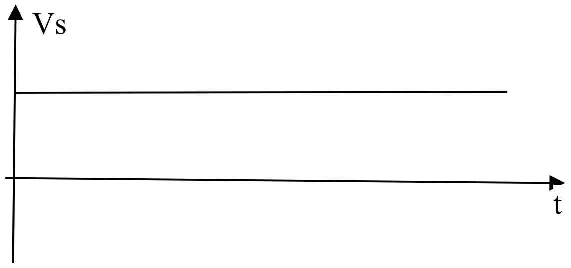
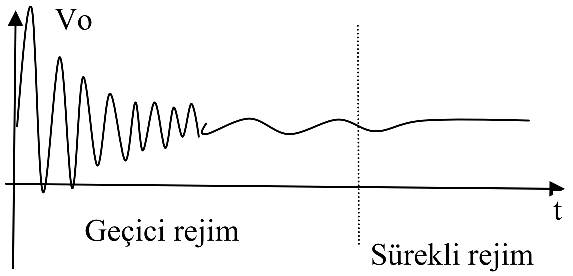
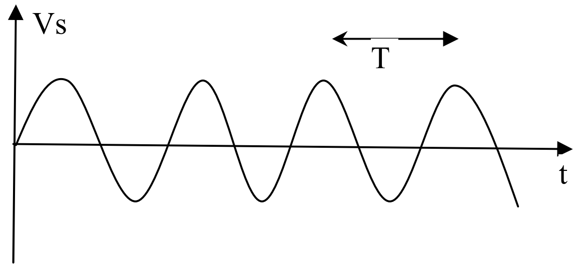
Frekans Cevabı Metodu.
(Frquency Response Methods)

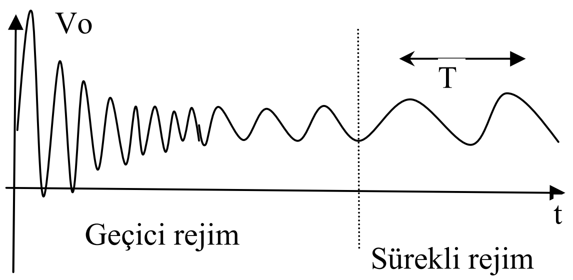
Bir sistemin (elektrik devresi, robot, uçak vs) girişine bir Kuvvet (elektrik akımı, döndürme momenti vs) uygulandığında çıkışta olan etki iki şekilde incelenir.
 1)geçici rejim
2)sürekli rejim
Mesela bir elektrik devresine birim basamak girişi uygulansa çıkış belli bir süre sonra sabit bir gerilim olur.





Aynı devreye sinuzoidal bir gerilim uygulansa çıkış belli bir süre sonra genliği ve fazı sabit olan, frekansı da giriş frekansına eşit olan bir sinuzoidal gerilim olur.



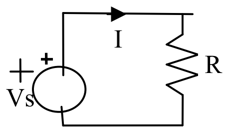


Frekans cevabı metodu, giriş geriliminin sinuzoidal olması durumunda sürekli rejimin incelenmesi olayıdır.

Hatırlatmalar

Devrelerin sürekli sinuzoidal halde çözümü
(Steady state analysis).

351) $\boldsymbol{v}$=20cos(8t+30) , R=10Ω ise I akımını hesaplayın.



Devrede

$\boldsymbol{v}$=20cos(8t+30) = Vm cow(wt+θv)

V_m=20,   w=8,   θv =30

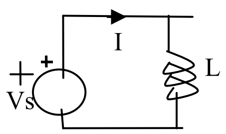
$\boldsymbol{i}$=Vs/R=20cos(8t+30)/10=2cos(8t+30)

= I_m cow(wt+θi)

I_m=2,   w=8,   θi=30

----

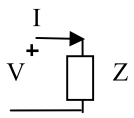
353) $\boldsymbol{v}$=20cos(5t+30) , L=2H ise I akımını hesaplayın.



a)Devrede

$\boldsymbol{v}$=20cos(5t+30) = Vm cow(wt+θv)

V_m=20,   w=5,   θv =30



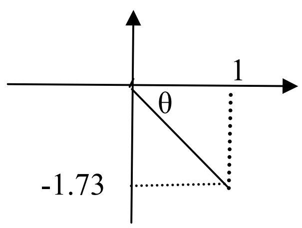
$\boldsymbol{v}$=20cos(5t+30) ==> $V=20\angle 30$, $V=20\ e^{j30}$,

Z=jwL=j 5 2=10j

$I = V/Z = 20\ e^{j30}/10j$

$$I=\frac{20\,(\cos 30+\sin 30)}{10j}=-2j\,(0.866+0.5j)$$

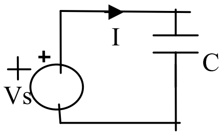
$$I=1\text{-}1.73j=\sqrt{1^2+1.73^2}\ e^{j\theta},\quad \theta=\arctan\left(\frac{-1.73}{1}\right)=-60$$



$I= 2\ e^{-j60} = 2\angle -60$ ==> $\boldsymbol{i}$=2 cos(5t-60)

I_m=2,   w=5,   θi=-60

355) $\boldsymbol{v}$=20cos(5t+30) , C=0.02F ise I akımını hesaplayın.

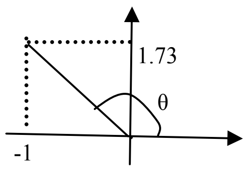


Z=1/jwC=1/(j 5 0.02)= - 10j

I = V/Z =20 e^{j30}/(-10j)

$$I=\frac{20\,(\cos 30+\sin 30)}{-10j}=2j\,(0.866+0.5j)$$

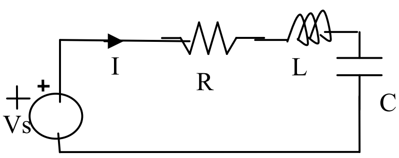
$$I=-1+1.73j=\sqrt{1^2+1.73^2}\ e^{j\theta},\quad \theta=\arctan(\frac{1.73}{-1})=120$$

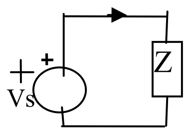


I= 2 e^{j120} = 2 ∠ 120 ==> ***i***=2 cos(5t+120)

I_m=2, w=5, θi=120

361) ***v***=20cos(5t+30) ,R=2Ω, L=2H, C=0.04F ise I akımını hesaplayın.





V_m=20, w=5, θv =30

Z=R + jwL + 1/jwC=2+j 5 2 + 1/(j 5 0.04)

= 2+10j-5j =2+5j

I = V/Z =20 e^{j30}/(2+5j)=2.9 - 2.3j

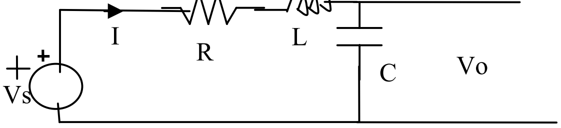
I= 3.7 $e^{-j38.2}$ ==> ***i***=3.7 cos(5t- 38.2)

I_m=3.7, w=5, θi=-38.2

-- ------------------

TRANSFER FONKSİYONU

Vo(jw)/Vs(jw) oranına devrenin frekans domeninde transfer fonksiyonu denir.



$$\frac{Vo(jw)}{Vs(jw)}=\frac{1/jwC}{R+jwL+1/jwC}=G(jw)$$

$$\frac{Vo(jw)}{Vs(jw)}=G(jw)$$

Vo(jw)=G(jw) Vs(jw)

Komplex sayılardan bilindiği gibi.
|Vo(jw)|=|G(jw)| |Vs(jw)|

<Vo(jw)=<G(jw)+< Vs(jw)

Veya

$$|\,Vo(jw)\,|=\left|\frac{Vo(jw)}{Vs(jw)}\right|\,|\,Vs(jw)\,|=|\,G(jw)\,|\ |\,Vs(jw)\,|$$

Hatırlatma

$$\frac{P}{Q}=\frac{(-6+10i)(-3+i)(10-4i)(7+3i)}{(5+3i)(3-7i)(-5+2i)(-9+3i)}=?$$

$-6+10i=\sqrt{6^2+10^2}\ e^{i\tan^{-1}\frac{10}{-6}}=11.66\ e^{121i}$

-3+i=3.16 e^{161i}

10-4i=10.77 $e^{-21.8i}$

7+3i=7.61 $e^{23.2i}$

Pay için

Amp=11.66 *x* 3.16 *x* 10.77 *x* 7.61=3019.8

Angle=121+161+(-21.8)+23.2=283.4

Payda

5+3i=5.83 $e^{30.9i}$

3-7i=7.61 $e^{-66.8i}$

-5+2i =5.38 e^{158i}

-9+3i 9.48 e^{161i}

payda için

Amp=5.83 7.61 5.38 9.48=2262.7

Angle=30.9+(-66.8)+(158)+161=283.1

$$\frac{P}{Q}=\frac{(-6+10i)(-3+i)(10-4i)(7+3i)}{(5+3i)(3-7i)(-5+2i)(-9+3i)}=2262.7\ e^{j283}$$

Genlikler bolunur. Acilar cikartilir

$$\frac{P}{Q}=\frac{3019.8\,e^{283.4i}}{2262.7\ e^{283.1i}}=\frac{3019.8}{2262.7}\ e^{(283.4-283.1)i}$$

=1.33 $e^{0.3i}$ = 1.33 (cos 0.3 +i sin 0.3)

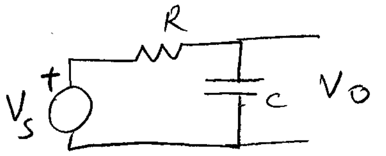
=1.329+0.0069i

e 223

Example Problem: Calculate output voltage

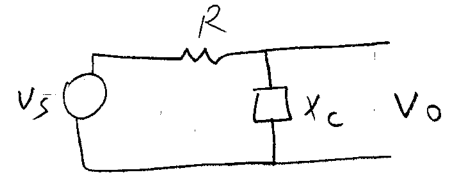
if a) $V_s = 10\cos t$ b) $V_s = 10\cos 2t$ c) $V_s = 10\cos(2t+30)$

d) $V_s = 10\sin 3t$ e) $V_s = 10\cos 10t$



$R = 2\Omega$ $C = 0.1\ f$

Solution:



$$V_o = \frac{X_c}{R + X_c} V_s$$ (Voltage divid-

$$\frac{V_o}{V_s} = \frac{X_c}{R + X_c} = \frac{\frac{1}{jwc}}{R + \frac{1}{jwc}} = \frac{1}{jwcR + 1} = \frac{1}{jw.0.1\times 2 + 1} = \frac{1}{jw0.2+1}$$

$V_s = 10\cos t \Rightarrow \underline{V_s} = 10e^{j0} = 10$ $\quad w = 1.$

$V_s = 10\cos 2t \Rightarrow V_s = 10$ $\quad w = 2$

$V_s = 10\cos 3t \Rightarrow V_s = 10$ $\quad w = 3$

⋮

$V_s = 10\cos 10t \Rightarrow V_s = 10$ $\quad w = 10$

$$\frac{V_o}{V_s} = \frac{1}{jw0.2+1} \qquad \left|\frac{V_o}{V_s}\right| = \frac{|1|}{|jw0.2+1|} = \frac{1}{\sqrt{(0.2w)^2 + 1^2}}$$

$$\angle\frac{V_o}{V_s} = \angle 1 - \angle jw0.2+1$$

$$= 0 - \tan^{-1}\frac{0.2w}{1}$$

$$w = 1 \Rightarrow \left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{(0.2\times 1)^2 + 1^2}} = \frac{1}{\sqrt{1.04}} = 0.98$$

$$\angle\frac{V_o}{V_s} = -\tan^{-1} 0.2w = -\tan 0.2 = -11.3^{\circ}$$

$$\left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{(0{,}2\,w)^2+1}}$$

$w=2 \quad \left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{(0{,}2\times 2)^2+1}} = \frac{1}{\sqrt{1.16}} = 0.928$

$$\angle\frac{V_o}{V_s} = -\tan^{-1}0{,}2w = -\tan^{-1}0{,}2\times2 = -\tan^{-1}0{,}4 = -21.8$$

$w=3. \quad \left|\frac{V_o}{V_s}\right| = \frac{1}{\sqrt{(0{,}2\times 3)^2+1}} = \frac{1}{\sqrt{1.36}} = 0{,}857$

$$\angle\frac{V_o}{V_s} = -\tan^{-1}0{,}2\times3 = -30{,}9^0$$

$w=4 \Rightarrow \left|\frac{V_o}{V_s}\right| = \qquad \angle\frac{V_o}{V_s} =$

$w=5 \Rightarrow \left|\frac{V_o}{V_s}\right| = \qquad \angle\frac{V_o}{V_s} =$

$\vdots$

$w=10 \Rightarrow \left|\frac{V_o}{V_s}\right| = \qquad \angle\frac{V_o}{V_s} =$

---

w	1	2	3	4
$\left\lvert\frac{V_o}{V_s}\right\rvert$	0,98	0,928	0,857	
$\angle\frac{V_o}{V_s}$	$-11{,}3$	$-21{,}8$	$-30{,}9$	

a) $V_s = 10\cos t \Rightarrow V_o = 10\times0{,}98\cos(t-11{,}9) = 9{,}8\cos(t-11{,}9)$

b) $V_s = 10\cos 2t \Rightarrow V_o = 10\times 0.928\cos(2t-21{,}8) = 9{,}28\cos(2t-21{,}8)$

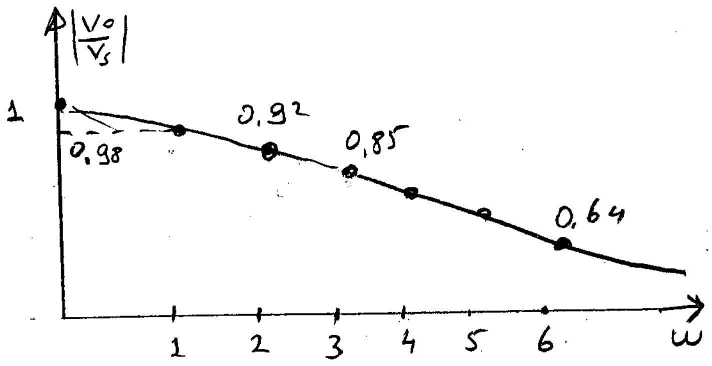
c) $V_s = 10\cos(2t+30) \Rightarrow V_o = 10\times0{,}928\cos(2t+30-21{,}8) = 9{,}28\cos(2t+8{,}2)$

d) $V_s = 10\sin 3t \Rightarrow V_o = 10\times 0{,}857\sin(3t-30{,}9) = 8{,}57\sin(3t-30{,}9)$

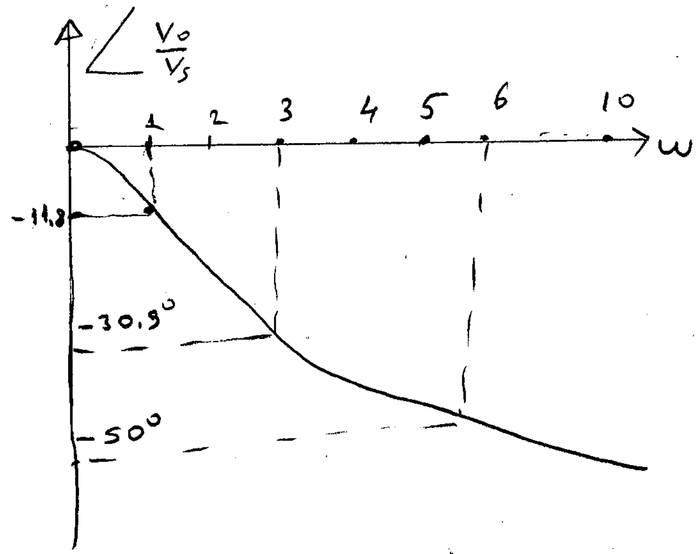
SPECTRUM

e225

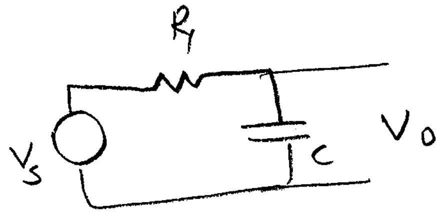
ω	0	1	2	3	4	5	6	8	10	100
$\left\lvert \frac{V_o}{V_s} \right\rvert$	1	0,98	0,92	0,85	0,78	0,707	0,64	0,53	0,44	0,05
$\angle \frac{V_o}{V_s}$	0	-11,9	-21,8	-30,9	-38	-45	-50	-54	-63	-87



Amplitude spectrum

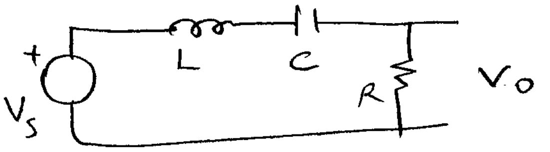


phase spectrum



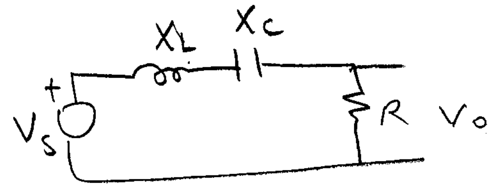
$$\frac{V_o}{V_s} = \frac{1}{j\omega CR + 1} = \frac{1}{0,2\omega j + 1}$$

