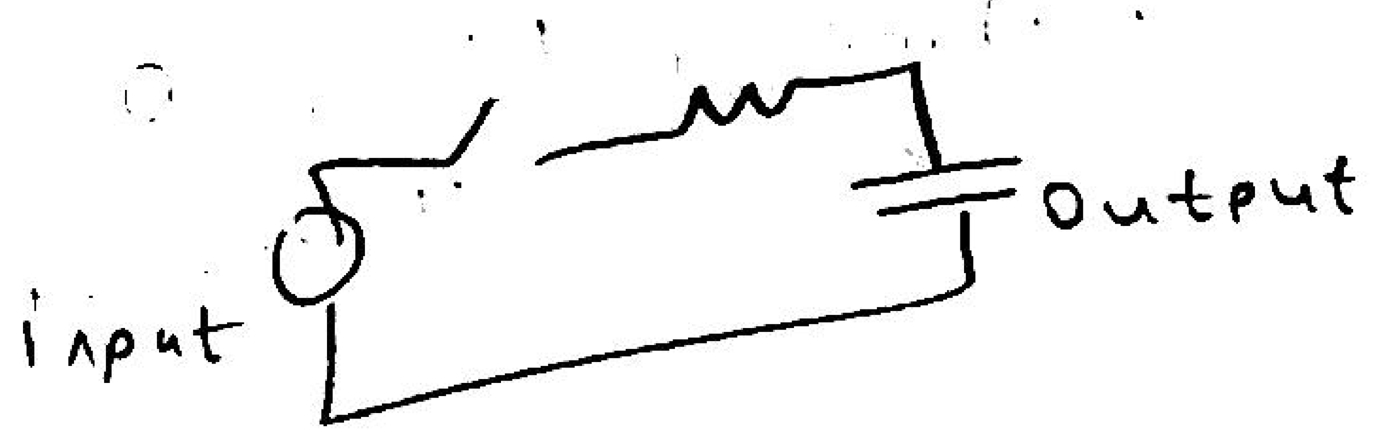
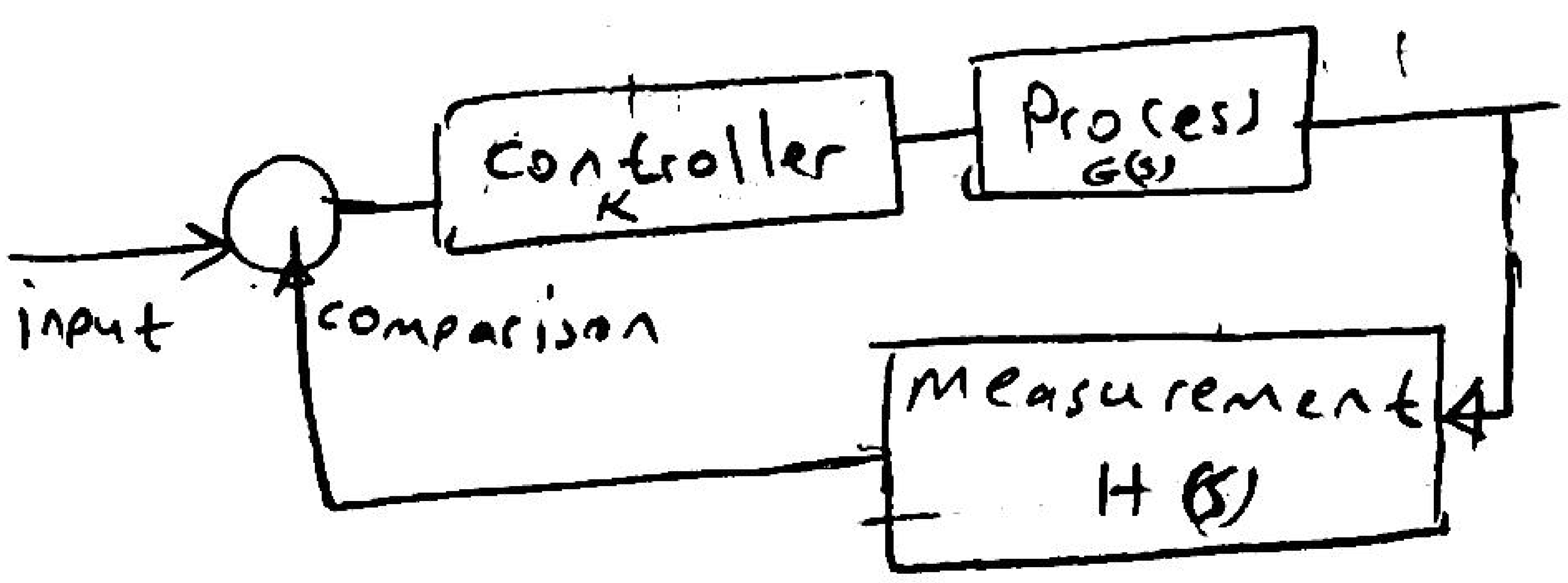
Open and closed loop Systems

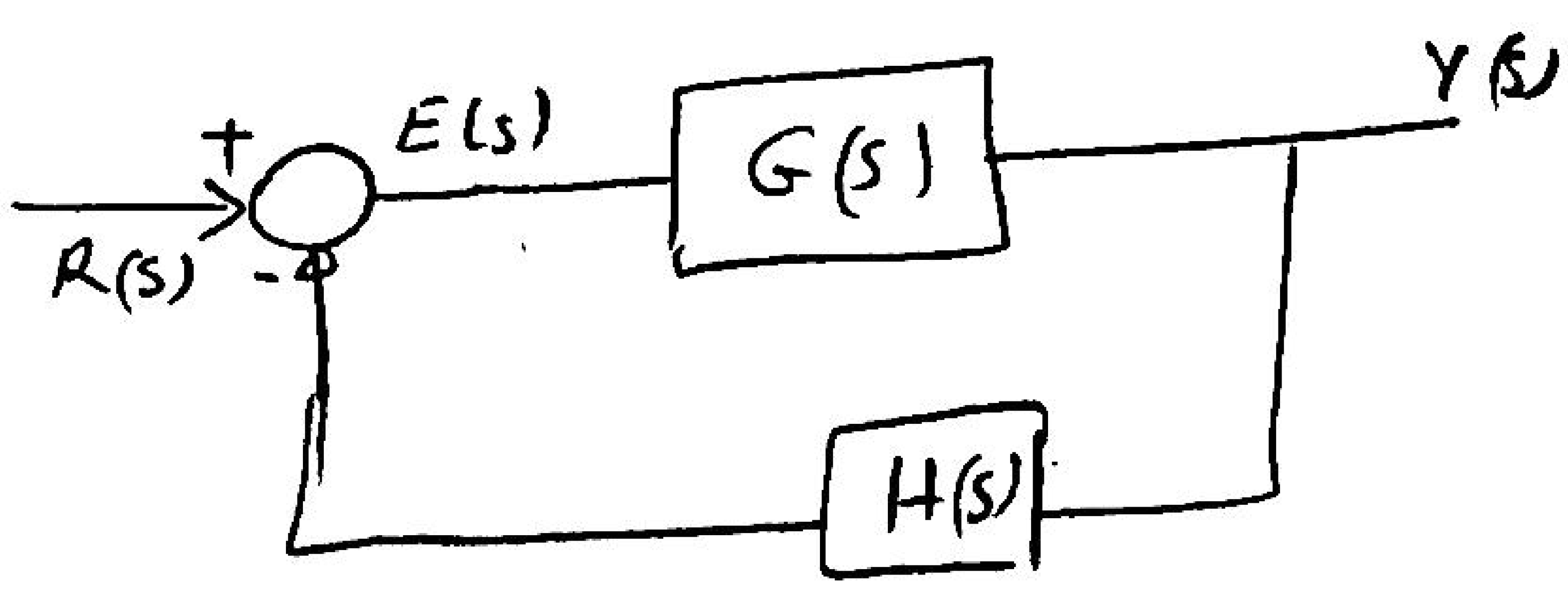


open loop system



closed loop system

(feedback control system)



$R(S)$ = reference input

$Y(S)$ = Process Output

$E(S)$ = error signal

$$E(S) = R(S) - H(S)\,Y(S)$$

$$Y(S) = G(S)\,E(S)$$

$$E(S) = R(S) - H(S)\,G(S)\,E(S)$$

$$E(S)\left[1 + G(S)\,H(S)\right] = R(S)$$

$$E(S) = \frac{1}{1+G(S)\,H(S)}\,R(S)$$

$$Y(S) = E(S)\,G(S) = \frac{G(S)}{1+G(S)H(S)}\,R(S)$$

In ideal case we want

$$Y(S) = R(S)$$

Output follows input and if $Y(S) = R(S)$ then

$E(S)$ is very small''

if $H(S) = 1$ and $Y(S) = R(S)$

then $E(S) = 0$

$$E(S) = \frac{1}{1+G(S)\,H(S)}$$

if we can select $G(S)H(S) \gg 1$

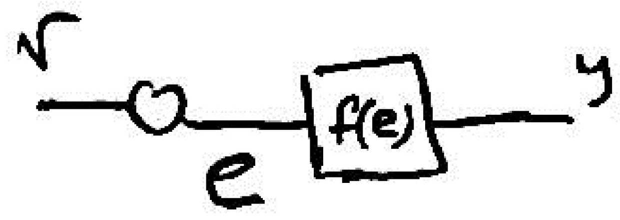
then $E(S) = \frac{1}{1+\infty} = 0$

and $Y(S) = \frac{G(S)}{1+G(S)H(S)}\,R(S)$

$$\cong \frac{G(S)}{G(S)\,H(S)}\,R(S) \cong \frac{1}{H(S)}\,R(S)$$

closed loop sistemi daha lineer yapar

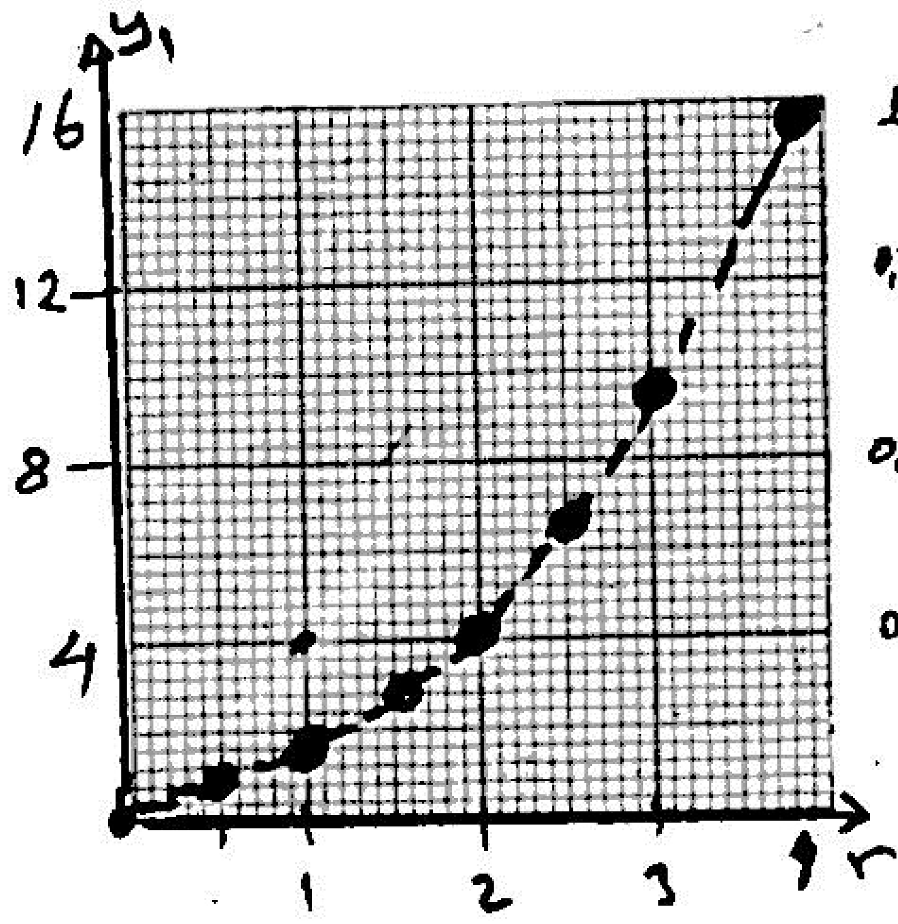
E 2.1 (Page 97)

