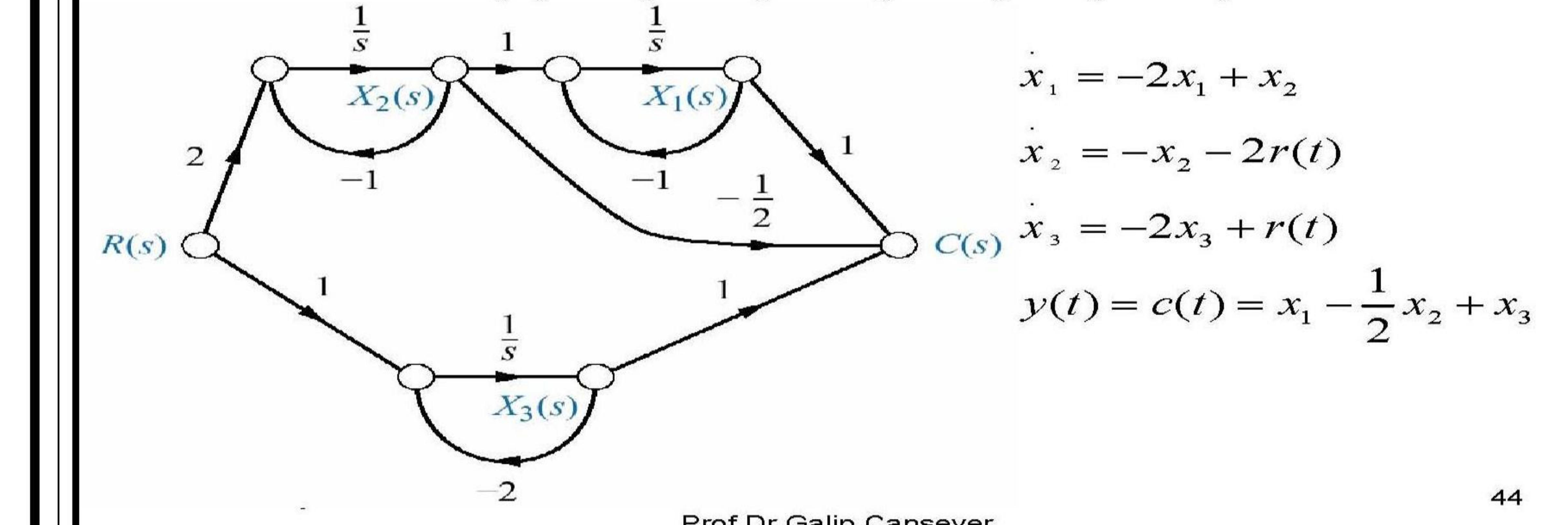
$$\dot{x}_3 = -4x_3 + 12r(t)$$

$$y(t) = c(t) = x_1 + x_2 + x_3$$

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Örnek: $\frac{C(s)}{R(s)} = \frac{(s+3)}{(s+1)^2(s+2)}$ işaret akış diyagramını çiziniz durum dinamiklerini yazınız.

$$\frac{C(s)}{R(s)} = \frac{2}{(s+1)^2} - \frac{1}{(s+1)} + \frac{1}{(s+2)}$$



43

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44

Geçişli Faz Değişken Gösterimi:

Sistem sıfırlara sahip olduğu durumdaki gösterimdir.

$$\frac{C(s)}{R(s)} = \frac{s^2 + 7s + 2}{s^3 + 9s^2 + 26s + 24}$$

Payı ve paydayı en yüksek derece olan s^3 'e bölelim,

$$\frac{C(s)}{R(s)} = \frac{\frac{1}{s} + \frac{7}{s^2} + \frac{2}{s^3}}{1 + \frac{9}{s} + \frac{26}{s^2} + \frac{24}{s^3}}$$

İçler dışlar çarpımı yapalım;

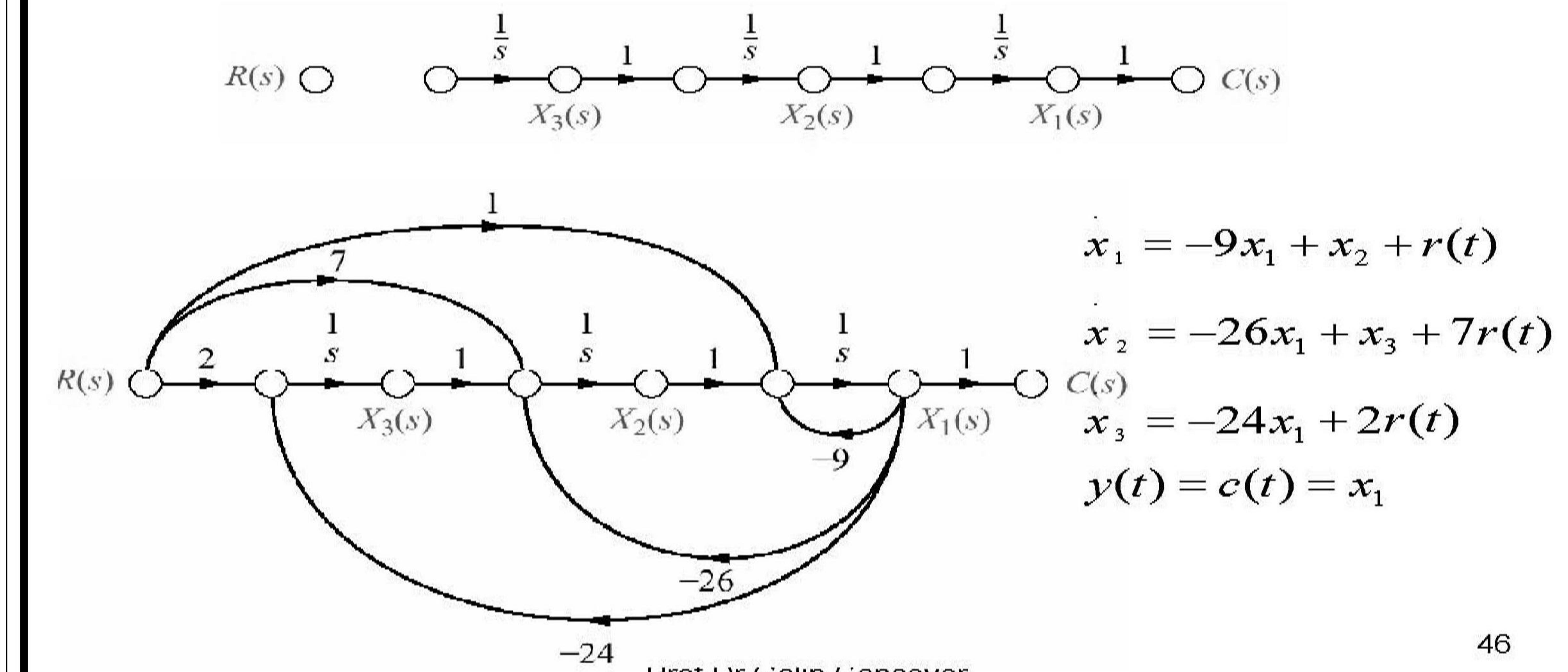
$$\left[\frac{1}{s} + \frac{7}{s^2} + \frac{2}{s^3} \right] R(s) = \left[1 + \frac{9}{s} + \frac{26}{s^2} + \frac{24}{s^3} \right] C(s)$$

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Aynı dereceli terimleri bir arada toplayalım:

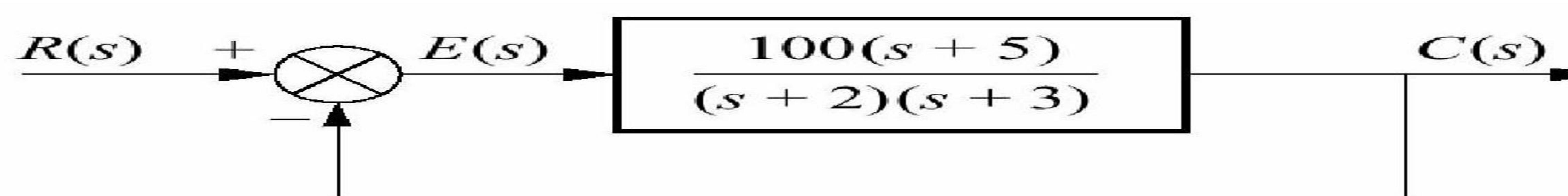
$$C(s) = \frac{1}{s} [R(s) - 9C(s)] + \frac{1}{s^2} [7R(s) - 26C(s)] + \frac{1}{s^3} [2R(s) - 24C(s)]$$



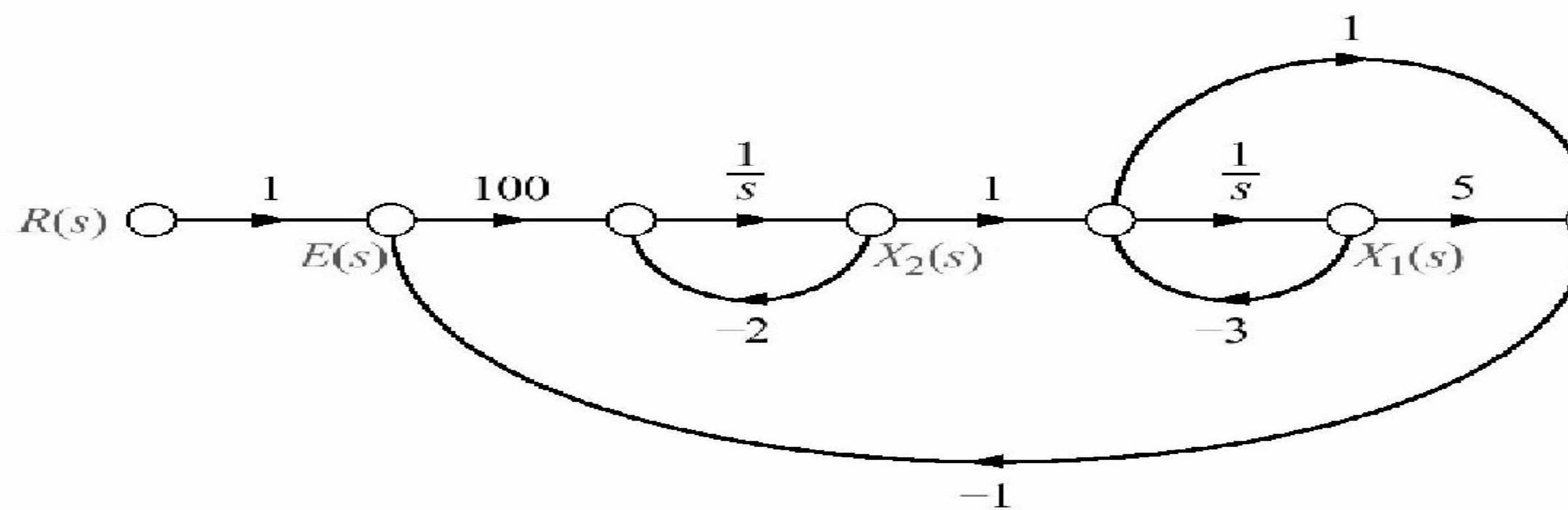
45

46

Örnek:



İşaret akış diyagramınızı
çiziniz durum dinamiklerini
yazınız.



$$\dot{x}_1 = -3x_1 + x_2$$

$$\dot{x}_2 = -2x_2 + 100(r - c)$$

$$c = 5x_1 + (x_2 - 3x_1) = 2x_1 + x_2$$

$$\dot{x}_1 = -3x_1 + x_2$$

$$\dot{x}_2 = -200x_1 - 102x_2 + 100r$$

$$y = c(t) = 2x_1 + x_2$$

26 February 2007

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Prof.Dr.Galip Cansever

47

Table 2.7. Block Diagram Transformations

Transformation	Original Diagram	Equivalent Diagram
1. Combining blocks in cascade		
2. Moving a summing point behind a block		
3. Moving a pickoff point ahead of a block		
4. Moving a pickoff point behind a block		
5. Moving a summing point ahead of a block		
6. Eliminating a feedback loop		

diagram shown in Fig. 2.22. This negative feedback control system is described by the equation for the actuating signal

$$\begin{aligned} E_a(s) &= R(s) - B(s) \\ &= R(s) - H(s)C(s). \end{aligned} \quad (2.81)$$

Because the output is related to the actuating signal by $G(s)$, we have

$$C(s) = G(s)E_a(s), \quad (2.82)$$

and therefore

$$C(s) = G(s)(R(s) - H(s)C(s)). \quad (2.83)$$

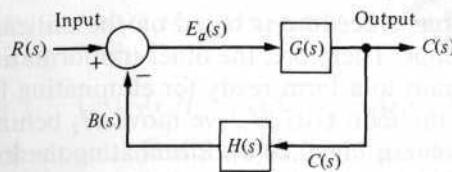


Figure 2.22. Negative feedback control system.

Solving for $C(s)$, we obtain

$$C(s)(1 + G(s)H(s)) = G(s)R(s). \quad (2.84)$$

Therefore the transfer function relating the output $C(s)$ to the input $R(s)$ is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}. \quad (2.85)$$

This *closed-loop transfer function* is particularly important because it represents many of the existing practical control systems.

The reduction of the block diagram shown in Fig. 2.22 to a single block representation is one example of several useful block diagram reductions. These diagram transformations are given in Table 2.7. All the transformations in Table 2.7 can be derived by simple algebraic manipulation of the equations representing the blocks. System analysis by the method of block diagram reduction has the advantage of affording a better understanding of the contribution of each component element than is possible to obtain by the manipulation of equations. The utility of the block diagram transformations will be illustrated by an example of a block diagram reduction.

■ Example 2.5 Block diagram reduction

The block diagram of a multiple-loop feedback control system is shown in Fig. 2.23. It is interesting to note that the feedback signal $H_1(s)C(s)$ is a positive feedback signal and the loop $G_3(s)G_4(s)H_1(s)$ is called a *positive feedback loop*. The

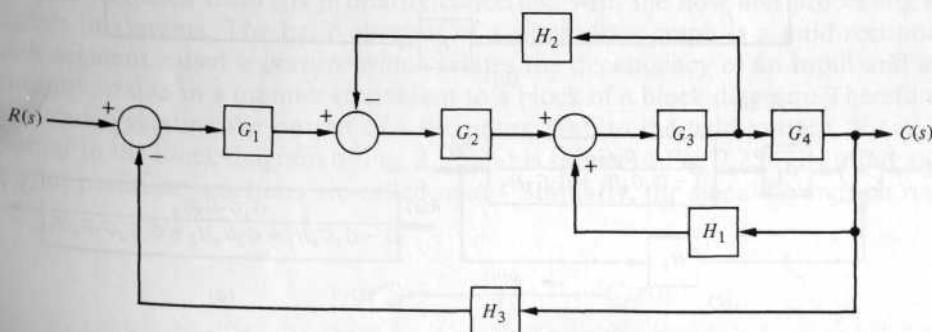


Figure 2.23. Multiple-loop feedback control system.

block diagram reduction procedure is based on the utilization of rule 6, which eliminates feedback loops. Therefore, the other transformations are used in order to transform the diagram to a form ready for eliminating feedback loops. First, in order to eliminate the loop $G_3G_4H_1$, we move H_2 behind block G_4 by using rule 4, and therefore obtain Fig. 2.24(a). Eliminating the loop $G_3G_4H_1$ by using rule 6, we obtain Fig. 2.24(b). Then, eliminating the inner loop containing H_2/G_4 , we obtain Fig. 2.24(c). Finally, by reducing the loop containing H_3 we obtain the closed-loop system transfer function as shown in Fig. 2.24(d). It is worthwhile to examine the form of the numerator and denominator of this closed-loop transfer function. We note that the numerator is composed of the cascade transfer function of the feedforward elements connecting the input $R(s)$ and the output $C(s)$. The denominator is comprised of 1 minus the sum of each loop transfer function. The sign of the loop $G_3G_4H_1$ is plus because it is a positive feedback loop, whereas

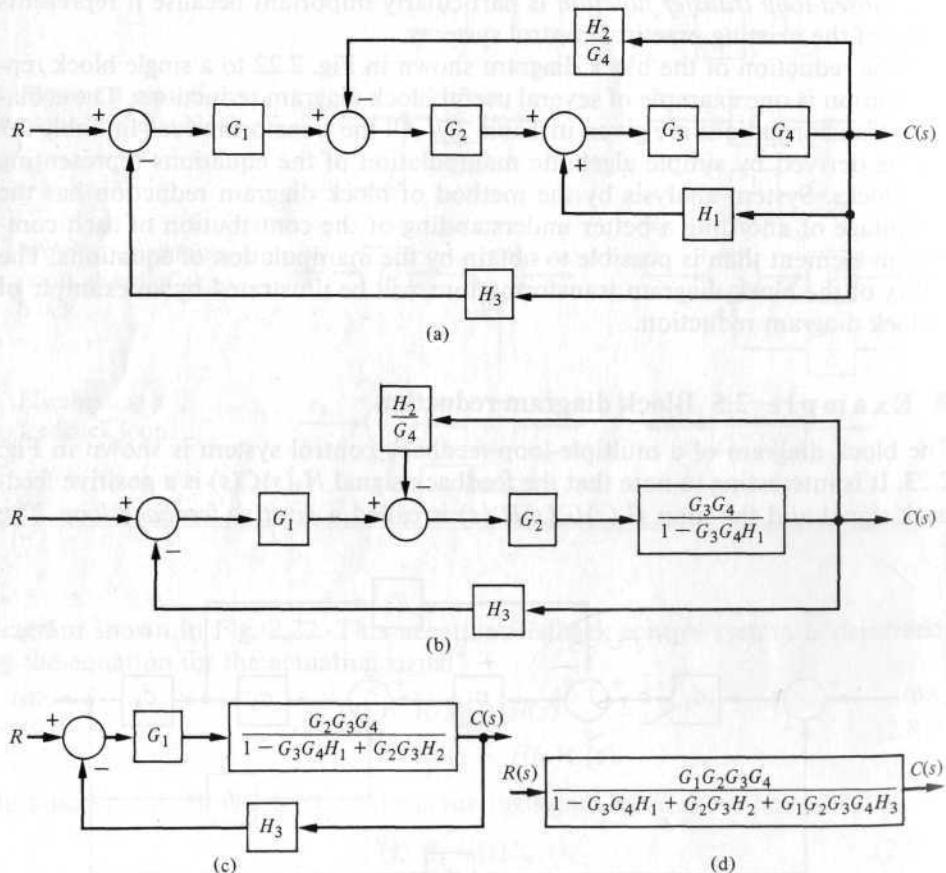
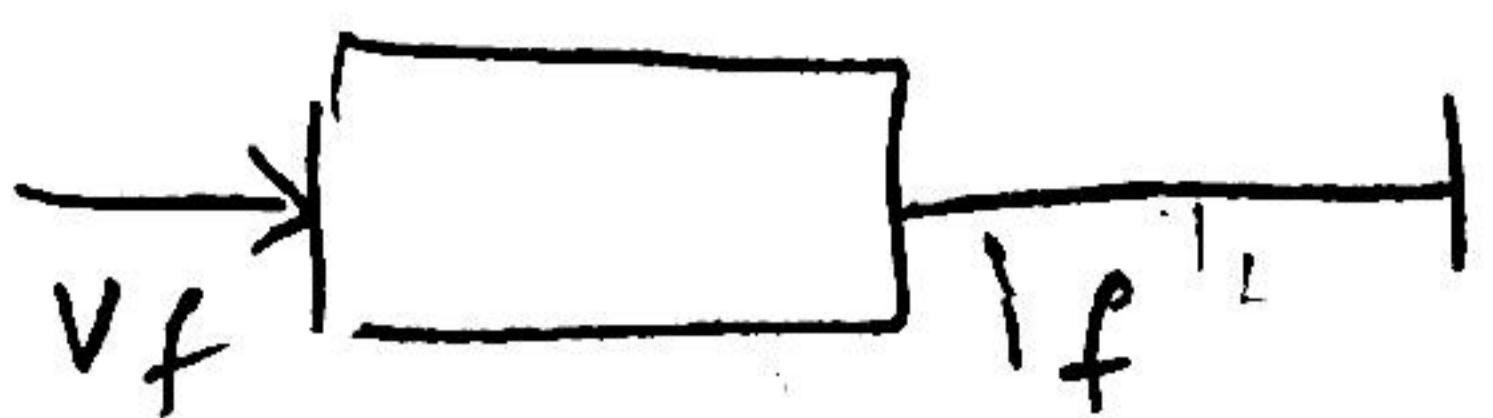


Figure 2.24. Block diagram reduction of the system of Fig. 2.23.

Block diagrams

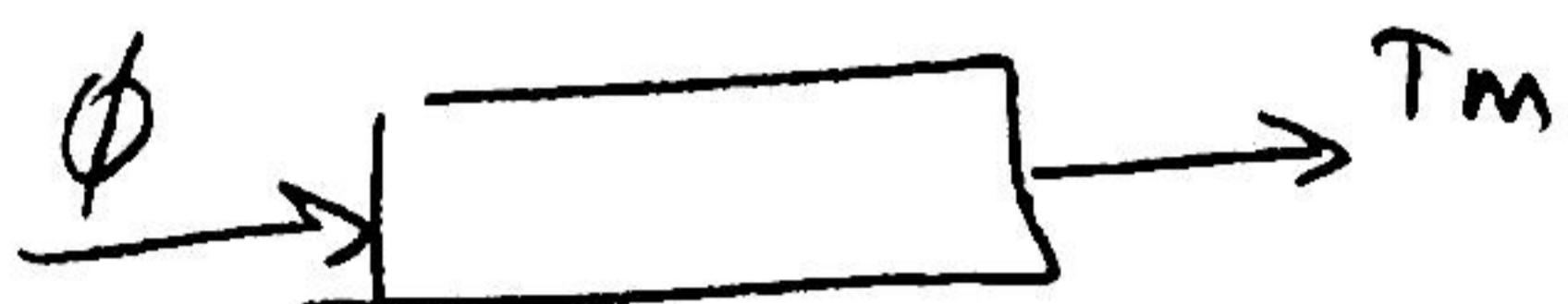
Cn44



$$\frac{i_f(s)}{v_f(s)} = \frac{1}{R_f + sL_f} = G_1(s)$$



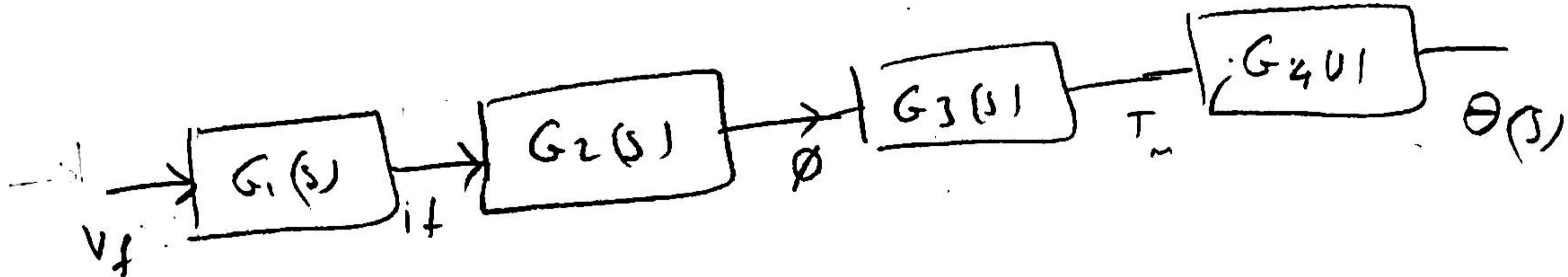
$$\frac{\phi(s)}{i_f(s)} = K_f = G_2(s)$$



$$\frac{T_m(s)}{\phi(s)} = K_I q = G_3(s)$$



$$\frac{\theta(s)}{T_m(s)} = \frac{1}{J s^2 + b s} = G_4(s)$$

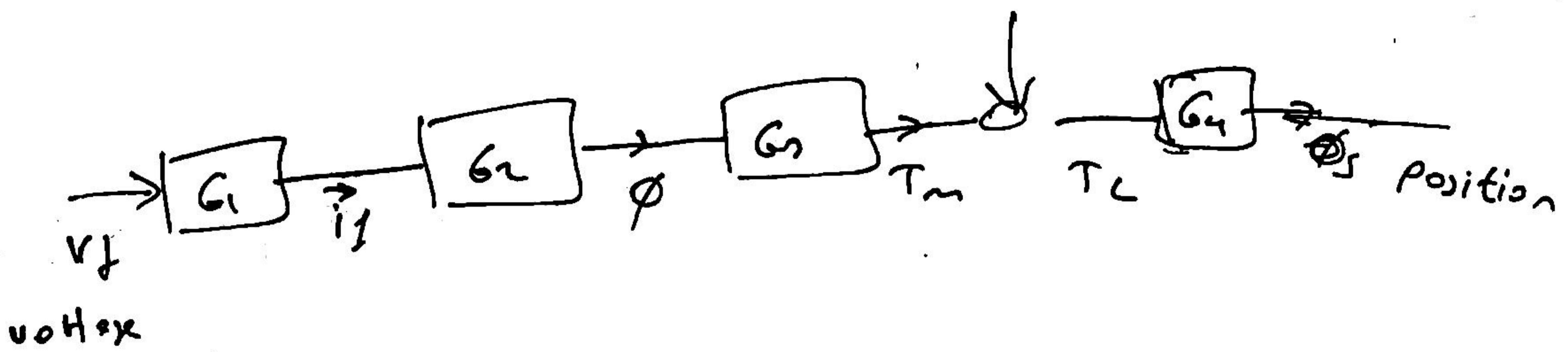


$$\frac{\theta(s)}{v_f(s)} = \frac{\theta(s)}{\tau_m(s)} \cdot \frac{\tau_m(s)}{\phi(s)} \cdot \frac{\phi(s)}{i_f(s)} \cdot \frac{i_f(s)}{v_f(s)}$$

$$= \frac{\theta(s)}{\tau_{df}(s)}$$

$$= G_1(s) \cdot G_2(s) \cdot G_3(s) \cdot G_4(s)$$

$$\frac{\theta(s)}{v_f(s)} = \frac{1}{R_f + sL_f} \cdot K_f \cdot K_I q \cdot \frac{1}{J s^2 + b s}$$



We assumed $T_m = T_L$

in practice $T_m = T_L + T_D$
 $T_m - T_D = T_L$

Theorem

Produced torque = load torque + disturbances
 disturbances = friction + electric leakage losses

$$\text{Efficiency} = 100\% \quad P_{produced} = P_{load}$$

in a car

in this type

$$\frac{T_m}{V_f} = G_1 G_2 G_3$$

of system)