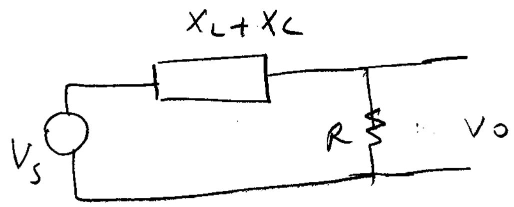
Example problem: Calculate transfer function e226
and draw amplitude and phase spectrum



$L = 0{,}1 \quad C = 0{,}4 \quad R = 10$

Solution:





$$\frac{V_o}{V_s} = \frac{R}{R + X_L + X_C} = \frac{R}{R + jwL + \frac{1}{jwC}} = \frac{R}{R + j\left(wL - \frac{1}{wC}\right)}$$

$$= \frac{10}{10 + j\left(0{,}1w - \frac{1}{0{,}4w}\right)}$$

$$\left|\frac{V_o}{V_s}\right| = \frac{10}{\sqrt{10^2 + \left(0{,}1w - \frac{1}{0{,}4w}\right)^2}}$$

$$\angle \frac{V_o}{V_s} = \angle 10 - \angle 10 + j\left(0{,}1w - \frac{1}{0{,}4w}\right)$$

$$\downarrow$$

$$= 0 - \tan^{-1}\left(\frac{0{,}1w - \frac{1}{0{,}4w}}{10}\right)$$

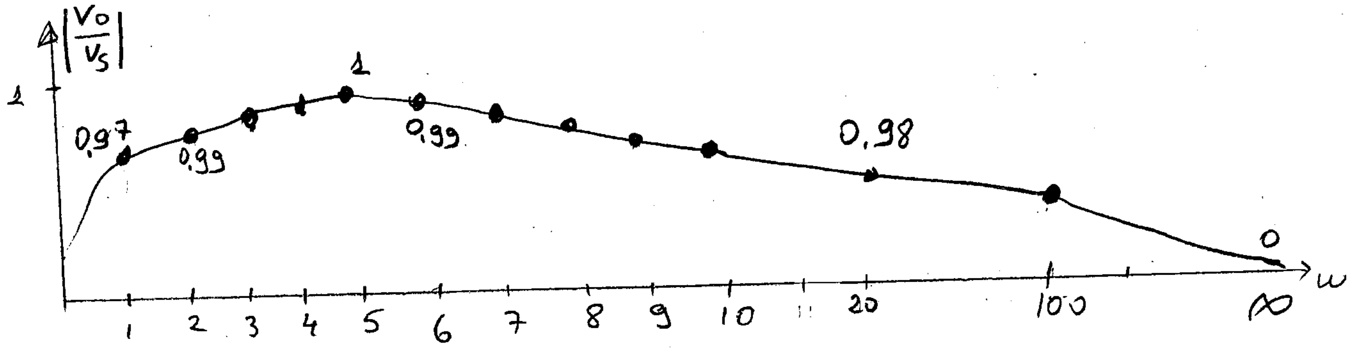
$$w = 1 \Rightarrow \left|\frac{V_o}{V_s}\right| = \frac{10}{\sqrt{10^2 + \left(0{,}1\times1 - \frac{1}{0{,}4\times1}\right)^2}} =$$

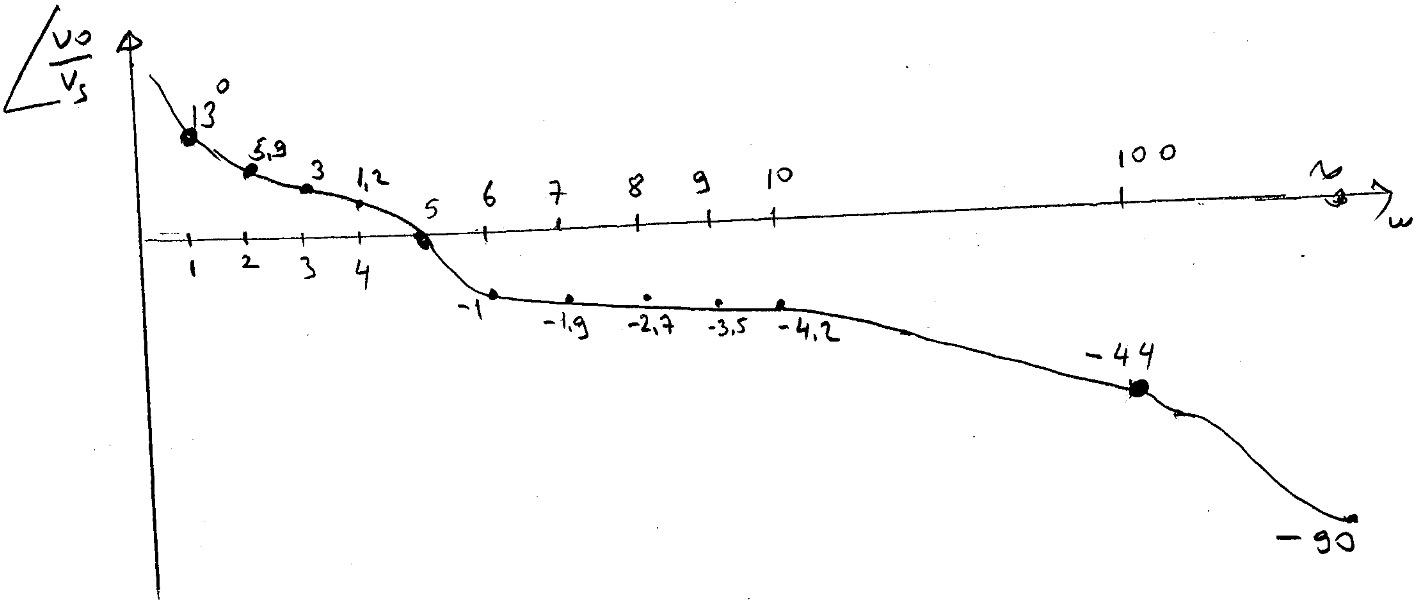
$$\angle \frac{V_o}{V_s} = -\tan^{-1}\left(\frac{0{,}1\times1 - \frac{1}{0{,}4\times1}}{10}\right) =$$

$$\omega = 2 \Rightarrow \left|\frac{V_o}{V_s}\right| = \frac{10}{\sqrt{10^2 + \left(0{,}1\omega - \frac{1}{0{,}4\omega}\right)^2}} = \frac{10}{\sqrt{10^2 + \left(0{,}1 \times 2 - \frac{1}{0{,}4 \times 2}\right)}} =$$

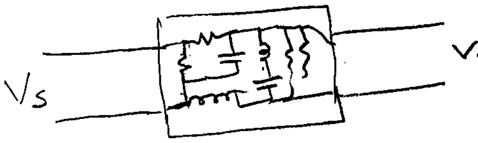
$$\angle \frac{V_o}{V_s} = -\tan^{-1}\left(0{,}1 \times 2 - \frac{1}{0{,}4 \times 2}\right) =$$

ω	1	2	3	4	5	6	7	8	9	10	20	30	40	50	100	∞
$\left\|\frac{V_o}{V_s}\right\|$	0.97	0.99	0,99	0,99	1	0,99	0,99	0,99	0,99	0.99	0,98	0.96	0.93	0,89	0,7	0
$\angle\frac{V_o}{V_s}$	13°	5.9°	3°	1,2°	0	-1°	-1.9°	-2.7°	-3,5	-4.2°	-10°	-16°	-21°	-26°	-44	-90





Example problem: Calculate the output

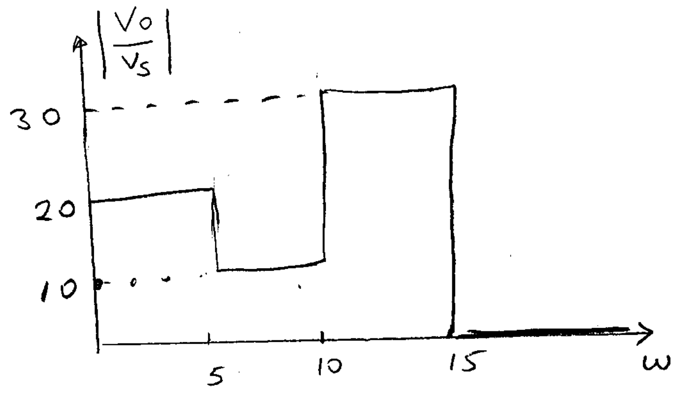


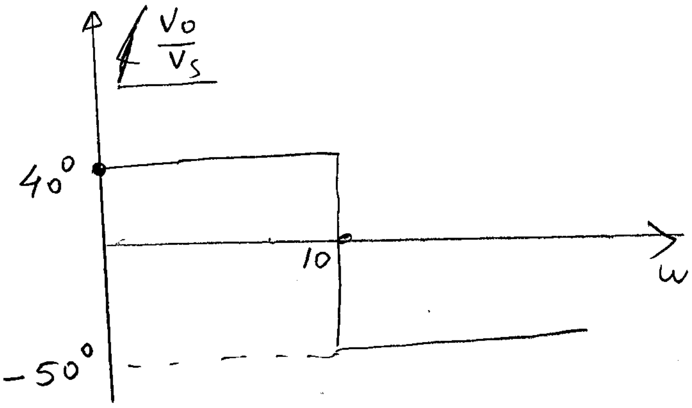
a) $V_s = 4 \cos 2t$

b) $V_s = 5 \cos(6t+20)$

c) $V_s = 2 \cos(13t+60)$

d) $V_s = 10 \cos 25t$





a) $V_s = 4 \cos 2t$ $\quad \omega = 2 \quad \left|\frac{V_o}{V_s}\right| = 20 \quad \angle \frac{V_o}{V_s} = 40^\circ$

$$V_o = 4 \times 20 \cos(2t + 40^\circ) = 80 \cos(2t+40)$$

b) $V_s = 5 \cos(6t+20)$ $\quad \omega = 6 \quad \left|\frac{V_o}{V_s}\right| = 10 \quad \angle \frac{V_o}{V_s} = 40^\circ$

$$V_o = 5 \times 10 \cos(6t + 20 + 40) = 50 \cos(6t+60)$$

c) $V_s = 2 \cos(13t+60)$ $\quad \omega = 13 \quad \left|\frac{V_o}{V_s}\right| = 30 \quad \angle \frac{V_o}{V_s} = -50^\circ$

$$V_o = 2 \times 30 \cos(13t + 60 - 50) = 60 \cos(13t+10)$$

d) $V_s = 10 \cos 25t$ $\quad \omega = 25 \quad \left|\frac{V_o}{V_s}\right| = 0 \quad \angle \frac{V_o}{V_s} = -50$

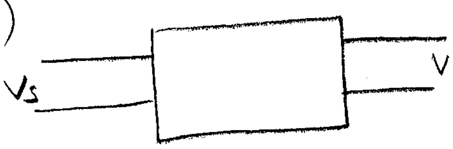
$$V_o = 10 \times 0 \cos(25t - 50) = 0$$

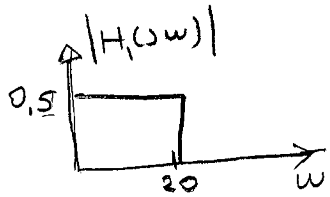
Filters

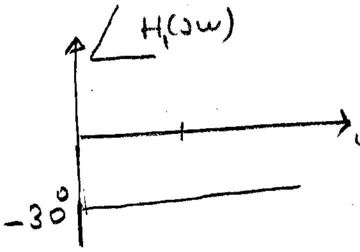
e229

$V_s = \cos 5t + \cos 50t + \cos 500t$

a)



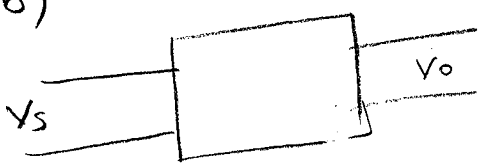


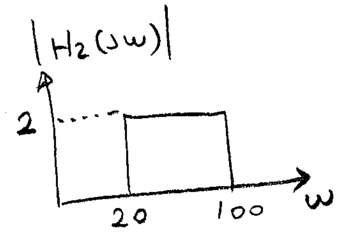


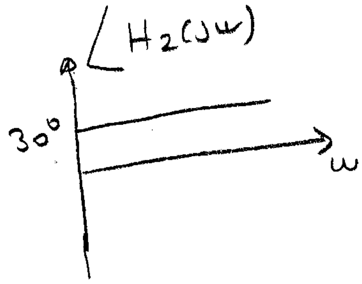
$H_1(jw) = \frac{V_o}{V_s}$

$V_o = 0.5 \cos(5t + 30) + 0 + 0$

b)



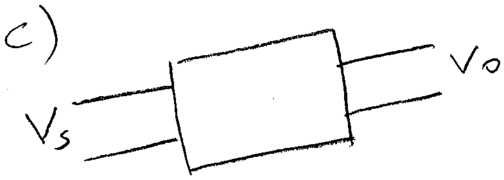


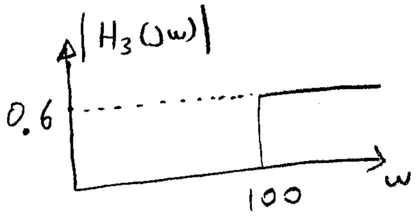


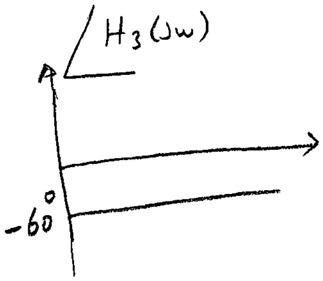
$H_2(jw) = \frac{V_o}{V_s}$

$V_o = 0 + 2 \cos(50t + 30) + 0$

c)







$H_3(jw) = \frac{V_o}{V_s}$

$V_o = 0 + 0 + 0.6 \cos(500t - 60)$

Result:

$$x(t) = A\sin\omega t \longrightarrow \boxed{G(s)} \longrightarrow y(t) = A\,|G(j\omega)|\,\sin(\omega t + \phi)$$

$$\phi = \angle G(j\omega)$$

Example:

$$x(t) = 2\sin 5t \longrightarrow \boxed{\frac{20}{s+10}} \longrightarrow y(t) = 2\times|G(j5)|\,\sin(5t+\phi)$$

$$G(j\omega) = \left.\frac{20}{s+10}\right|_{s=j\omega} = \frac{20}{j\omega+10} \qquad |G(j5)| = \left|\frac{20}{j5+10}\right| = \frac{20}{\sqrt{5^2+10^2}} = 1.78$$

$$\angle G(j5) = \angle 20 - \angle j5+10 = 0 - \tan^{-1}\frac{5}{10} = -26.5^\circ$$

$$y(t) = 2\times 1.78 \cdot \sin(5t + (-26.5^\circ)) = 3.56\sin(5t - 26^\circ)$$

Frequency Response plots.