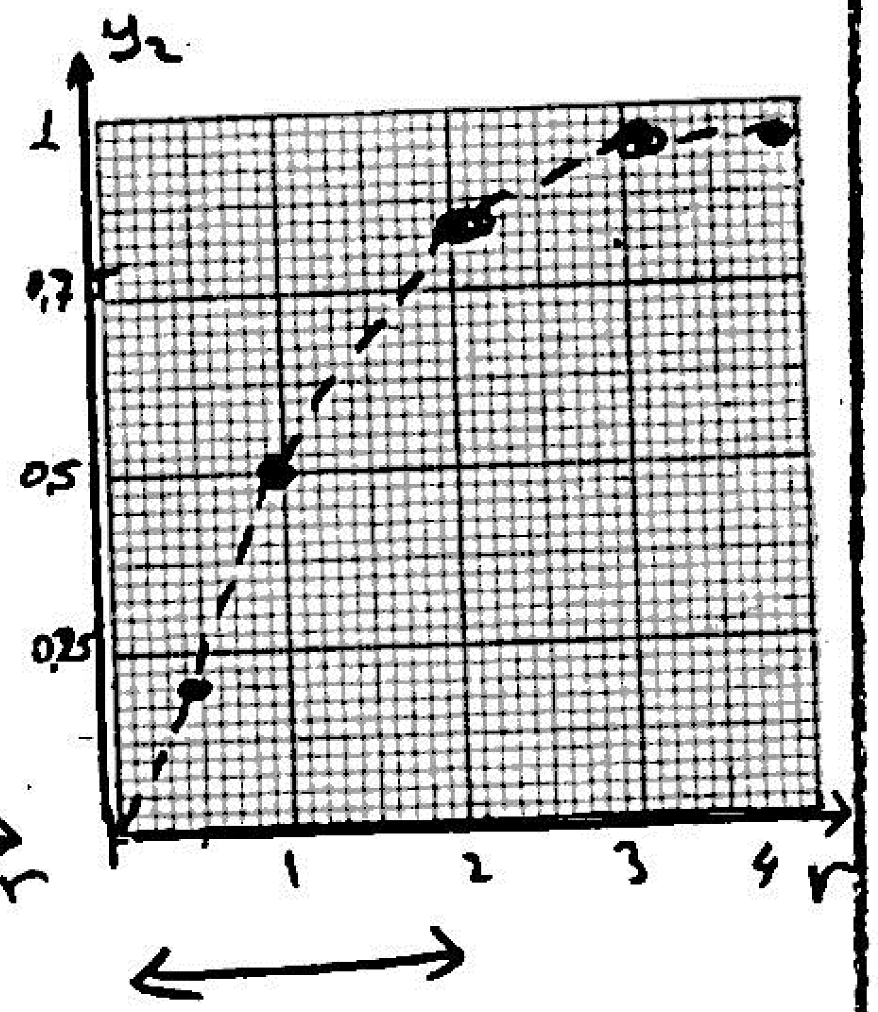


Open loop $y_1 = r^2$

closed loop $y_2 = \frac{r^2}{1+r^2}$

r	0	0.5	1	1.5	2	2.5	3	4
$y_1 = r^2$	0	0.25	1	2.25	4	6.25	9	16
$y_2 = \frac{r^2}{1+r^2}$	0	0.2	0.5	0.7	0.8	0.86	0.9	0.94





more linear

E2.2

$$R = R_0 e^{-0.1T}$$

$$f(x) = f(x_0) + f'(x_0)[x - x_0]$$

$$f(x) = R_0 e^{-0.1x}$$

$$f'(x) = -0.1 R_0 e^{-0.1x}$$

$$f(x_0) = R_0 e^{-0.1 \times 20} = R_0\, 0.1353 \cong 13.53$$

$$f'(x_0) = -0.1 R_0 e^{-0.1 \times 20} = -1.353$$

$$f(x) = 13.53 + (-135.3)[x - 20]$$

$$= \underbrace{-135.3}_{\text{slope}} x + 4053$$

$$\text{Slope} = \frac{\Delta R}{\Delta T} = -135.3$$

2.4) $r(t) = u(t) \Rightarrow R(s) = \frac{1}{s}$

$$Y(s) = \frac{5(s+100)}{s^2+60s+500} \cdot R(s) = \frac{5(s+100)}{s(s^2+60s+500)}$$

$$s^2 + 60s + 500 = 0 \rightarrow s_1 = -50$$
$$s_2 = -10$$

$$\frac{5(s+100)}{s^3+60s^2+500s} = \frac{A}{s} + \frac{B}{s+50} + \frac{C}{s+10}$$

$A = 1$ $B = 0.0125$ $C = -0.125$

$$y(t) = u(t) + 0.0125 e^{-50t} - 0.125 e^{-10t}$$

$$y(\infty) = 1 + 0 - 0 = 1$$

We can calculate $y(\infty)$ from $Y(s)$.

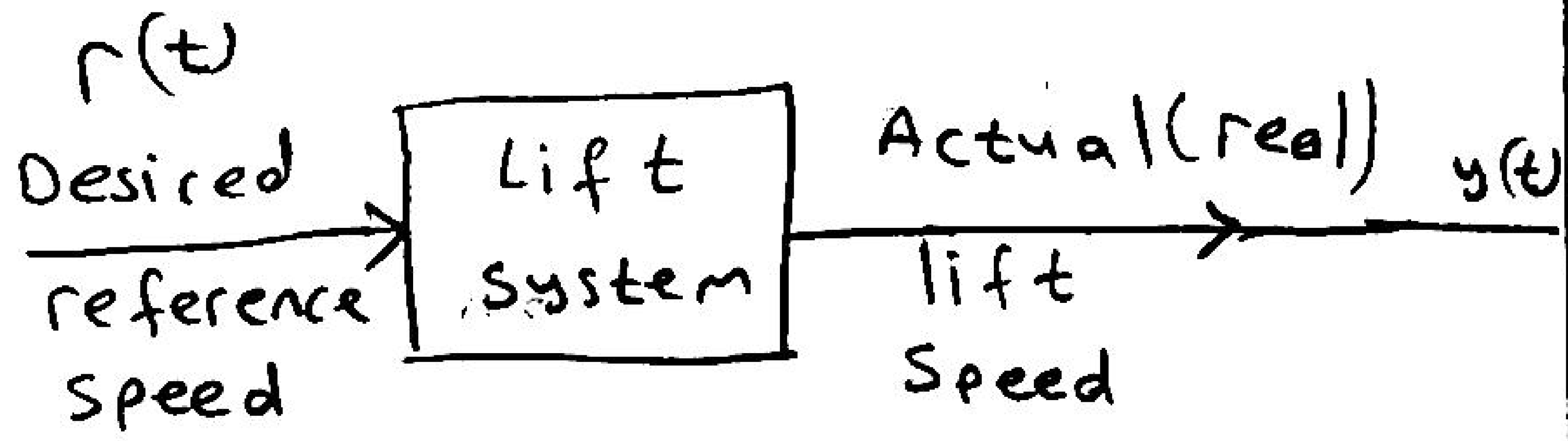
Final value theorem

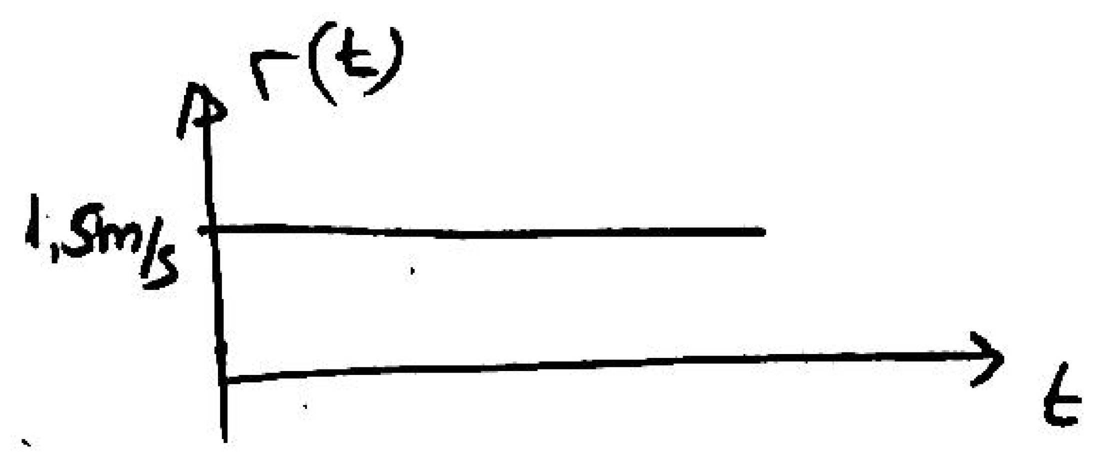
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y(s)$$

$$= \lim_{s\to 0} s \frac{5(s+100)}{s(s^2+60s+500)} = \frac{500}{500} = 1$$

Note: The book assumes $r(t) = 100\, u(t)$. Thus the answers are different.

Steady state Error





steady state error $1.5-1.6=-0.1$

Open loop system

$$E(s) = R(s) - Y(s)$$
$$= R(s) - G(s)R(s) = [1-G(s)]R(s)$$

steady state error is

$$\lim_{t\to\infty} e(t) = e(\infty)$$

Laplace Transform final Value theorem:

$$\mathcal{L}\{x(t)\} = X(s)$$

$$\lim_{t\to\infty} x(t) = x(\infty) = \lim_{s\to 0} s\,X(s)$$

Thus

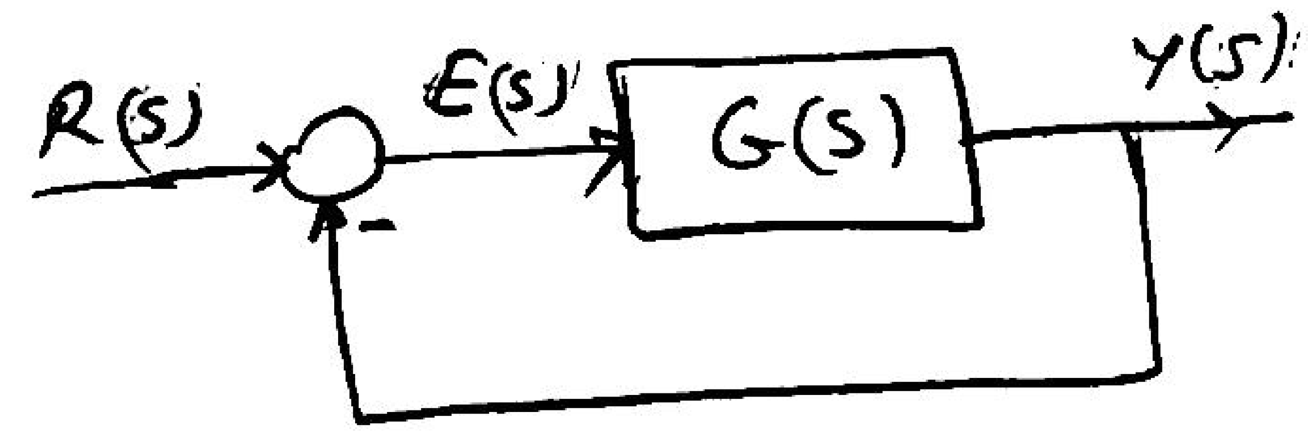
$$e(\infty) = \lim_{s\to 0} s\,E(s) = L_1$$

$$e(\infty) = \lim_{s\to 0} s(1-G(s))R(s)$$

We assume $r(t) = u(t)$ (step input)

$$R(s) = \frac{1}{s}$$

$$e(\infty) = \lim_{s\to 0} s[1-G(s)]\frac{1}{s} = 1 - G(0)$$



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$Y(s) = \frac{G(s)}{1+G(s)} R(s)$$

$$E(s) = R(s) - Y(s) = \left[1 - \frac{G(s)}{1+G(s)}\right] R(s)$$

$$E(s) = \frac{1}{1+G(s)} R(s)$$

$$e(\infty) = \lim_{s\to 0} s\,E(s) = \frac{1}{1+G(0)}$$

$G(0)$ = DC gain and usually greater than 1.