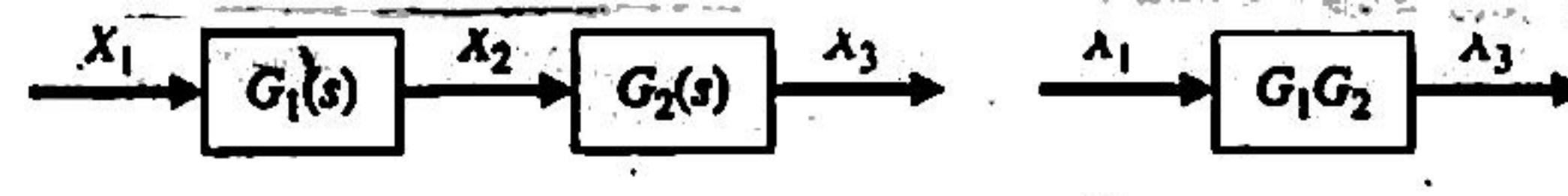
$$T_L = T_m - T_D$$

we define
 two transfer
 functions

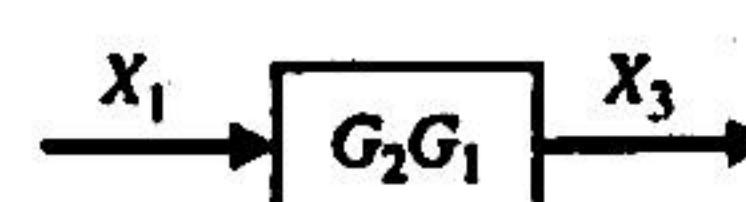
$$\Theta = G_4 T_L = G_4 T_m - G_4 T_D$$

$$\boxed{\Theta = G_4 G_1 G_2 G_3 V_f - G_4 T_D}$$

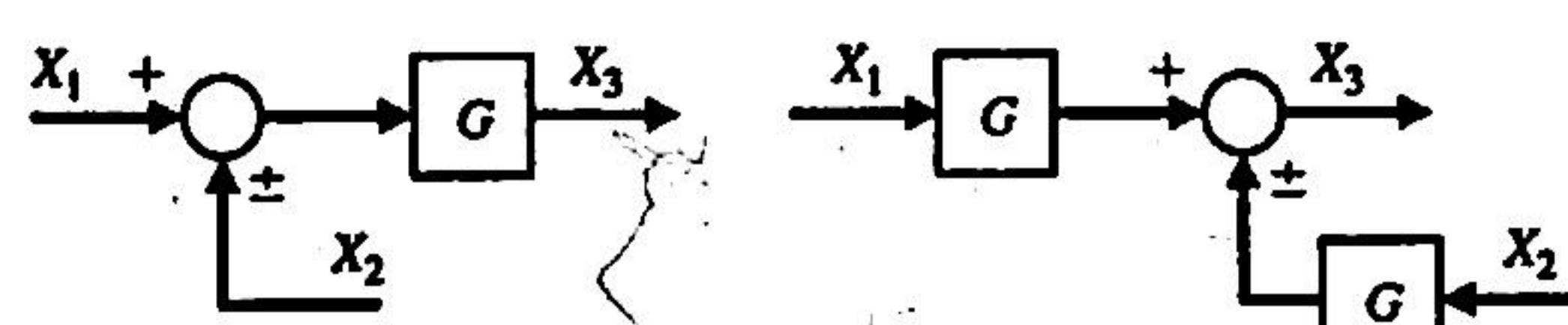
1. Combining blocks in cascade



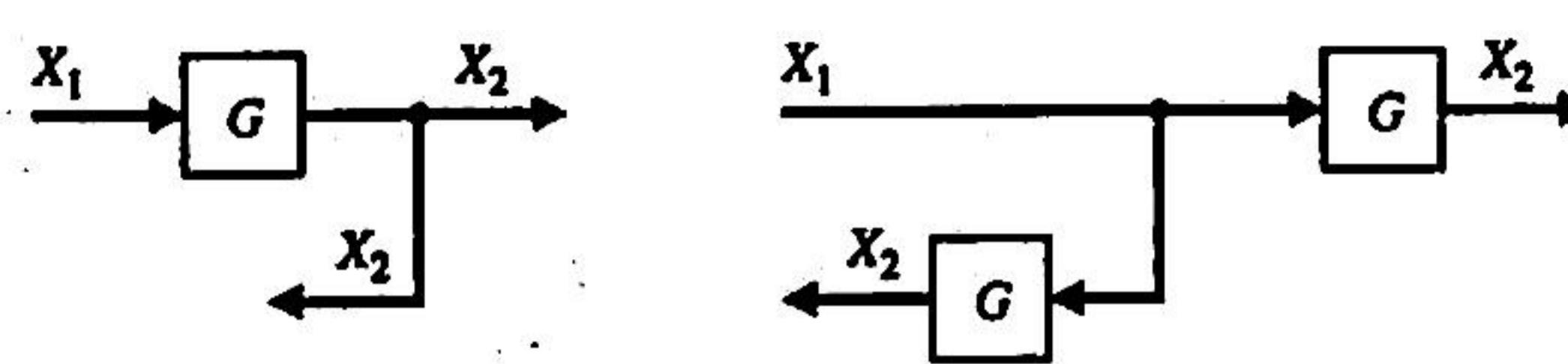
or



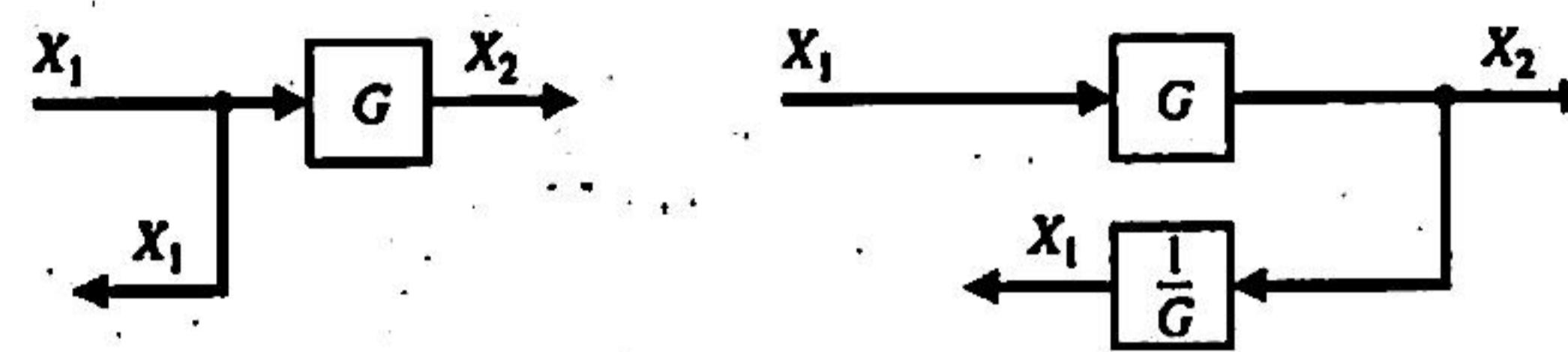
2. Moving a summing point behind a block



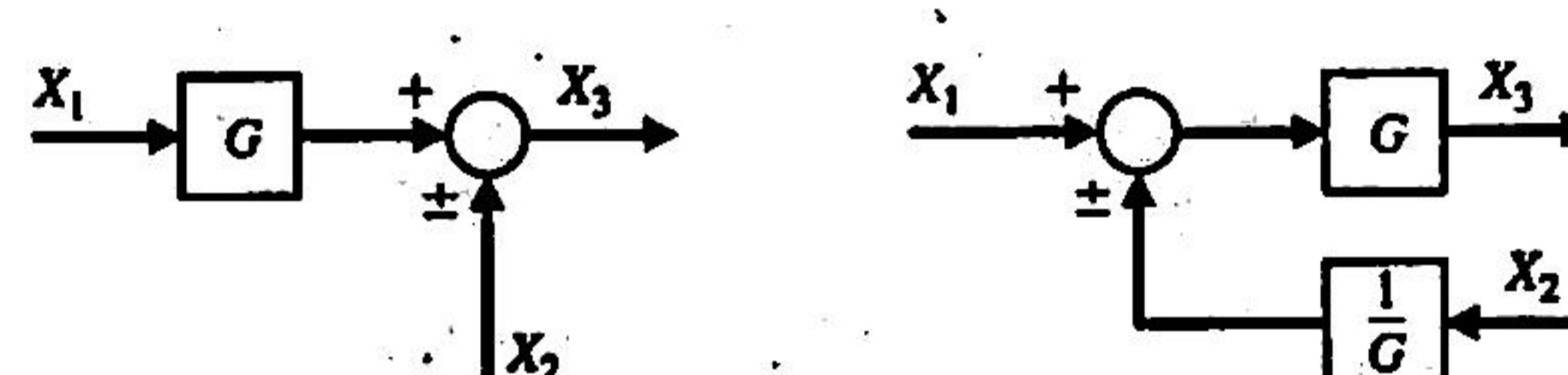
3. Moving a pickoff point ahead of a block



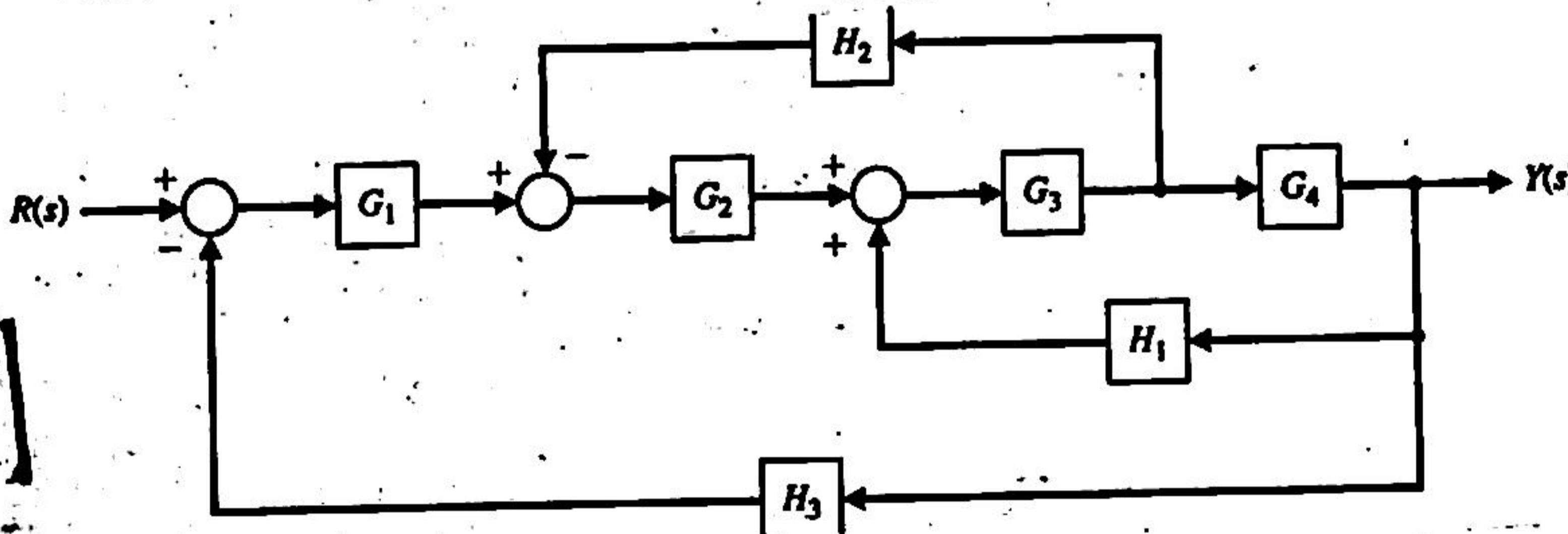
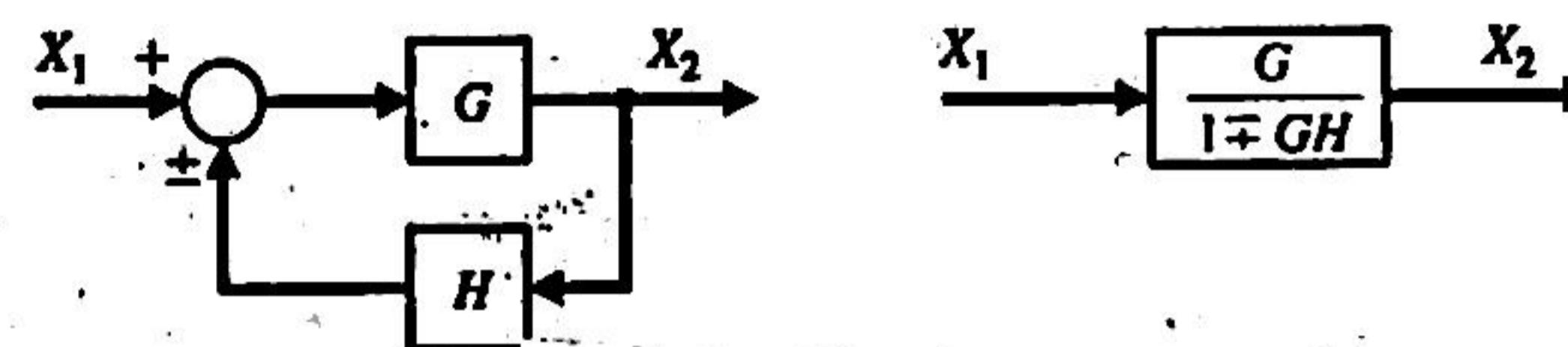
4. Moving a pickoff point behind a block



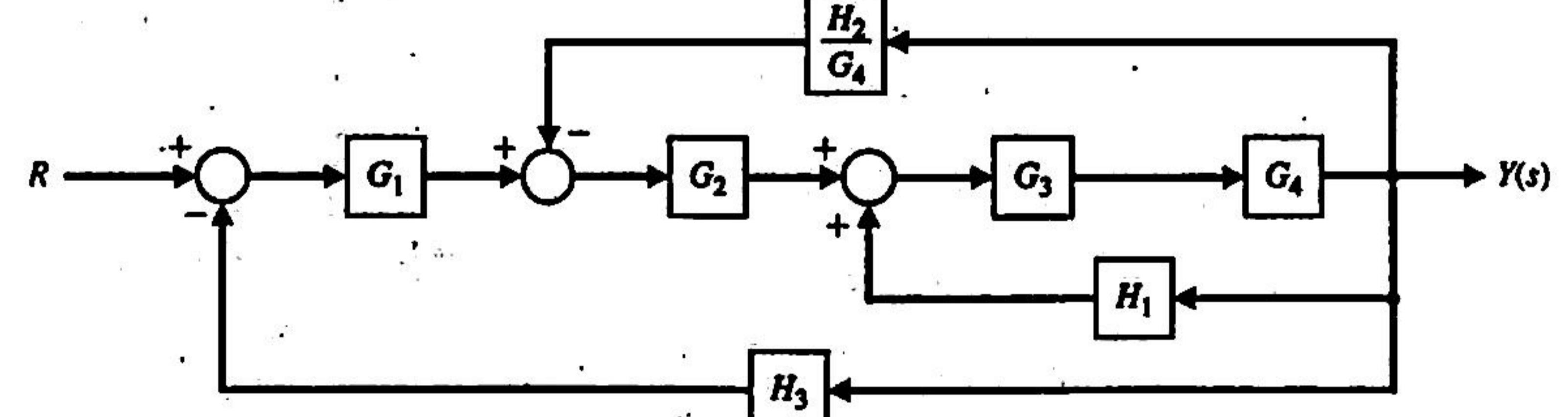
5. Moving a summing point ahead of a block



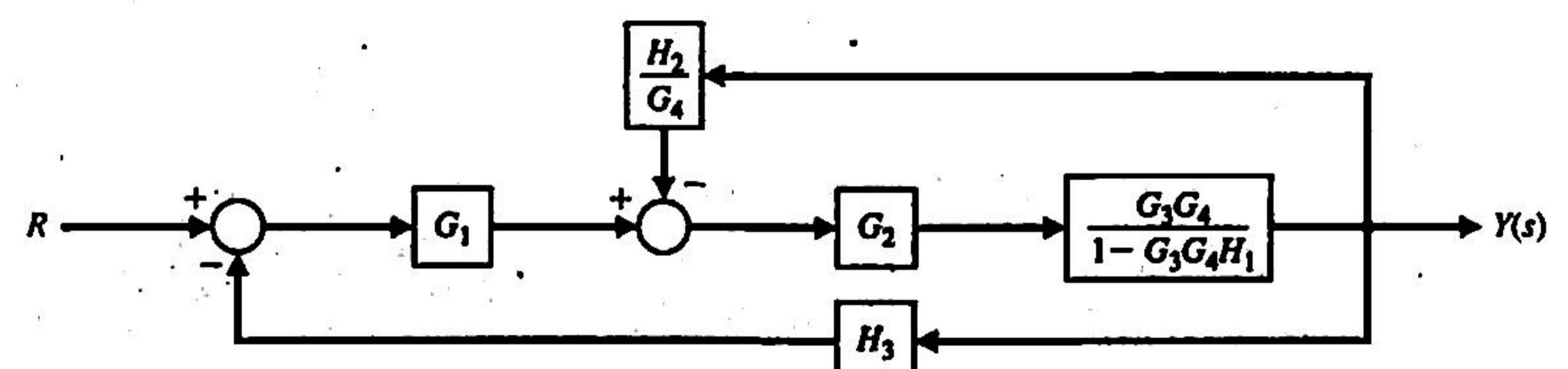
6. Eliminating a feedback loop



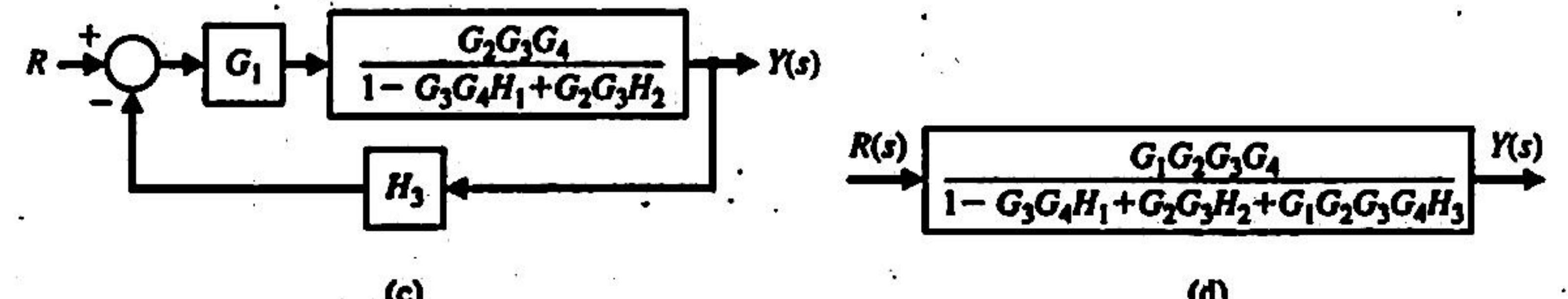
(a)



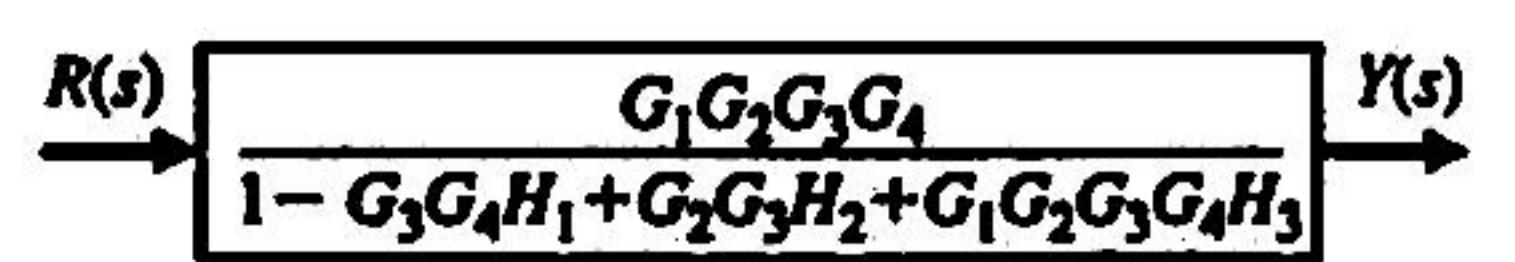
(a)



(b)



(c)



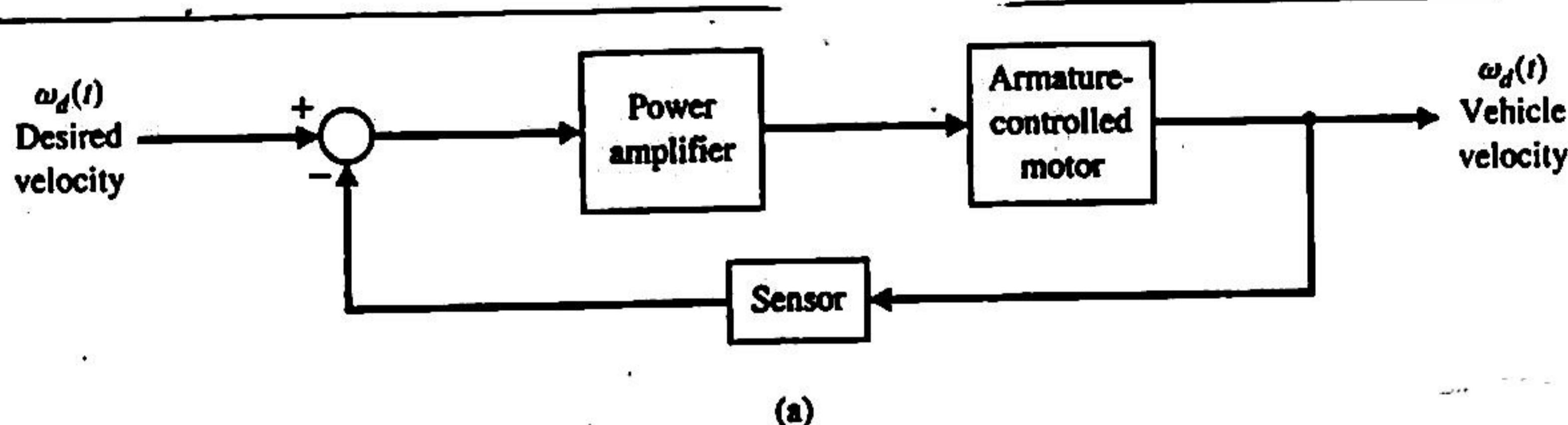
(d)

FIGURE 2.25
Block diagram reduction of the system of Fig. 2.24.

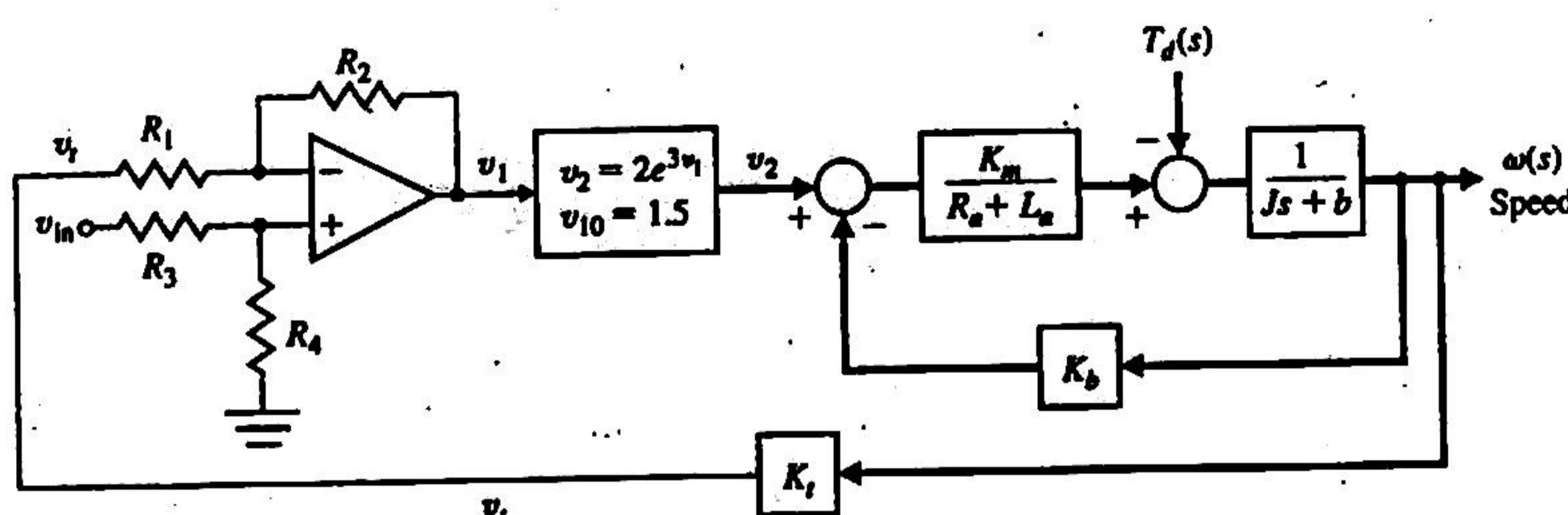
EXAMPLE 2.11 Electric traction motor control

A majority of modern trains and local transit vehicles utilize electric traction motors. The electric motor drive for a railway vehicle is shown in block diagram form in Fig. 2.33(a), incorporating the necessary control of the velocity of the vehicle. The goal of the design is to obtain a system model and the closed-loop transfer function of the system, $\omega(s)/\omega_d(s)$, select appropriate resistors R_1, R_2, R_3 , and R_4 , and then predict the system response.

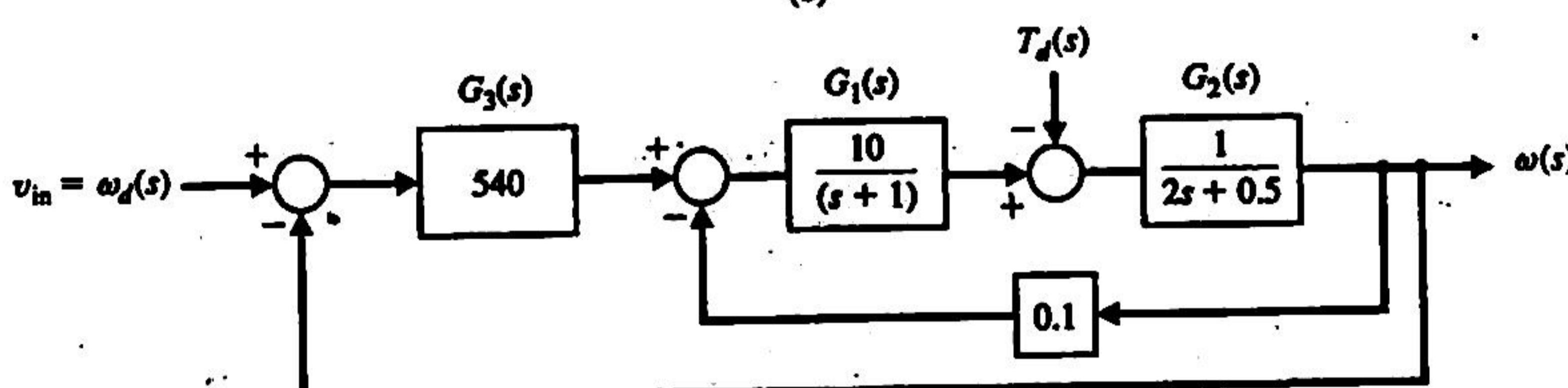
FIGURE 2.33
Speed control of
an electric traction
motor.