A simple plot is $|G(j\omega)|$ versus ω and $\angle G(j\omega)$ versus ω

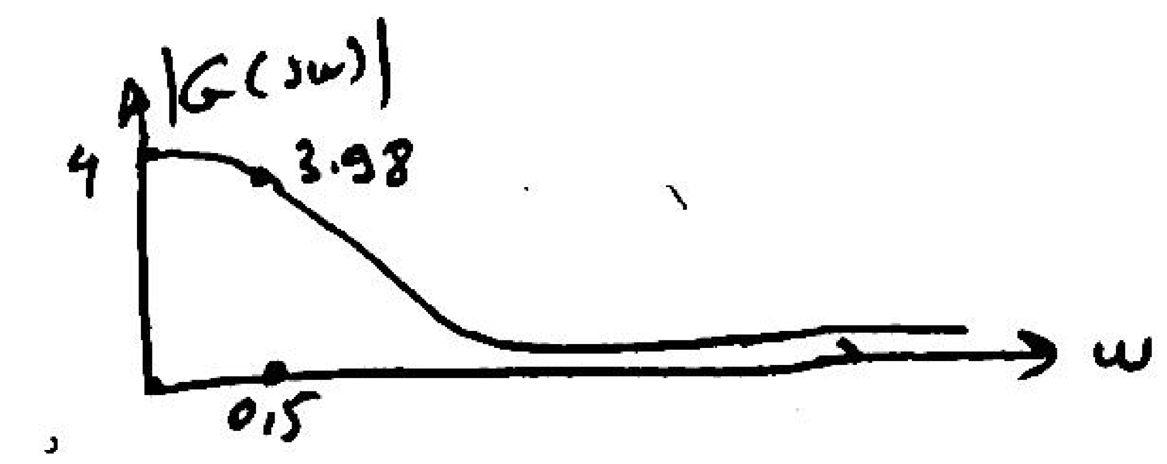
Example: plot frequency response of $G(s) = \frac{30}{s+5}$

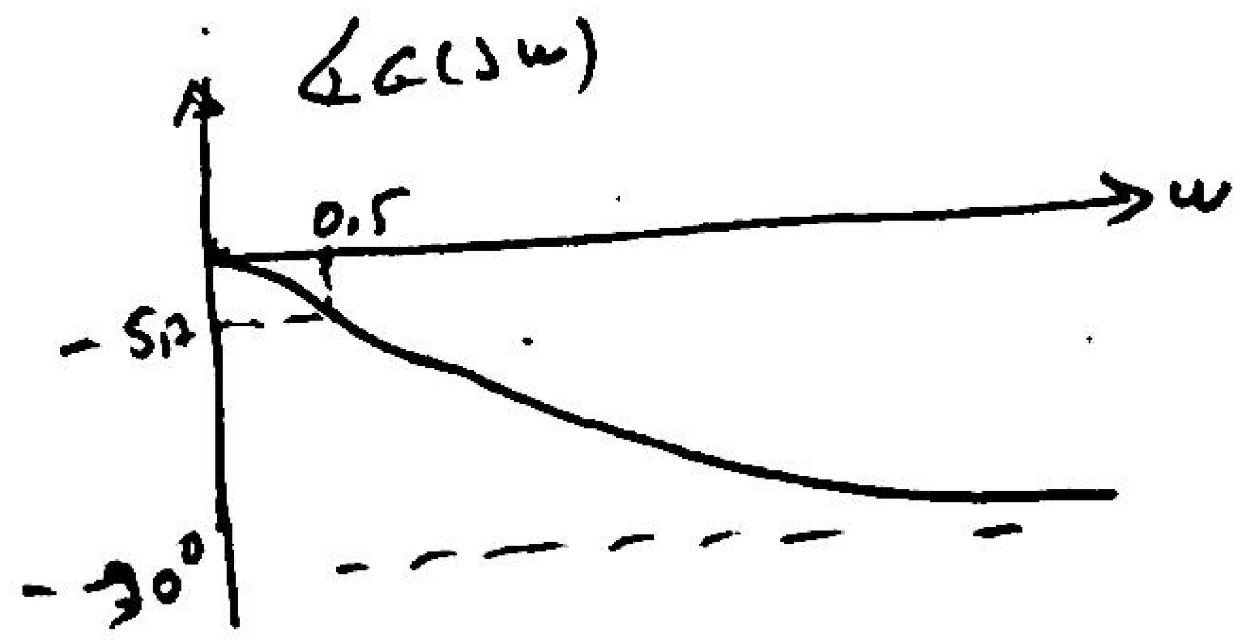
$$G(j\omega) = \frac{30}{j\omega+5} \qquad |G(j\omega)| = \frac{30}{\sqrt{\omega^2+5^2}} \qquad \angle G(j\omega) = 0 - \tan^{-1}\frac{\omega}{5}$$

$$\omega = 0 \longrightarrow |G(j0)| = \frac{30}{\sqrt{0^2+5^2}} = 6 \qquad \angle G(j0) = 0 - 0 = 0$$

$$\omega = 0.5 \longrightarrow |G(j0.5)| = \frac{30}{\sqrt{0.5^2+5^2}} = 5.97 \qquad \angle G(j0.5) = -5.7^\circ$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	6	0
0.5	5.97	-5.7°
1	5.88	-11°
2	5.57	-22°
5	4.24	-45°
6	3.84	-50
10	2.68	-63
20	1.45	-75
100	0.29	-87
1000	0.03	-89.7
∞	0	-90°

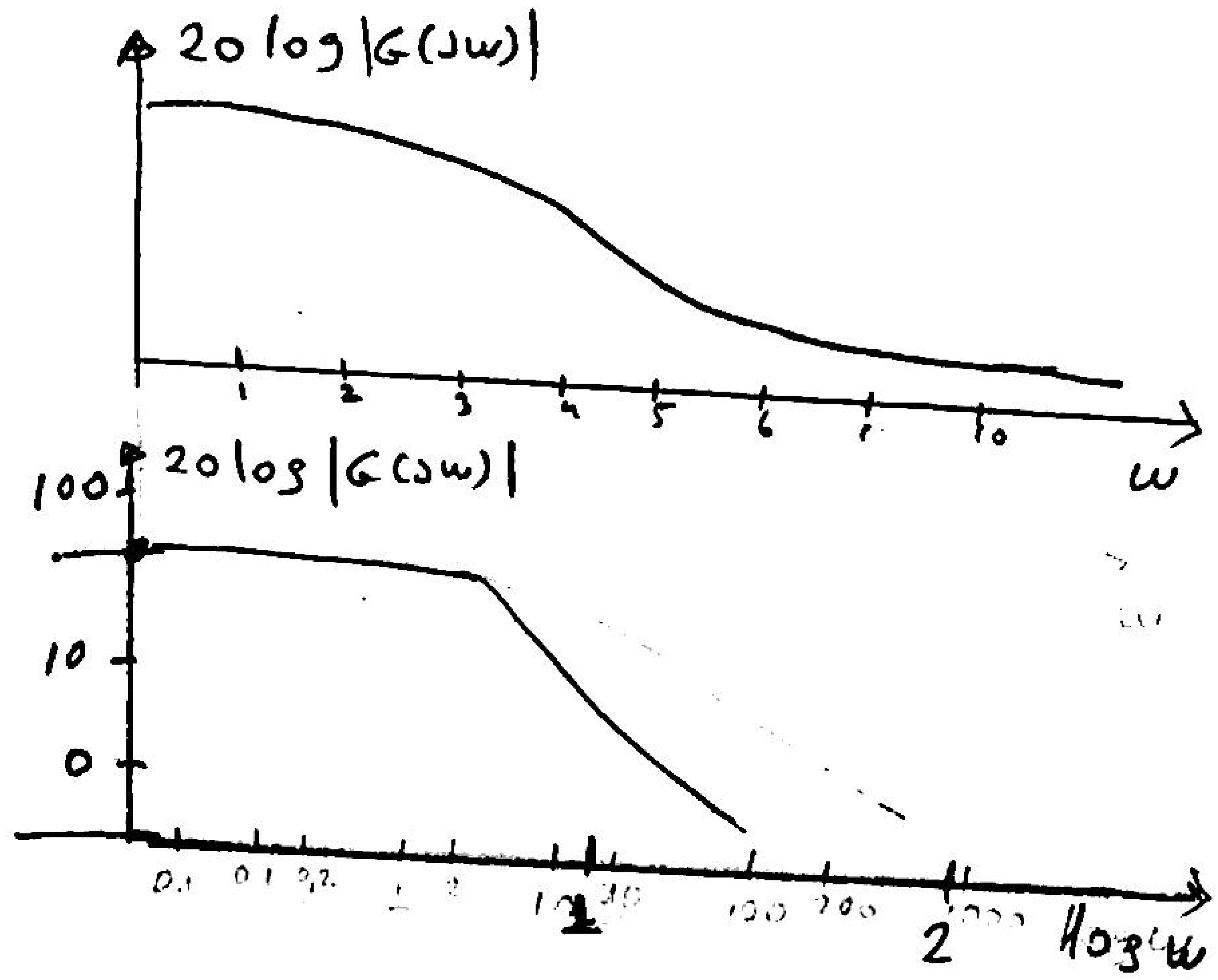




Böde plot : $20 \log |G(jw)|$ versus $\log w$ plot

Exomple plot Bode diagrom of $G(s) = \frac{30}{s+5}$

w	$\lvert G(jw) \rvert$	$\log(w)$	$20\log\lvert G(jw)\rvert$
0	6	[illegible]	15.63
0.5	5.97	−10.3	15.51
1	5.88	0	15.39
2	5.57	0.3	14.9
5	4.24	0.7	12.55
6	3.84	0.77	11.68
10	2.68	1	8.57
20	1.4552	1.3	3.25
29.5803	1	1.47	0
30	0.98	−1.477	−0.11
50	0.59	−1.69	−4.48
100	0.29	2	−10.4
1000	0.03	3	−30
∞	0	−∞	−∞



Advantages of Bode Diogrom

1) We can see whole grophics in a simple plot
2) We can guess easily the shape of new graphics if we add a zero or pole or increase K.
3) Drawing Bode plot is easier for complex systems
4) Experimental Results can be analysed by Bode diagram

The effects of Poles, zeros, Gain on Bode plot

$$G(s) = \frac{K\,(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)(s+p_3)} \qquad |G(jw)| = \frac{K\sqrt{w^2+z_1^2}\,\sqrt{w^2+z_2^2}}{\sqrt{w^2+p_1^2}\,\sqrt{w^2+p_2^2}\,\sqrt{w^2+p_3^2}}$$

$$20\log|G(jw)| = 20\log K + 20\log\sqrt{w^2+z_1^2} + 20\log\sqrt{w^2+z_2^2} - 20\log\sqrt{w^2+p_1^2} - 20\log\sqrt{w^2+p_2^2} - 20\log\sqrt{w^2+p_3^2}$$

$$= 20\log K + 10\log(w^2+z_1^2) + 10\log(w^2+z_2^2) - 10\log(w^2+p_1^2) - 10\log(w^2+p_2^2) - 10\log(w^2+p_3^2)$$

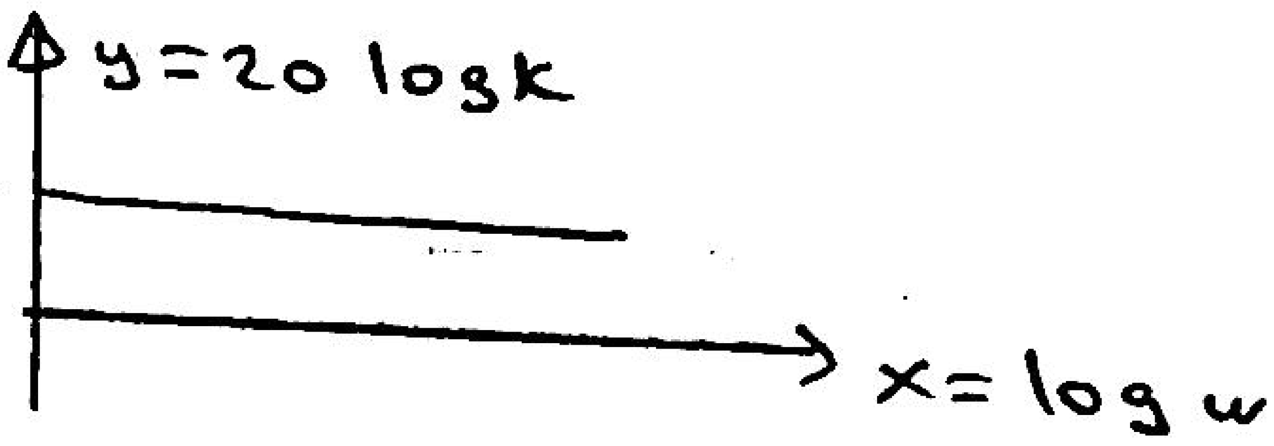
multiplications in $|G(jw)|$ become addition in logorithm.

∴ There are three types of terms

1) Gain 2) zero 3) pole

Gain: $20 \log k$

this is a straight line



Zero at $s = z_1$ $\quad G(s) = k\frac{(s+z_1)(\ \ )}{(\ \ )(\ \ )}$

$20 \log \sqrt{w^2 + z_1^2} = \Big\{$ low frequencies $w \ll z_1$

$y = 20 \log \sqrt{w^2+z_1^2} = 20 \log \sqrt{z_1^2} = 20 \log z_1$

high frequencies $w \gg z_1$

$y = 20 \log \sqrt{w^2+z_1^2} = 20 \log w$

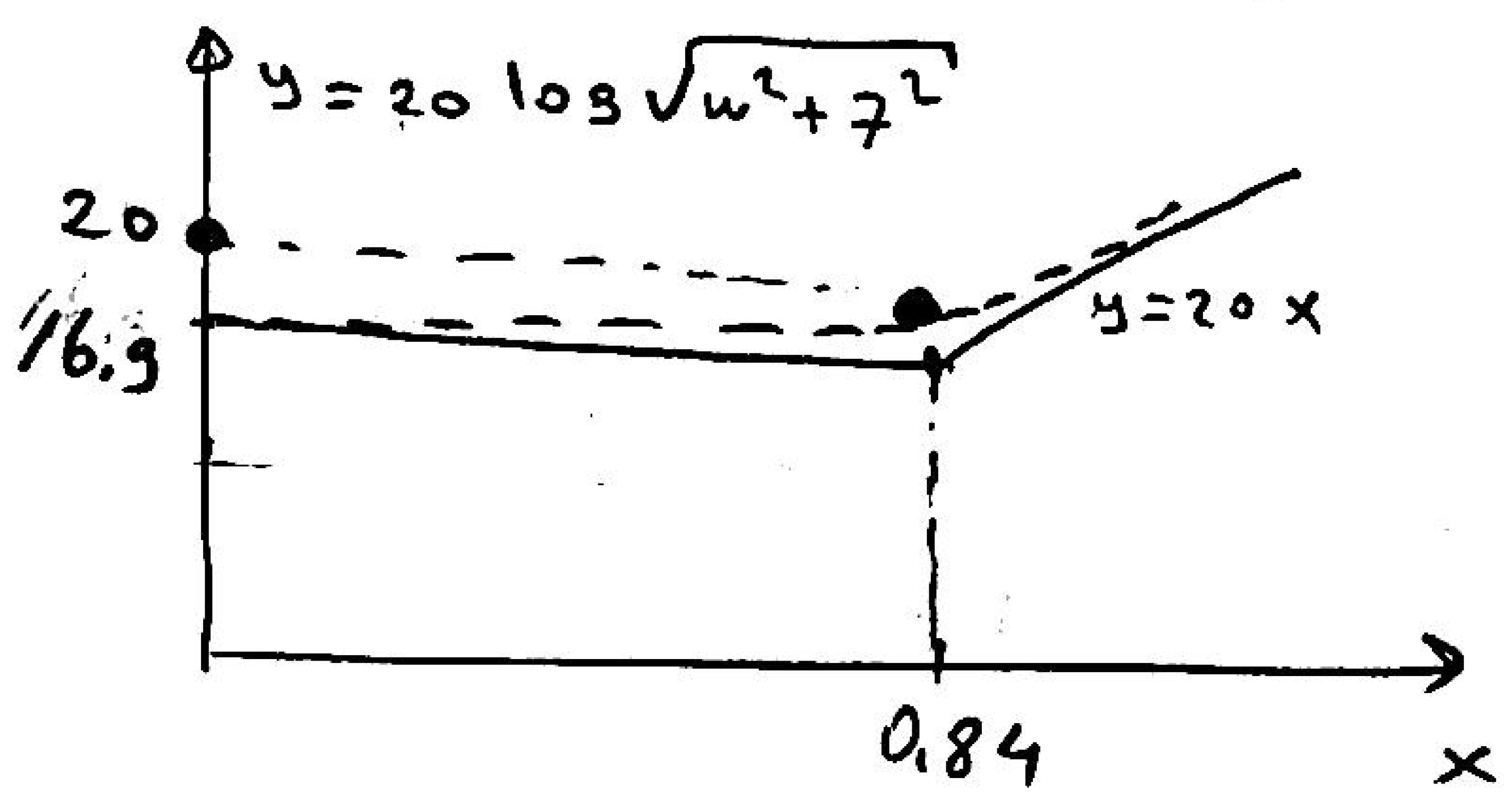
$= 20\ x$

Exomple $\quad G(s) = \frac{(s+7)(\ \ )}{(\ \ )(\ \ )} \qquad 20 \log \sqrt{w^2+7^2}$