Example $G(0) = 2$

Open loop steady state error

$$1 - G(0) = 1 - 2 = -1$$

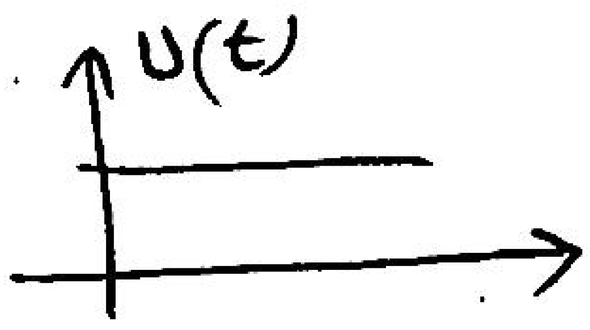
closed loop steady state error

$$\frac{1}{1+G(0)} = \frac{1}{1+2} = 0.33$$

A feedback system reduces the steady state error.

Performance of feedback control system.

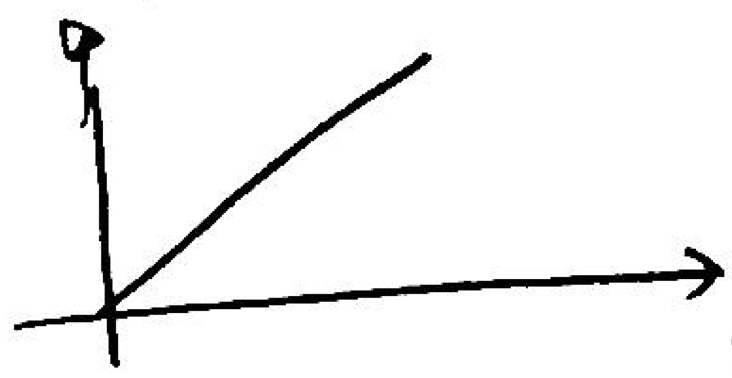
Test input signals



Unit step

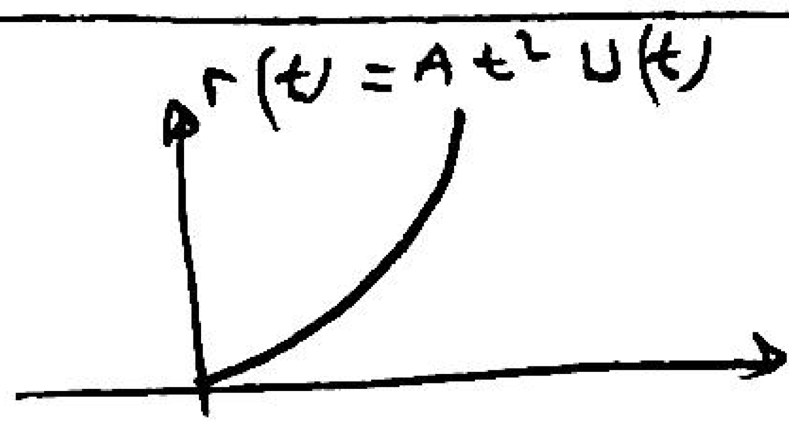
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases} \qquad R(s) = \frac{1}{s}$$

$r(t) = A t\, u(t)$



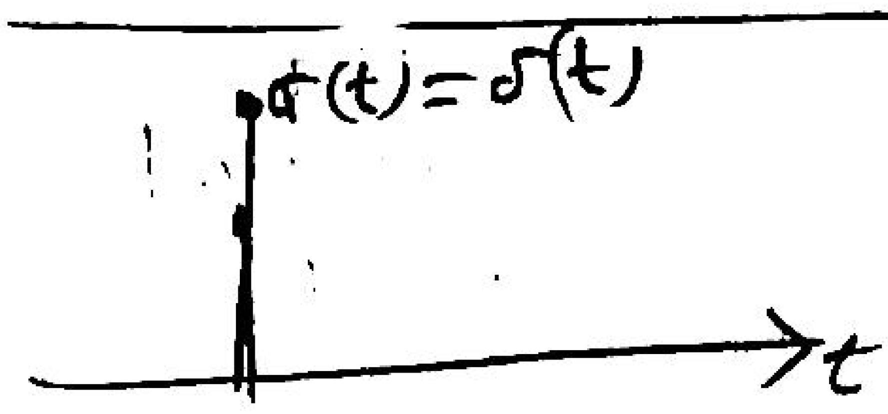
ramp

$$r(t) = \begin{cases} 0 & t < 0 \\ At & t \geq 0 \end{cases} \qquad R(s) = \frac{1}{s^2}$$



Parabolic input

$$r(t) = \begin{cases} 0 & t < 0 \\ At^2 & t \geq 0 \end{cases} \qquad R(s) = \frac{1}{s^3}$$

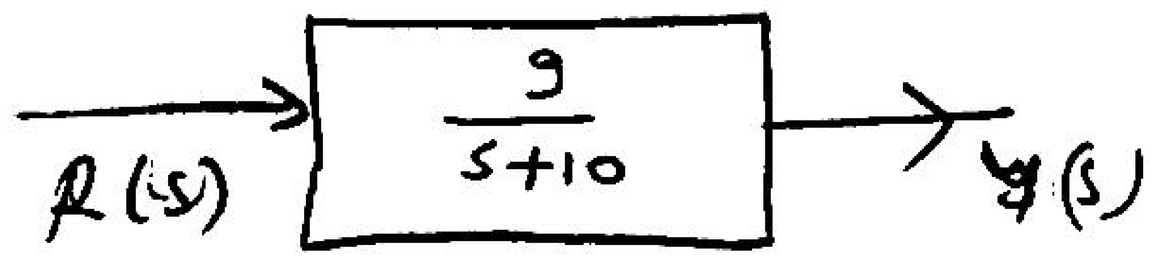


Impulse

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \neq 0 & t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) f(t) = f(0)$$

Example problem:



Calculate $y(t)$ if

a) $r(t) = \delta(t)$

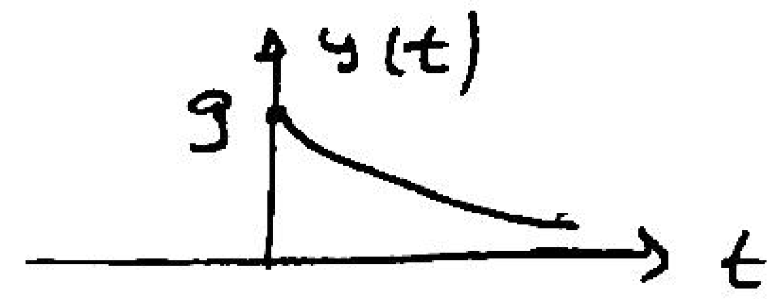
b) $r(t) = u(t)$

c) $r(t) = t\, u(t)$

a) $Y(s) = \frac{9}{s+10} R(s)$

$= \frac{9}{s+10} \cdot 1$

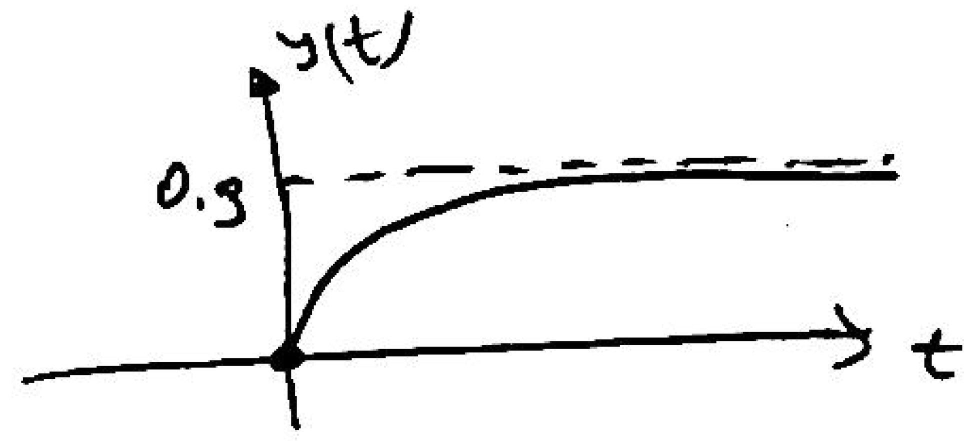
$y(t) = 9 e^{-10t}$



$\boxed{e_{ss} = 0}$

b) $Y(s) = \frac{9}{s+10} \frac{1}{s} = \frac{0.9}{s} - \frac{0.9}{s+10}$

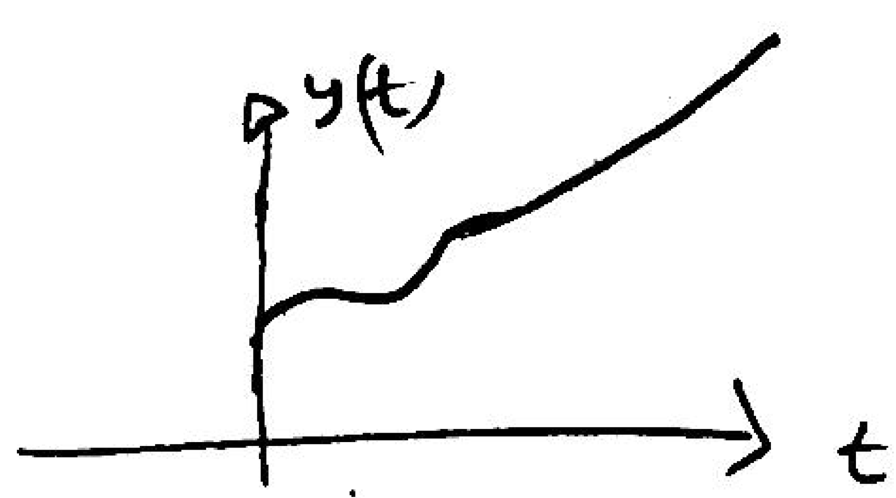
$y(t) = 0.9 - 0.9 e^{-10t}$



$e_{ss} = 1 - 0.9 = 0.1$

c) $Y(s) = \frac{9}{s+10} \cdot \frac{1}{s^2} = \frac{A}{s} + \frac{0.9}{s^2} + \frac{0.09}{s+10}$

$y(t) = -0.09 + 0.9t + 0.09 e^{-10t}$



$e_{ss} = y(\infty) - r(\infty) = \lim_{t \to \infty} (0.9 \times t - t)$

$= \lim_{t \to \infty} (0.9 - 1) t = -\infty$

We can also calculate e_{ss} using final value theorem

$$Y(s) = \frac{9}{s+10} \cdot \frac{1}{s^2} \qquad R(s) = \frac{1}{s^2}$$

$$E(s) = Y(s) - R(s) = \frac{1}{s^2}\left(\frac{9}{s+10} - 1\right)$$

$$\boxed{e(\infty) = e_{ss} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \frac{1}{s}\left(\frac{9}{s+10} - 1\right) = \mp\infty}$$

Performance of A Second Order system

$$R(s) \rightarrow \boxed{\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}} \rightarrow Y(s)$$

$R(s) = \frac{1}{s}$ $\quad r(t) = u(t)$

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

roots are

$$s_1 = 0$$

$$s_{2,3} = \frac{-2\xi\omega_n \mp \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \mp 2\xi\sqrt{\xi^2 - 1}}{2}$$