(a)

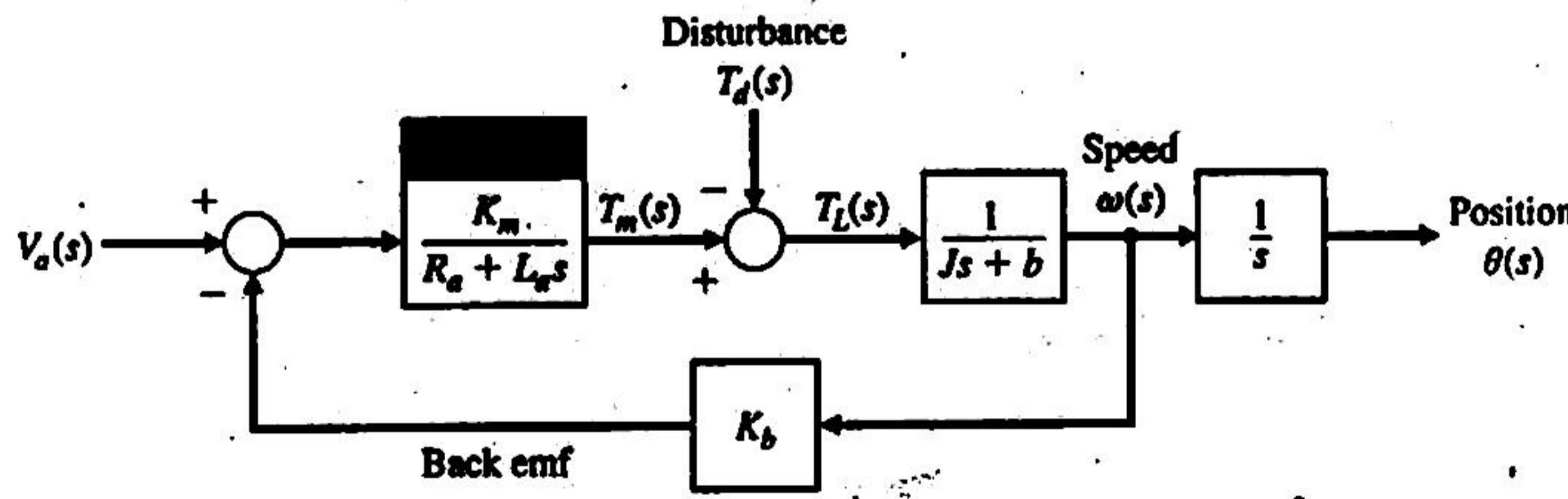


(b)



(c)

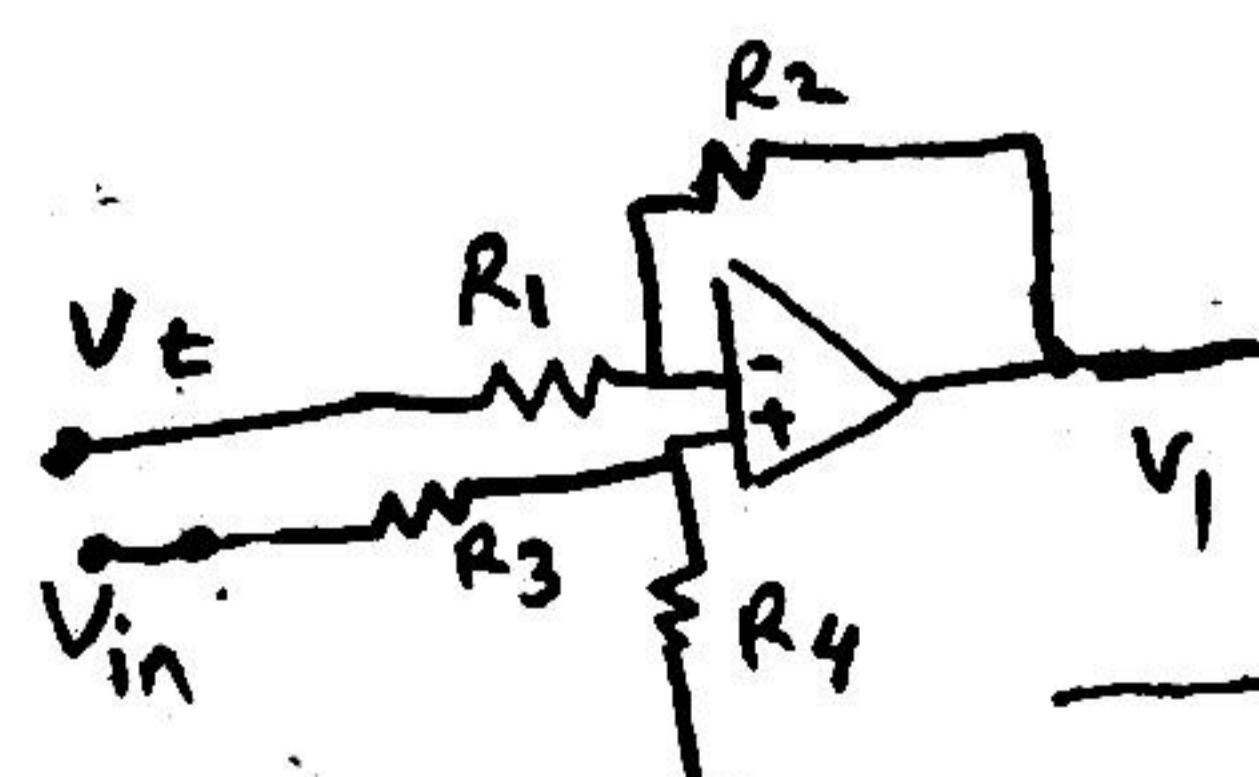
FIGURE 2.18
Armature-controlled
dc motor.



$$I_1 = I_2$$

$$\frac{V_L}{R_1} = -\frac{V_1}{R_2}$$

$$V_1 = -\frac{R_2}{R_1} V_L$$



$$V_{in} = 0$$



$$I_2 = I_1 = \frac{V_1}{R_2 + R_1}$$

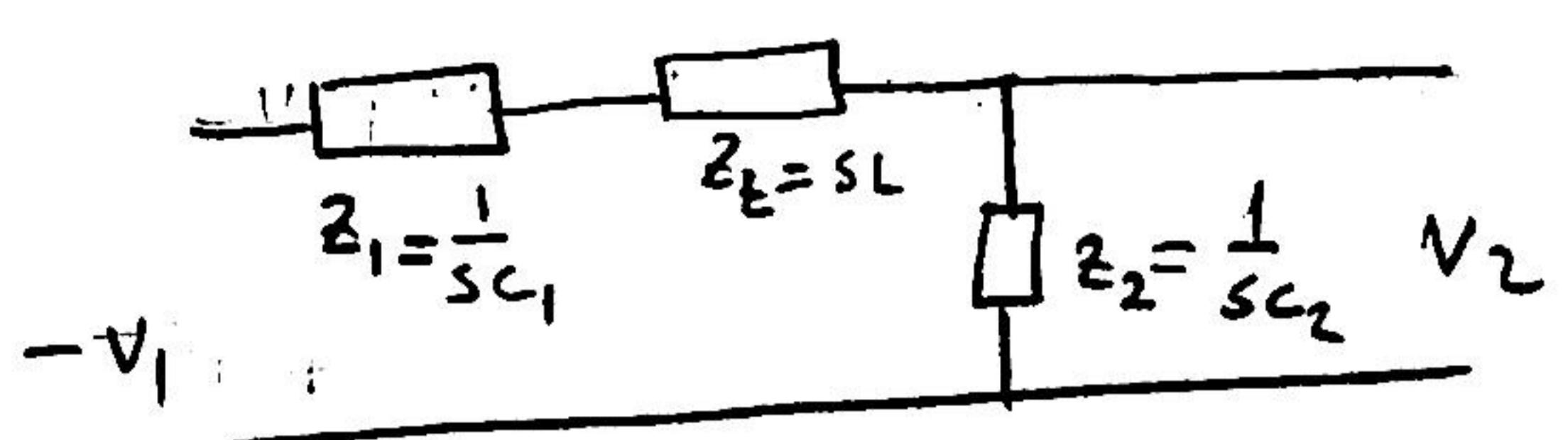
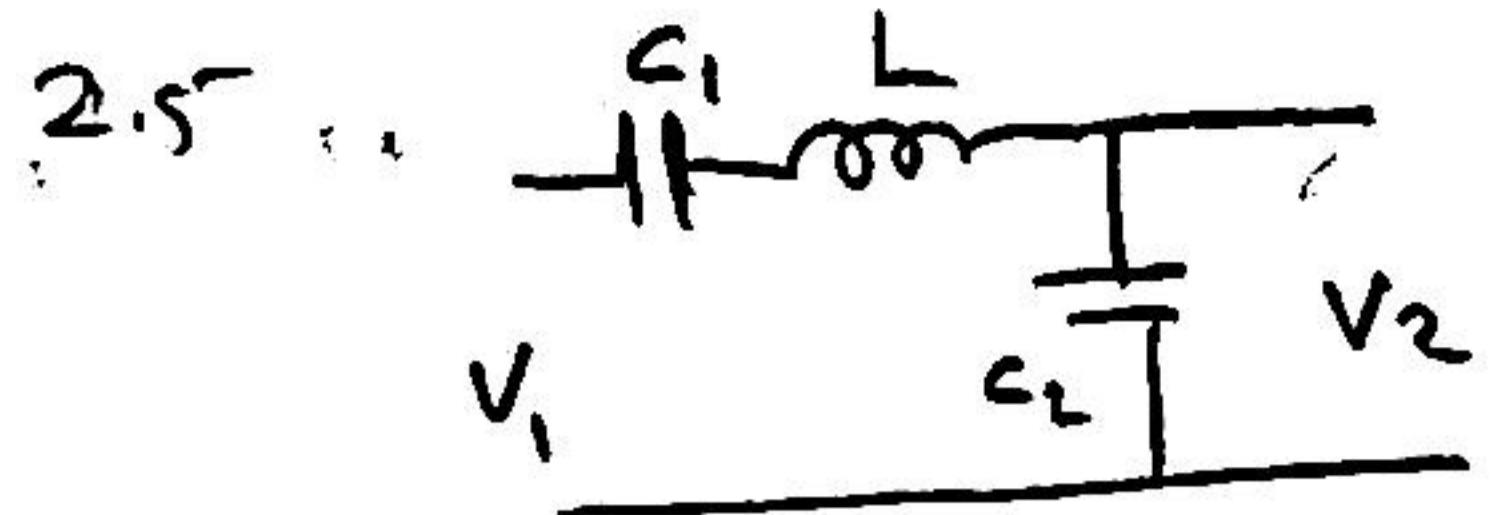
$$V_L = R_1 I_1 = R_1 \frac{V_1}{R_1 + R_2}$$

$$V_+ = \frac{R_4}{R_3 + R_4} V_{in} \quad \text{since } V_- = V_+$$

$$V_1 = \frac{R_1 + R_2}{R_1} \frac{R_4}{R_3 + R_4} V_{in}$$

Superposition

$$V_1 = -\frac{R_2}{n} V_L + \frac{R_3 + R_2}{n} \cdot \frac{R_4}{R_1 + R_2} V_{in}$$



$$-V_1 + Z_1 I + Z_L I + Z_2 I = 0$$

$$I = \frac{V_1}{Z_1 + Z_L + Z_2}$$

$$V_2 = Z_2 I = Z_2 \frac{V_1}{Z_1 + Z_L + Z_2}$$

$$\frac{V_2}{V_1} = \frac{Z_2}{Z_1 + Z_L + Z_2} = \frac{\frac{1}{sC_2}}{\frac{1}{sC_1} + sL + \frac{1}{sC_2}}$$

$$= \frac{sC_1}{sC_2 + (sL)(sC_1)(sC_2) + sC_1}$$

$$= \frac{sC_1}{s^3 C_1 C_2 L + s(C_1 + C_2)}$$

$$= \frac{C_1}{C_1 C_2 L s^2 + (C_1 + C_2)}$$

Note

$$I = C \frac{dV}{dt}$$

$$I(s) = C s V(s)$$

$$Z = \frac{V(s)}{I(s)} = \frac{1}{sC}$$

$$V = L \frac{dI}{dt}$$

$$V(s) = sL I$$

$$Z = sL$$

2.6

$$y = f(x) = x^{\frac{1}{2}} \quad x_0 = \frac{1}{2}$$

$$\frac{\Delta y}{\Delta x} = ?$$

$$f(x) = f(x_0) + f'(x_0) [x - x_0]$$

$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$

$$f\left(\frac{1}{2}\right) = \sqrt{\frac{1}{2}} = 0.707$$

$$f'(x) = \frac{1}{2} \frac{1}{\sqrt{x}} = 0.707$$

$$f(x) = 0.707 + 0.707(x - 0.5)$$

$$= 0.707x + 0.35$$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = 0.707$$

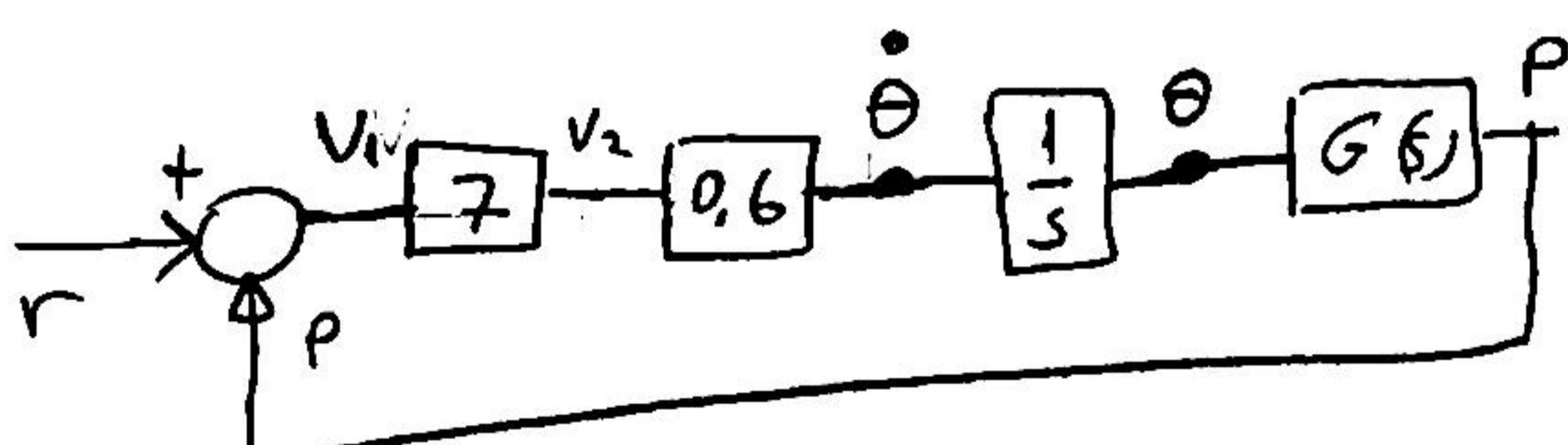
2.15 (Page 100)

$$\frac{d^2 p}{dt^2} + 2 \frac{dp}{dt} + 4p = 0$$

$$V_1 = r - p$$

$$\frac{d\theta}{dt} = 0.6 V_2$$

$$V_2 = 7 V_1$$



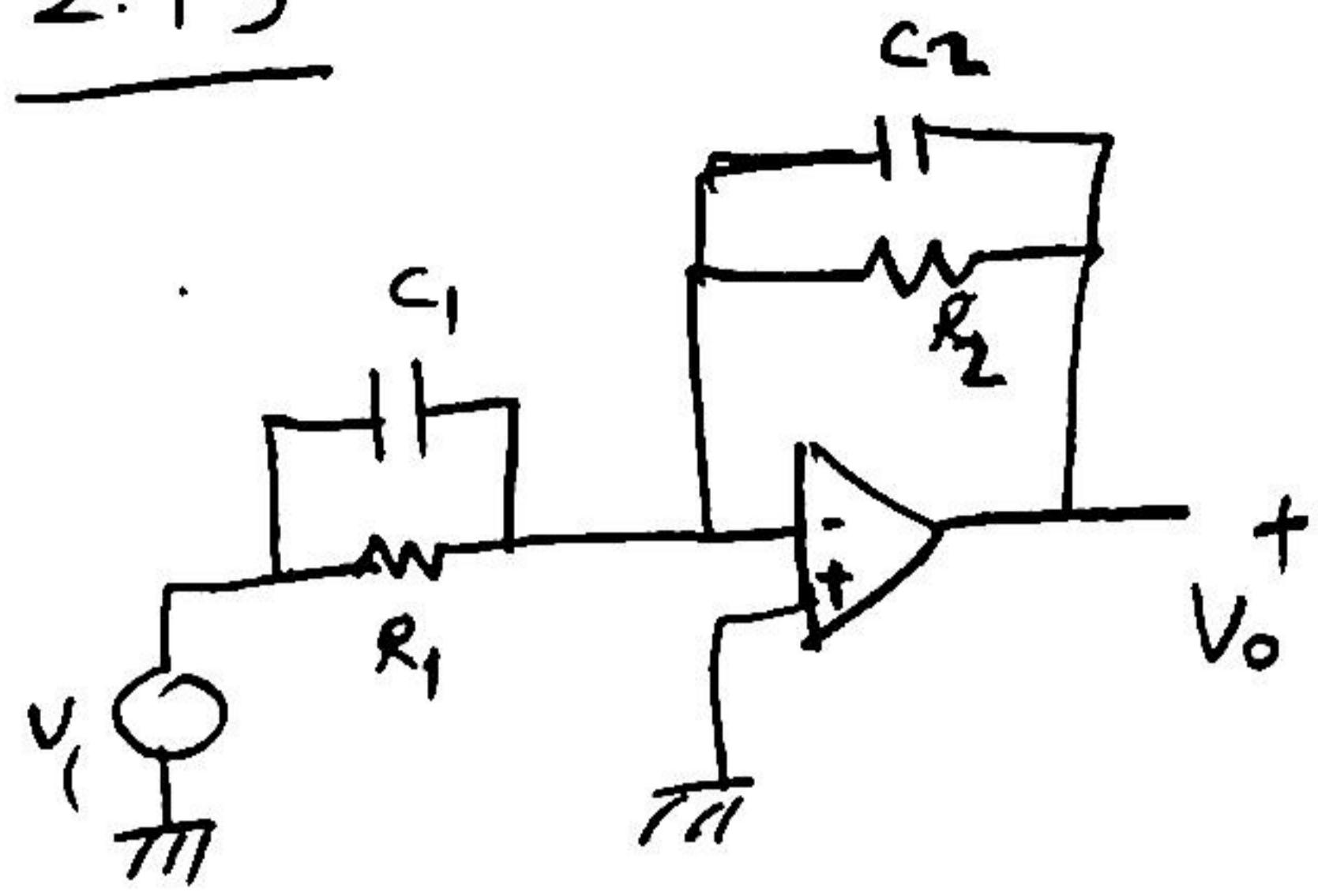
$$G(s) = \frac{p(s)}{\theta(s)} = \frac{1}{s^2 + 2s + 4}$$



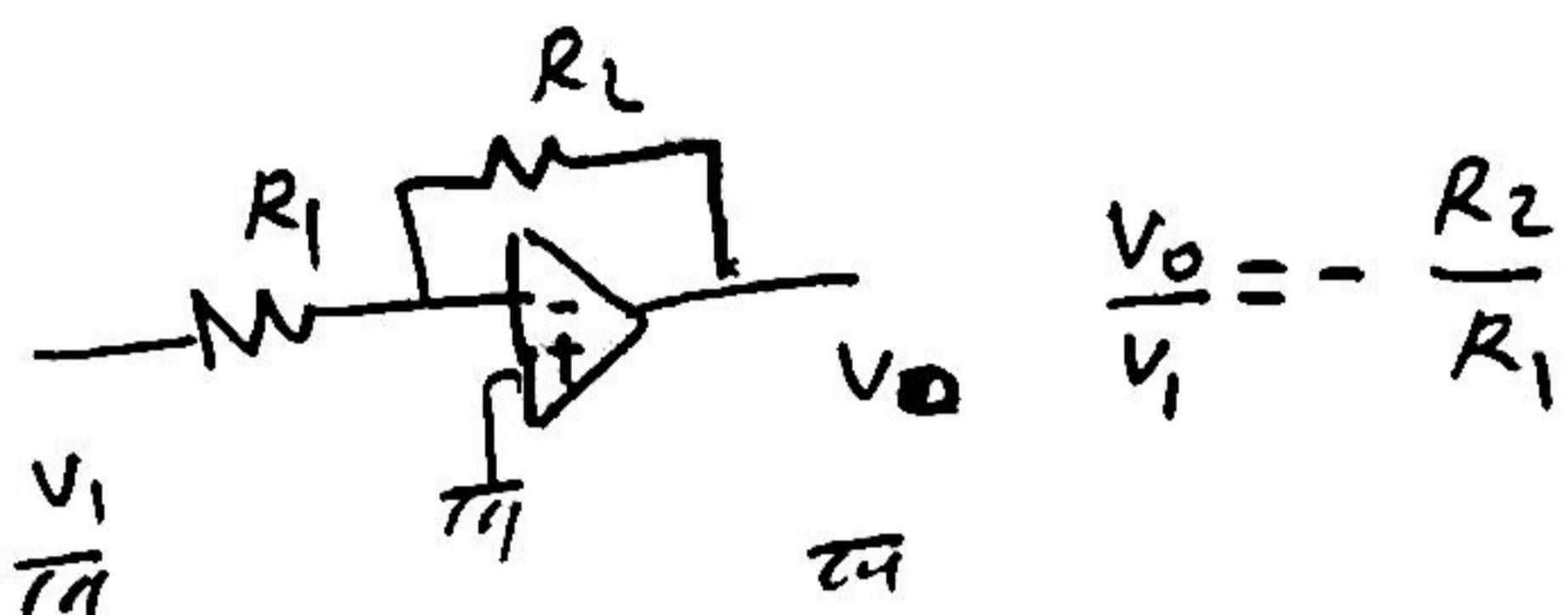
$$N(s) = 7 \times 0.6 \times \frac{1}{s} G(s)$$

$$\frac{p(s)}{R(s)} = \frac{N}{1+N} = \frac{\frac{4.2}{s^2 + 2s + 4}}{1 + \frac{4.2}{s^2 + 2s + 4}} = \frac{4.2}{s^3 + 2s^2 + 4s + 4}$$

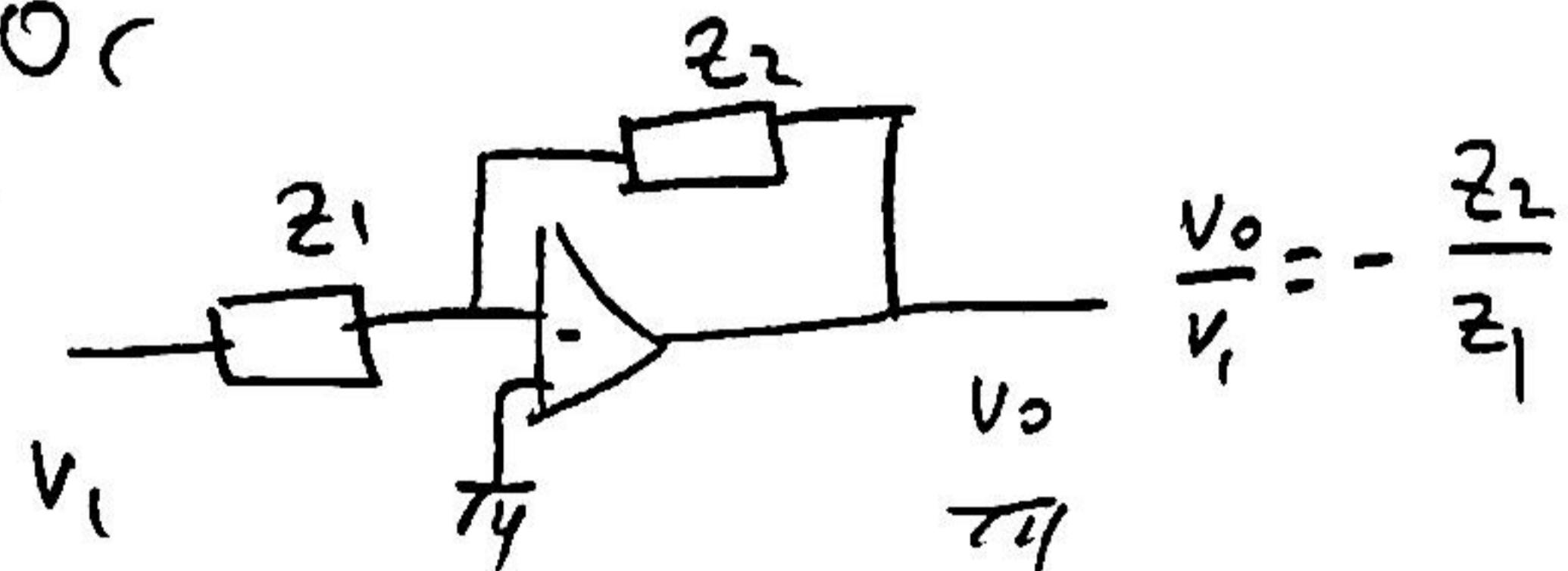
2.19



In Opam circuits



or



Thus

$$Z_1 = R_1 \parallel \frac{1}{sC_1} = \frac{R_1 \cdot \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{sC_1 R_1 + 1}$$

$$Z_2 = R_2 \parallel \frac{1}{sC_2} = \frac{R_2}{sC_2 R_2 + 1}$$

$$\frac{V_0}{V_1} = -\frac{Z_2}{Z_1} = -\frac{R_2(sC_1 R_1 + 1)}{R_1(sC_2 R_2 + 1)}$$