$\log 7 = 0.84$

Low frequencies $\quad y = 20 \log 7 = 20 \times 0.84 = 16.9$

high frequencies $\quad y = 20 \log w = 20x$



real value at $w=7$

$20 \log \sqrt{7^2+7^2} = 20$

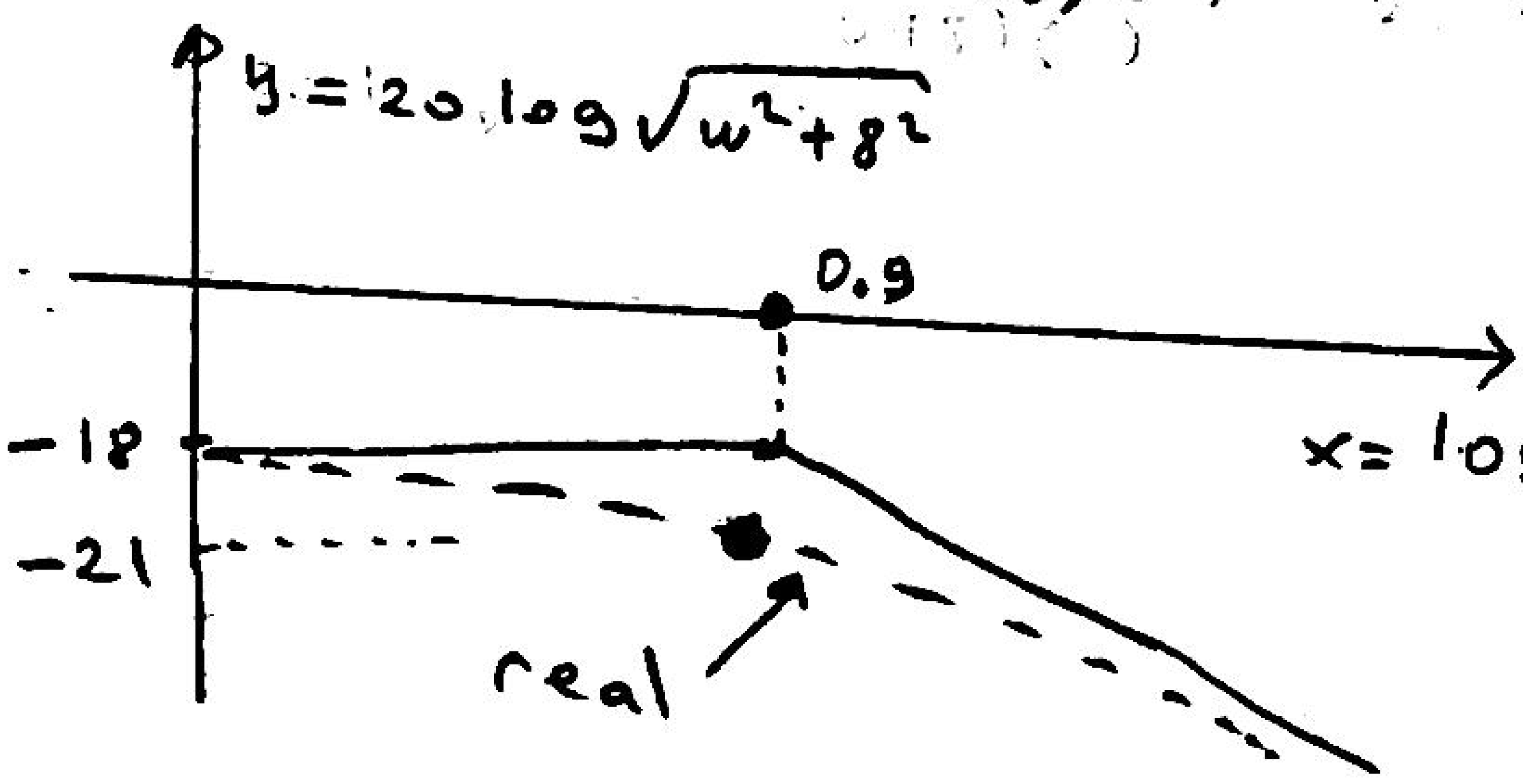
w	0,1	0,5	1	2	5	6	7	8	10	20	50
$x = \log w$	−1	−0,3	0	0,3	0,69	0,78	0,84	0,9	1	1,3	1,7
$y = 20 \log \sqrt{w^2+7^2}$	16,9	16,9	16,98	17,2	18,7	19,3	20	20,5	21.7	26,5	34.6
approximation	16.9	16,9	16.9	16,9	16.9	16.9	16,9	18	20	26	34

pole at $s = p_1$ $\quad G(s) = \dfrac{(\ )}{(s+p_1)(s+p_2)(\ )}$ $\quad -20\log\sqrt{w^2+p_1^2}$