Case 1: $\xi > 1$

$$\sqrt{\xi^2 - 1} > 0$$

All roots are real

$$Y(s) = \underbrace{\frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b}}_{\text{real numbers}}$$

$$y(t) = A + Be^{-at} + Ce^{-bt}$$

Case 2 $\quad \xi < 1$

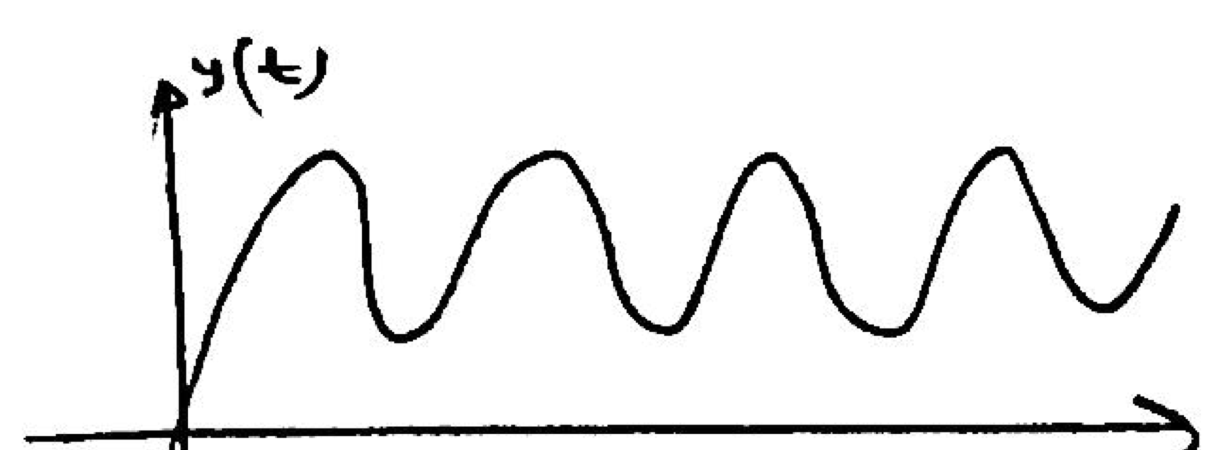
Two roots are complex

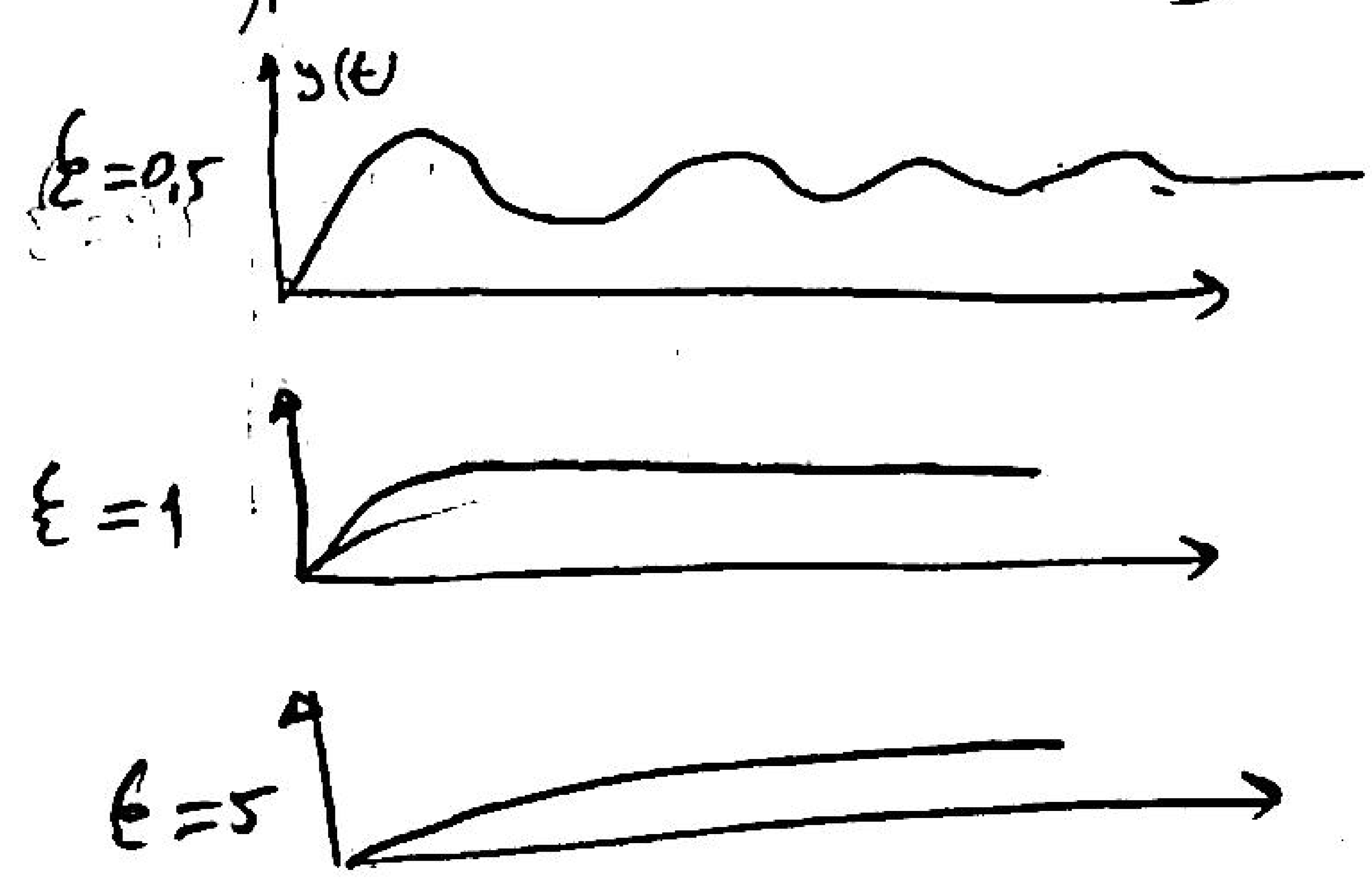
$$Y(s) = \underbrace{\frac{A}{s} + \frac{B}{s+a} + \frac{C}{s+b}}_{\text{complex numbers}}$$

$$y(t) = 1 - \frac{1}{\beta} e^{-\xi\omega_n t} \sin(\omega_n \beta t + \theta)$$

$$\beta = \sqrt{1 - \xi^2} \qquad \theta = \cos^{-1}\xi$$

$\xi = 0$





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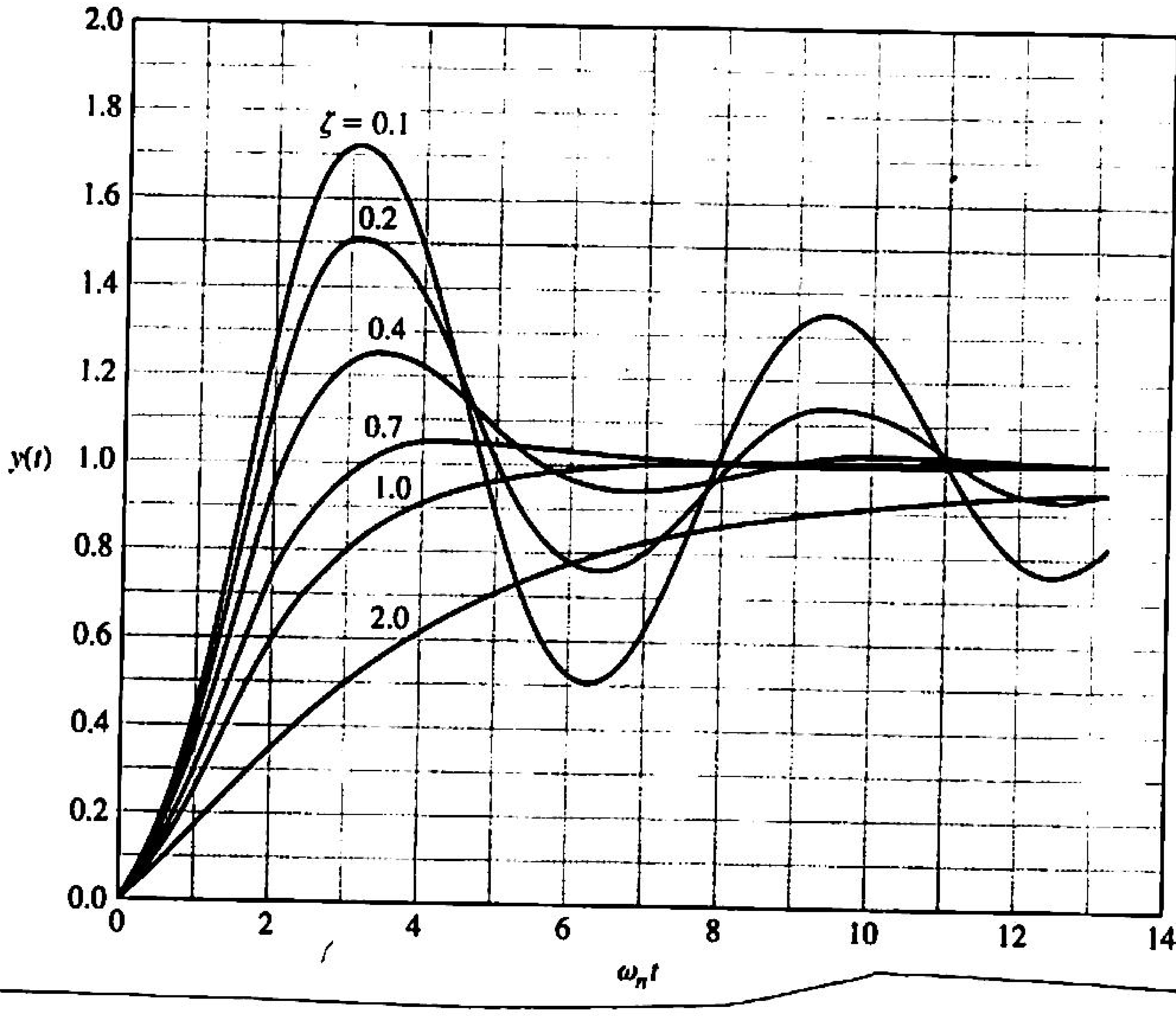
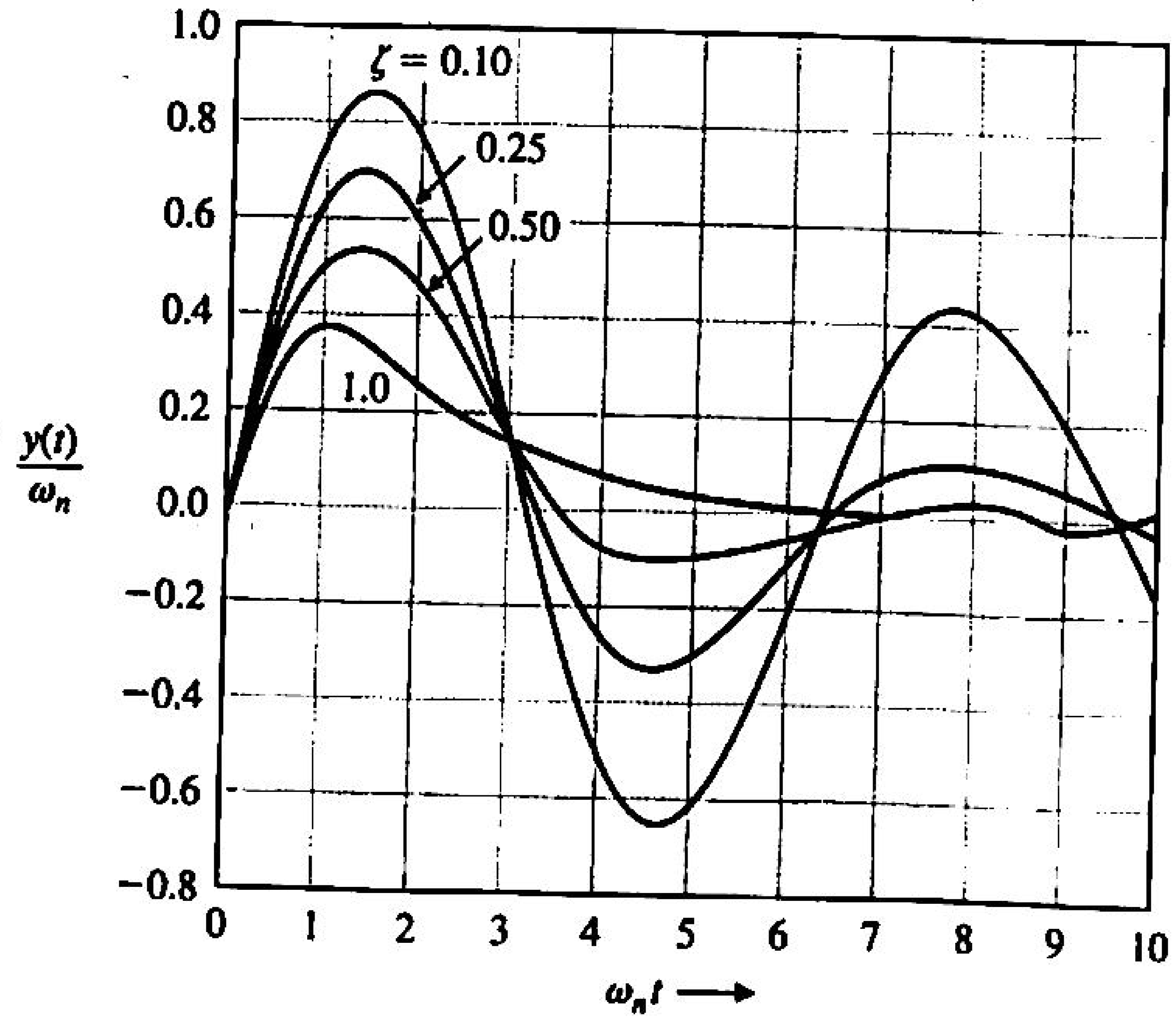
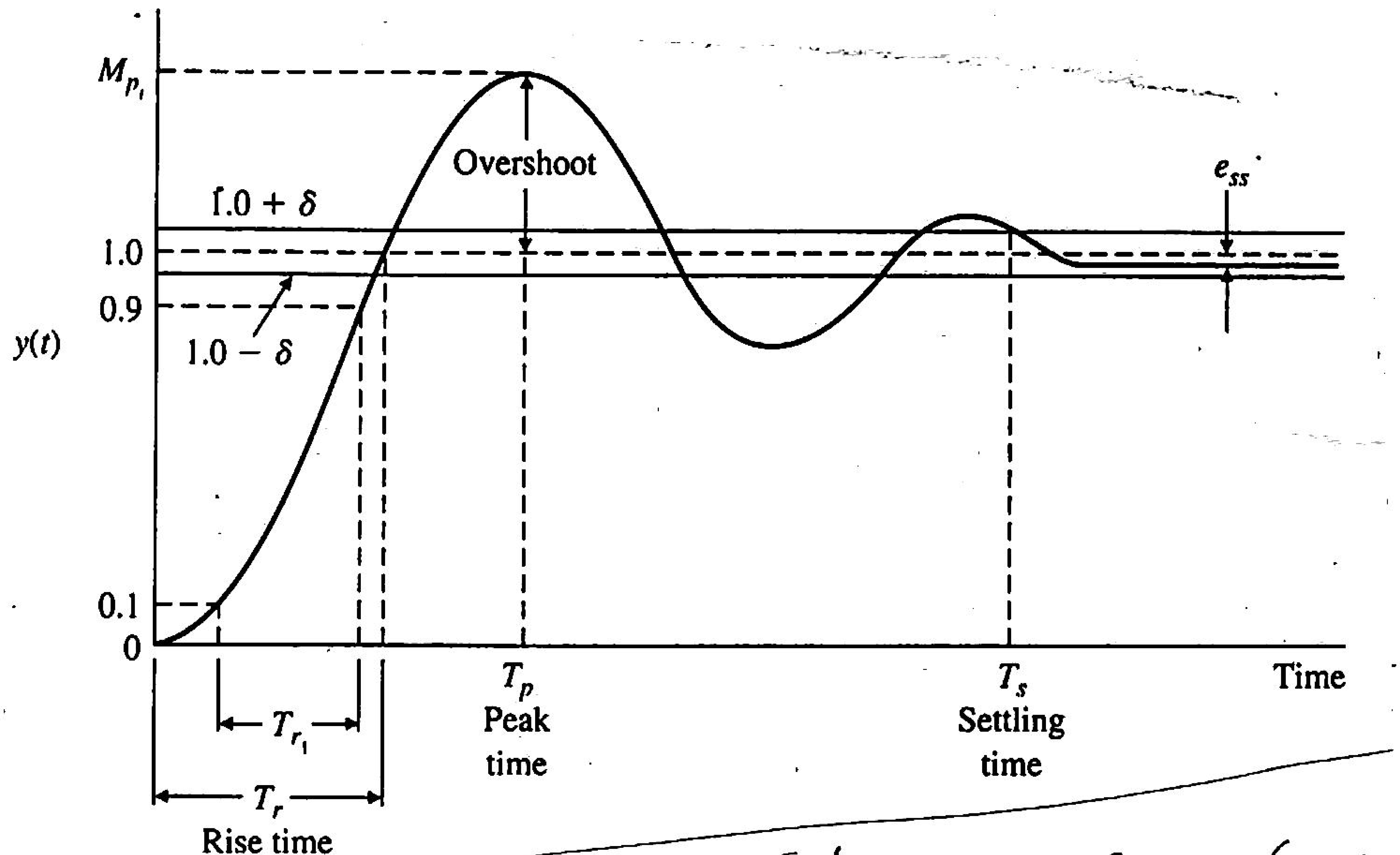