2.21

$$\frac{w(s)}{\Delta L(s)} = \frac{2.5(s+2)}{(s+5)(s+1)^2}$$

$$\Delta L(s) = \frac{1}{4s}$$

$$w(s) = \frac{2.5(s+2)}{(s+5)(s+1)^2} \cdot \frac{1}{4s}$$

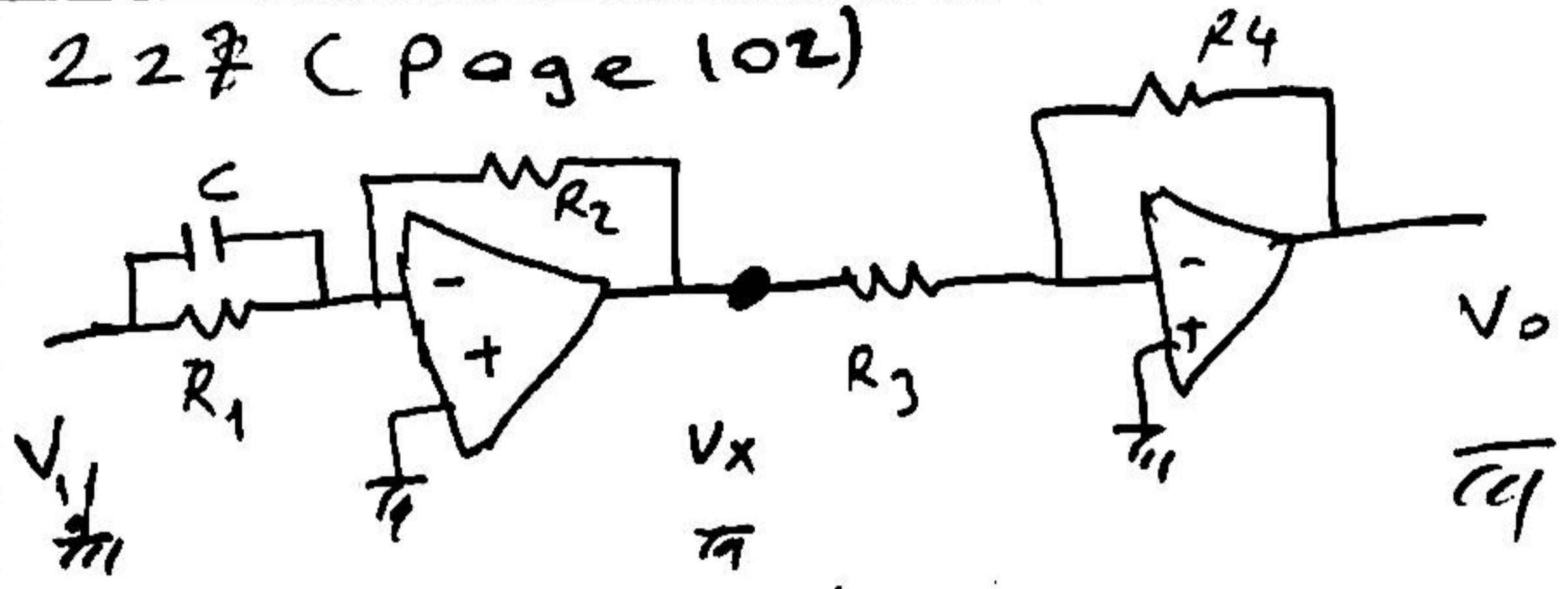
CS31

$$w(s) = \left(\frac{A}{s} + \frac{B}{s+5} + \frac{C}{s+1} + \frac{D}{(s+1)^2} \right)$$

$$A = 0.25 \quad B = 0.023 \quad C = -0.273 \quad D = -0.156$$

$$w(t) = 0.25 + 0.023 e^{-5t} - 0.273 e^{-t} - 0.156 t e^{-t}$$

2.22 (Page 102)



$$\frac{V_x}{V_1} = -\frac{R_2}{Z_1}$$

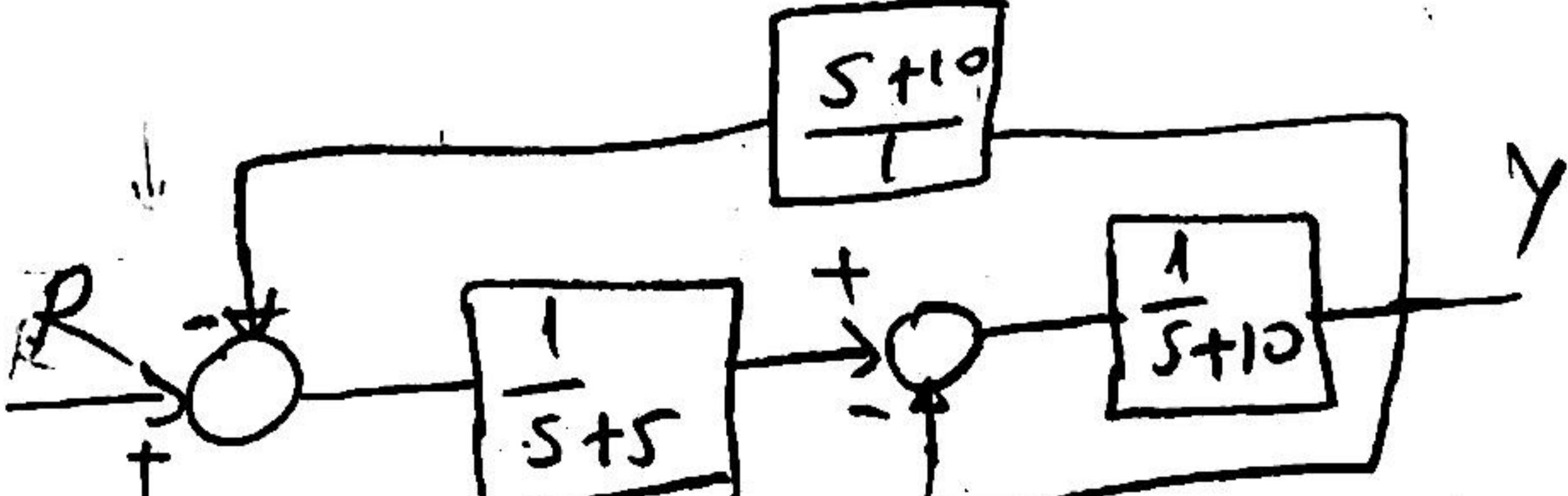
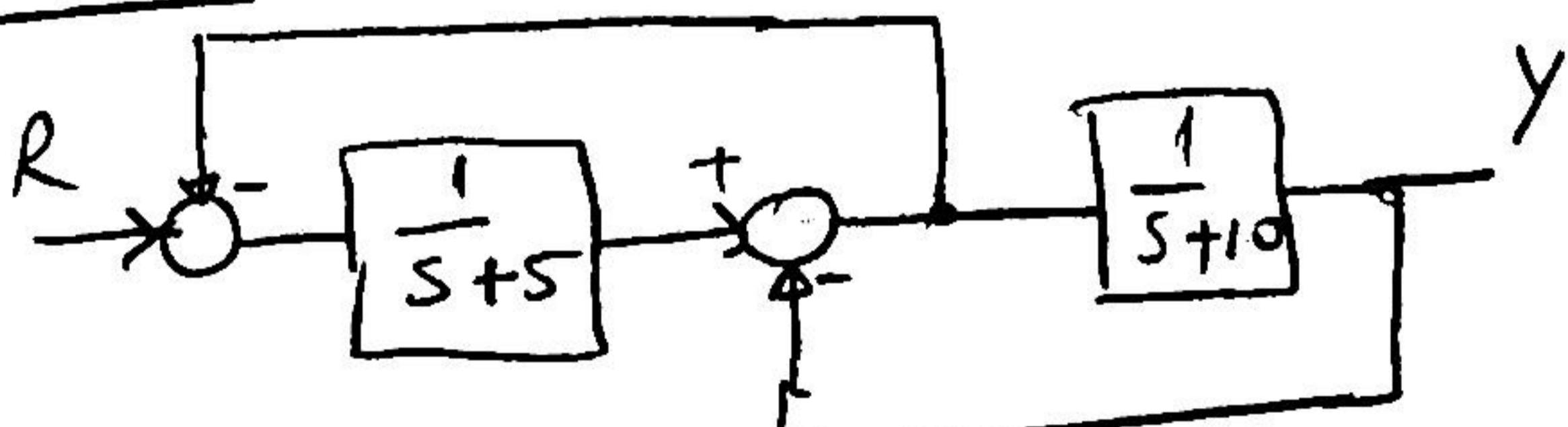
$$\frac{V_0}{V_x} = -\frac{R_4}{R_3}$$

$$Z_1 = R_1 \parallel \frac{1}{sC_1} = \frac{R/sC_1}{R + \frac{1}{sC_1}} = \frac{R}{R/sC_1 + 1} = \frac{R}{RC_1 + 1}$$

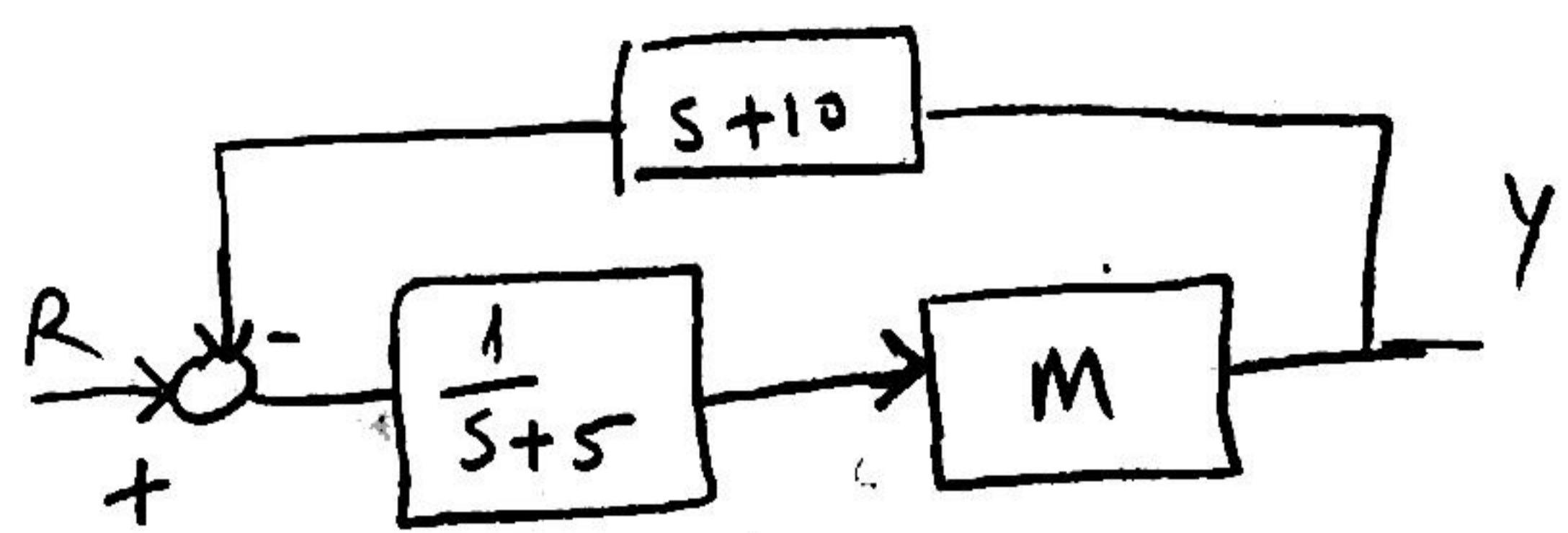
$$\frac{V_0}{V_1} = \frac{V_0}{V_x} \cdot \frac{V_x}{V_1} = -\frac{R_4}{R_3} \left(-\frac{R_2}{Z_1} \right)$$

$$\frac{V_0}{V_1} = +\frac{R_4}{R_3} \frac{\frac{R_2}{R}}{\frac{R}{RC_1 + 1}} = \frac{R_2 R_4 (RC_1 + 1)}{R R_3}$$

2.28

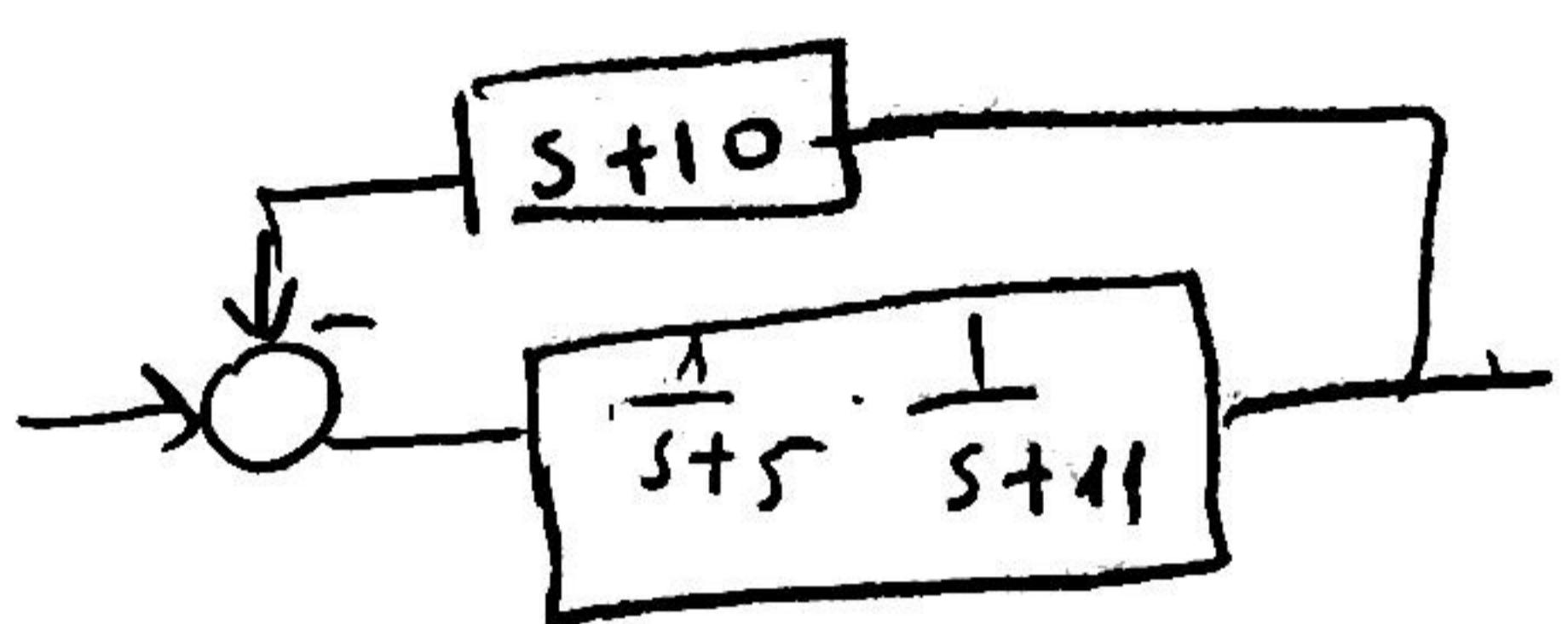


2.28 (continued)



$$M = \frac{1}{s+5} = \frac{1}{s+11}$$

$$D = \frac{s+10}{1 + \frac{1}{s+10}} = \frac{s+10}{s+11}$$



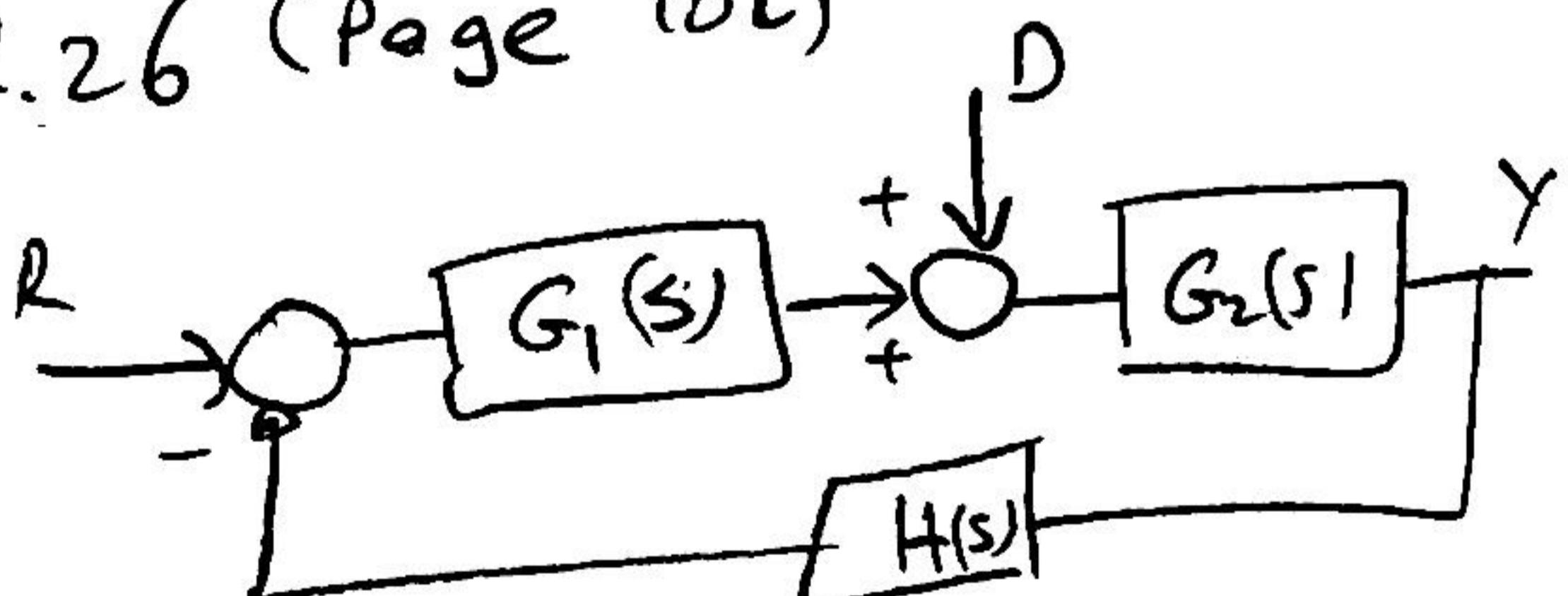
$$G(s) = \frac{1}{(s+5)(s+11)}$$

$$H(s) = s+10$$

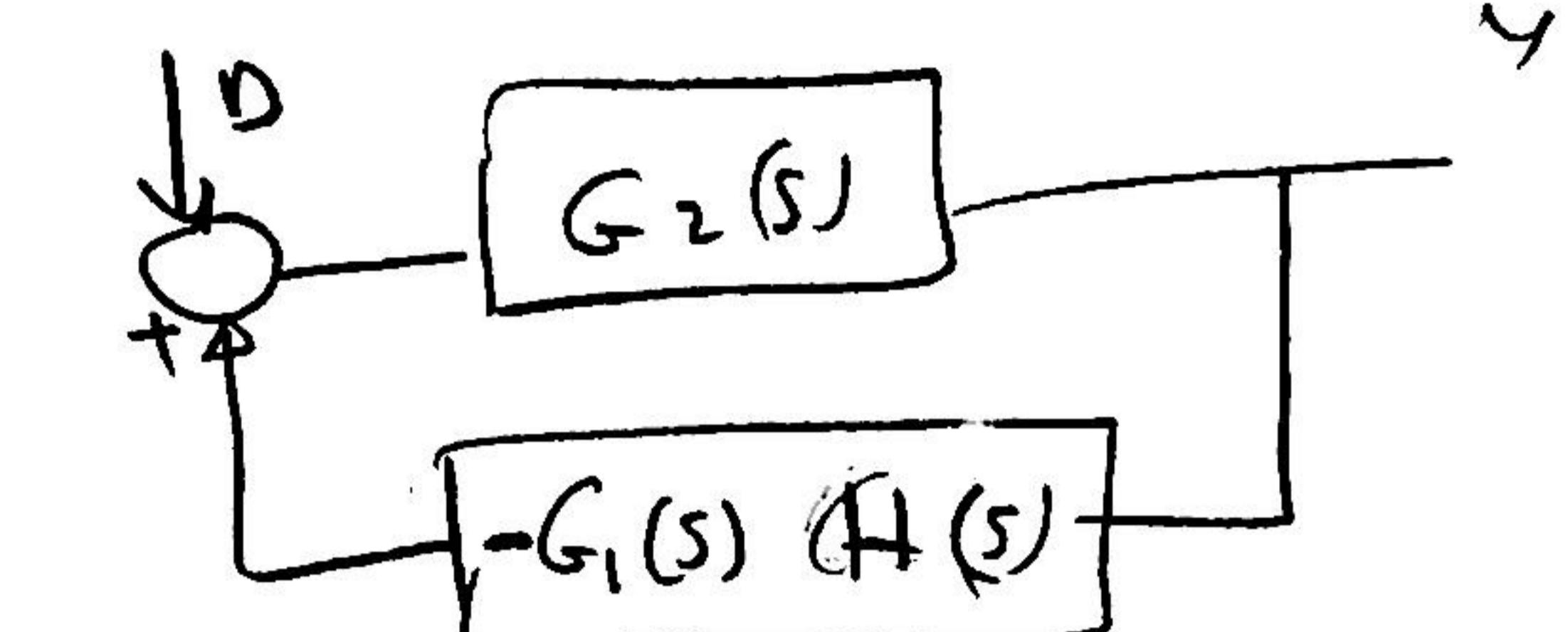
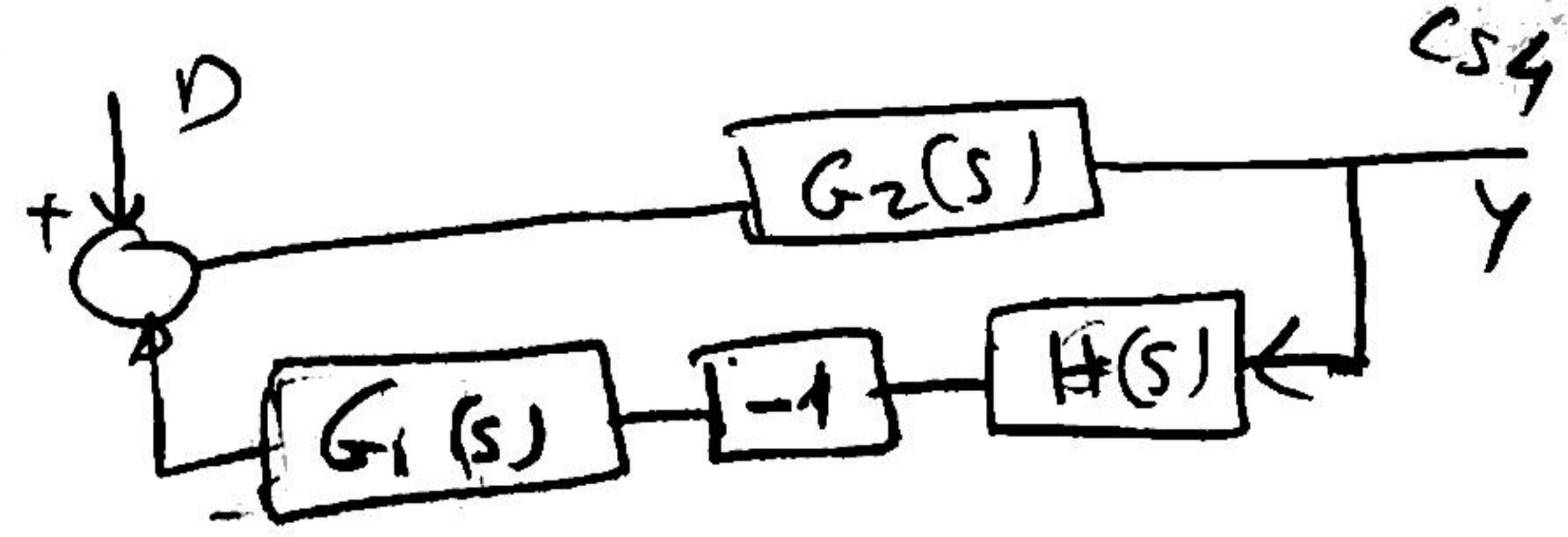
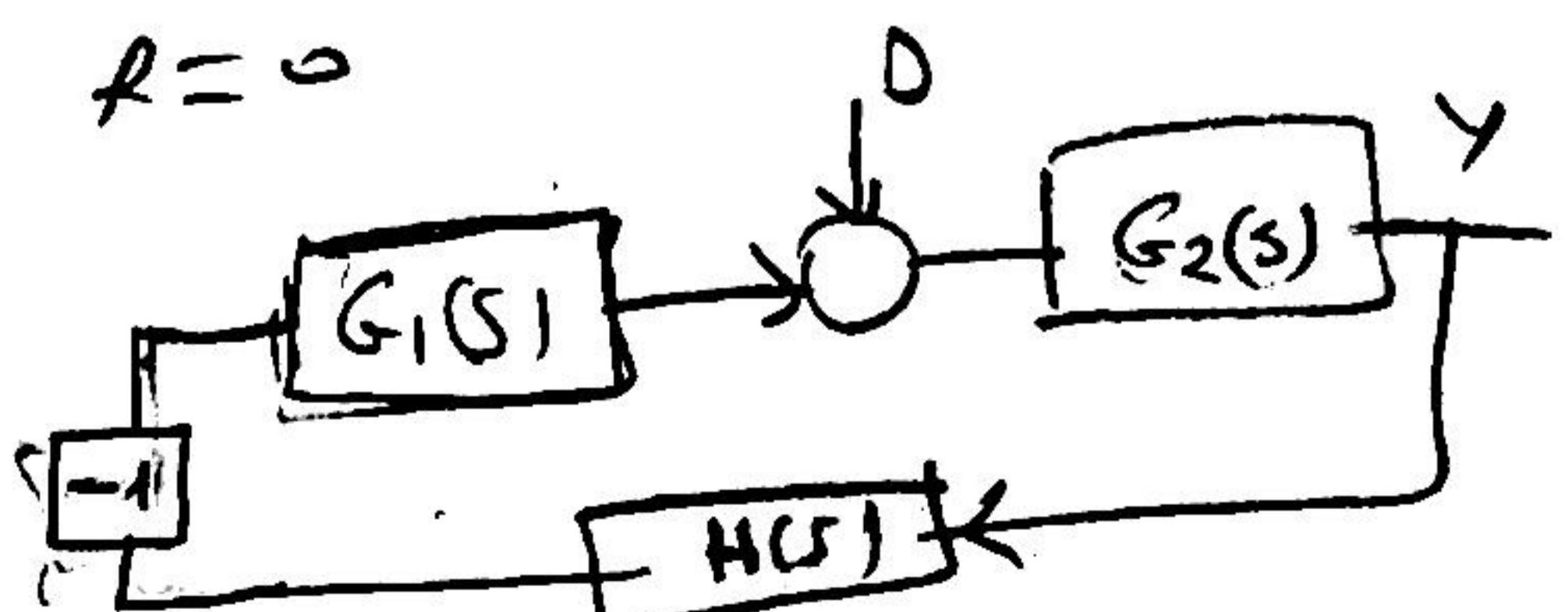
$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+GH} = \frac{\frac{1}{(s+5)(s+11)}}{1 + \frac{s+10}{(s+5)(s+11)}} =$$

$$= \frac{1}{(s+5)(s+11) + s+10} = \frac{1}{s^2 + 7s + 15}$$

2.26 (Page 102)