at low frequencies $w \ll p_1$ $\quad -20\log\sqrt{w^2+p_1^2} = -20\log p_1$

at high frequencies $w \gg p_1$ $\quad -20\log\sqrt{w^2+p_1^2} = -20\log w$

Example $G(s) = \dfrac{(\ )}{(s+8)(\ )}$



low frequencies $\quad \log 8 = 0.9$

$y = -20\log 8 = -18$

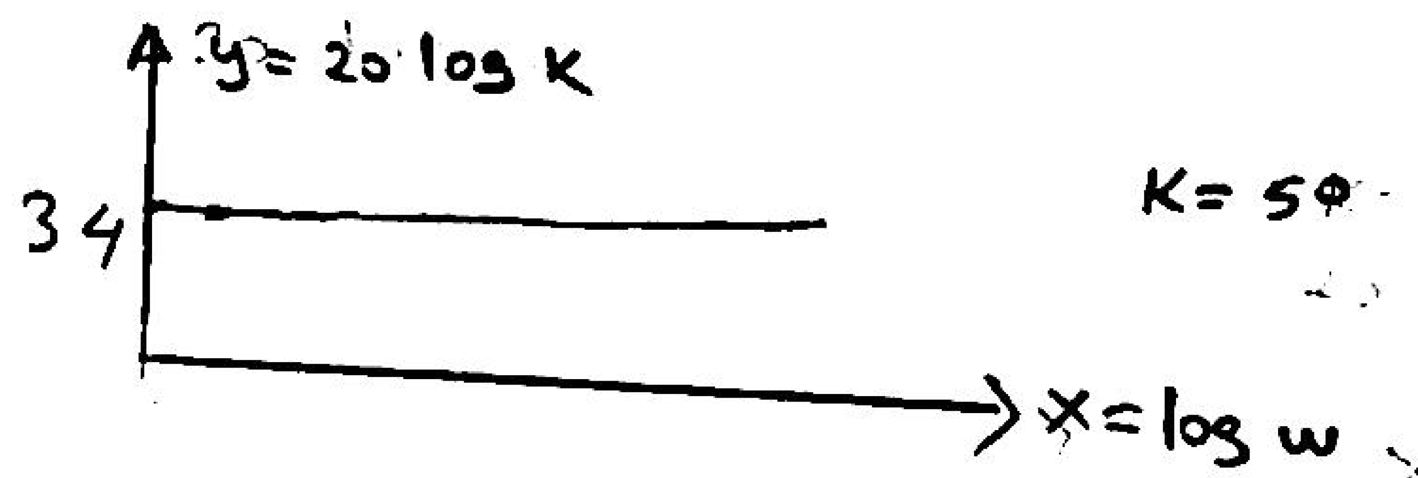
high frequencies

$y = -20\log w = -20x$

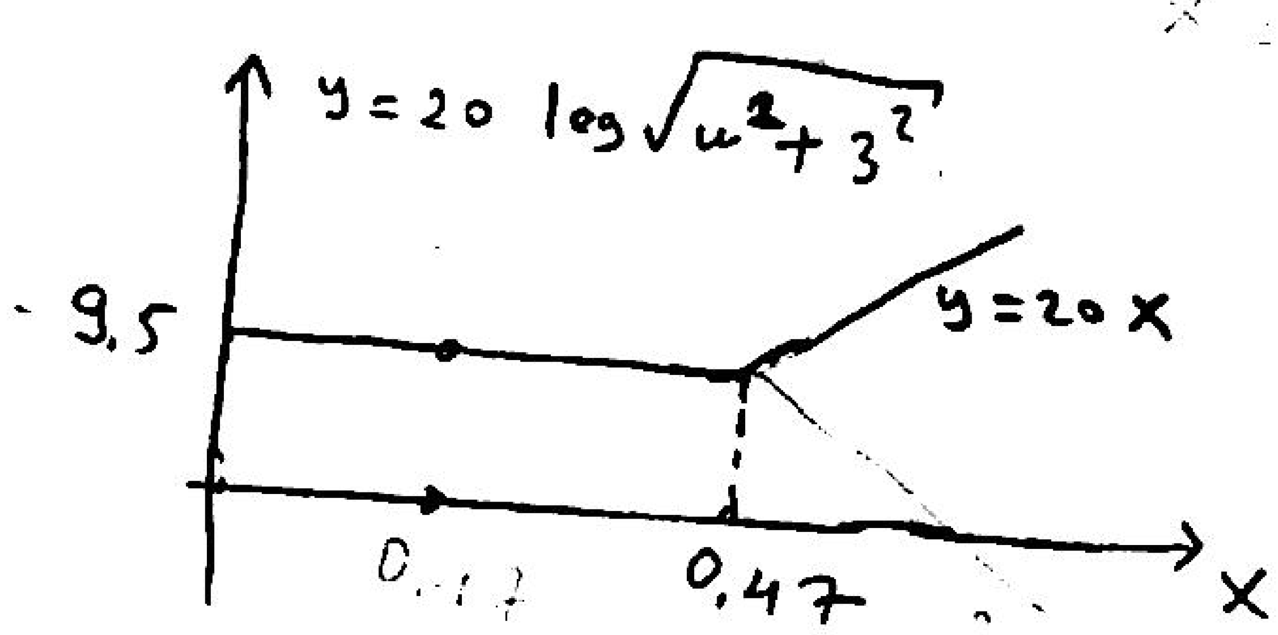
real value at $w = 8$

$-20\log\sqrt{8^2+8^2} = -21$

Example: Draw Bode diagram for $\quad G(s) = \dfrac{50(s+3)}{(s+2)(s+5)}$



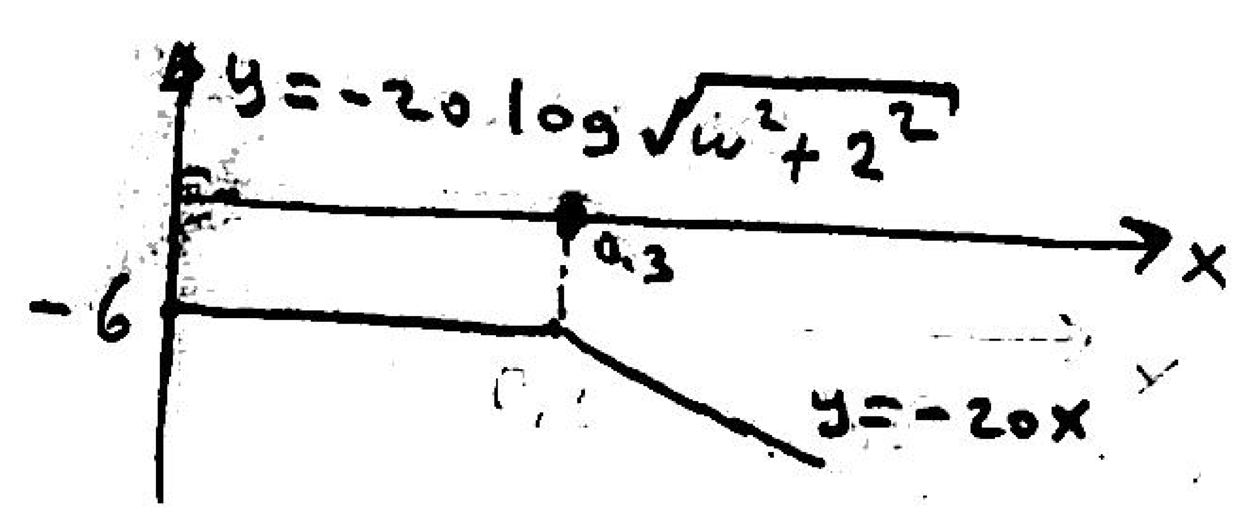
$K = 50 \quad 20\log 50 = 34$



low frequencies $y = 20\log 3 = 9.5$

high frequencies $y = 20\log w = 20x$

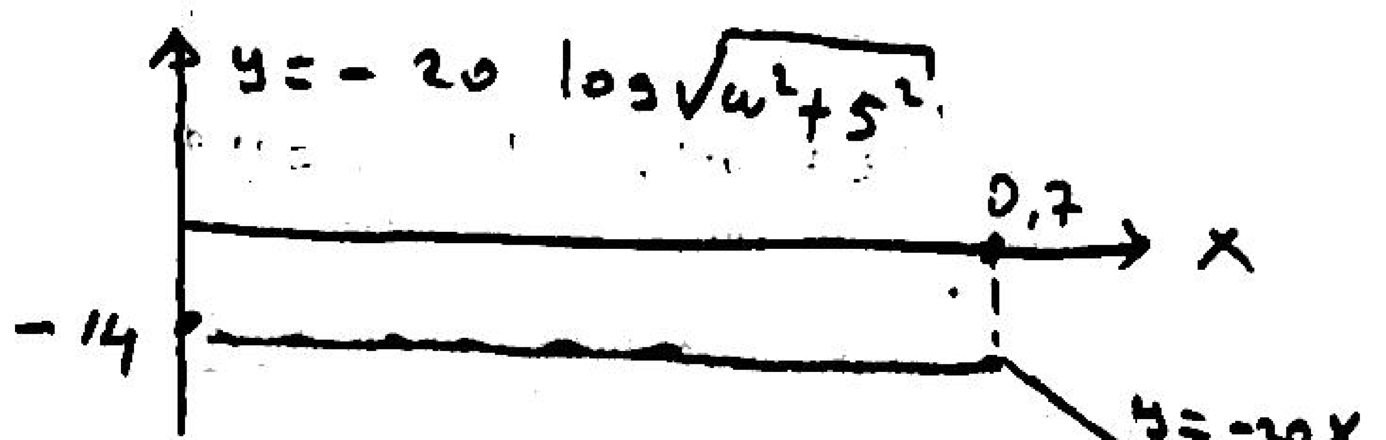
$0.47 = \log 3$



low frequencies $y = -20\log 2 = -6$

high frequencies $y = -20\log w = -20x$

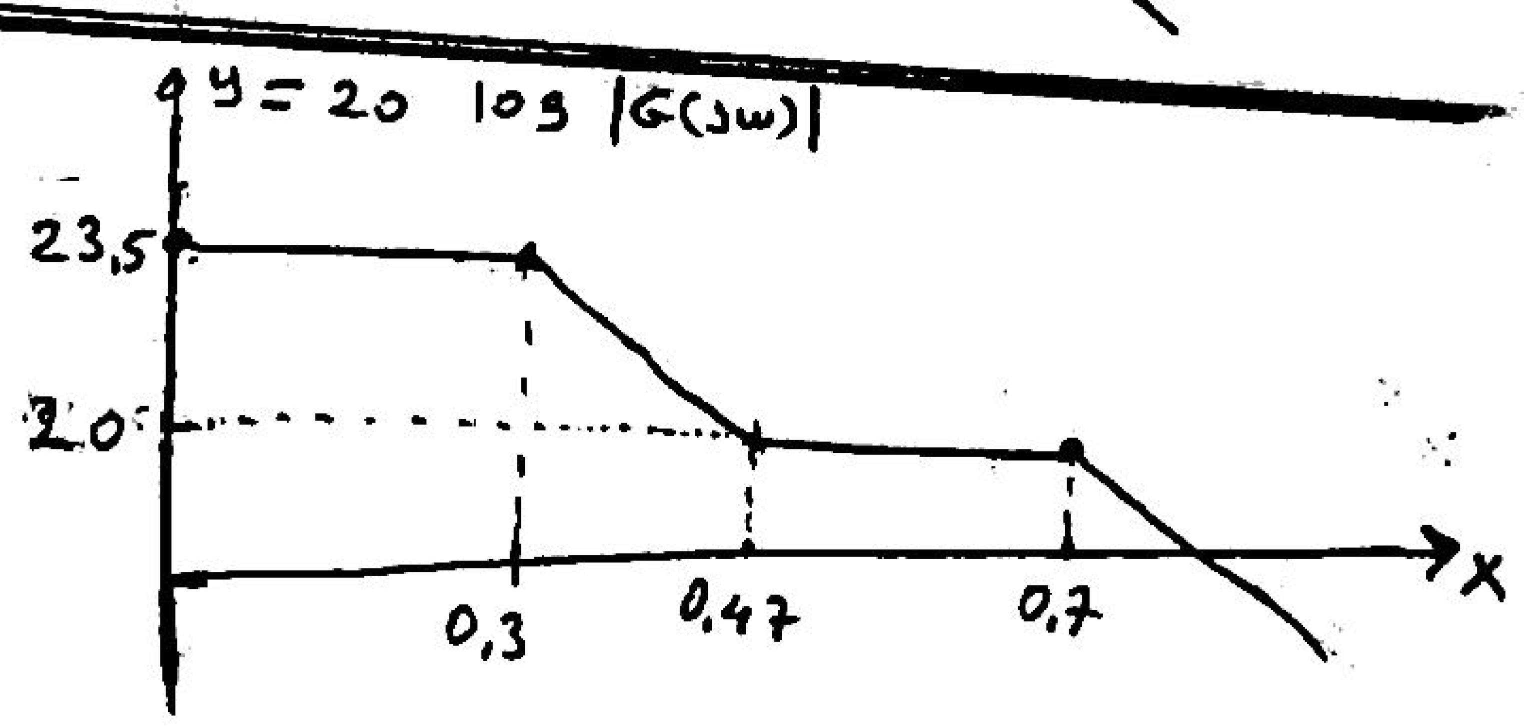
$0.3 = \log 2$



low frequencies $y = -20\log 5 = -14$

high frequencies $y = -20\log w$

$0.7 = \log 5$



low frequencies

$34 + 9.5 - 6 - 14 = 23.5$

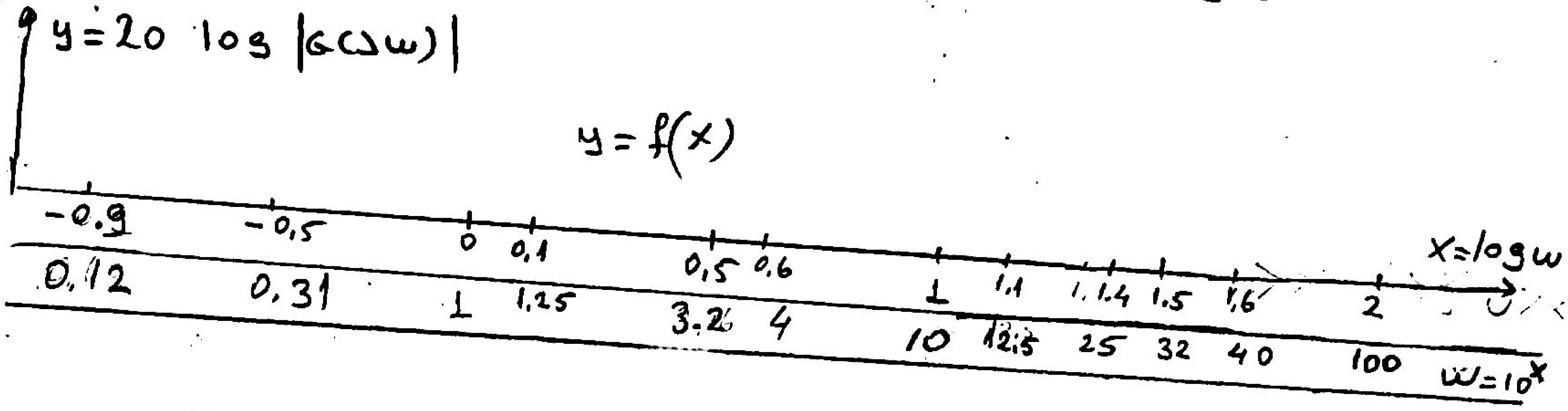
high frequencies

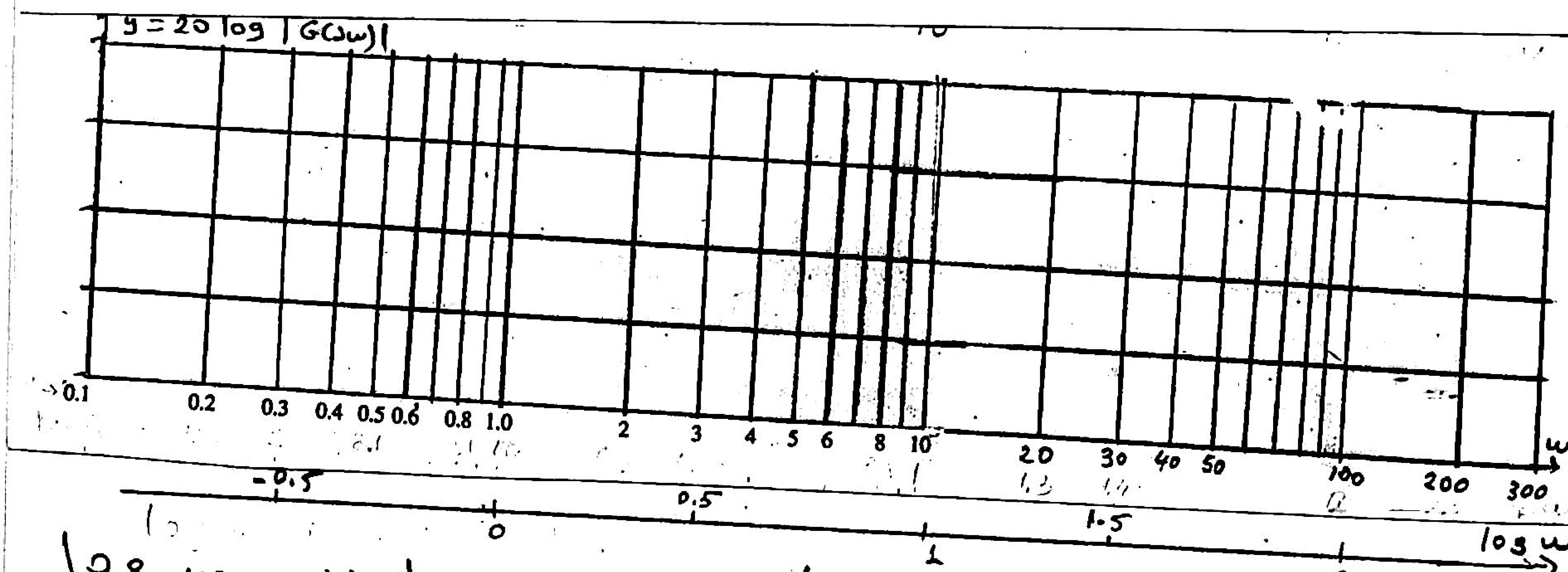
$34 + 20x - 20x - 20x = 34 - 20x$

between $0.47 - 0.7$

$34 + 20x - 20x - 14 = 20$

Horizontal axis are numbered in w. not log w.

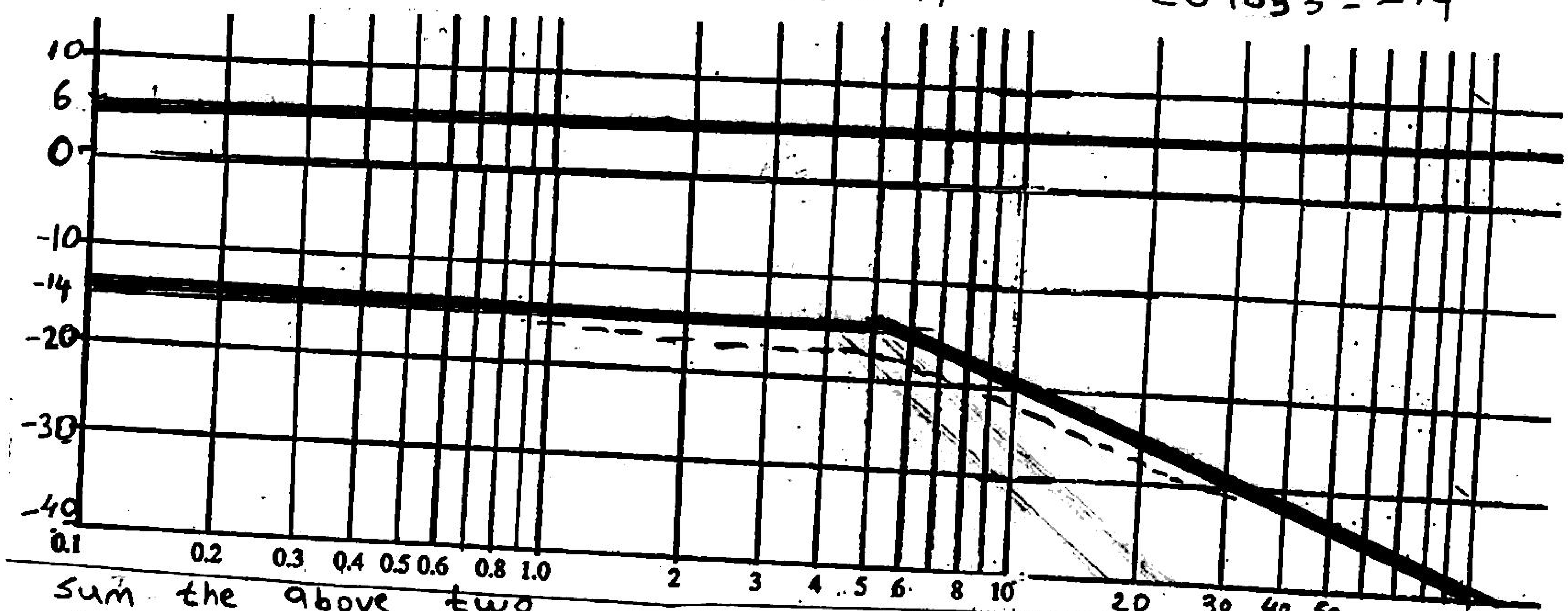




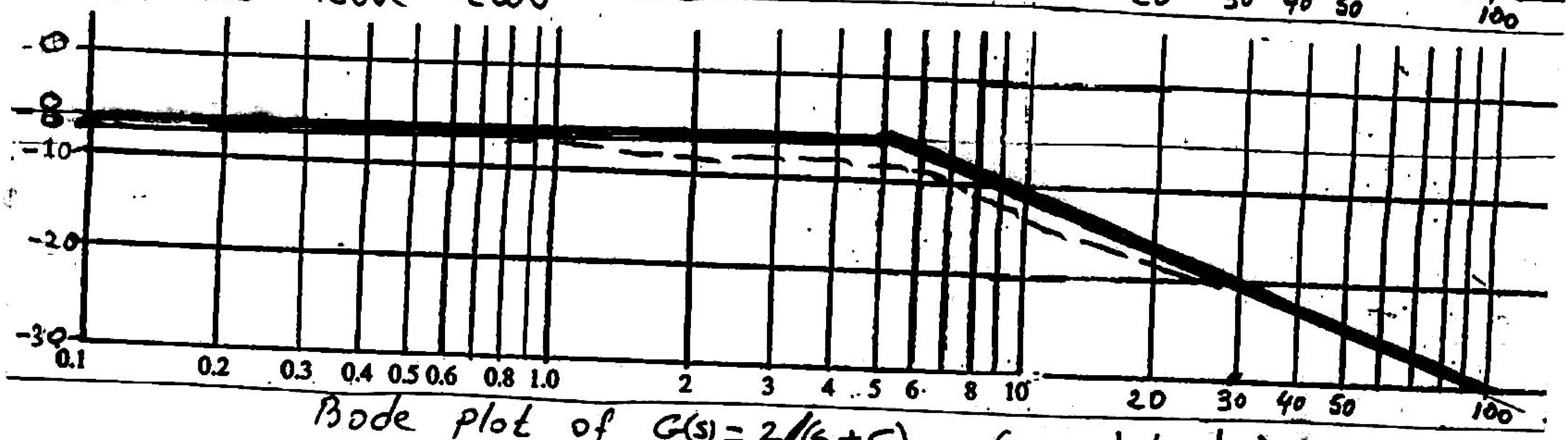
log w values are not shown on graphs.

Draw $G(s) = \frac{2}{s+5}$ on Bode diagram.

$20 \log 2 = 6$ $\qquad \log 5 = 0.7$ $\qquad -20 \log 5 = -14$



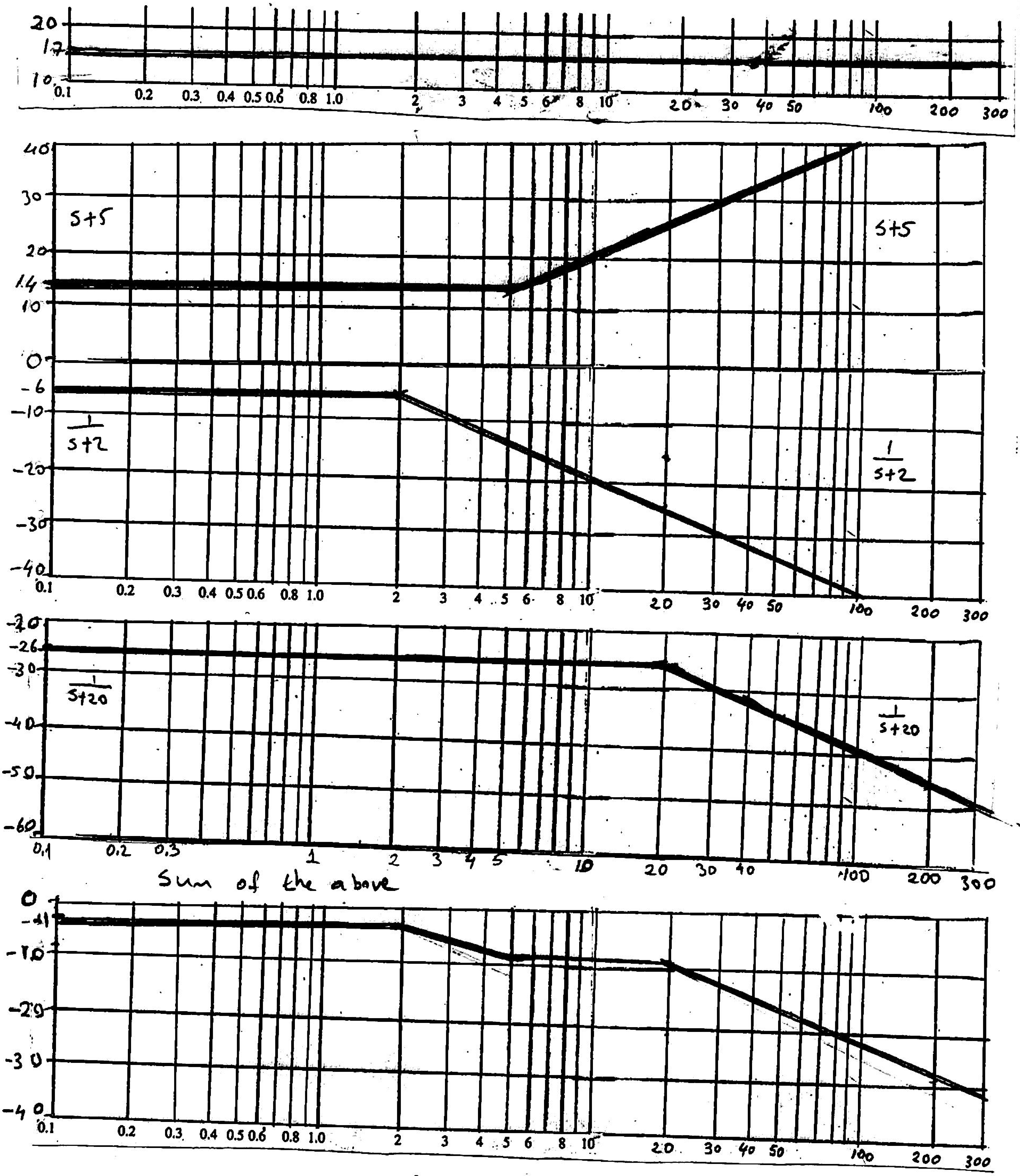
sum the above two



Bode plot of $G(s) = 2/(s+5)$ (amplitude)

Example $G(s) = \frac{7(s+5)}{(s+2)(s+20)}$ $\quad 20\log 5 = 14 \quad 20\log 7 = 17 \quad -20\log 2 = -6$

$-20\log 20 = -26$



Bode plot of $G(s) = \frac{7(s+5)}{(s+2)(s+20)}$ (amplitude)

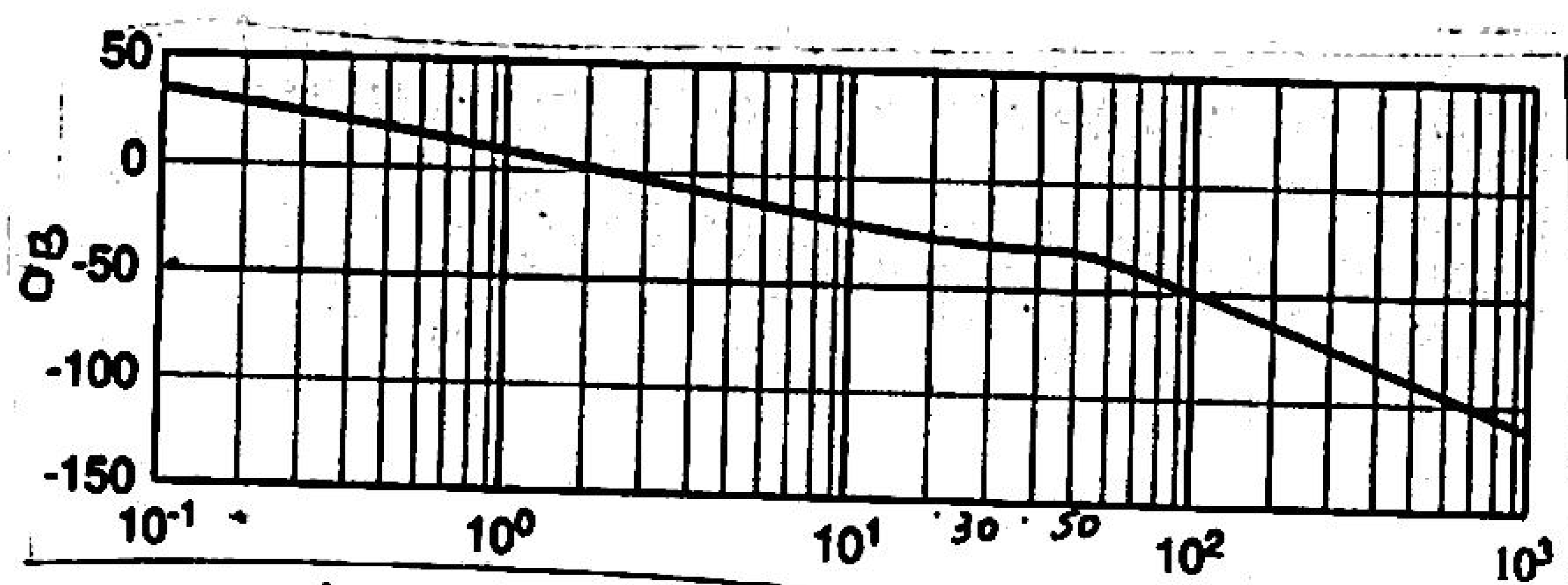
Example problem: $G(s) = \dfrac{5(s+0.1s)}{s(1+0.5s)(1+\frac{0.6}{50}s+\frac{1}{50^2}s^2)}$
(Page 447)