Im puls response

Step response



Step response $\xi < 1$

<u>Rise time</u> has two definitions

Tr = from $t=0$ to t_1 $y(t_1) = 1$. 1 is the reference step input value

Tr_1 = from t_1 $y(t_1) = 0{,}1$ to t_2 $y(t_2) = 0{,}9$

Ts = settling time = from $t=0$ to t_1 where

$1-\delta < y(t_1) < 1+\delta$ δ is usually 0.1.

$$e^{-\xi \omega_n T_s} < 0.02$$

$$\ln(e^{-\xi \omega_n T_s}) \cong \ln 0.02$$

$$\xi \omega_n T_s = 4$$

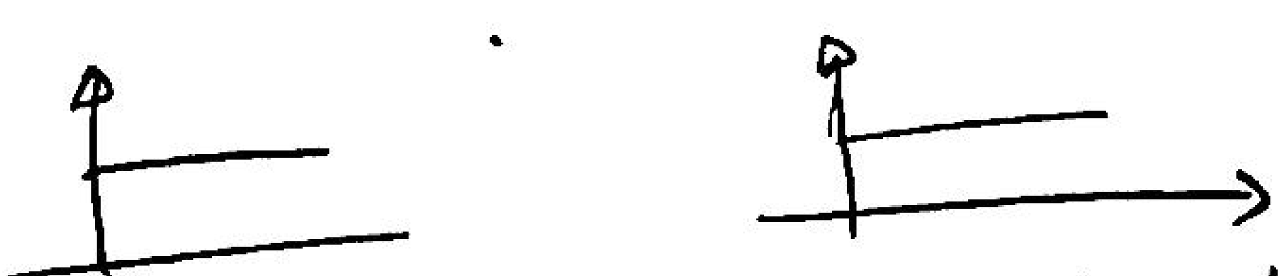
$$\boxed{T_s = \frac{4}{\xi \omega_n}} \quad \text{(setling time)}$$

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \quad \text{(Peak time}$$

$$M_p = 1 + e^{-\xi\pi/\sqrt{1-\xi^2}} \quad \text{(overshoot}$$

$$T_{r_1} \cong \frac{2.16\,\xi + 0.6}{\omega_n} \quad \text{(rise time)}$$

In the ideal case



input

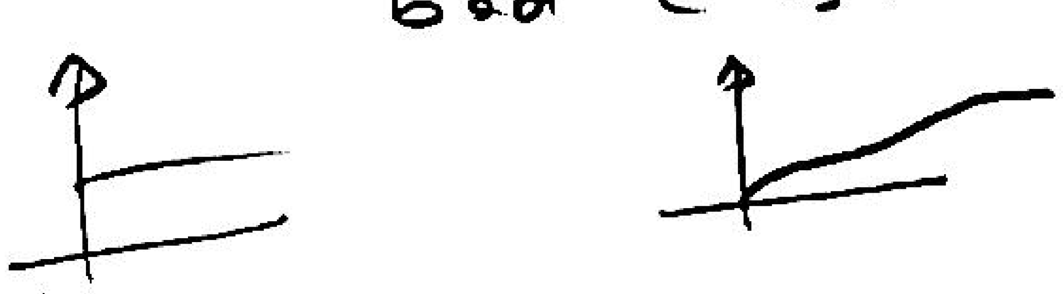
output

good case

input

output

bad case



input

Good case

T_r must be small

M_p must be 1

T_s must be small

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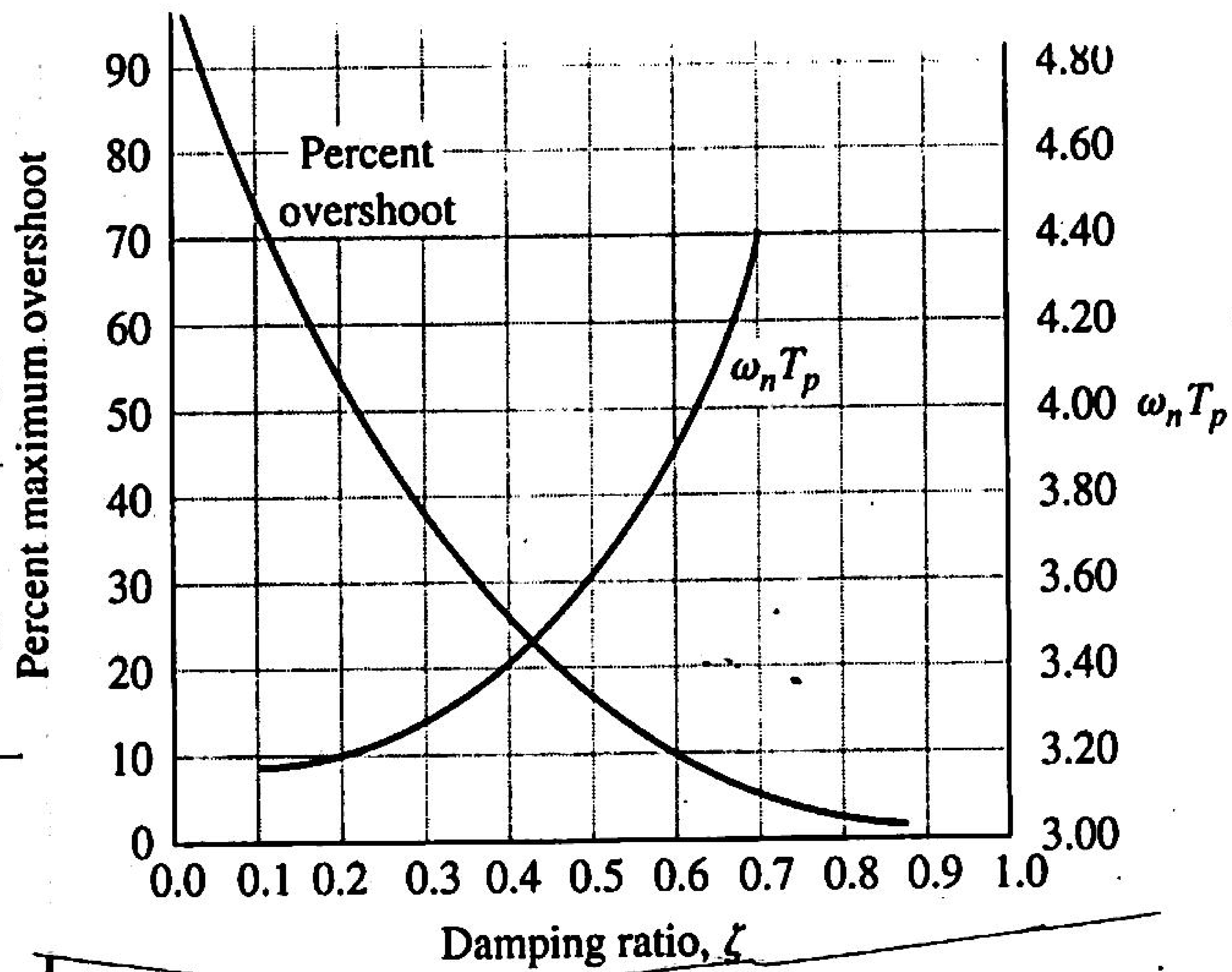
$T_s = \frac{4}{\xi \omega_n}$ is small. $\xi \omega_n$ is large

$T_r = \frac{2.16\,\xi + 0.6}{\omega_n}$ is small $\Rightarrow \xi$ is small ω_n is large

$T_p = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$ is small ξ must be close to 1.