Set $R=0$



$$\frac{Y}{D} = \frac{G_2(s)}{1 + G_1(s) \{-G_2(s) H(s)\}}$$

$$\frac{Y(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s) G_2(s) H(s)}$$



$$G(s) = \frac{24}{s^2 + 30s + 176}$$

$$r(t) = u(t) \quad R(s) = \frac{1}{s}$$

$$\frac{Y(s)}{R(s)} = \frac{G}{1+G} = \frac{24/(s^2 + 30s + 176)}{1 + 24/(s^2 + 30s + 176)}$$

$$\frac{Y(s)}{R(s)} = \frac{24}{s^2 + 30s + 176 + 24}$$

$$Y(s) = \frac{24}{s^2 + 30s + 200} \cdot \frac{1}{s} = \frac{24}{s(s+10)(s+20)}$$

$$= \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+20}$$

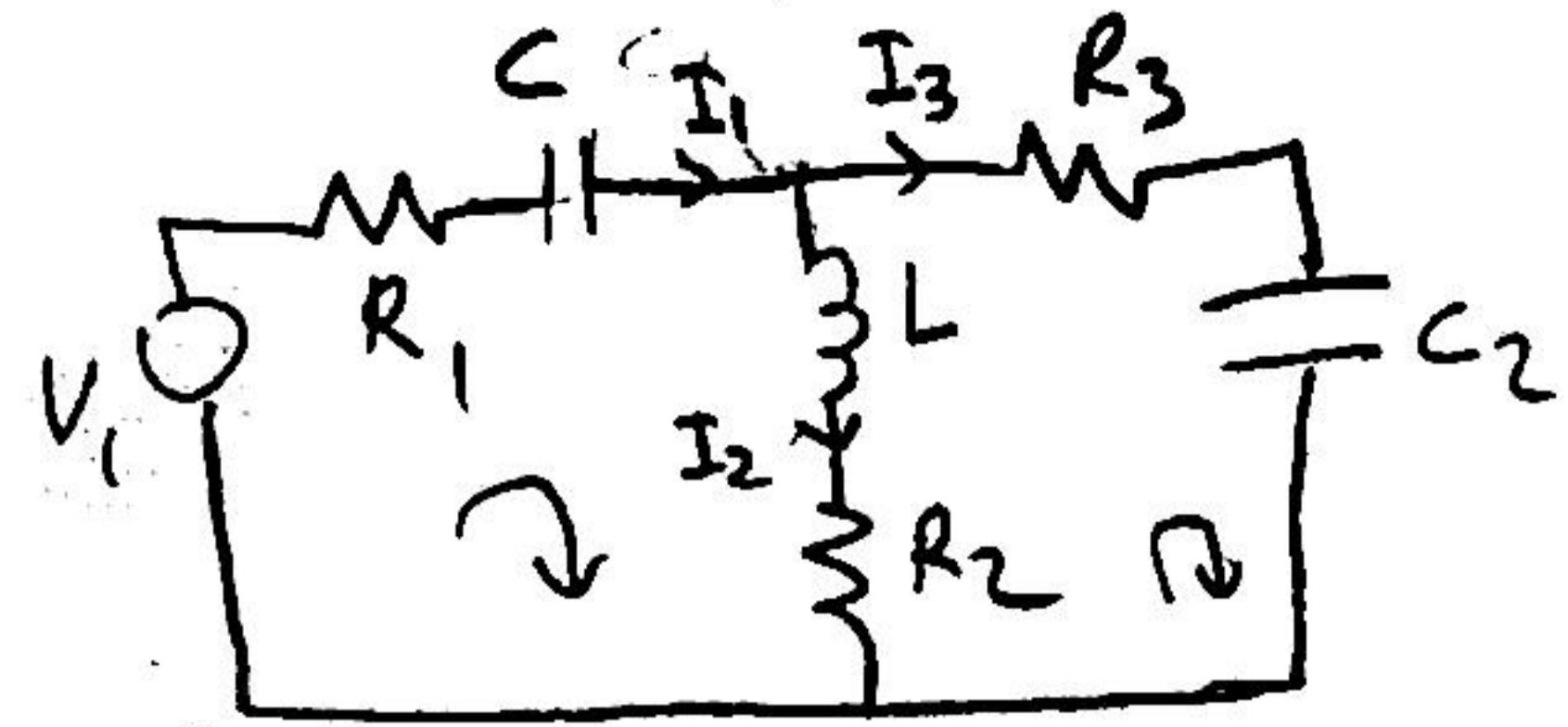
$$A = 0.12$$

$$B = -0.24$$

$$C = 0.12$$

$$y(t) = 0.12 - 0.24 e^{-10t} + 0.12 e^{-20t}$$

p2.6: (page 103)



$$-V_1 + R_1 I_1 + V_{C_1} + V_L + R_2 I_2 = 0 \quad (1)$$

$$-R_2 I_2 - V_L + R_3 I_3 + V_{C_2} = 0 \quad (2)$$

$$I_1 = C_1 \frac{dV_{C_1}}{dt} \quad I_1(s) = C_1 [sV_{C_1} - V_{C_1}(0)]$$

$$I_3 = C_2 \frac{dV_{C_2}}{dt} \quad I_3(s) = C_2 [sV_{C_2} - V_{C_2}(0)]$$

$$V_L = L \frac{dI_2}{dt} \quad V_L(s) = L (sI_2 - I_2(0))$$

$$I_1 = I_2 + I_3 \quad (3)$$

$$V_{C_1}(0) = 0 \quad I_2(0) = 0 \quad V_{C_2}(0) = 10$$

$$V(t) = 0$$

$$0 = R_1 I_1 + \frac{I_1}{sC_1} + sL I_2 + R_2 I_2 \quad (1.b)$$

$$-R_2 I_2 - sL I_2 + R_3 I_3 + V_{C_2} = 0 \quad (2.b)$$

$$sC_2 V_{C_2} = I_3 - C_2 V_{C_2}(0)$$

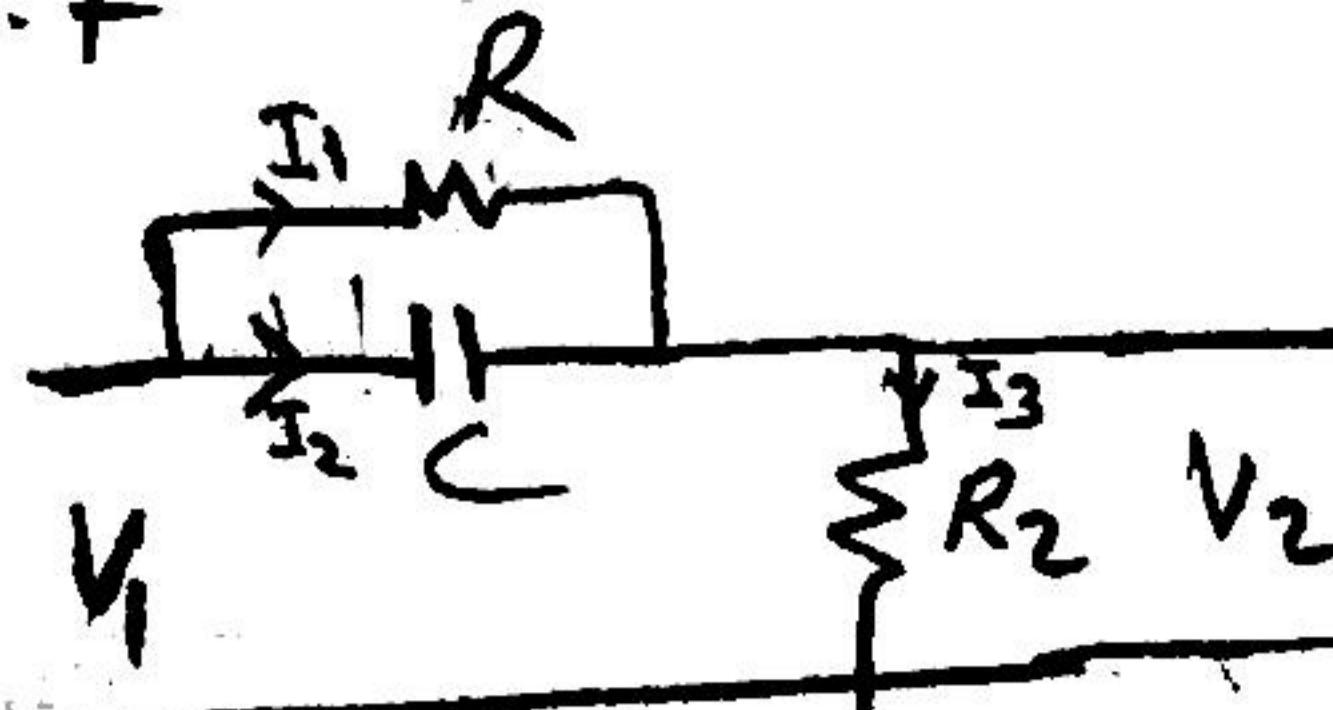
$$V_{C_2} = \frac{I_3}{sC_2} - \frac{1}{s} \cdot 10$$

$$-R_2 I_2 - sL I_2 + R_3 I_3 + \frac{I_3}{sC_2}$$

$$- \frac{10}{s} = 0 \quad (2.c)$$

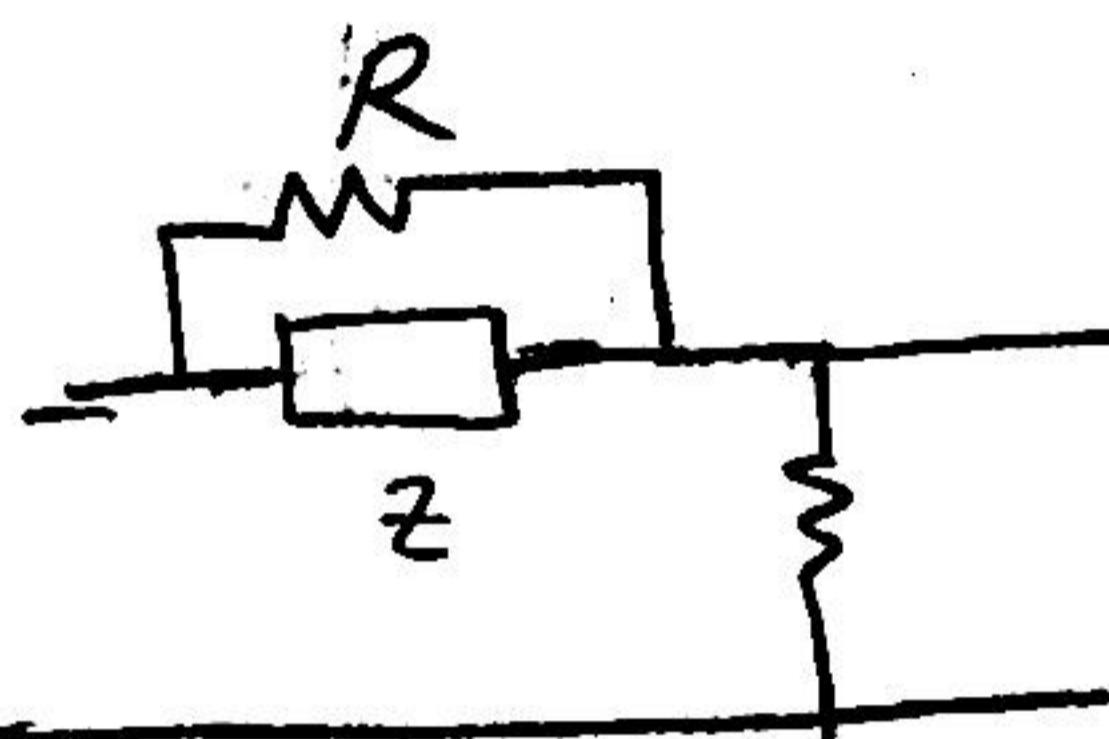
(1.b, 2.c, 3) Three equations
Three unknowns I_1, I_2, I_3

2.7



For transfer function
all initial conditions
are set to zero.

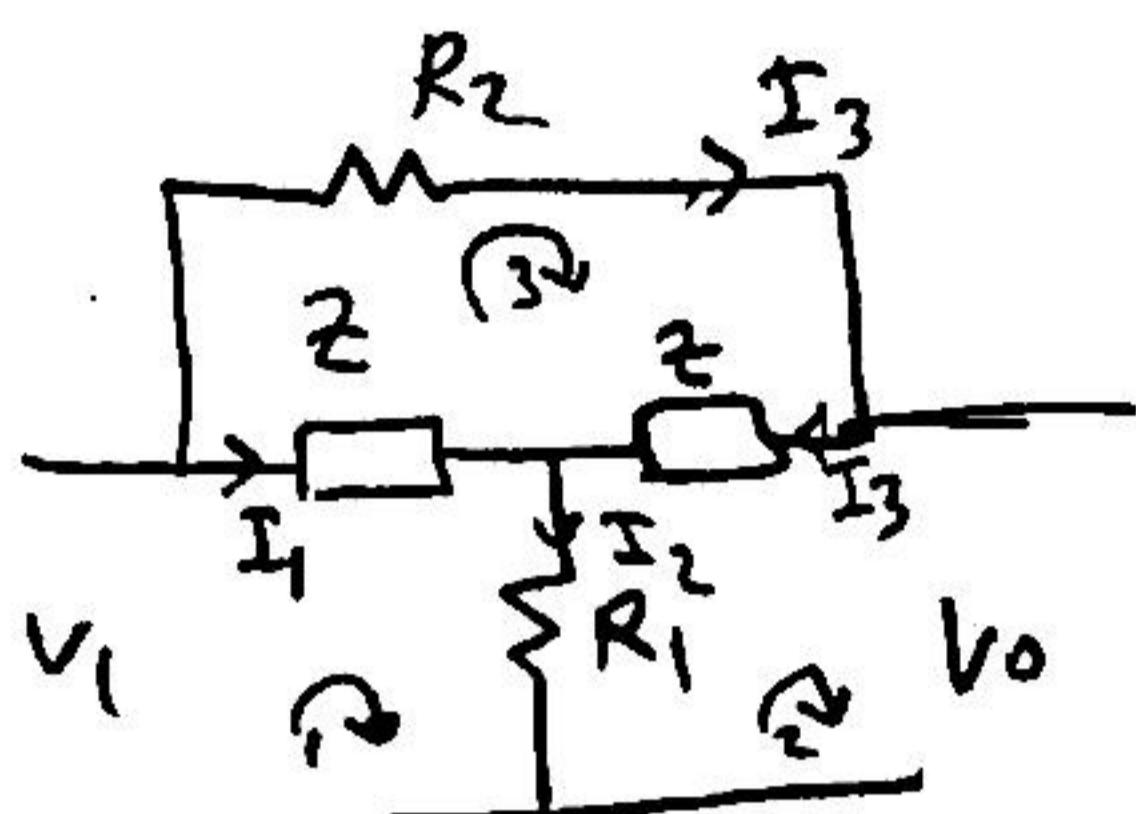
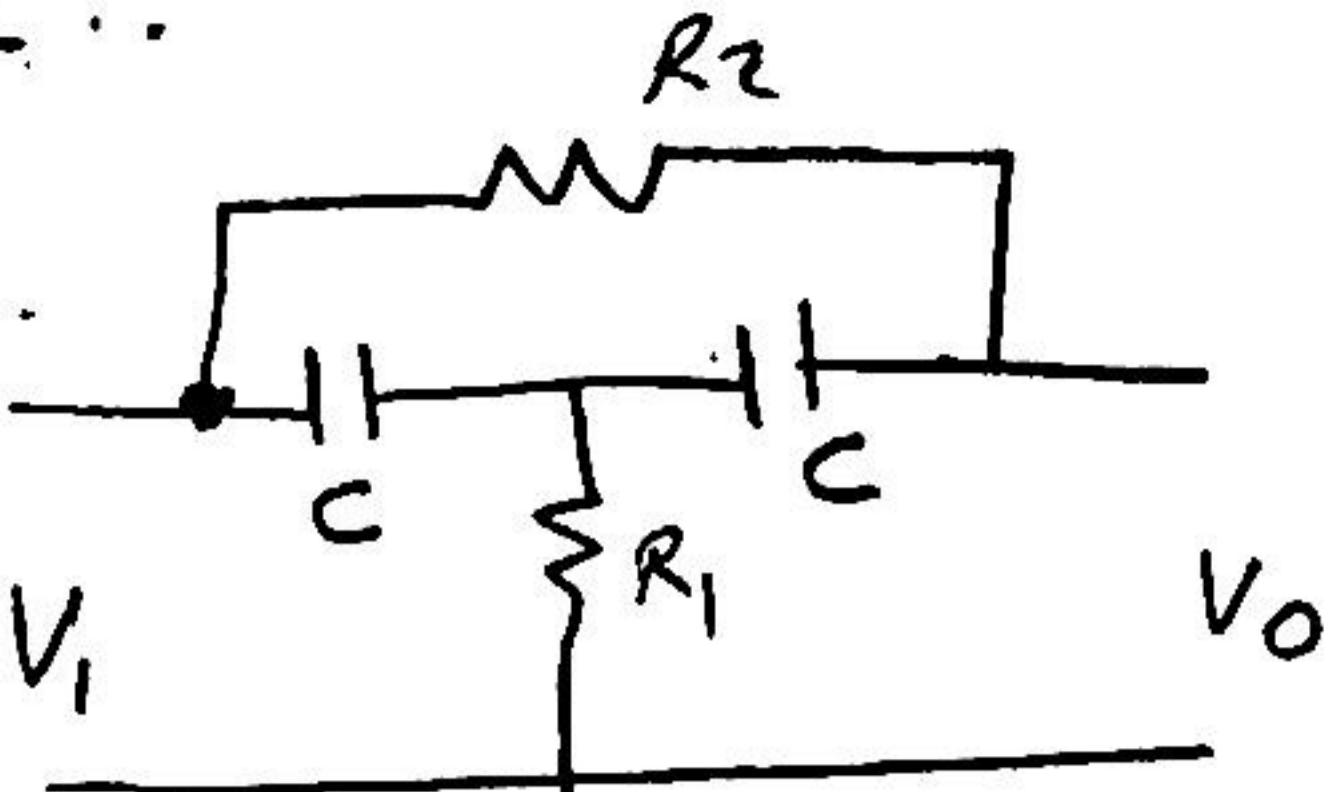
$$\frac{V_C}{I_2} = \frac{1}{Z} \quad Z = \frac{1}{sC}$$



$$Z_2 = R//Z = \frac{R^2}{R+Z} = \frac{R \cdot \frac{1}{sC}}{R + \frac{1}{sC}} = \frac{R}{RCS + 1}$$

$$\frac{V_2}{V_1} = \frac{R_2}{R_2 + Z_2} = \frac{R_2}{R_2 + \frac{R}{RCS + 1}} = \frac{R_2(RCS + 1)}{(RCS + 1)R_2 + R}$$

2.8 ..



$$I_2 = I_1 + I_3 \quad (1)$$

$$-V_1 + ZI_1 + R_1 I_2 \quad (2)$$

$$-R_1 I_2 + ZI_3 + V_0 = 0 \quad (3)$$

$$-ZI_1 + R_2 I_3 + ZI_3 = 0 \quad (4)$$

$$-ZI_1 + (R_2 + Z)I_3 = 0 \quad (4)$$

from 4 $I_3 = \frac{Z}{R_2 + Z} I_1 \quad I_1 = M I_1 \quad (4.b)$

$$(1) \rightarrow I_2 = I_1 + M I_1 = (1+M) I_1 \quad (1.b)$$

$$(2) \rightarrow -V_1 + ZI_1 + R_1(1+M)I_1 = 0$$

$$-V_1 + \left[Z_1 + \underbrace{R_1(1+M)}_N \right] I_1 = 0$$

$$-V_1 + N I_1 = 0$$

$$I_1 = \frac{V_1}{N} \quad (2.b)$$

$$4.b) \rightarrow I_3 = M I_1 = M \frac{V_1}{N} \quad (4.c)$$

$$1.b) \rightarrow I_2 = (1+M) I_1 = (1+M) \frac{V_1}{N}$$

CS6

Replace 4.b, 1.b into (3)

$$-R_1 I_2 + Z I_3 + V_0 = 0 \quad (3)$$

$$-R_1(1+M) \frac{V_1}{N} + Z \frac{M}{N} V_1 + V_0 = 0$$

$$\left[R_1 \frac{1+M}{N} + Z \frac{M}{N} \right] V_1 + V_0 = 0$$

$$\frac{V_0}{V_1} = \frac{1}{-R_1 \frac{1+M}{N} + Z \frac{M}{N}}$$

$$\boxed{\frac{V_0}{V_1} = \frac{N}{R_1(1+M) + ZM}}$$

$$\text{Replace } Z = \frac{1}{sc}$$

$$M = \frac{Z}{R_2 + Z} = \frac{1/sc}{R_2 + 1/sc} = \frac{1}{R_2 sc + 1}$$

$$N = Z + R_1(1+M)$$

$$= \frac{1}{sc} + R_1 \left(1 + \frac{1}{R_2 sc + 1} \right)$$

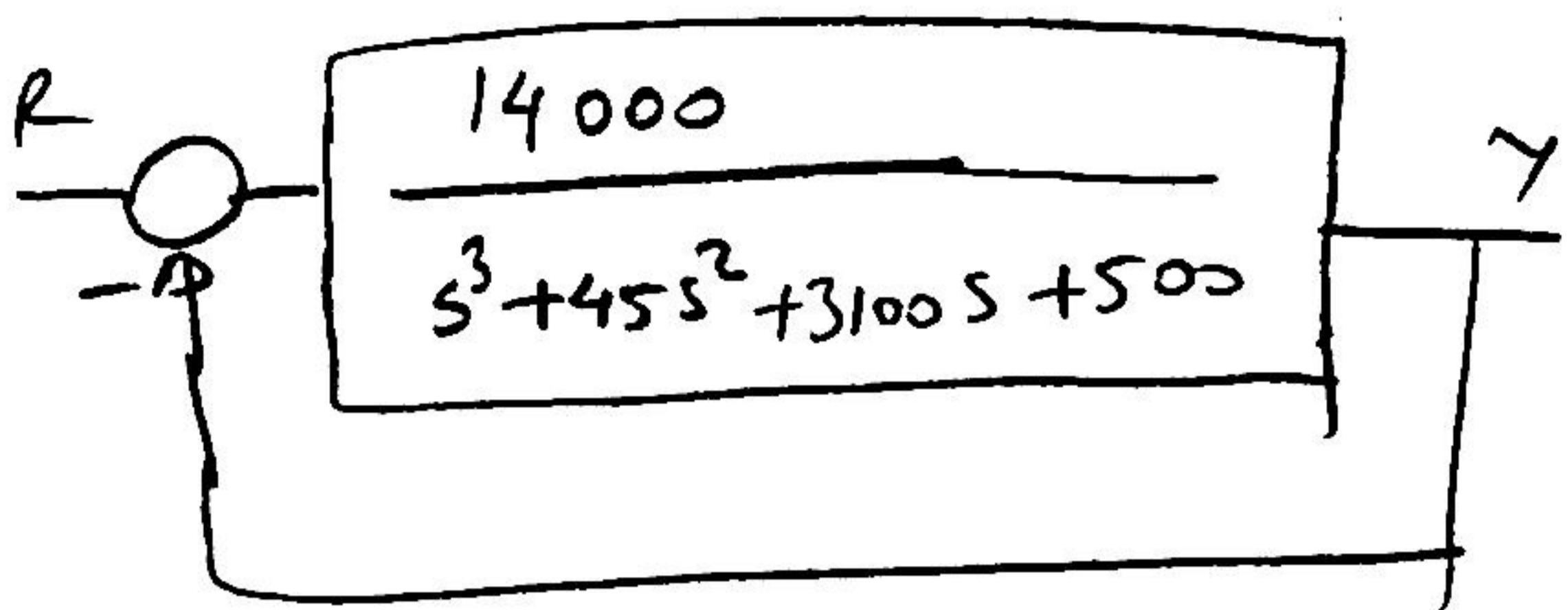
$$N = \frac{1}{sc} + R_1 + \frac{R_1}{R_2 sc + 1}$$

$$= \frac{R_2 sc + 1 + R_1 sc (R_2 sc + 1) + R_1 sc}{sc (R_2 sc + 1)}$$

$$\text{Denominating term } (R_2 sc + 1) \\ R_1(1+M) + ZM = R_1 \left(1 + \frac{1}{R_2 sc + 1} \right) + \frac{1}{sc} \left(\frac{1}{R_2 sc + 1} \right)$$

$$= R_1 + \frac{R_1}{R_2 sc + 1} + \frac{1}{sc(R_2 sc + 1)}$$

2.5.1



$$\frac{Y}{R} = \frac{G}{1+G} = \frac{A/B}{1+\frac{A}{B}} = \frac{A}{B+A} = \frac{14000}{s^3 + 45s^2 + 3100s + 14500}$$

$$R = \frac{1}{s}$$

$$Y = \frac{14000}{s(s^3 + 45s^2 + 3100s + 14500)}$$

$$= \frac{A}{s} + \frac{B}{s+20+50i} + \frac{C}{s+20-50i} + \frac{D}{s+5}$$

$$A = 0.36 \quad B = 0.031 - 0.039i$$

$$C = 0.031 + 0.039i$$

$$D = -1.02$$

$$y(t) = A u(t) + (0.031 - 0.039i) e^{(-20+50i)t} + (0.031 + 0.039i) e^{(-20-50i)t} - 1.02 e^{-5t}$$

$$(p+qi)e^{(a+bi)t} + (p-qi)e^{(a-bi)t} =$$

$$e^{at} \left\{ 2p \cos bt - 2q \sin bt \right\}$$

$$p = 0.031 \quad q = -0.039 \quad a = -20$$

$$b = -50$$

$$e^{-20t} \left[2 \times 0.031 \cos 50t + 2(-0.039) \sin(-50t) \right]$$

$$= e^{-20t} (0.062 \cos 50t - 0.08 \sin 50t)$$

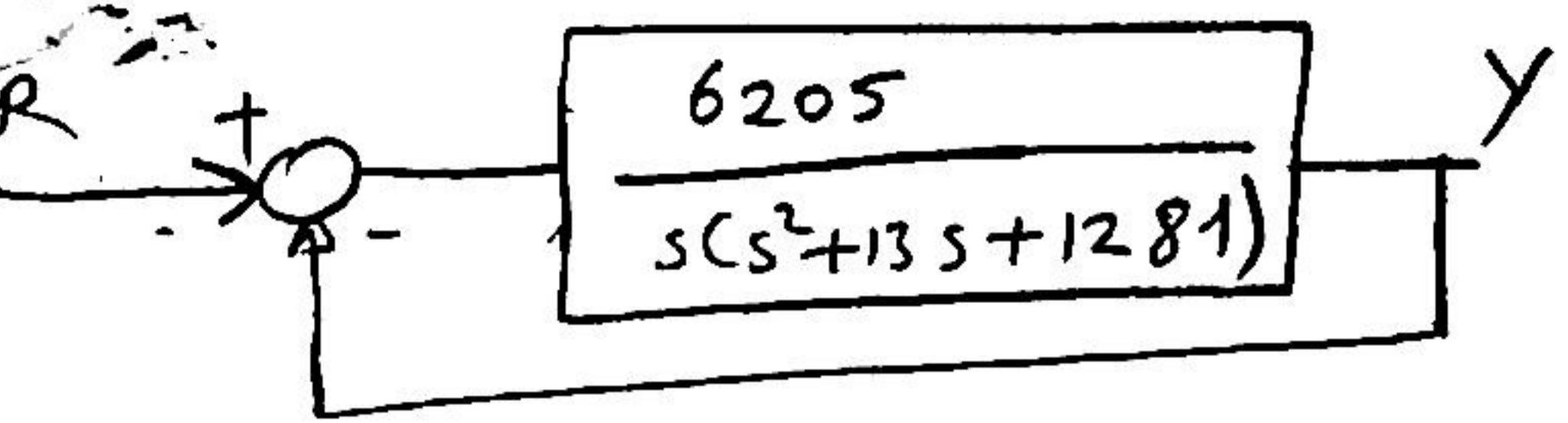
$$y(t) = 0.36 u(t)$$

$$+ e^{-20t} (0.062 \cos 50t - 0.08 \sin 50t)$$

$$+ 1.02 e^{-5t}$$

$$y(\infty) = 0.36 + 0 + 0 + 0$$

f2.5Q



$$9) \frac{Y}{R} = \frac{G}{1+G} = \frac{\frac{A}{B}}{1+\frac{A}{B}} = \frac{A}{B+A} = \frac{6205}{s(s^2+13s+1281)+6205}$$

$$= \frac{6205}{s^3+13s^2+1281s+6205}$$

$$s^3 + 13s^2 + 1281s + 6205 = 0$$

$$s_1 =$$

$$s_2 =$$

$$s_3 =$$

$$R(s) = \frac{1}{s}$$

$$Y(s) = \frac{6205}{s(s^3+13s^2+1281s+6205)}$$

$$= \frac{A}{s} + \frac{B}{s+4+35i} + \frac{C}{s+4-35i} + \frac{D}{s+5}$$

$$A=1 \quad B=0.061-0.071i \quad C=0.01+0.071i \quad D=-1$$

Note e =

$$(p+qi)e^{(p+qi)t} + (p-qi)e^{(p-qi)t}$$

$$= (p+qi)e^{pt}e^{bit} + (p-qi)e^{pt}e^{-bit}$$

$$= e^{pt} [(p+qi)(\cos bt + i \sin bt) + (p-qi)(\cos bt - i \sin bt)]$$

$$= e^{pt} [p \cos bt + pi \sin bt + qi \cos bt - q \sin bt + p \cos bt - pi \sin bt - qi \cos bt - q \sin bt]$$