$$G(s) = \frac{5\times2\times50^2}{10}\;\frac{(s+10)}{s(s+2)(s^2+30s+50^2)}$$

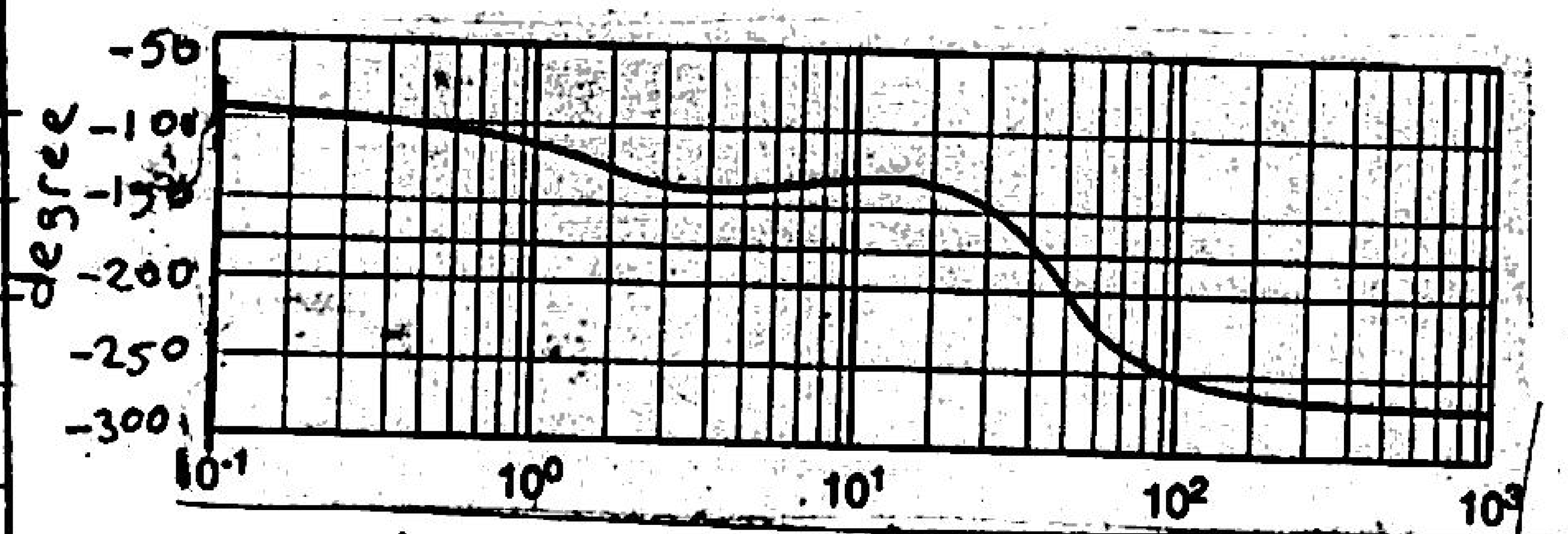
$$G(jw) = \frac{2500\,(jw+10)}{jw\,(jw+2)\left((jw)^2+30jw+2500\right)} \qquad |G(jw)| = \frac{2500\sqrt{w^2+10^2}}{w\sqrt{w^2+2^2}\sqrt{(2500-w^2)^2+(30w)^2}}$$

$$\measuredangle G(jw) = \tan^{-1}\frac{w}{10} - 90^\circ + \tan^{-1}\frac{2}{w} - \tan^{-1}\frac{2500-w^2}{30\,w}$$

w	$\lvert G(jw)\rvert$	$20\log\lvert G(jw)\rvert$	$\measuredangle G(jw)$
0.1	49.9	33.9	−92
0.2	24.8	28	−95
1	4.5	13	−111
2	1.8	5.1	−125
5	0.41	−7.5	−135
10	0.14	−16.8	−130
20	0.06	−24	−126
40	0.04	−27	−154
50	0.03	−29	−189
60	0.02	−34	−218
100	0.003	−50	−252
1000	0.00001	−112	−268

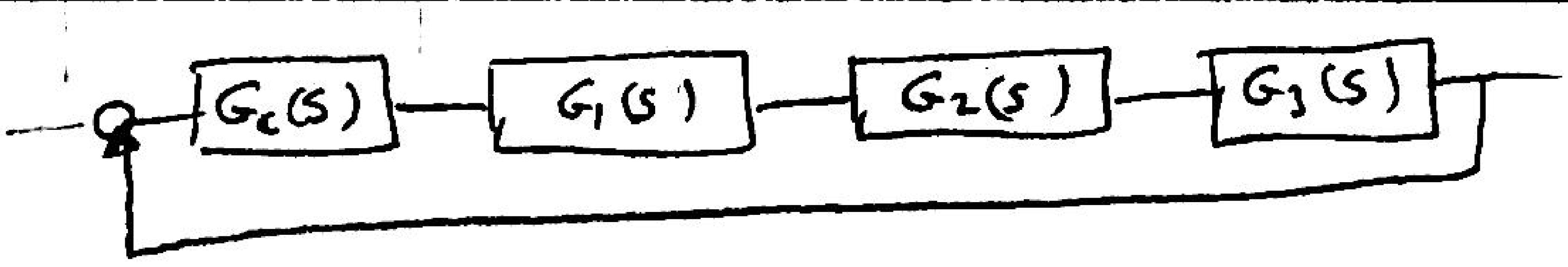


Bode Amplitude diagram



Bode phase diagram

Example problem:
(Page 453)



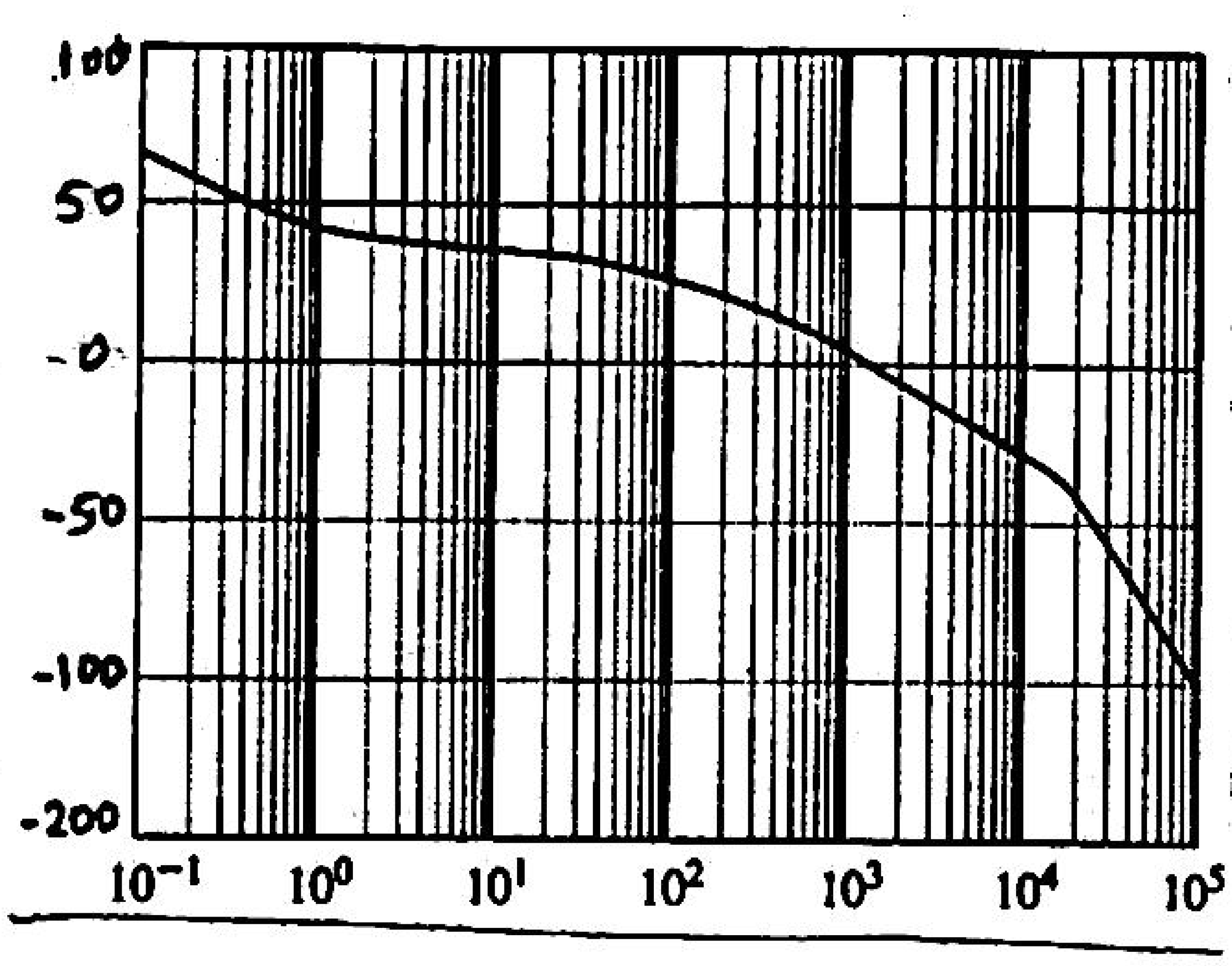
$G_c(s) = 400(s+1)$

$G_1(s) = \dfrac{1000}{s+1000}$

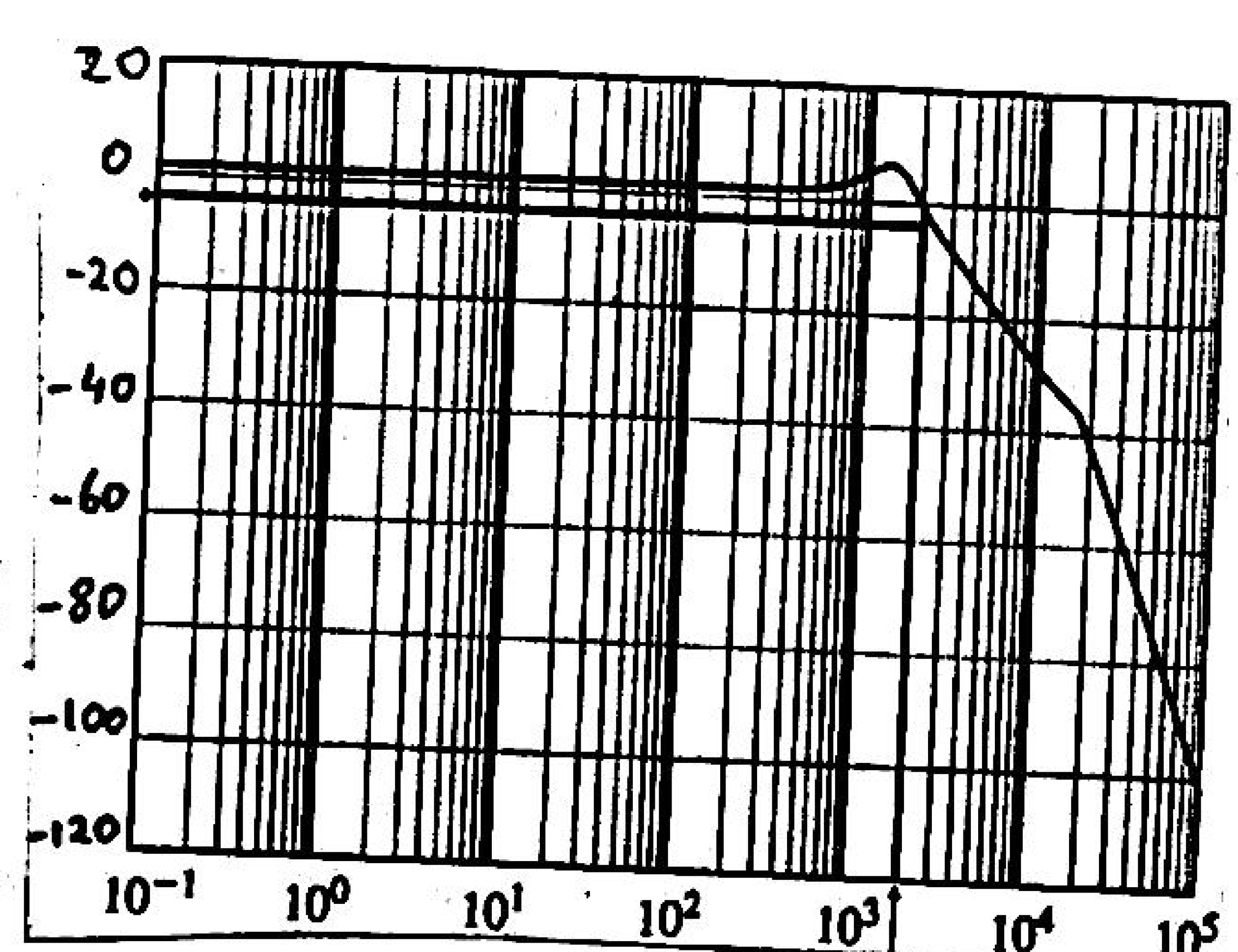
$G_2(s) = \dfrac{1}{s(s+20)}$

$G_3(s) = \dfrac{w_n^2}{s^2+0.6\,w_n+w_n^2}$

$(w_n = 18.8\ 10^3)$



Open loop Bode Amplitude



Closed loop Bode Amplitude

Bode diagram for complex conjugate poles

$$G(s) = \frac{1}{s^2 + as + b} \qquad G(jw) = \frac{1}{(jw)^2 + a(jw) + b}$$

$$|G(jw)| = \frac{1}{\sqrt{(-w^2+b)^2 + (aw)^2}}$$

$$20 \log |G(jw)| = 20 \log \frac{1}{\sqrt{\;}} = 20 \log 1 - 20 \log \sqrt{\;}$$

$$-20 \log \sqrt{(b-w^2)^2 + a^2w^2} = -20 \cdot \frac{1}{2} \log\left((b-w^2)^2 + a^2w^2\right)$$

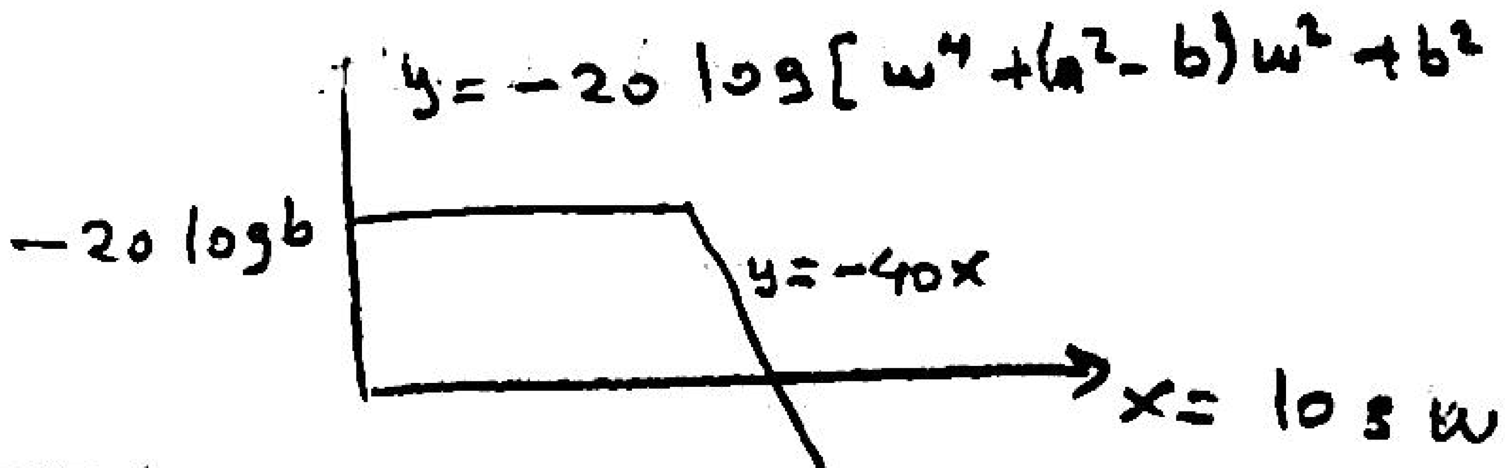
$$(b-w^2)^2 + a^2w^2 = b^2 - 2bw^2 + w^4 + a^2w^2 = w^4 + (a^2-2b)w^2 + b^2$$