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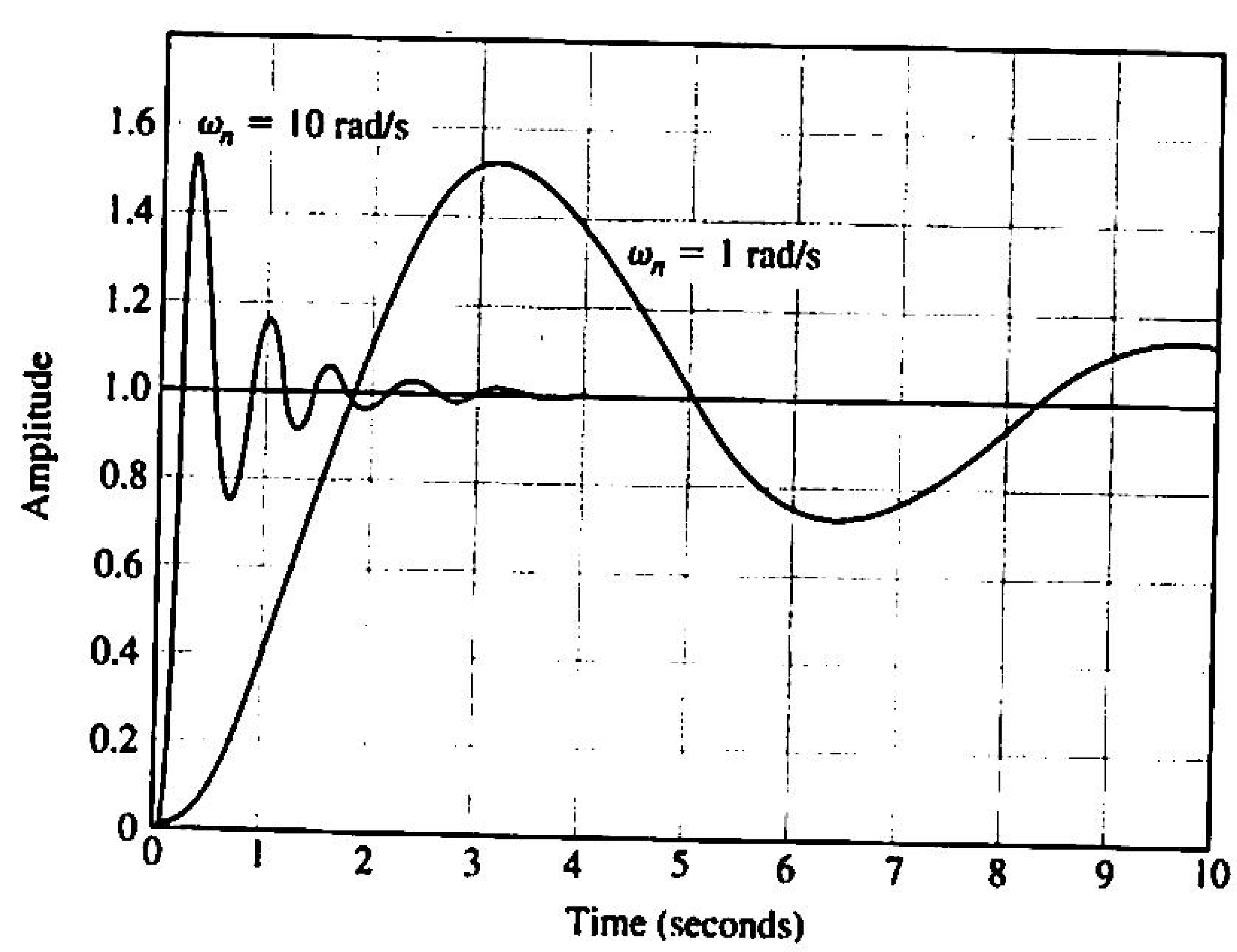


$K_p = \lim\limits_{s\to 0} G(s)$ Position error constant

$K_v = \lim\limits_{s\to 0} sG(s)$ velocity " "

$K_a = \lim\limits_{s\to 0} s^2 G(s)$ Acceleratin " "

Type zero $G(s) = \dfrac{(s+\ )(\ )}{(s+\ )(\ )(\ )}$

Type 1 system $G(s) = \dfrac{(s+\ )}{s(s+\ )}$

s in denominator

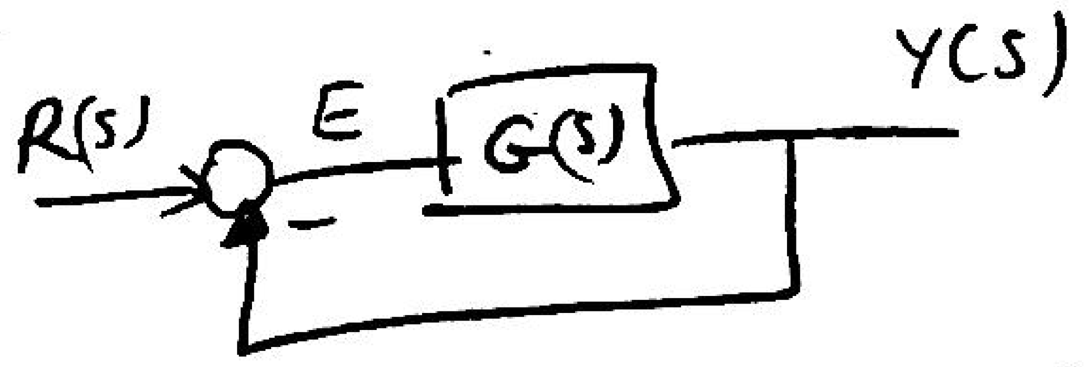
Type 2 system $G(s) = \dfrac{(\ )}{s^2(s+\ )}$

s^2 in denominator

Step input $r(t) = u(t)$ $R(s) = \frac{1}{s}$

ramp " $r(t) = t\,u(t)$ $R(s) = \frac{1}{s^2}$

Acceleration " $r(t) = \frac{t^2}{2} u(t)$ $R(s) = \frac{1}{s^3}$



$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} \qquad \frac{E(s)}{R(s)} = \frac{1}{1+G(s)}$$

$$\boxed{E(s) = \frac{1}{1+G(s)} R(s)}$$

$$\boxed{e_{ss} = \lim_{s\to 0} sE(s)}$$

$$\boxed{e_{ss} = \lim_{s\to 0} s \frac{1}{1+G(s)} R(s)}$$

Type zero system (No s in the denominator)

$R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+G(s)} \frac{1}{s} = \frac{1}{1+G(0)}$$

$$= \frac{1}{1+K_p}$$

$R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+G(s)} \frac{1}{s^2} = \frac{1}{1+G(0)} \frac{1}{0} = \infty$$

$\uparrow \infty$

$R(s) = \frac{1}{s^3}$

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+G(s)} \frac{1}{s^3} = \frac{1}{1+G(0)} \cdot \frac{1}{0^2} = \infty$$

Type 1 System.

$R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+\frac{1}{s}\frac{(\ )(\ )}{(\ )(\ )}} \frac{1}{s} =$$

$$= \lim_{s\to 0} \frac{s}{s + \frac{(\ )(\ )}{(\ )(\ )}} = \frac{0}{0+?} = 0$$

$R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+\frac{1}{s}\frac{(\,)(\,)}{(\,)(\,)}} \frac{1}{s^2}$$

$$= \lim_{s\to 0} \frac{1}{s + s\frac{(\,)(\,)}{(\,)(\,)}} = \frac{1}{0+K_v} = \frac{1}{K_v}$$

$R(s) = \frac{1}{s^3}$

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+\frac{1}{s}\frac{(\,)(\,)}{(\,)(\,)}} \frac{1}{s^3}$$

$$= \lim_{s\to 0} \frac{1}{s^2 + s\frac{(\,)(\,)}{(\,)(\,)}} = \frac{1}{0+0} = \infty$$

Type 2 system

$R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+\frac{1}{s^2}\frac{(\,)(\,)}{(\,)(\,)}} \cdot \frac{1}{s}$$

$$= \lim_{s\to 0} \frac{s^2}{s^2 + \frac{(\,)(\,)}{(\,)(\,)}} = \frac{0}{0+\ldots} = 0$$

$R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+\frac{1}{s^2}\frac{(\,)(\,)}{(\,)(\,)}} \frac{1}{s^2}$$

$$= \lim_{s\to 0} \frac{s}{s + \frac{(\,)(\,)}{(\,)(\,)}} = \frac{0}{0+(\,)} = 0$$

$R(s) = \frac{1}{s^3}$

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+\frac{1}{s^2}\frac{(\,)(\,)}{(\,)(\,)}} \frac{1}{s^3} =$$

$$= \lim_{s\to 0} \frac{1}{s^2 + \frac{(\,)(\,)}{(\,)(\,)}} = \frac{1}{K_a}$$

Type	step	ramp	Parabola
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$

Example $G(s) = \frac{1}{s} \frac{(s+1)}{s+3}$

Calculate e_{ss} for

a) $R(s) = \frac{1}{s}$ b) $R(s) = \frac{1}{s^2}$ c) $R(s) = \frac{1}{s^3}$

a)

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+\frac{s+1}{s(s+3)}} \cdot \frac{1}{s} =$$

$$= \lim_{s\to 0} \frac{s}{s + \frac{(s+1)}{s+3}} = \frac{0}{0+\frac{1}{3}} = 0$$

b)

$$e_{ss} = \lim_{s\to 0} s \cdot \frac{1}{1+\frac{s+1}{s(s+3)}} \cdot \frac{1}{s^2}$$

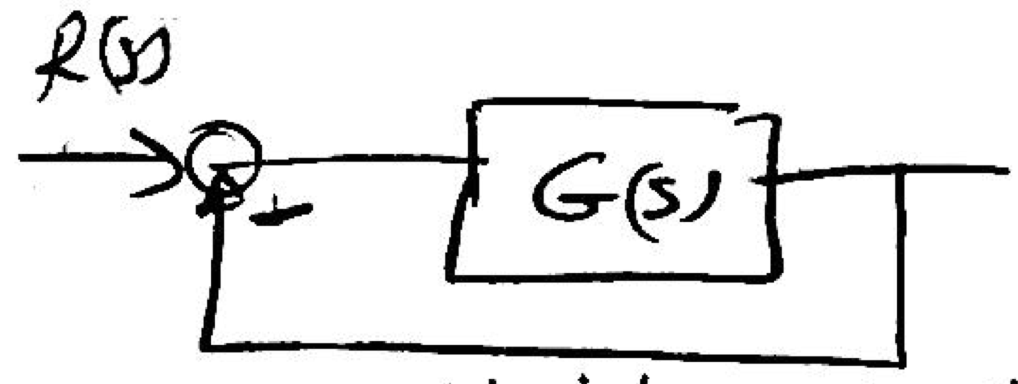
$$= \frac{1}{s + \frac{s+1}{s+3}} = \frac{1}{0+\frac{1}{3}} = 3$$

c)

$$e_{ss} = \lim_{s\to 0} s \frac{1}{1+\frac{s+1}{s(s+3)}} \frac{1}{s^3}$$

$$= \lim_{s\to 0} \frac{1}{s^2 + s\frac{(s+1)}{(s+3)}} = \frac{1}{0+0} = \infty$$