$$= e^{at} [2p \cos bt - 2q \sin bt]$$

No complex Number

$$(p+qi)e^{(p+qi)t} + (p-qi)e^{(p-qi)t}$$

$$= e^{at} [2p \cos bt - 2q \sin bt]$$

$$y(t) = A_1 u(t) + B e^{(-4-35i)t} + C e^{(-4+35i)t} + D e^{-5t}$$

$$= (0.061-0.071i)e^{(-4-35i)t} + (0.061+0.071i)e^{(-4+35i)t}$$

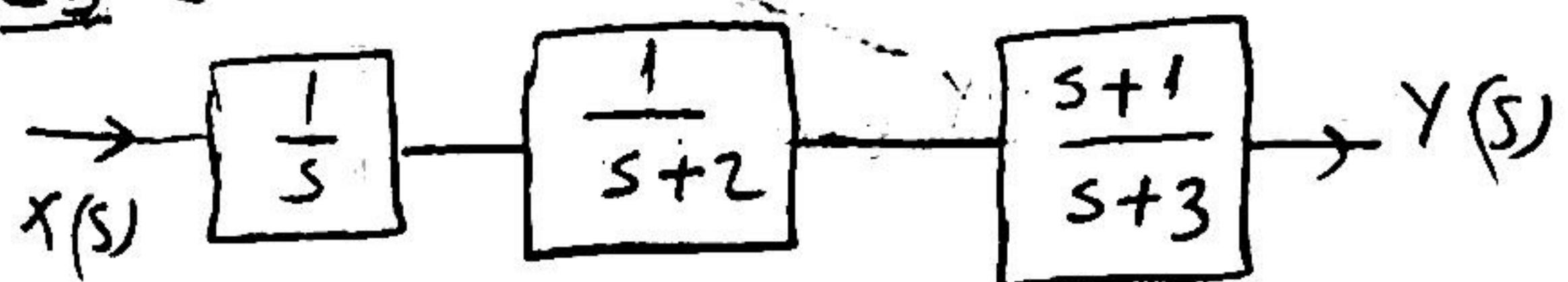
$$p=0.061 \quad a=-4$$

$$q=-0.071 \quad b=-35$$

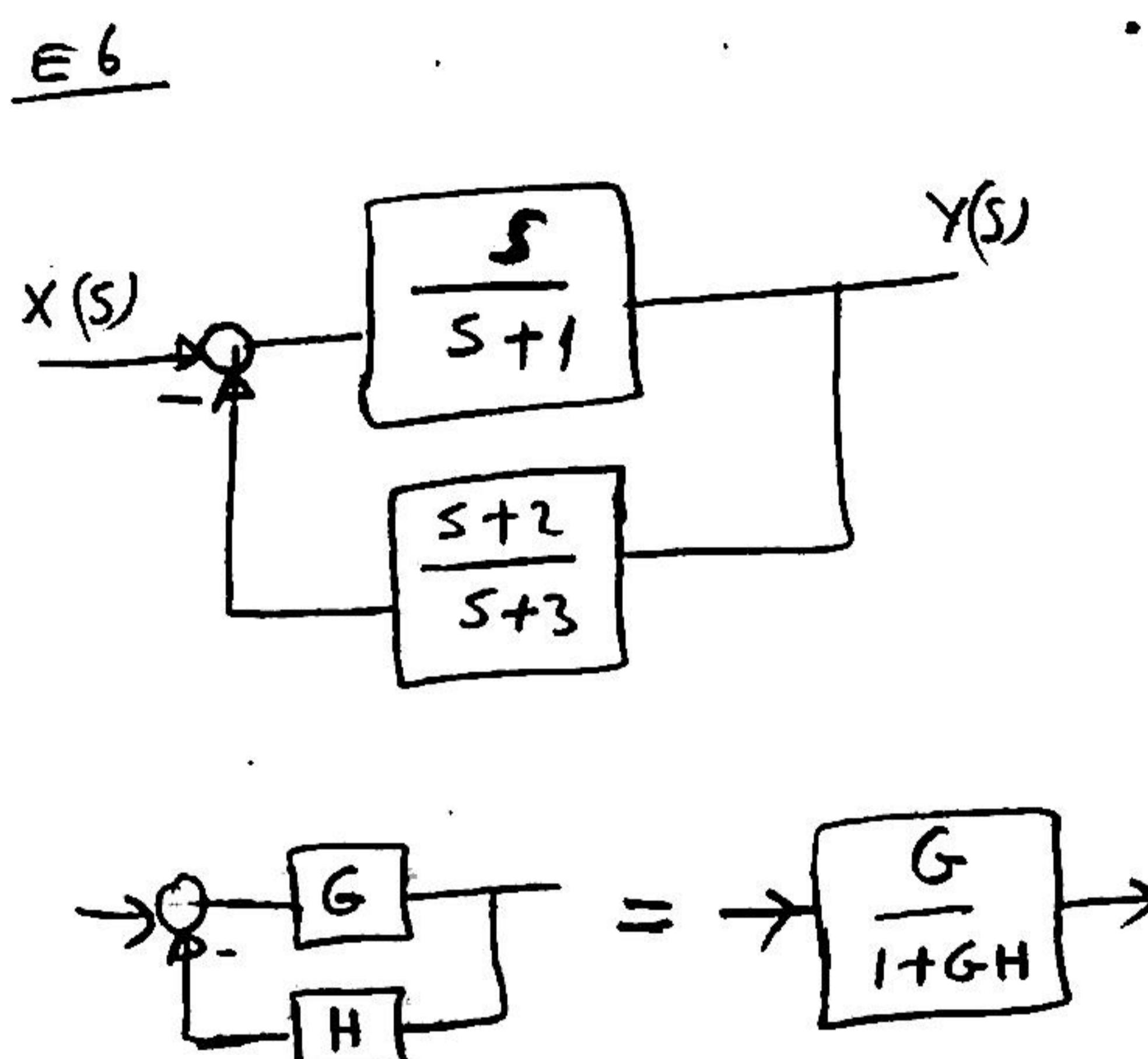
$$T = e^{-4t} (2 \times 0.061 \cos 35t - 2 \times (-0.071) \sin 35t)$$

$$= e^{-4t} (0.121 \cos 35t - 0.141 \sin 35t)$$

$$Y(t) = u(t) + e^{-4t} (0.121 \cos 35t - 0.141 \sin 35t) - e^{-5t}$$



$$X(s) \rightarrow \frac{s+1}{s(s+2)(s+3)} \rightarrow Y(s)$$

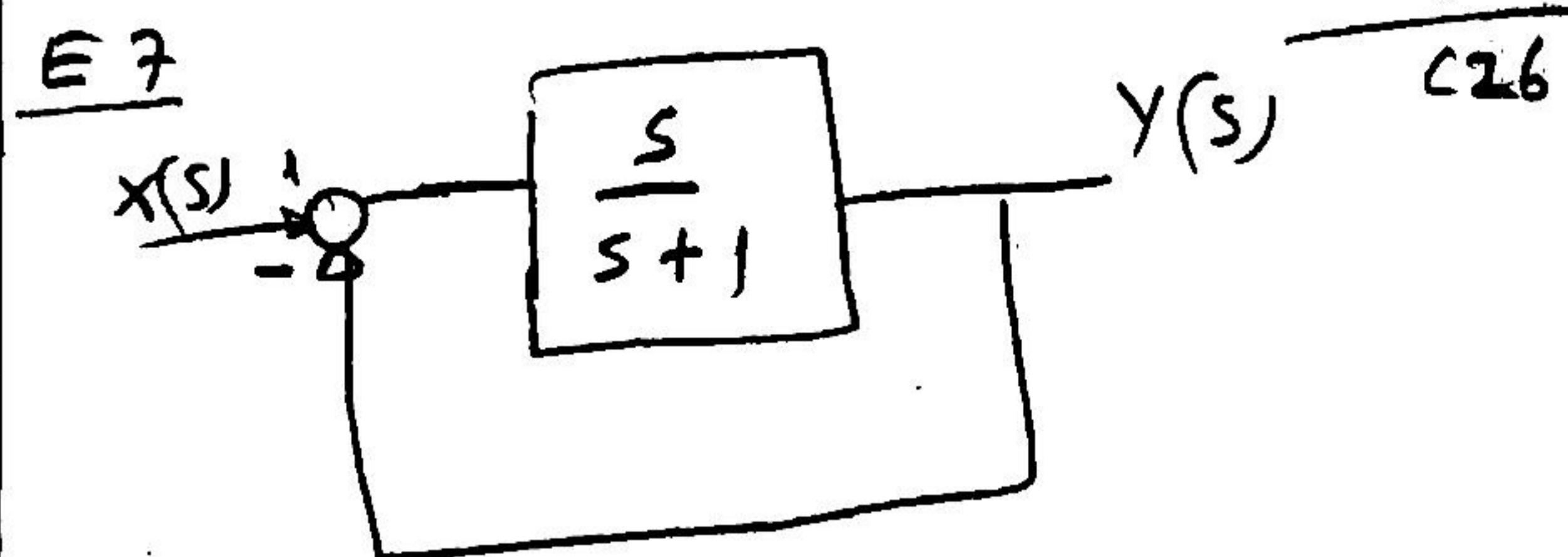


$$G = \frac{s}{s+1} \quad H = \frac{s+2}{s+3}$$

$$\frac{G}{1+GH} = \frac{\frac{s}{s+1}}{1 + \frac{s}{s+1} \cdot \frac{s+2}{s+3}} = \frac{s(s+3)}{(s+1)(s+3) + s(s+2)}$$

$$= \frac{s^2 + 3s}{2s^2 + 6s + 3}$$

$$X(s) \rightarrow \frac{s^2 + 3s}{2s^2 + 6s + 3} \rightarrow Y(s)$$



$$G = \frac{1}{s+1} \quad H = -1$$

$$\frac{G}{1+GH} = \frac{\frac{s}{s+1}}{1 + \frac{s}{s+1} \cdot -1} = \frac{s}{s+1+s} = \frac{s}{2s+1}$$

$$\frac{Y(s)}{X(s)} = \frac{s}{2s+1}$$

$$X(s) \rightarrow \frac{s}{2s+1} \rightarrow Y(s)$$

E8

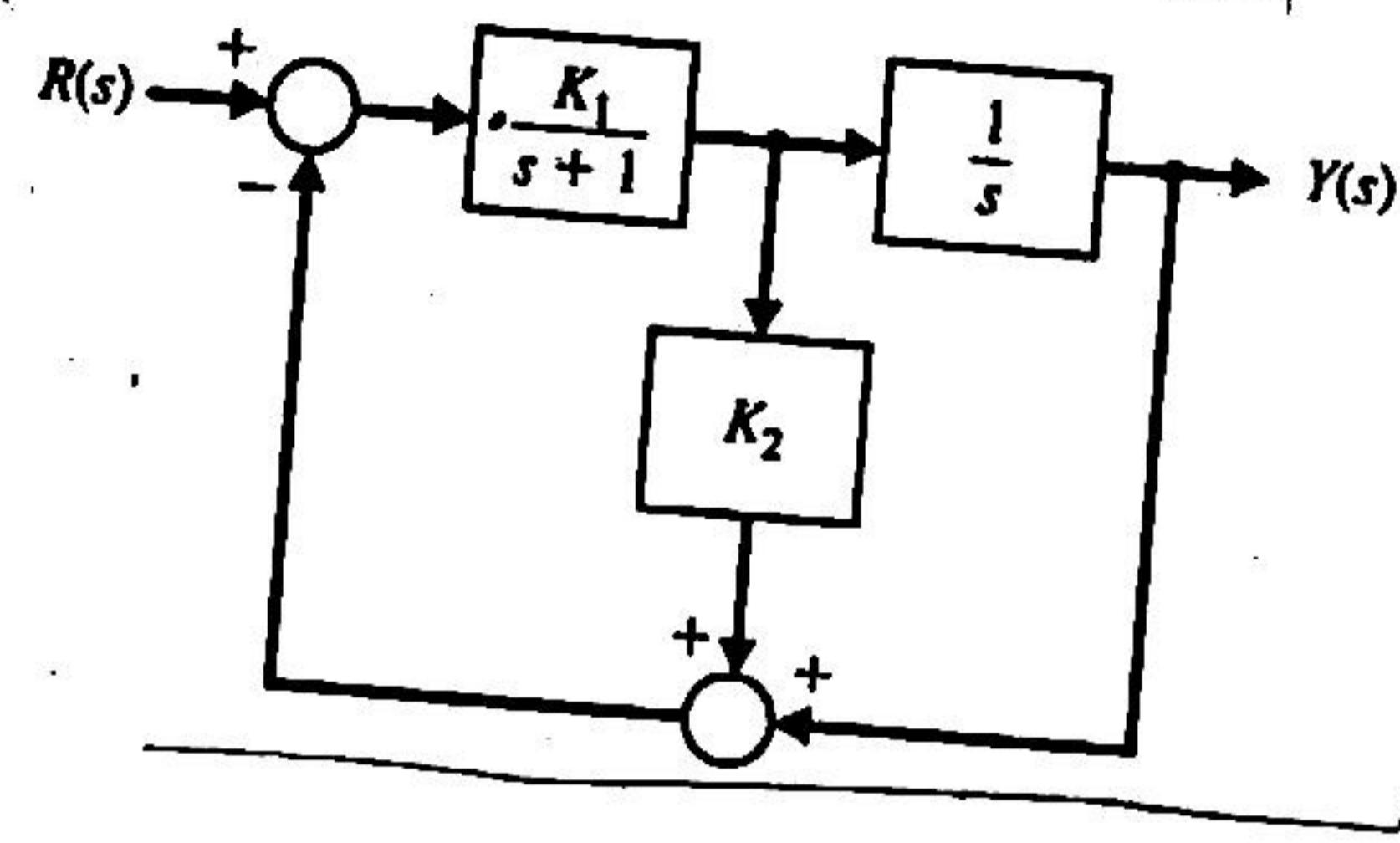
$$X(s) \rightarrow \frac{s}{s+1} \rightarrow Y(s)$$

$$G = \frac{s}{s+1} \quad H = K$$

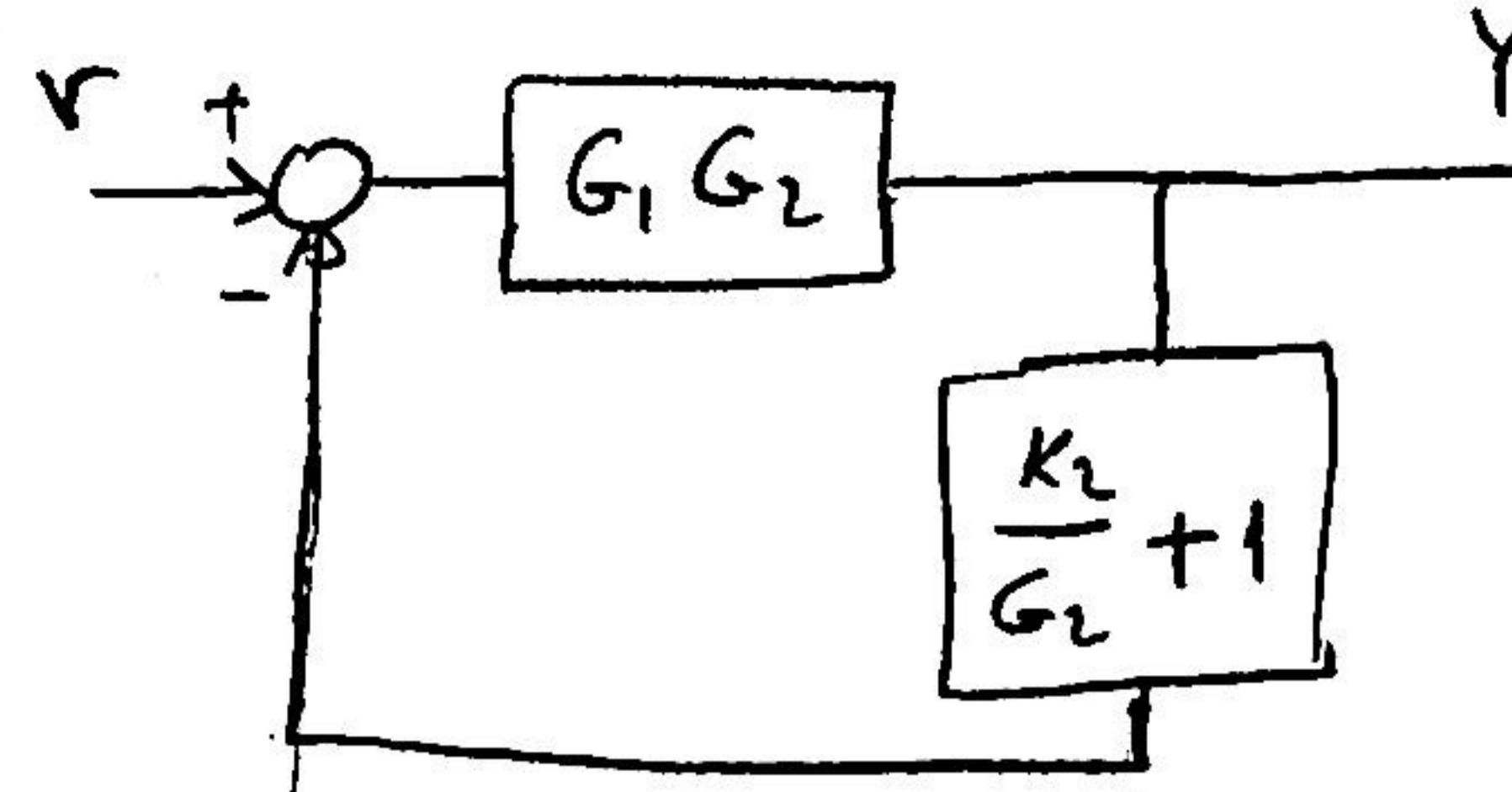
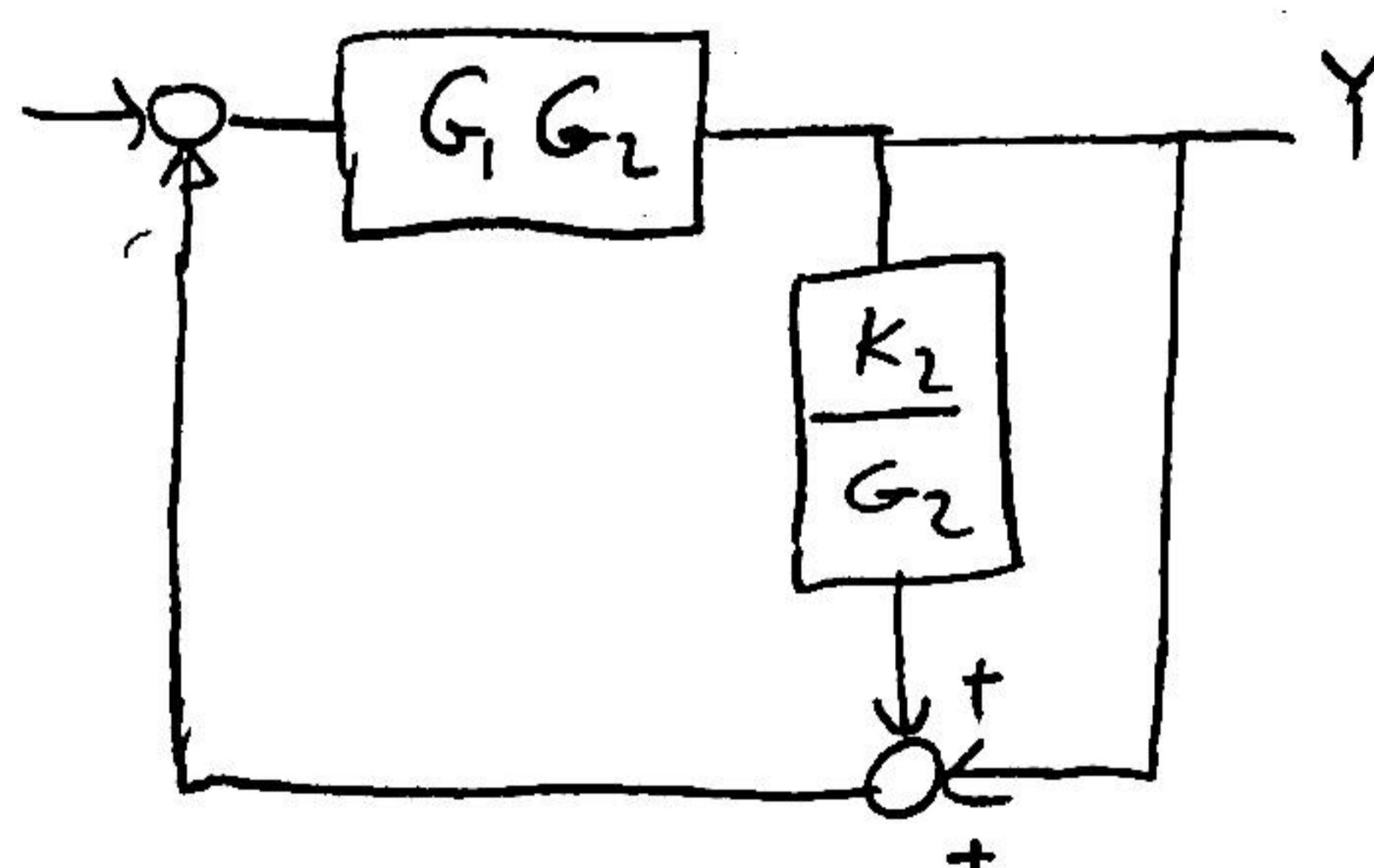
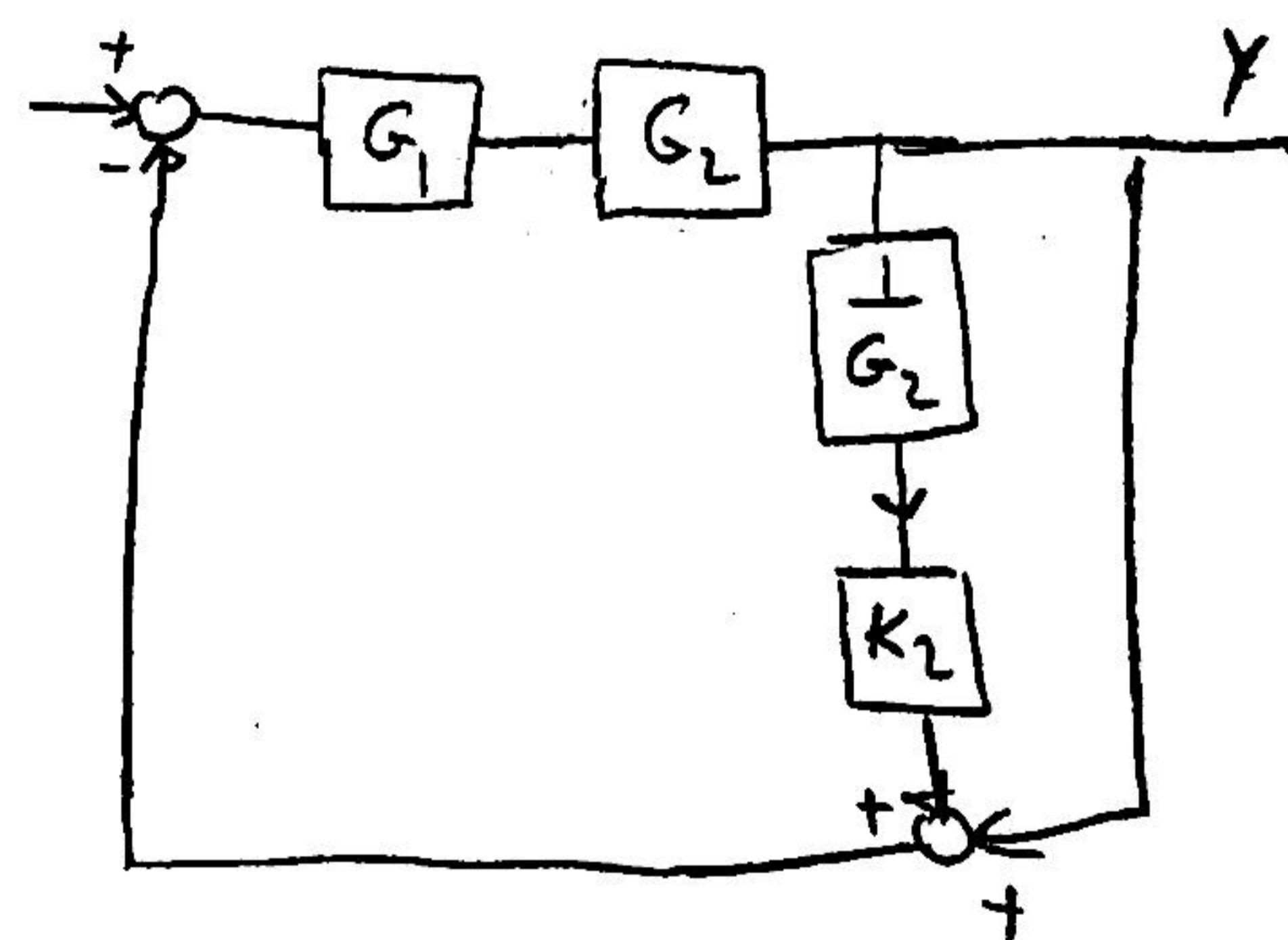
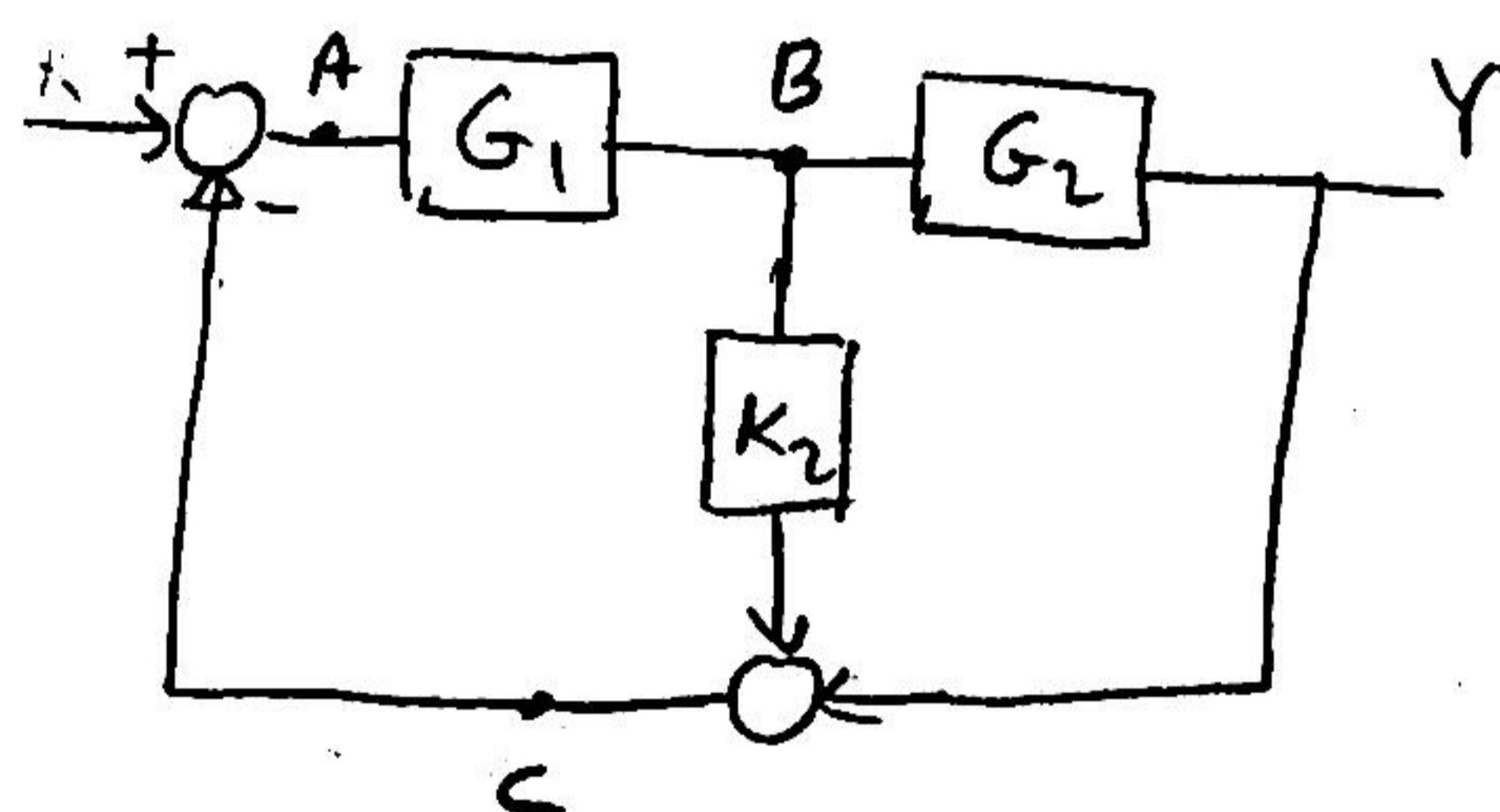
$$\frac{G}{1+GH} = \frac{\frac{s}{s+1}}{1 + \frac{s}{s+1} \cdot K} = \frac{s}{s+1+ks}$$