for small values of w

$$-10 \log\{w^4 + (a^2-2b)w^2 + b^2\} \cong -10 \log b^2 = -20 \log b$$

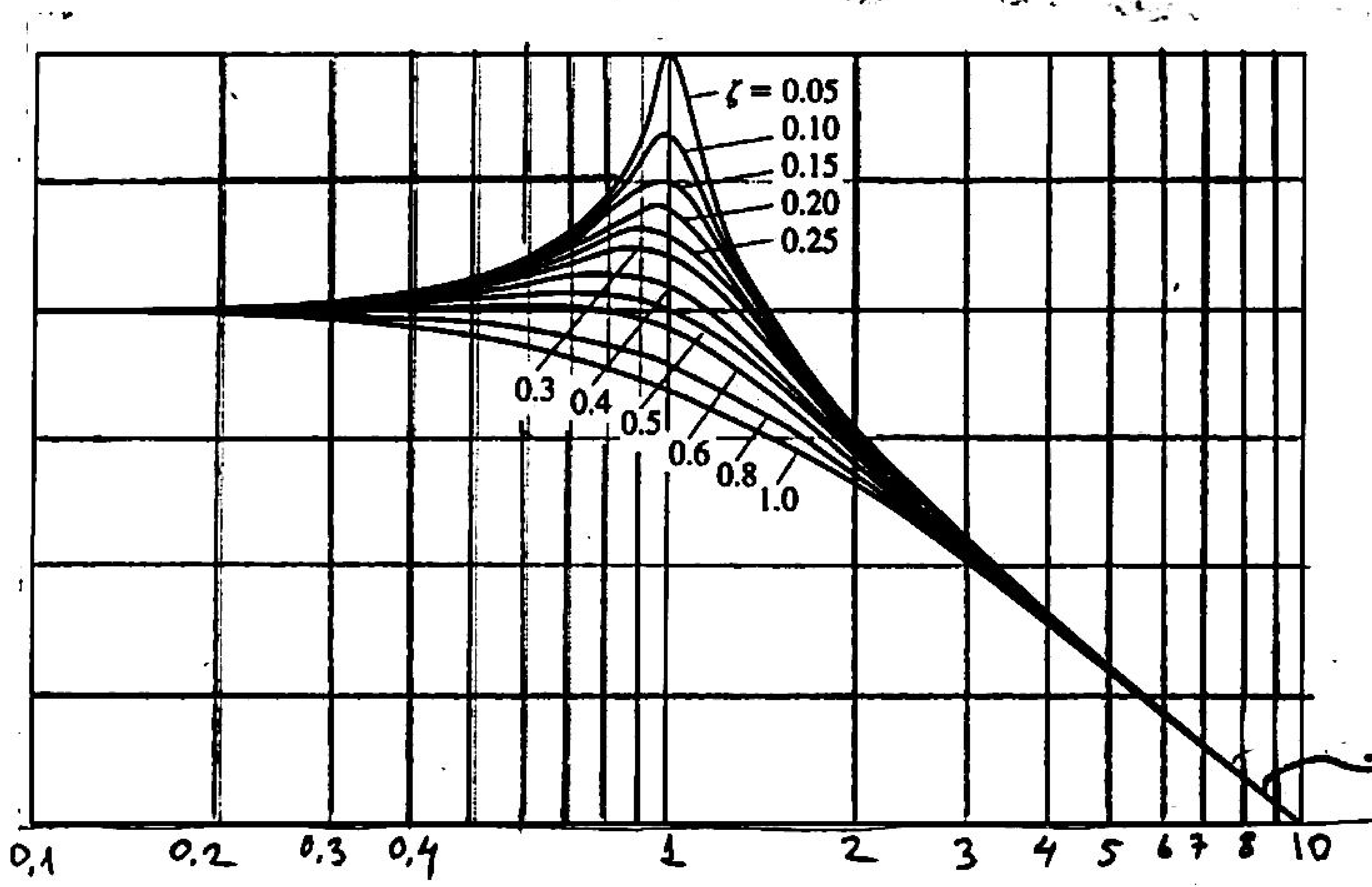
for big values of w

$$-10 \log\{w^4 + (a^2-2b)b^2 + b^2\} \cong -10 \log w^4 = -40 \log w$$



$$G(s) = \frac{1}{s^2+as+b} = \frac{1}{s^2 + 2\xi w_n s + w_n^2}$$

-20 log b



slope = - 40

plot of $-20 \log |G(jw)|$ for $w_n = 1$

$$G(s) = \frac{1}{s^2 + 2\xi s + 1}$$

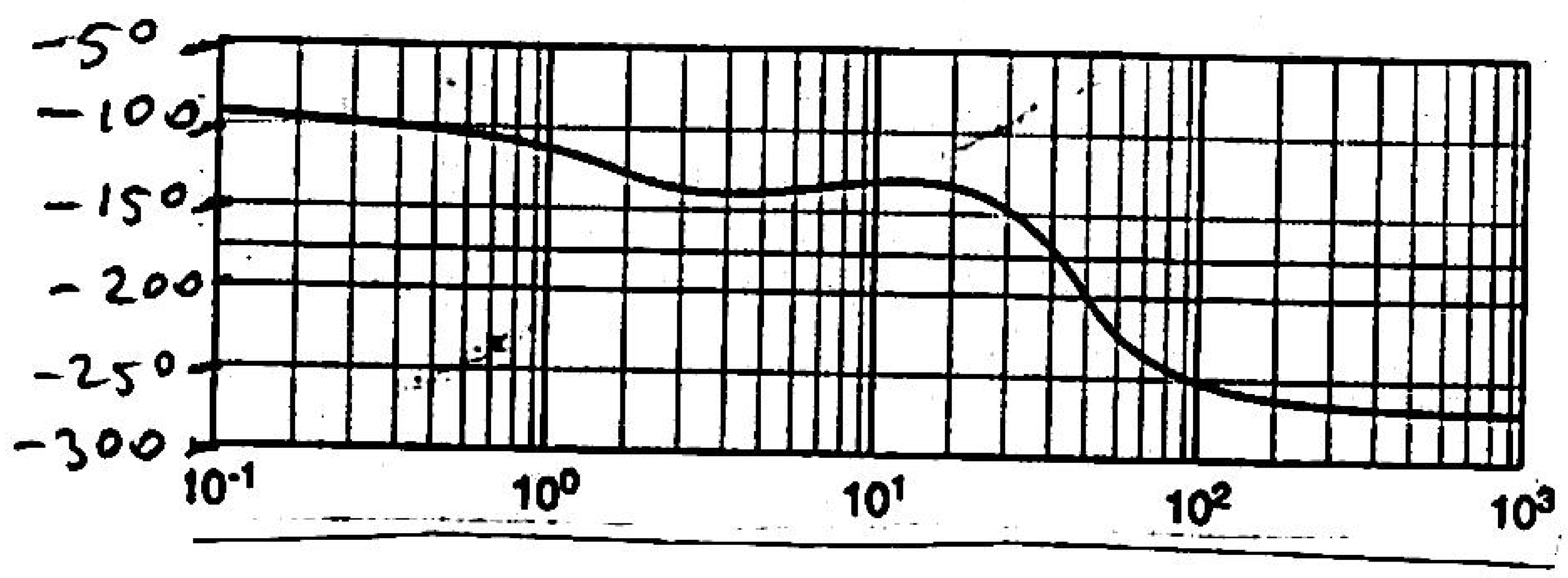
Example (page 447)

$$G(s) = \frac{5(1+0.1s)}{s(1+0.5s)\left(1+\frac{0.6}{50}s+\frac{1}{50^2}s^2\right)} = \frac{5\times 2\times 50^2}{10}\,\frac{s+10}{s(s+2)(s^2+30s+2500)}$$

$$= \frac{2500(s+10)}{s(s+2)(s^2+30s+2500)}$$

w	$20\log\lvert G(j\omega)\rvert$

Bode amplitude diagram



Bode phase diagram

fig 8.42 sh 457 2 tee

open loop

closed loop

847 sh 455 freq response

Frequency Response (Polar pilot)

$$G(j\omega) = |G(j\omega)|\, e^{j\phi(\omega)} = Re\{G(j\omega)\} + j\, Imag\{G(j\omega)\}$$

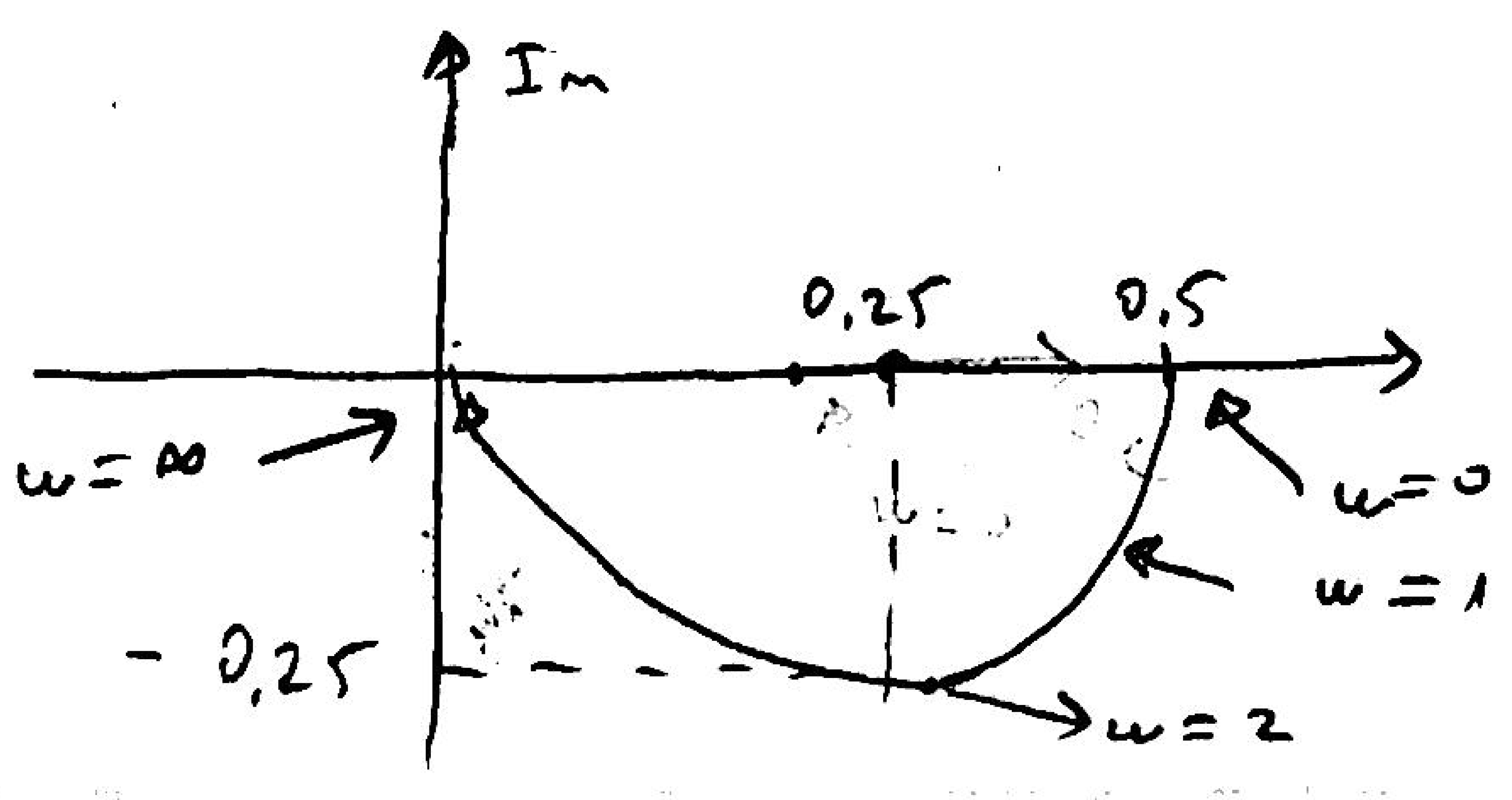
Example

$$G(s) = \frac{1}{s+2} \qquad G(j\omega) = \frac{1}{j\omega+2} = \frac{2-j\omega}{(2+j\omega)(2-j\omega)} = \frac{2-j\omega}{2^2+\omega^2}$$

$$= \frac{2}{2^2+\omega^2} + j\,\frac{-\omega}{2^2+\omega^2}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+2^2}}, \quad \angle G(j\omega) = 0 - \tan^{-1}\frac{\omega}{2}$$

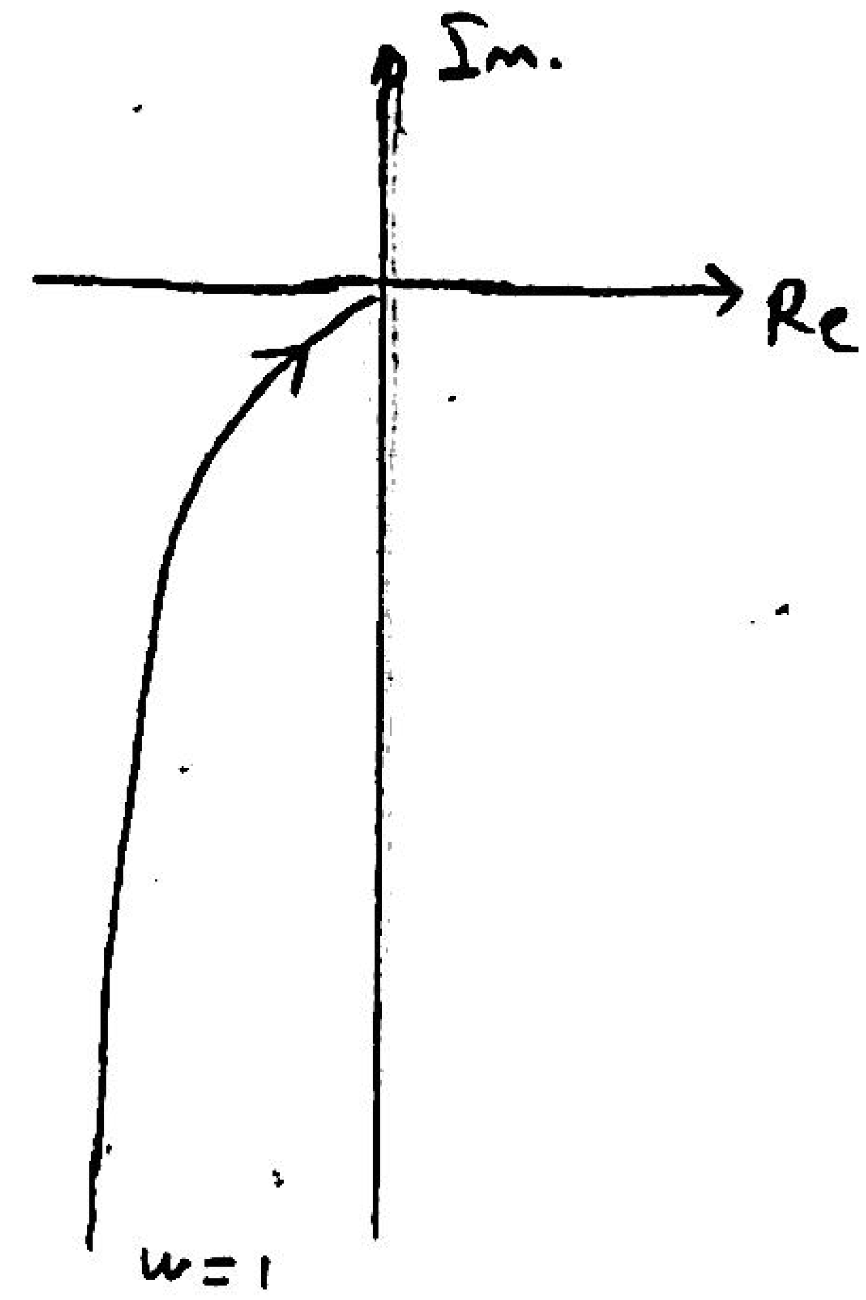
ω	$G(j\omega)$	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$	$Re\{G(j\omega)\}$	$Im\{G(j\omega)\}$
0	0.5	0.5	0	0.5	0
0.1	0.49 − 0.02j	0.49	−2.8°	0.49	−0.02
0.2	0.49 − 0.04j	0.49	−5.7°	0.49	−0.04
0.5	0.47 − 0.12j	0.48	−14.04°	0.47	−0.12
1	0.4 − 0.2j	0.44	−26.5°	0.4	−0.2
1.5	0.32 − 0.24j	0.4	−36.8°	0.32	−0.24
1.8	0.27 − 0.24j	0.37	−42°	0.27	−0.24
2	0.25 − 0.25j	0.35	−45°	0.25	−0.25
2.5	0.19 − 0.24j	0.31	−51.°	0.19	−0.24
3	0.15 − 0.23j	0.27	−56	0.15	−0.23
5	0.07 − 0.17j	0.18	−68°	0.07	−0.17
10	0.02 − 0.09j	0.09	−78°	0.02	−0.09
50	0.0008 − 0.02j	0.02	−87°	0.0008	−0.02
100	0.0002 − 0.01j	0.01	−88	0.0002	−0.01
∞	0	0	−90	0	0



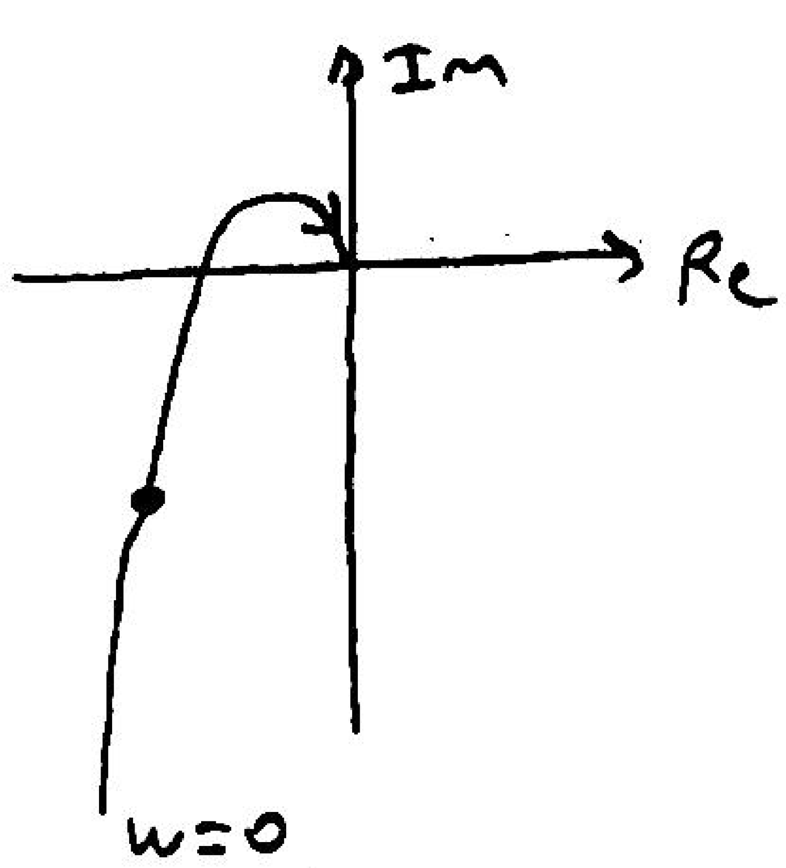
Example $G(s) = \dfrac{10}{s(s+2)}$ $G(jw) = \dfrac{10}{jw(jw+2)}$