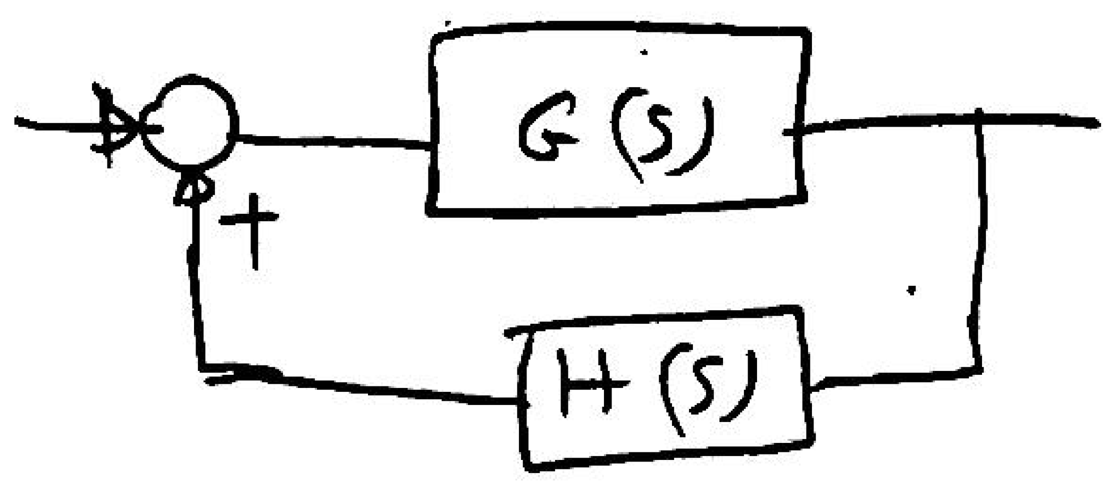
Non-Unity feedback



$E(s) = R(s) - Y(s)$

Unity feedback

$$E(s) = \frac{1}{1+G(s)}$$

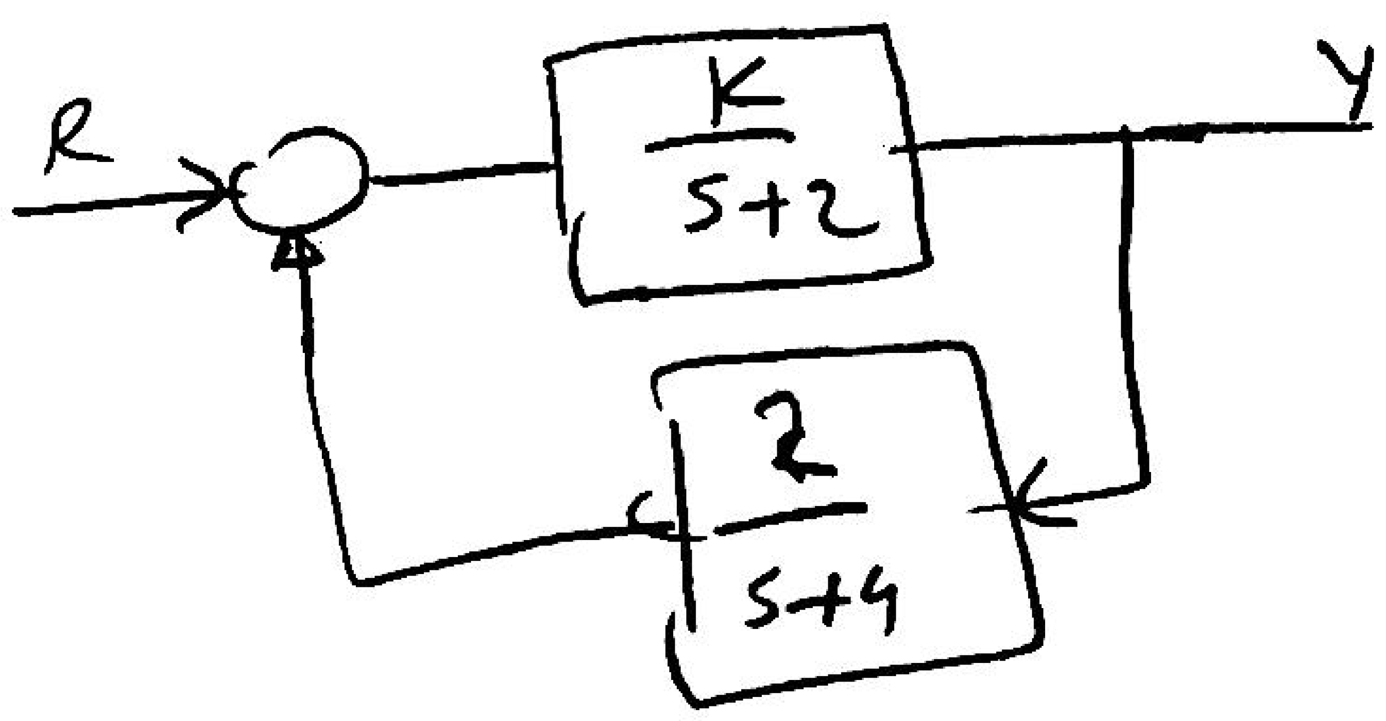


$E(s) = R(s) - Y(s)$

$$= R(s) - \frac{G(s)}{1+G(s)} R(s)$$

Non-Unity feedbak

Example 5.5 (page 254)



Calculate K to obtain zero steady state for step input.

$$\frac{Y}{R} = \frac{G}{1+GH} = \frac{K/s+2}{1+\frac{K}{s+2}\cdot\frac{2}{s+4}} = \boxed{\frac{K(s+4)}{(s+2)(s+4)+2K} = T}$$

$$E(s) = R(s) - Y(s) = R(s) - T\, R(s) = [1-T]\, R(s)$$

$$e_{ss} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s.[1-T]\,\frac{1}{s}$$

$$= \lim_{s\to 0}[1-T] = \lim_{s\to 0}\left[1 - \frac{K(s+4)}{(s+2)(s+4)+2K}\right]$$

$$= 1 - \frac{4K}{8+2K} = e_{ss}$$

$$e_{ss} = 0$$

$$1 - \frac{4K}{8+2K} = 0$$

$$4K = 8 + 2K$$

$$\boxed{K = 4}$$

EX-115

$$Y(s) = \frac{s^2}{s^2+1}$$

calculate $y(\infty)$

$$y(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} s\,Y(s) =$$

$$= \lim_{s\to 0} s \cdot \frac{s^2}{s^2+1} = 0\,\frac{0^2}{0^2+1} = 0$$

EX-116

$$E(s) = \frac{s+1}{s(s+2)}$$

calculate $e(\infty)$

$$e(\infty) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s\,\frac{s+1}{s(s+2)} = \frac{0+1}{0+2} = \frac{1}{2}$$

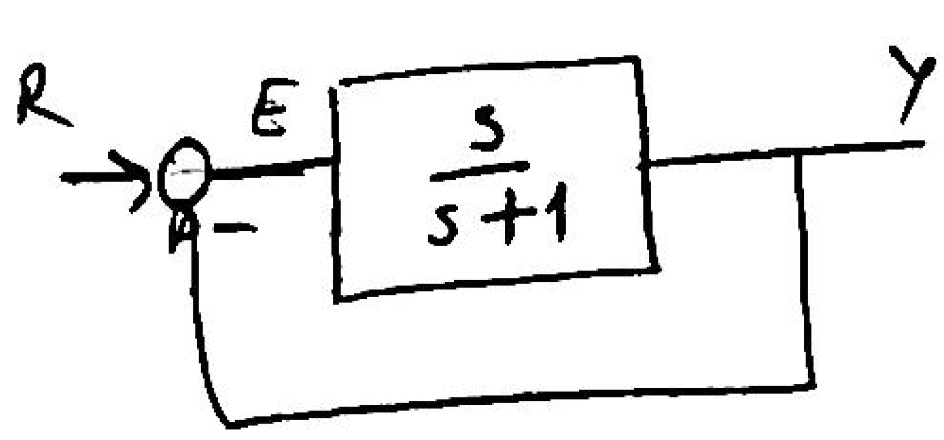
EX-117

$$E(s) = \frac{s+1}{s^2}$$

calculate $e(\infty)$

$$e(\infty) = \lim_{s\to 0} s\,\frac{s+1}{s^2} = \lim_{s\to 0} \frac{s+1}{s} = \infty$$

EX-118



calculate $y(\infty)$ and $e(\infty)$

a) $r(t) = \delta(t)$

b) $r(t) = u(t)$

c) $r(t) = t\,u(t)$

$$\frac{Y(s)}{R(s)} = \frac{s/(s+1)}{s/(s+1)+1} = \frac{s}{2s+1}$$

<N36

$$E(s) = R(s) - Y(s) = R(s) - \frac{s}{2s+1}R(s)$$

$$E(s) = \left[1 - \frac{s}{2s+1}\right] R(s)$$

a) $r(t) = \delta(t)$ $\quad R(s) = 1$

$$Y(s) = \frac{s}{2s+1}$$

$$y(\infty) = \lim_{s\to 0} s \cdot \frac{s}{2s+1} = 0$$

$$e(\infty) = \lim_{s\to 0} s\left[1 - \frac{s}{2s+1}\right] = 0$$

b) $r(t) = u(t)$ $\quad R(s) = \frac{1}{s}$

$$Y(s) = \frac{s}{2s+1} \cdot \frac{1}{s} = \frac{1}{2s+1}$$

$$y(\infty) = \lim_{s\to 0} s \cdot \frac{1}{2s+1} = 0$$

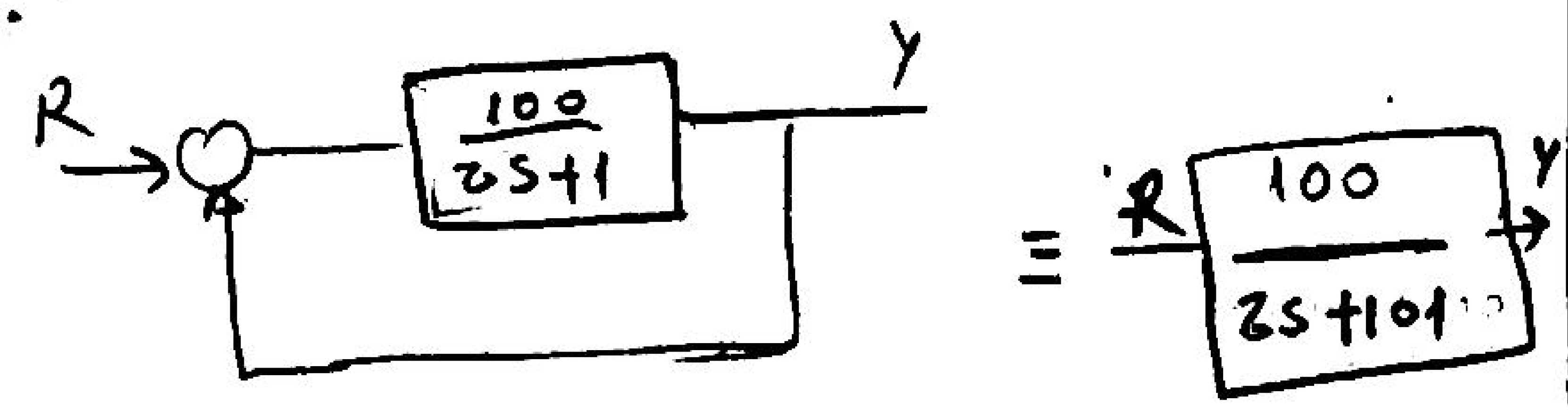
$$e(\infty) = \lim_{s\to 0} s \cdot \left[1 - \frac{s}{2s+1}\right]\frac{1}{s} = 1$$

c) $r(t) = t\,u(t)$ $\quad R(s) = \frac{1}{s^2}$

$$y(\infty) = \lim_{s\to 0} s \cdot \frac{s}{2s+1} \cdot \frac{1}{s^2} = 1$$

$$e(\infty) = \lim_{s\to 0} s \cdot \left[1 - \frac{s}{2s+1}\right]\frac{1}{s^2} = \infty$$

E4.1 Page 212



$\frac{A}{\tau s+1}$ $\tau \to$ time constant

$\frac{A}{8s+1}$ time constant is 8

$\frac{A}{7s+1}$ time constant is 7

$\frac{A}{3s+4} = \frac{A/4}{\frac{3}{4}s+1}$ time constant is $\frac{3}{4}$

$\frac{100}{\tau s+101} = \frac{100/101}{\frac{\tau}{101}s+1}$ time Constant: $\frac{\tau}{101}$

$$T = \frac{100}{\lambda s+101}$$

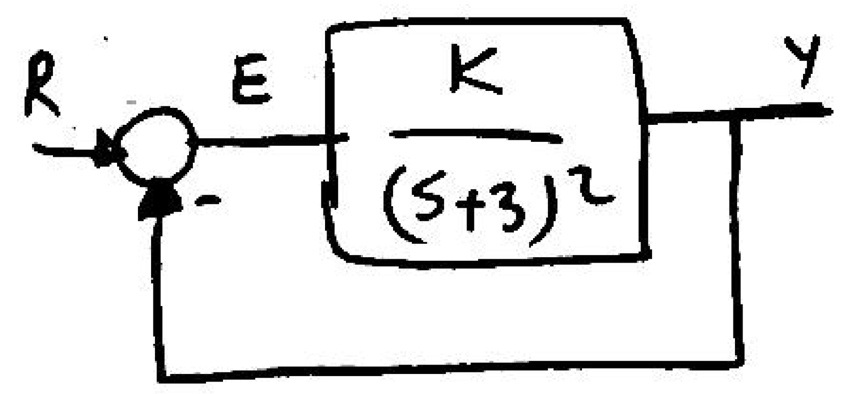
$$S_\lambda^T = \frac{\partial T}{\partial \lambda}\frac{\lambda}{T} = \frac{-s\,100}{(\lambda s+101)^2}\cdot\frac{\lambda}{100/(\lambda s+101)}$$

$$= \frac{-\lambda s}{\lambda s+101}$$

$\tau = 3$

$$\boxed{S_\lambda^T = \frac{-3s}{3s+101}}$$

E4.3 Page 212



$$Y(s) = \frac{K}{(s+3)^2+K}R(s)$$

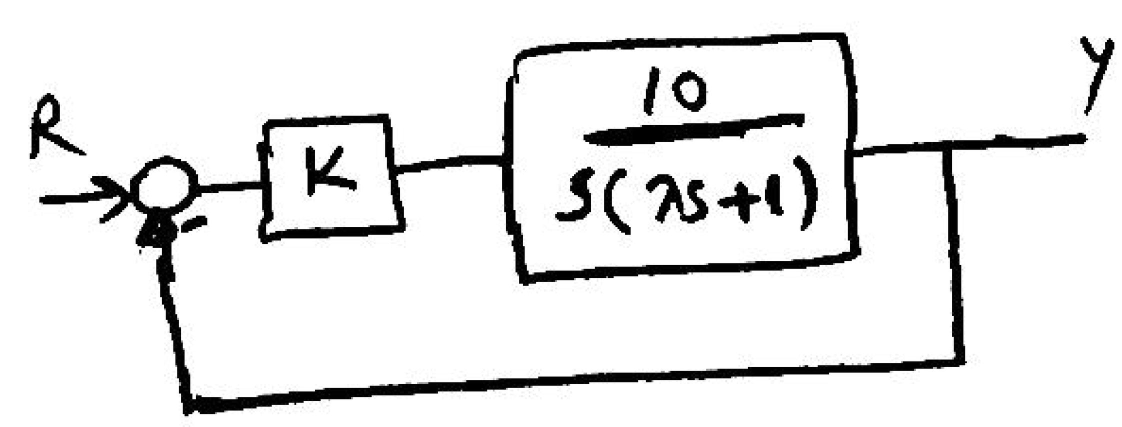
$$E(s) = R(s) - Y(s) = \frac{(s+3)^2}{(s+3)^2+K}R(s)$$