$$X(s) \rightarrow \frac{s}{(k+1)s+1} \rightarrow Y(s)$$

2.36 (page 111)



$$G_1 = \frac{K_1}{s+1} \quad G_2 = \frac{1}{s}$$



$$Y = \frac{G_1 G_2}{1 + G_1 G_2 \left(\frac{K_2}{G_2} + 1 \right)}$$

$$Y = \frac{G_1 G_2}{1 + G_1 K_2 + G_1 G_2} \quad Y$$

$$A = r - c$$

$$B = G_1 A$$

$$Y = G_2 B = G_2 G_1 A \Rightarrow A = \frac{1}{G_1 G_2} Y$$

$$C = K_2 B + Y = K_2 (G_1 A) + Y$$

$$A = r - c = r - (K_2 G_1 A + Y)$$

$$A = r - K_2 G_1 A - Y$$

$$A + K_2 G_1 A + Y = r$$

$$A(1 + K_2 G_1) + Y = r$$

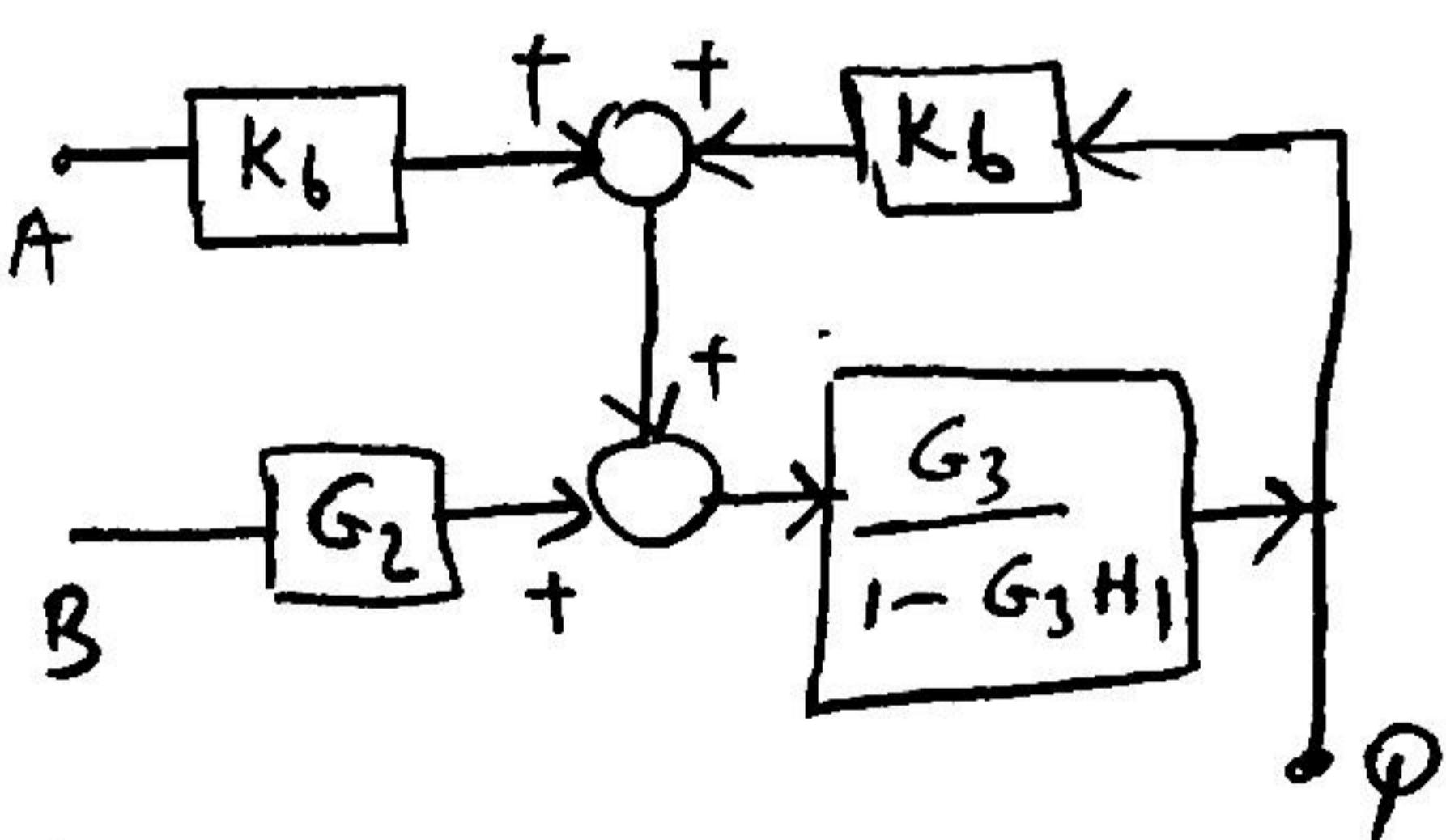
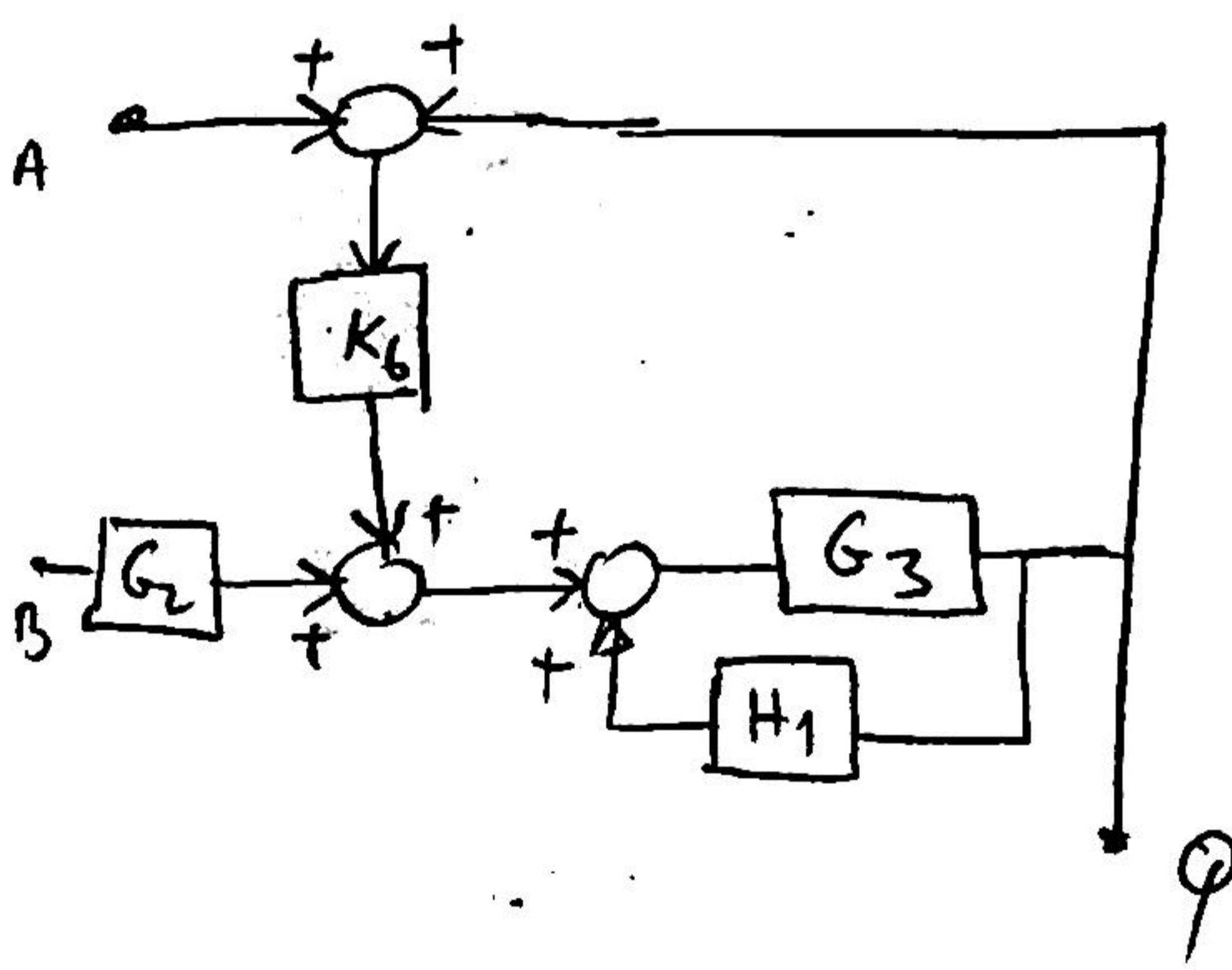
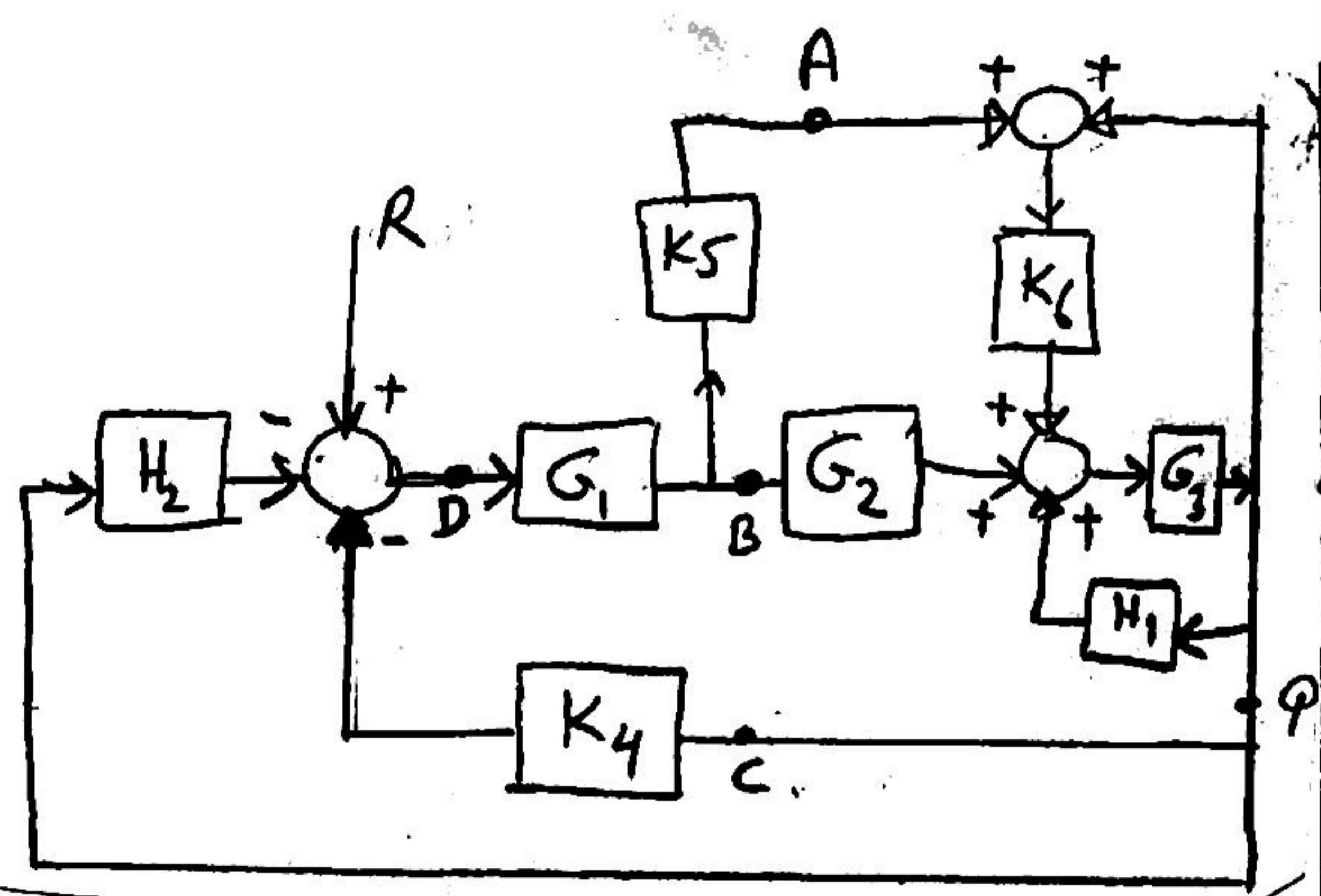
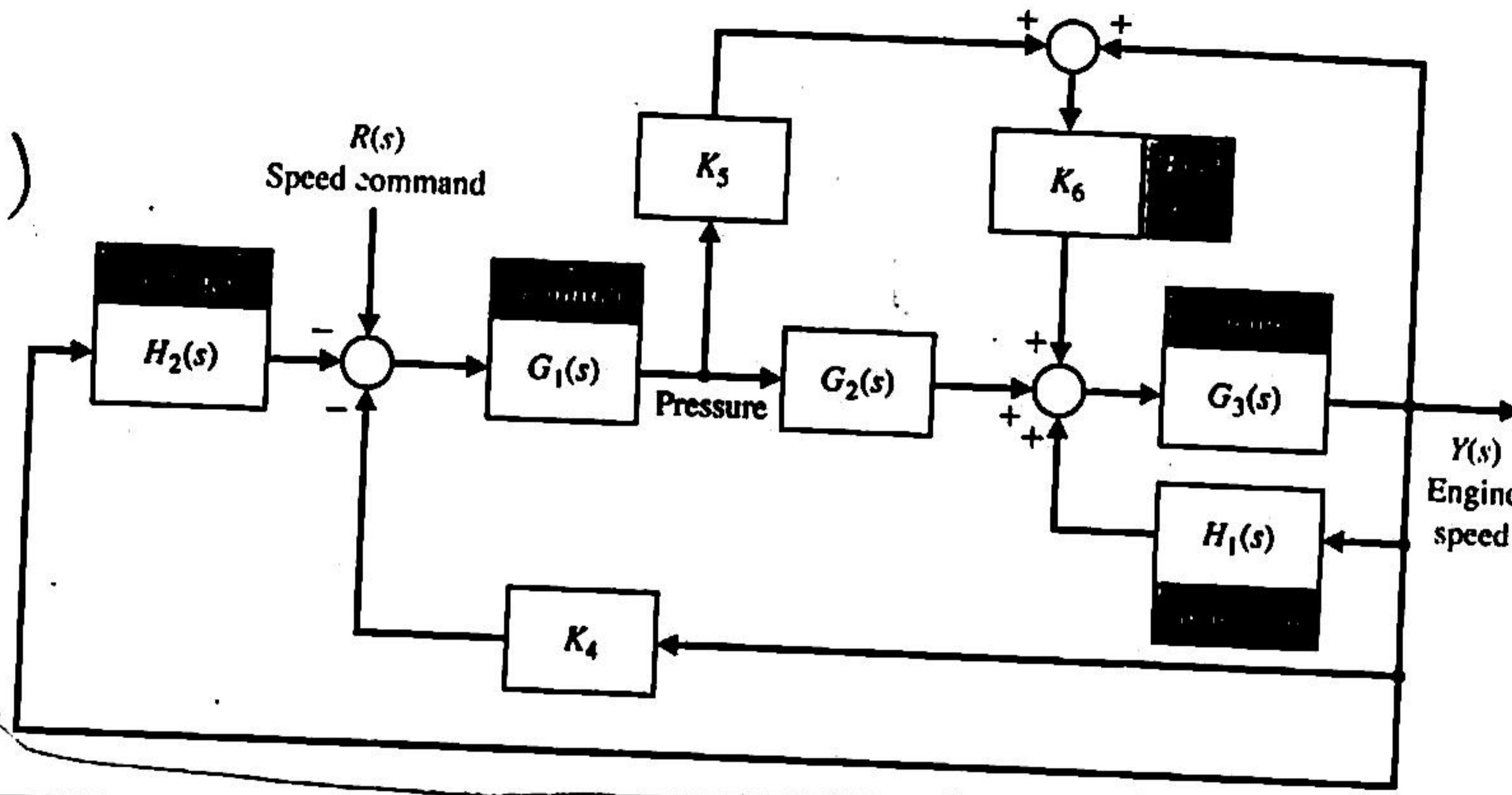
$$\frac{Y}{G_1 G_2} (1 + K_2 G_1) + Y = r$$

$$\left[\frac{1}{G_1 G_2} (1 + K_2 G_1) + 1 \right] Y = r$$

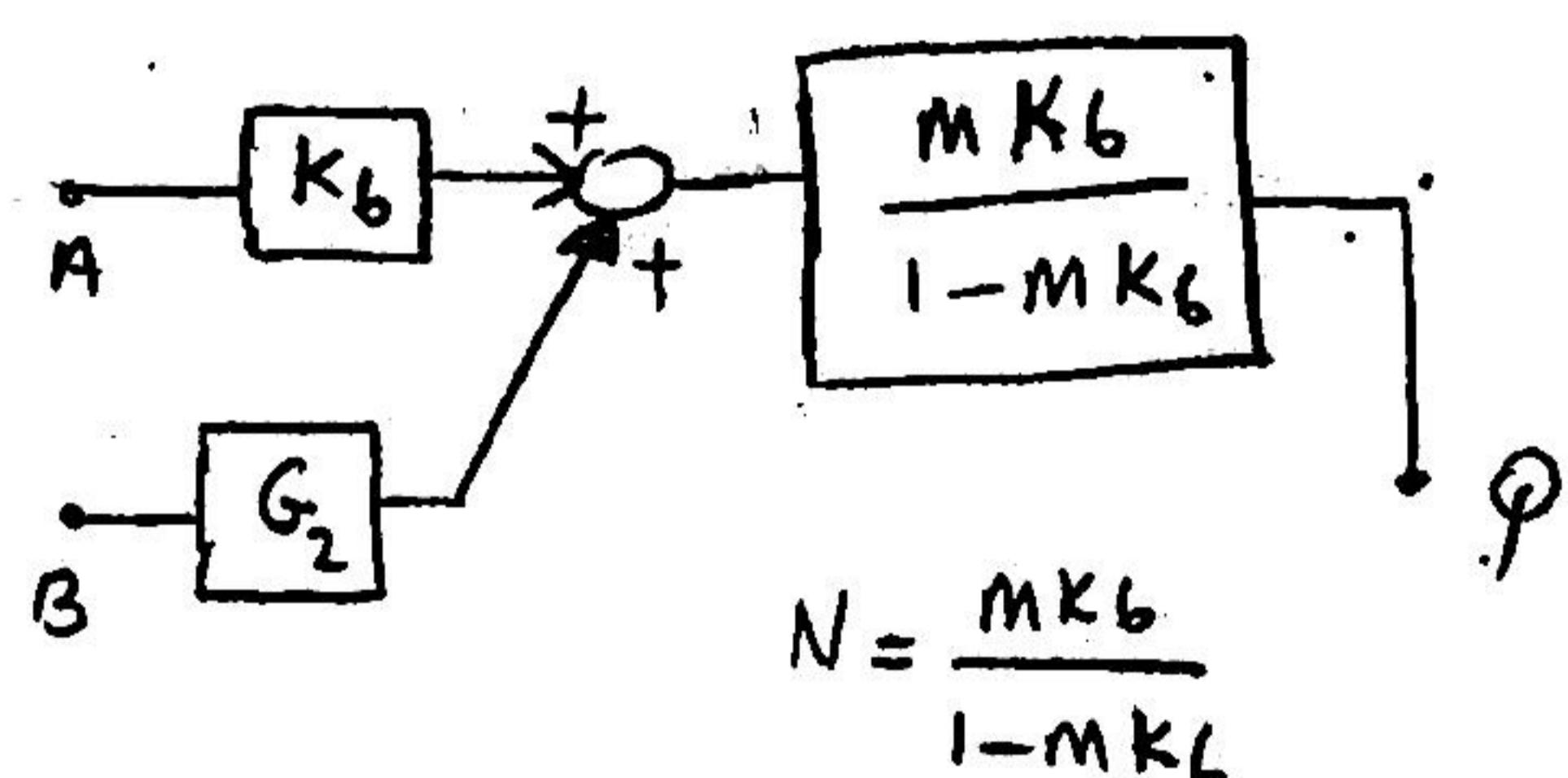
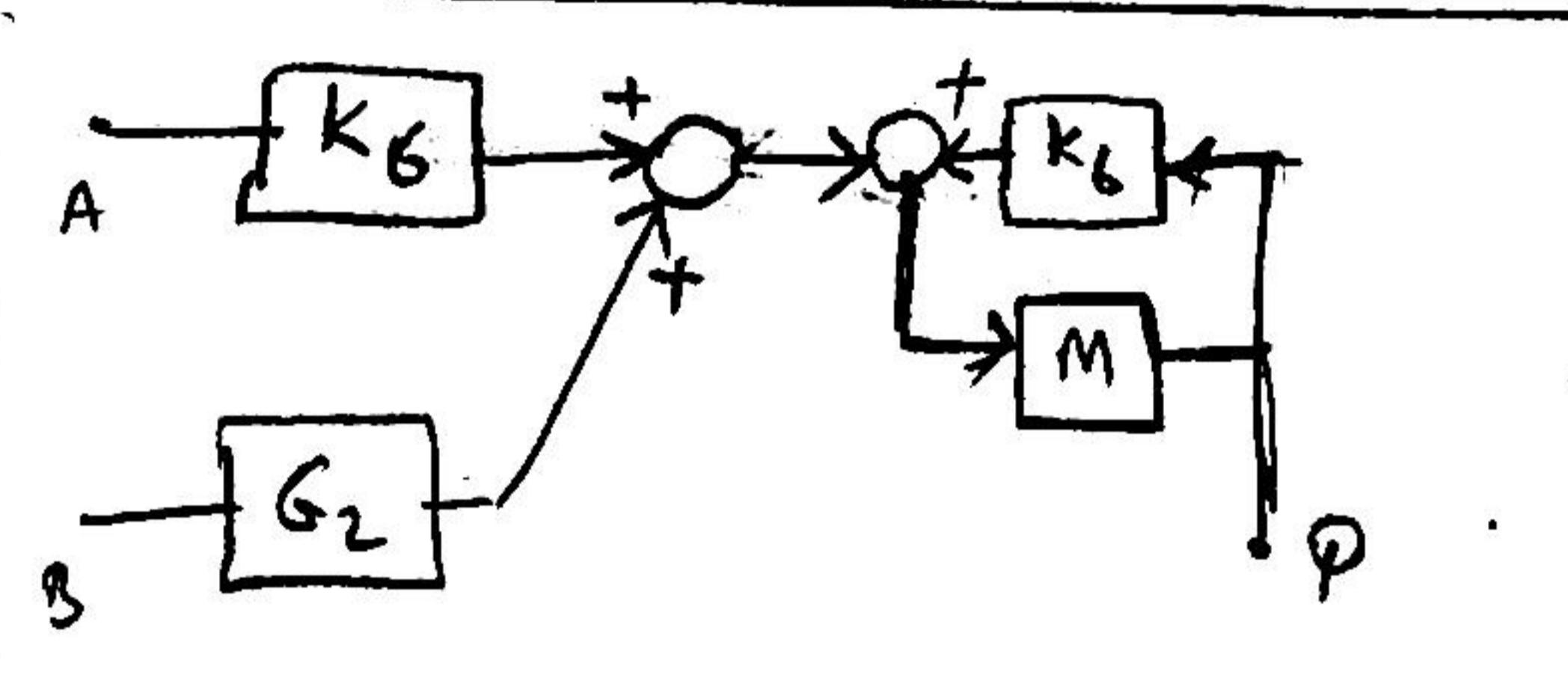
$$\frac{r}{Y} = \frac{1}{\frac{1}{G_1 G_2} (1 + K_2 G_1) + 1} = \frac{G_1 G_2}{1 + K_2 G_1 + G_1 G_2}$$