$r(t) = A\,u(t)$ $R(s) = \frac{A}{s}$

$$e_{ss} = e(\infty) = \lim_{s\to 0} s\frac{(s+3)^2}{(s+3)^2+K}\cdot\frac{A}{s} = \frac{9A}{9+K}$$

$$e_{ss} = \frac{A}{1+\frac{K}{9}}$$

E4.4



$$E(s) = \frac{1}{1+G(s)}R(s) \qquad G(s) = \frac{10K}{s(\lambda s+1)}$$

$$R = \frac{1}{s}$$

$$E(s) = \frac{s(\lambda s+1)}{s(\lambda s+1)+10K}\frac{1}{s}$$

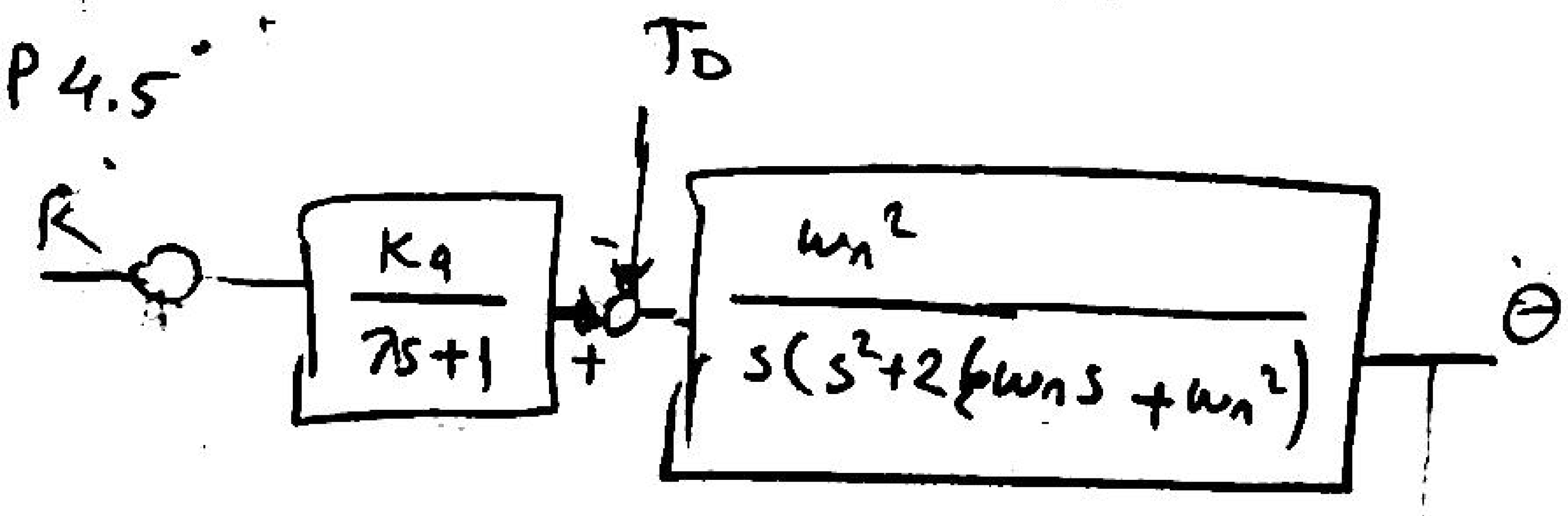
$$e_{ss} = e(\infty) = \lim_{s\to 0} sE(s) = 0$$

b) $R(s) = \frac{10\ \text{cm/s}}{s} = \frac{100\ \text{mm/s}}{s}$

$$e_{ss} = \lim_{s\to 0} s\cdot\frac{s(\lambda s+1)}{s(\lambda s+1)+10K}\cdot\frac{100}{s^2} = \frac{100}{10K} = \frac{10}{K}$$

$e_{ss} = 0.1$ $\quad 0.1 = \frac{10}{K} \Rightarrow \boxed{K = 100}$

P 4.5



$\tau = 0.15$ $\xi = 0.707$ $\omega_n = 15$

a) $$\frac{\theta(s)}{R(s)} = \frac{K_a \omega_n^2}{(\tau s+1)s(s^2+2\xi\omega_n s+\omega_n^2)} = T$$

$$S^{T}_{K_a} = \frac{\partial T}{\partial K_a}\cdot\frac{K_a}{T}$$

$$\frac{\partial T}{\partial K_a} = \frac{\omega_n^2 \Delta - 0}{\Delta^2} = \frac{\omega_n^2}{\Delta}$$

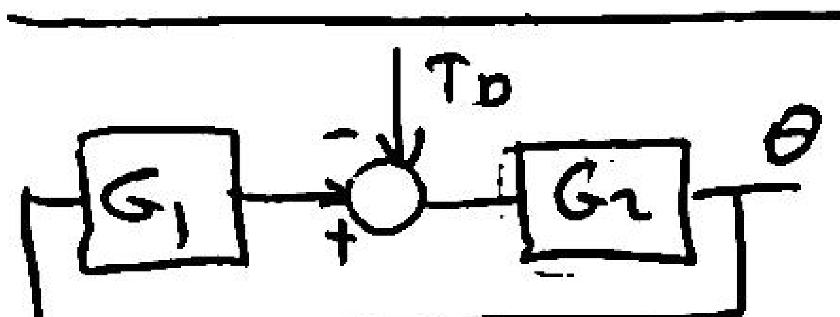
$$\Delta = s(\tau s+1)(s^2+2\xi\omega_n s+\omega_n^2)$$

$$S^{T}_{K_a} = \frac{\omega_n^2}{\Delta}\cdot\frac{K_a}{T} = \frac{\omega_n^2}{\Delta}\cdot\frac{K_a}{\frac{K_a\omega_n}{\Delta}} = \omega_n = 15$$

b) $R(s) = 0$ $T_D = \frac{10}{s}$

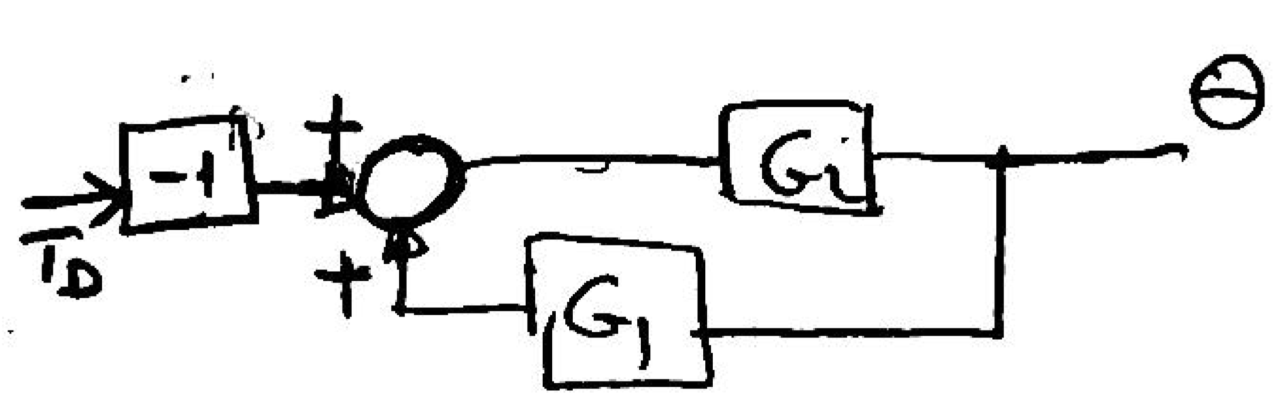
error = output − input = $\theta(t) - r(t)$
= output − 0
= output

$R(s) = 0$ the output is the error



$$G_1(s) = \frac{K_a}{\tau s+1}$$

$$G_2(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n+\omega_n^2}$$



$$\frac{\theta}{T_D} = -\frac{G_2}{1-G_1G_2} =$$

$$\theta(s) = -\frac{G_2}{1+G_1G_2}\,\frac{1}{s}$$

$$e_{ss} = \theta(\infty) = \lim_{s\to 0} s\cdot\frac{G_2}{1-G_1G_2}\cdot\frac{1}{s}$$

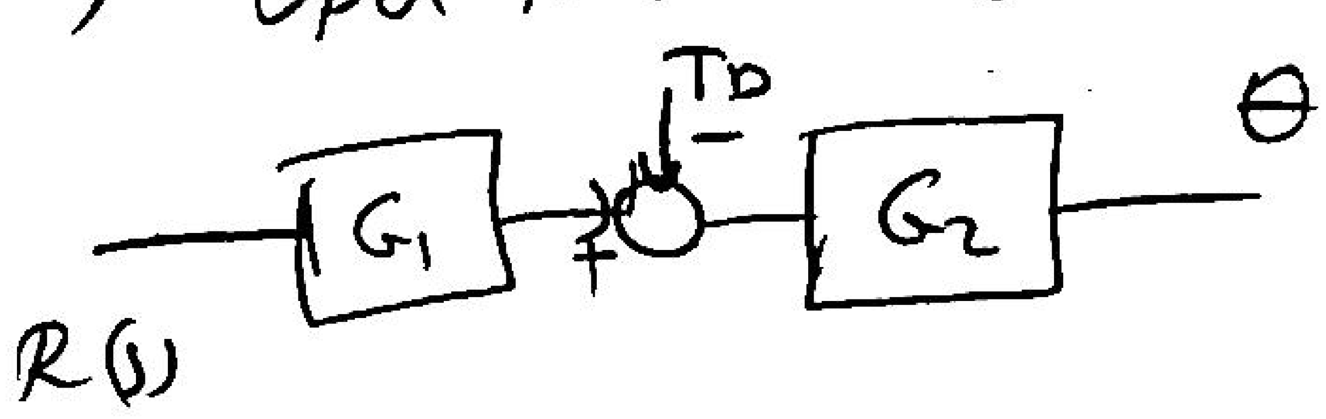
$$\frac{G_2}{1+G_1G_2} = \frac{\omega_n^2/\Delta}{1-\frac{K_a}{\tau s+1}\cdot\frac{\omega_n^2}{\Delta}} = \frac{\omega_n^2}{\Delta - \frac{K_a\omega_n^2}{\tau s+1}}$$

$$\Delta = s(s^2+2\xi\omega_n+\omega_n^2)$$

$$\left.\frac{G_2}{1-G_1G_2}\right|_{s=0} = \frac{\omega_n^2}{0-\frac{K_a\omega_n^2}{0+1}} = \frac{1}{-K_a}$$

$$\boxed{e_{ss} = -\frac{1}{-K_a} = \frac{1}{K_a}}$$

c) *Open loop case*



$R(s) = 0$ $\theta = -T_D G_2$

$$\theta(\infty) = \lim_{s\to 0} s(-T_D G_2)$$

$$= -s\,\frac{10}{s}\,G_2\Big|_{s=0} = \infty$$