227

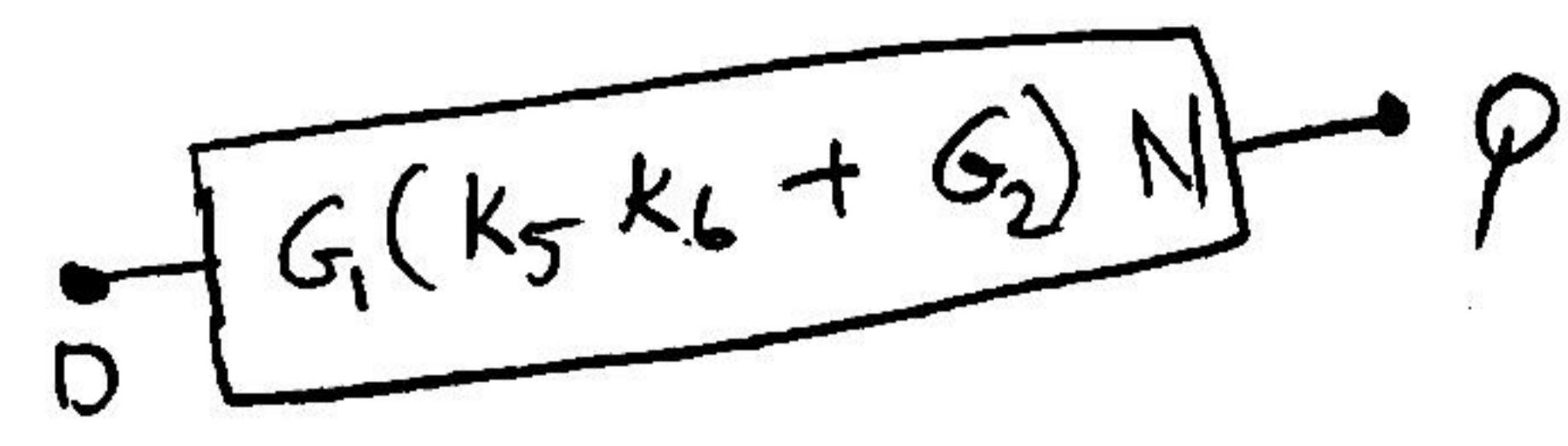
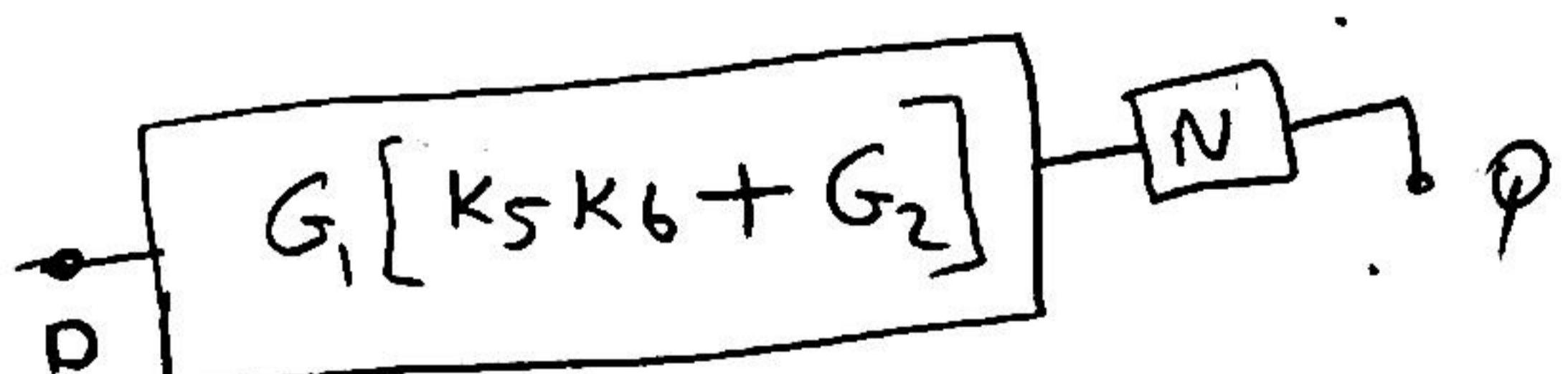
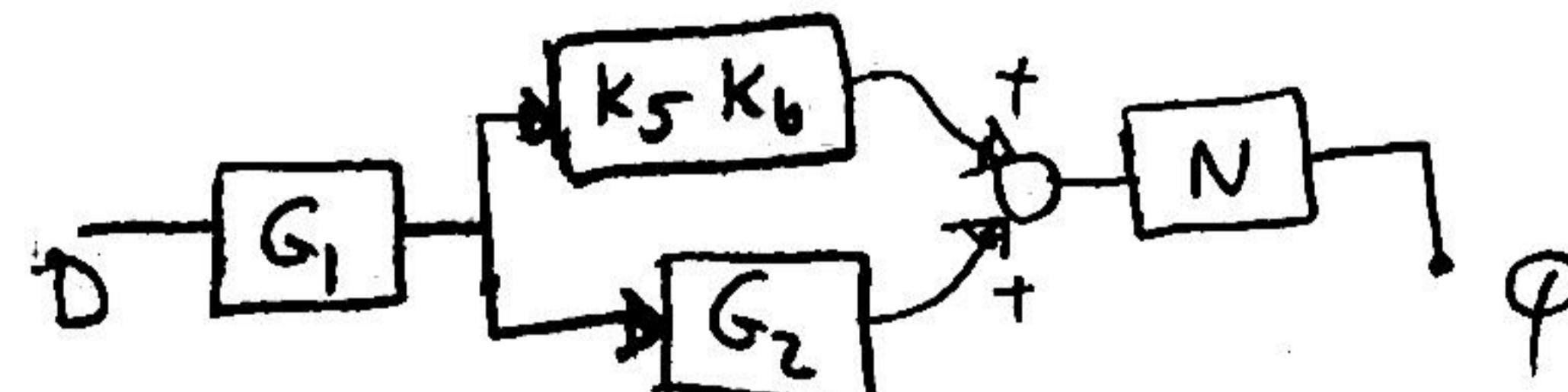
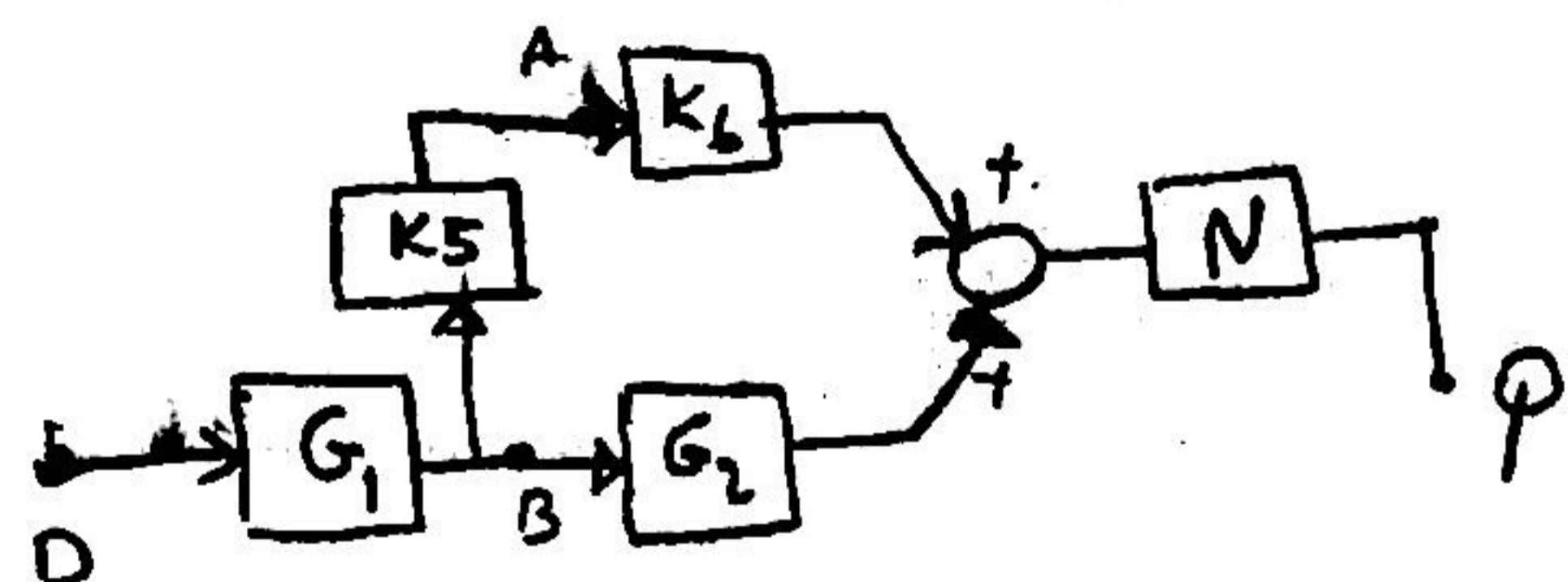
2.34
(Page 111)



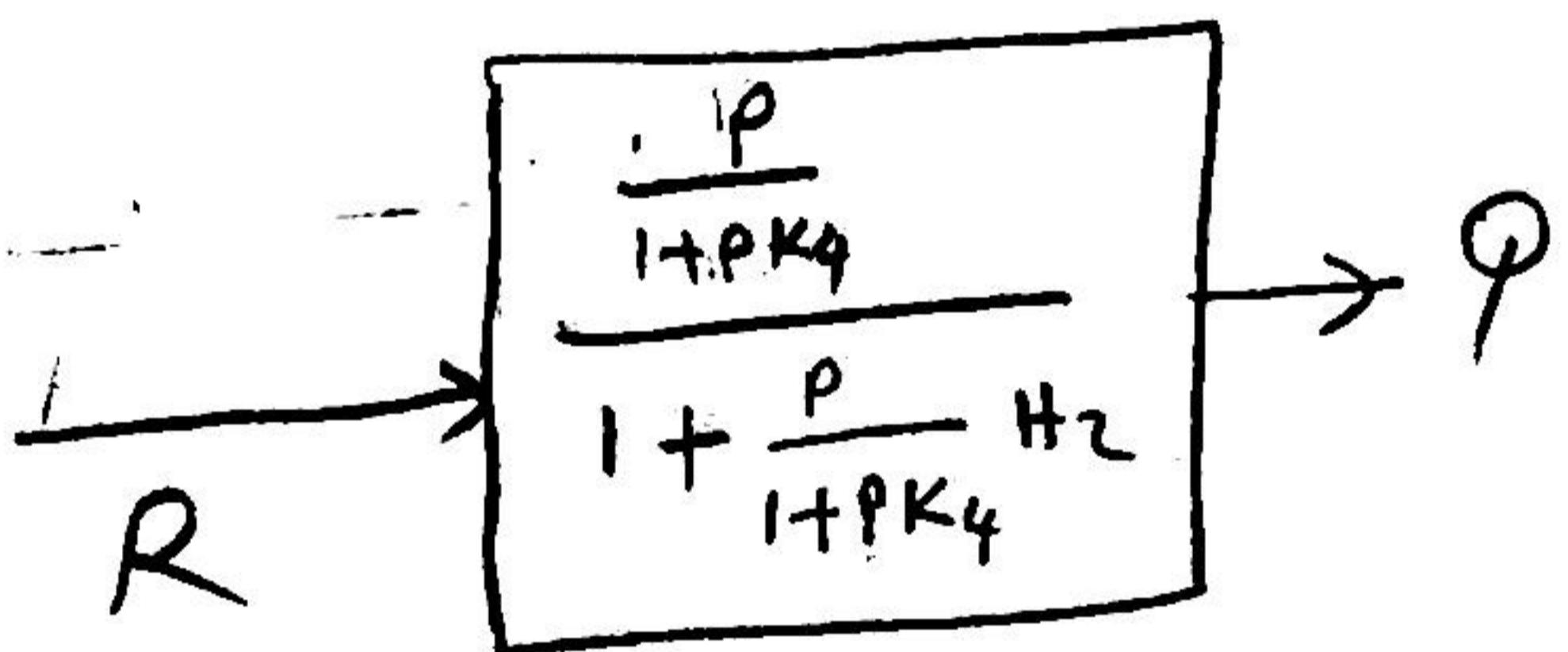
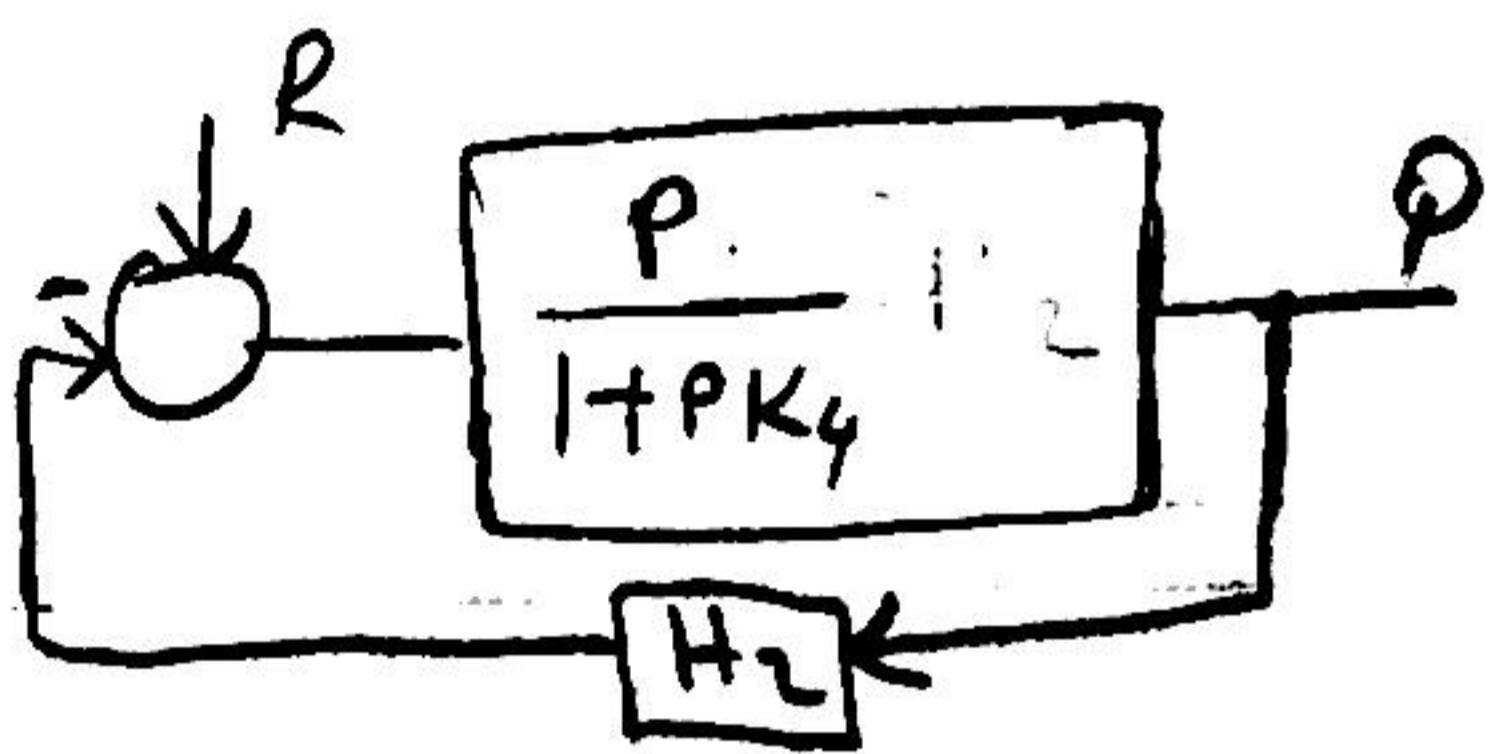
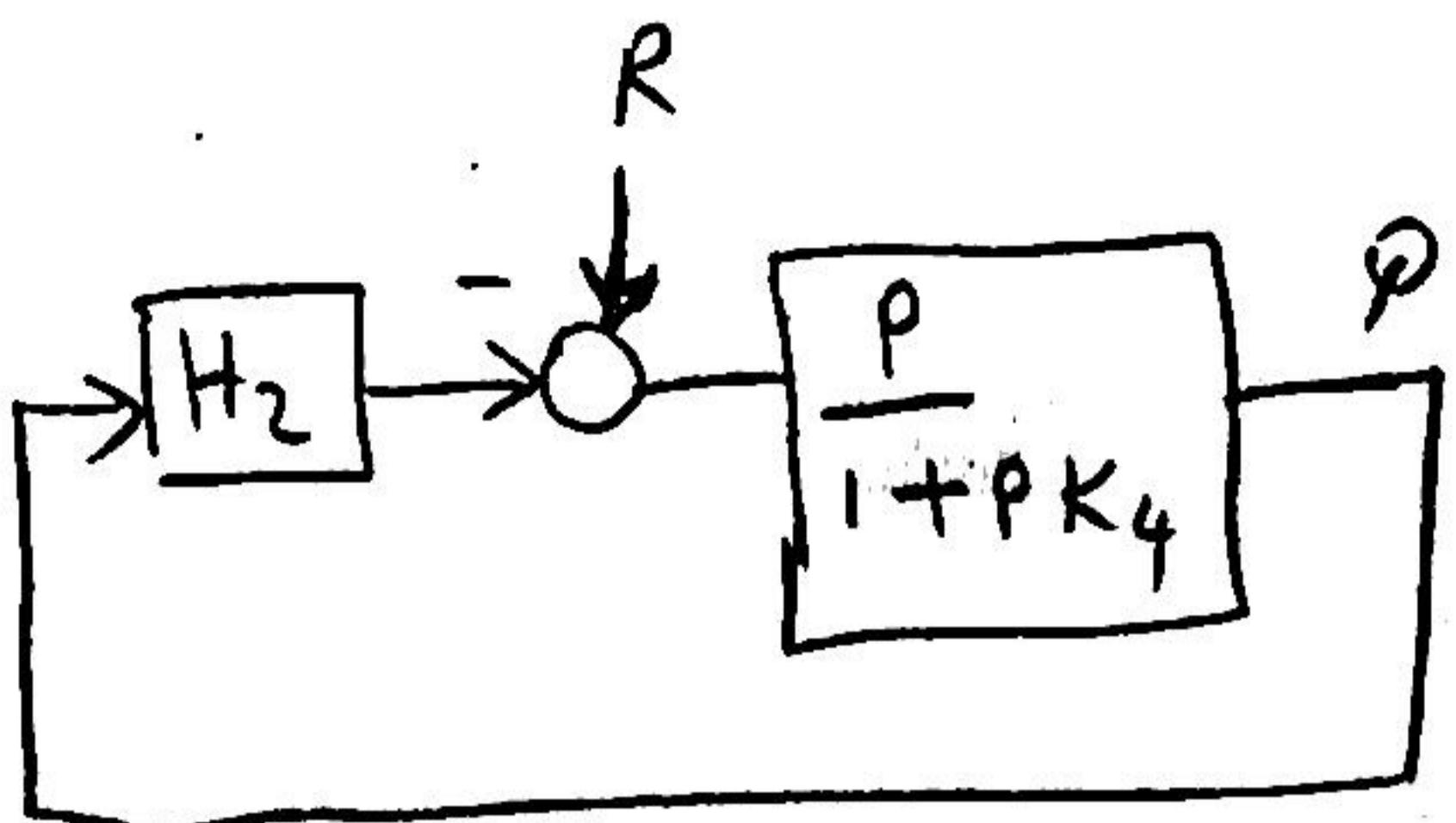
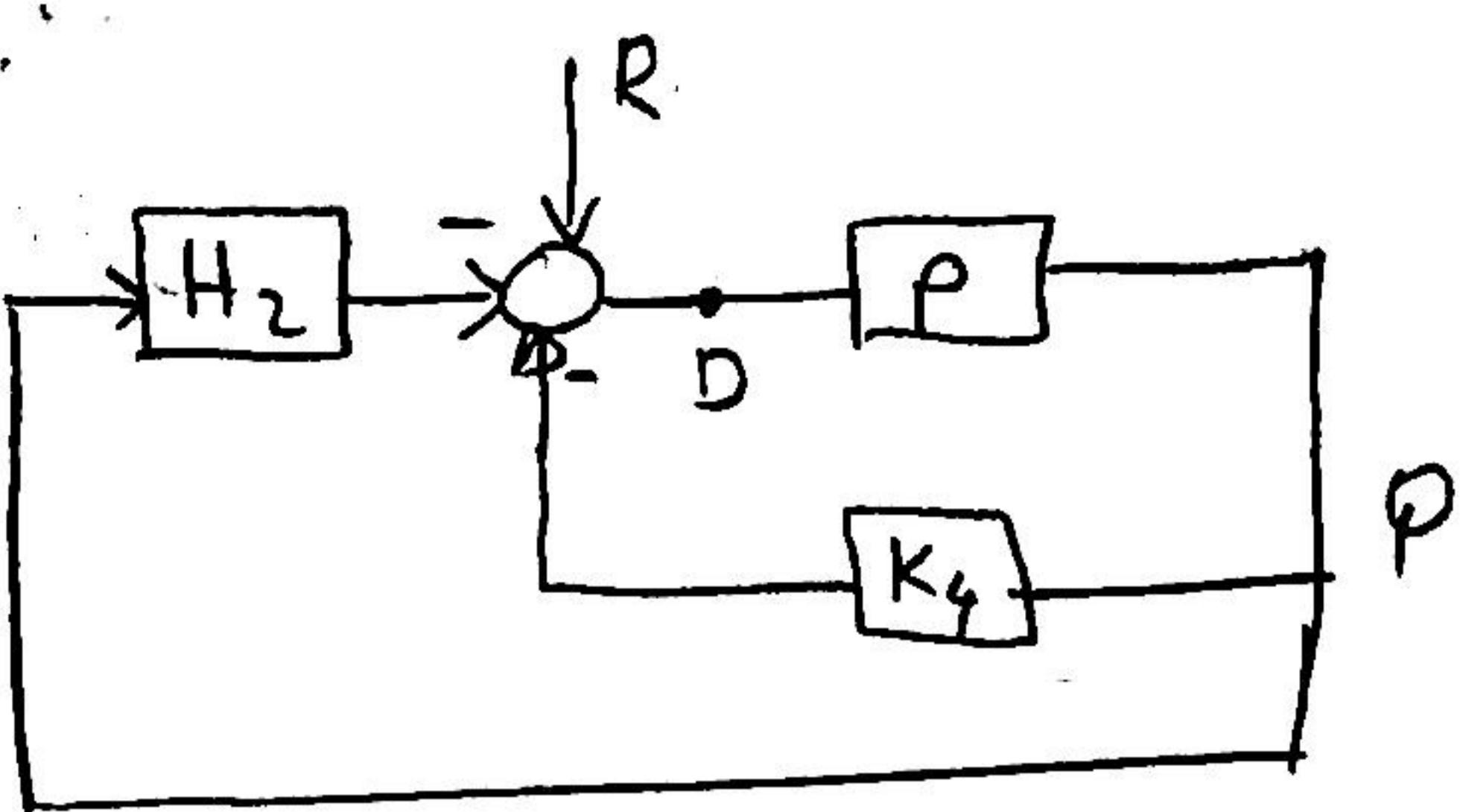
$$M = \frac{G_3}{1 - G_3 H_1}$$



$$N = \frac{MK_6}{1 - MK_6}$$



$$\rho = G_1 (K_5 K_6 + G_2) N$$



$$P = G_1(K_5 K_6 + G_2) N$$

$$N = \frac{MK_6}{1-MK_6}$$

$$M = \frac{G_3}{1-G_3 H_1}$$

Transfer function of DC motor

$$\phi = K_f i_f \text{ if } i_f \text{ flux current}$$

$K_f \rightarrow \text{constant}$

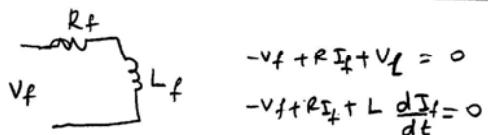
$\phi \rightarrow \text{magnetic flux}$

$$T_m = K_1 \phi i_a$$

$T_m \rightarrow \text{Torque (produced by currents)}$

$i_a \rightarrow \text{armature current}$

$\phi \rightarrow \text{magnetic flux}$



$$-V_f + R I_f + V_L = 0$$

$$-V_f + R I_f + L \frac{dI_f}{dt} = 0$$

$$-V_f(s) + R I_f(s) + L_f s I_f(s) = 0$$

$$V_f(s) = I_f(s)(R + sL_f)$$

$I_f(s) = \frac{1}{R + sL_f} V_f(s)$

$T_L = \text{load torque}$

$$T_L = J \frac{d\theta^2}{dt^2} + b \frac{d\theta}{dt}$$

$$T_L(s) = J s^2 \theta(s) + b s \theta(s)$$

$$= (J s^2 + b s) \theta(s)$$

$\theta \rightarrow \text{angle to be controlled.}$

Produced torque = load torque

$$T_m = T_L$$

$$K_1 \phi i_a = (J s^2 + b s) \theta(s)$$

flux controlled motor $i_a = \text{constant}$ if variable

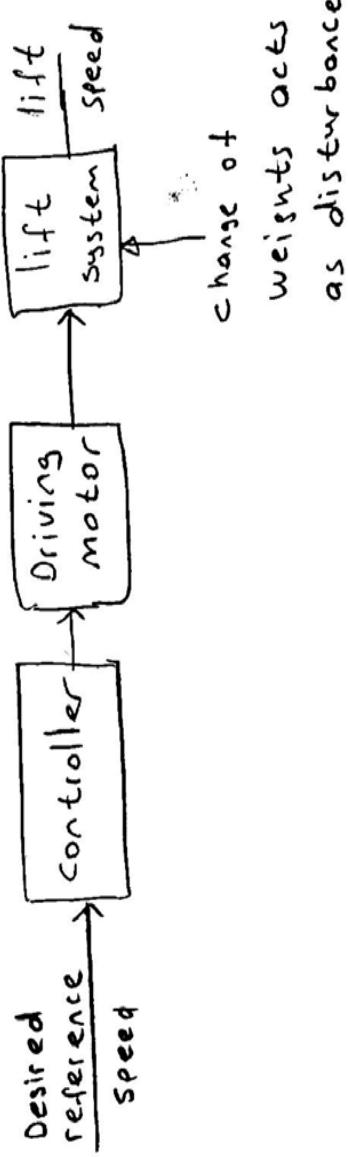
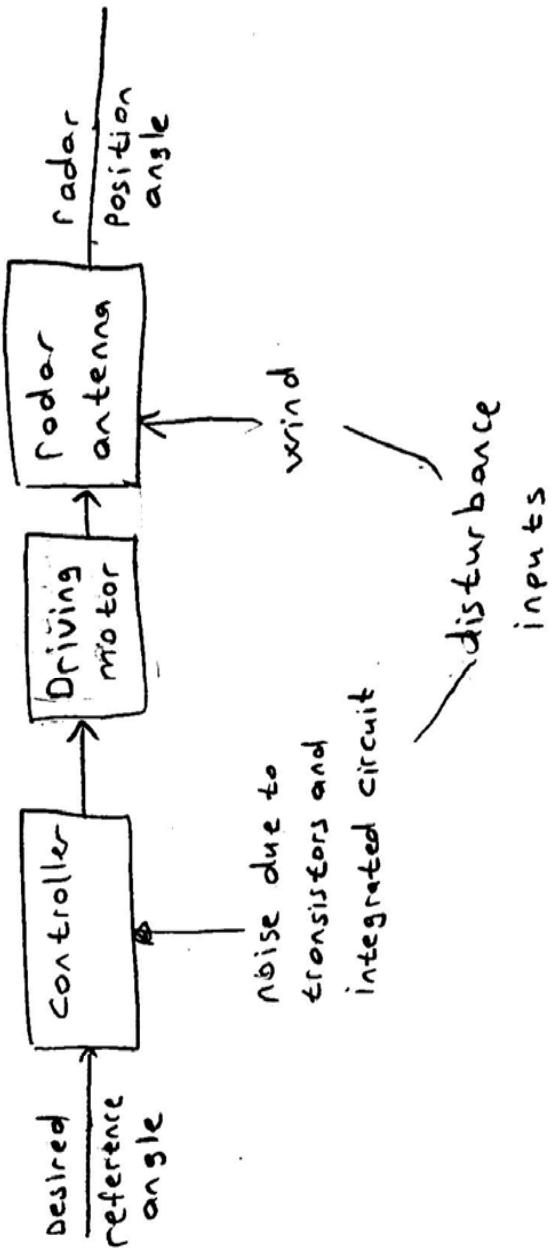
$$K_1 K_f i_f I_a = (J s^2 + b s) \theta(s)$$

$$K_1 K_f \frac{V_f(s)}{R_f + sL_f} I_a = (J s^2 + b s) \theta(s)$$

$$\theta(s) = \frac{K_1 K_f I_a}{V_f(s) (J s^2 + b s) (R_f + sL_f)}$$



Examples



The lift system is designed for 500 kg
if the weight is 600 kg $600 - 500 = 100 \text{ kg}$ is disturbance
if the weight is 400 kg $400 - 500 = -100 \text{ kg}$ is disturbance.

Do not confuse reliability and disturbance