$|G(jw)| = \dfrac{10}{w\sqrt{w^2+2^2}}$ $\measuredangle G(jw) = 0 - 90 - \tan^{-1}\frac{w}{2}$

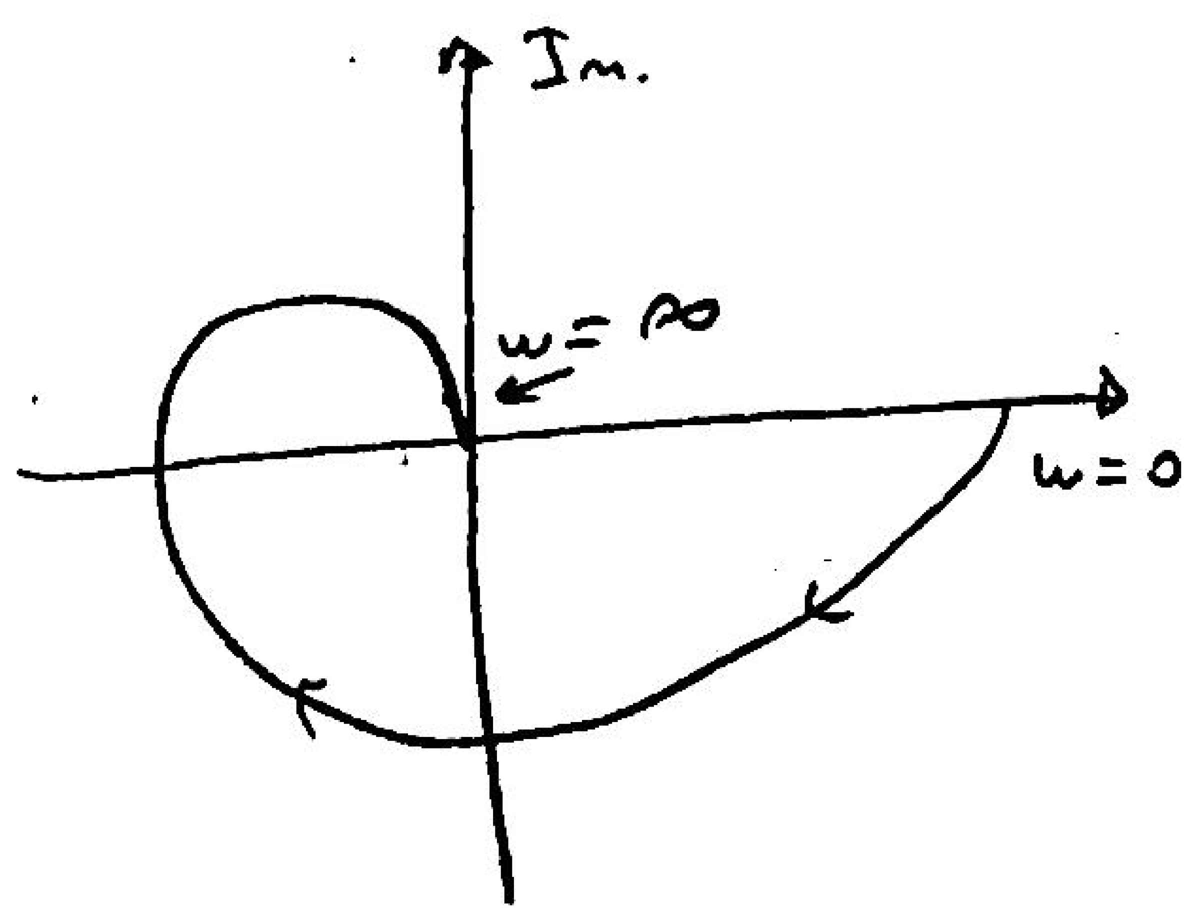
w	G(jw)	|G(jw)|	∡G(jw)	Re G	Im
0	∞	∞	-90		
0.0001	$-2-50000$	50000	-90.002°	-2	-50000
0.1	$-2{,}49-49j$	49,9	$-92{,}8^\circ$	$-2{,}49$	-49
0,2	$-2{,}47-24j$	24,8	$-95{,}7^\circ$	$-2{,}47$	-24
1	$-2-4j$	4,4	-116.5°	-2	-4
1,20	$-1{,}25-1{,}25j$	1,76	-135°	$-1{,}25$	$-1{,}25$
10	$-0{,}086-0{,}01$	0,098	-168°	$-0{,}086$	$-0{,}01$
1000	$-0{,}000009-0{,}0000$	0	-178°	0	0
∞	$0-0$	0	-180°	0	0



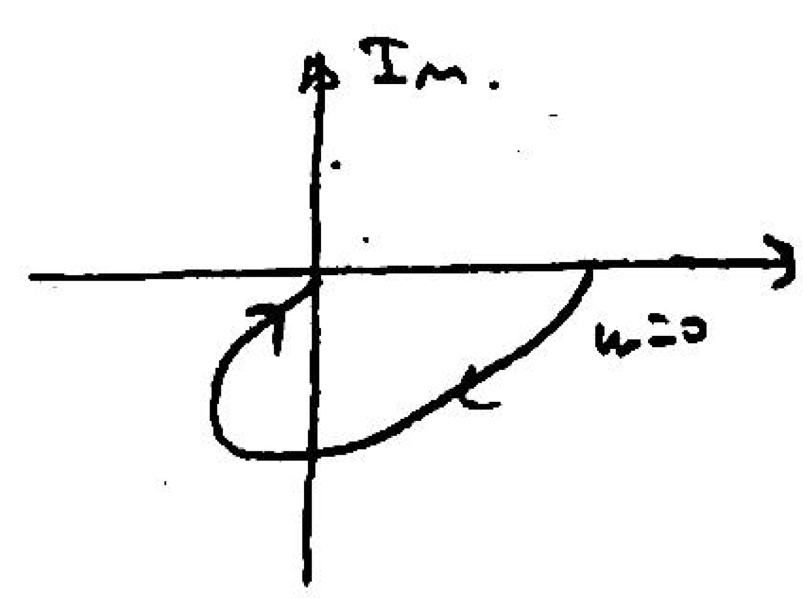
$G(s) = \dfrac{K}{s(s+5)(s+10)}$



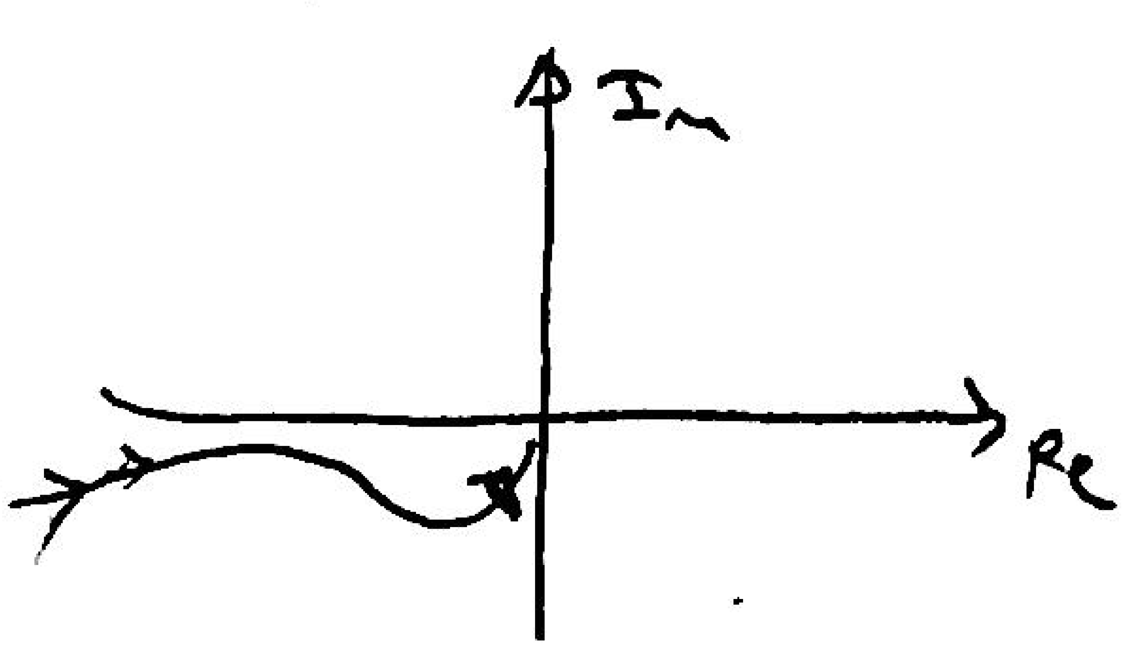
$G(s) = \dfrac{K}{(s+2)(s+5)(s+10)}$



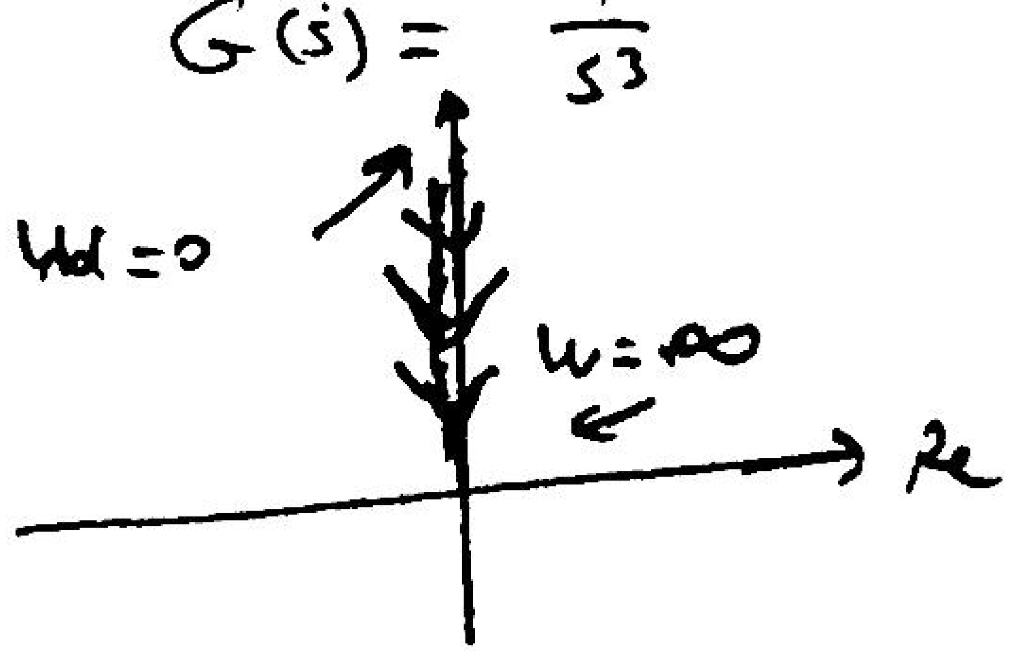
$G(s) = \dfrac{K}{(s+2)(s+5)}$



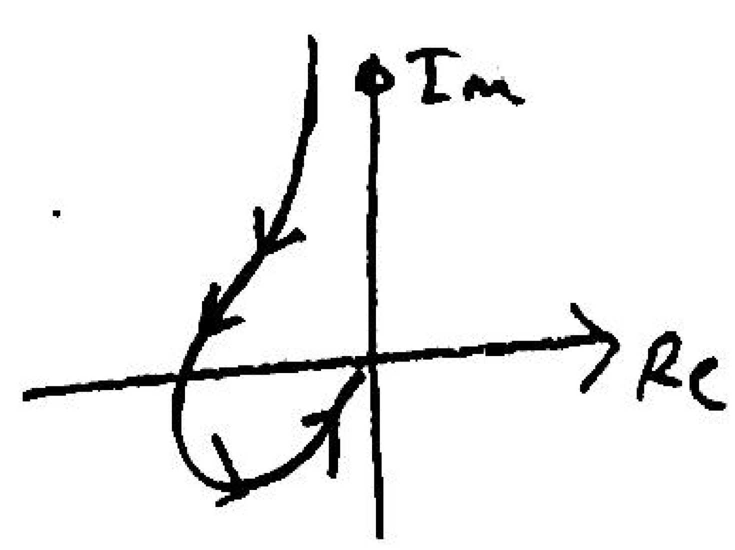
$G(s) = \dfrac{K\ s+3}{s^2(s+5)}$



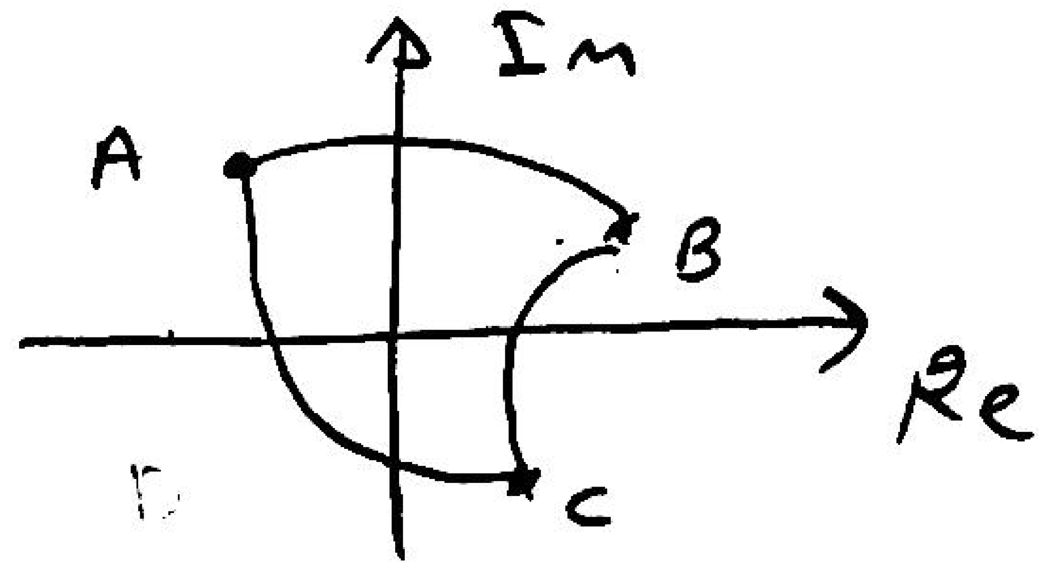
$G(s) = \dfrac{1}{s^3}$



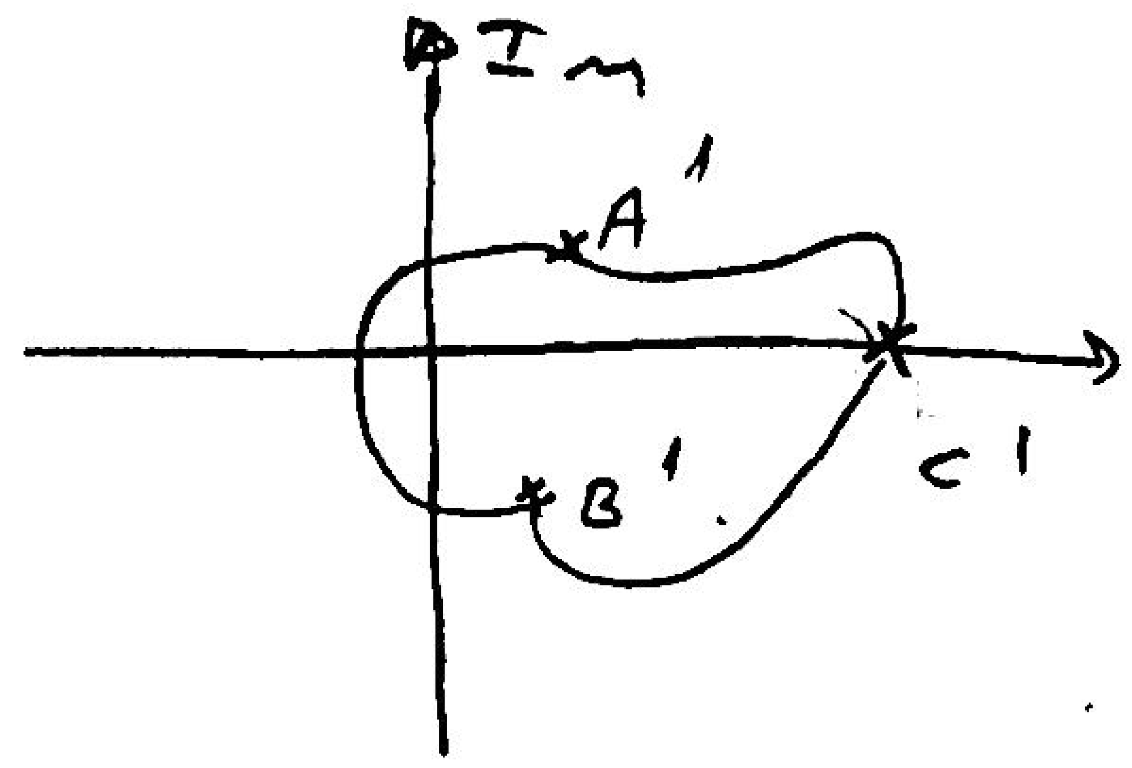
$G(s) = \dfrac{K(s+1)(s+3)}{s^3}$



Mapping contours: Mapping a closed curve from s domain to F(s) domain



s domain



F(s) domain

ABC curve mapped into f(S) domain A'B'C'

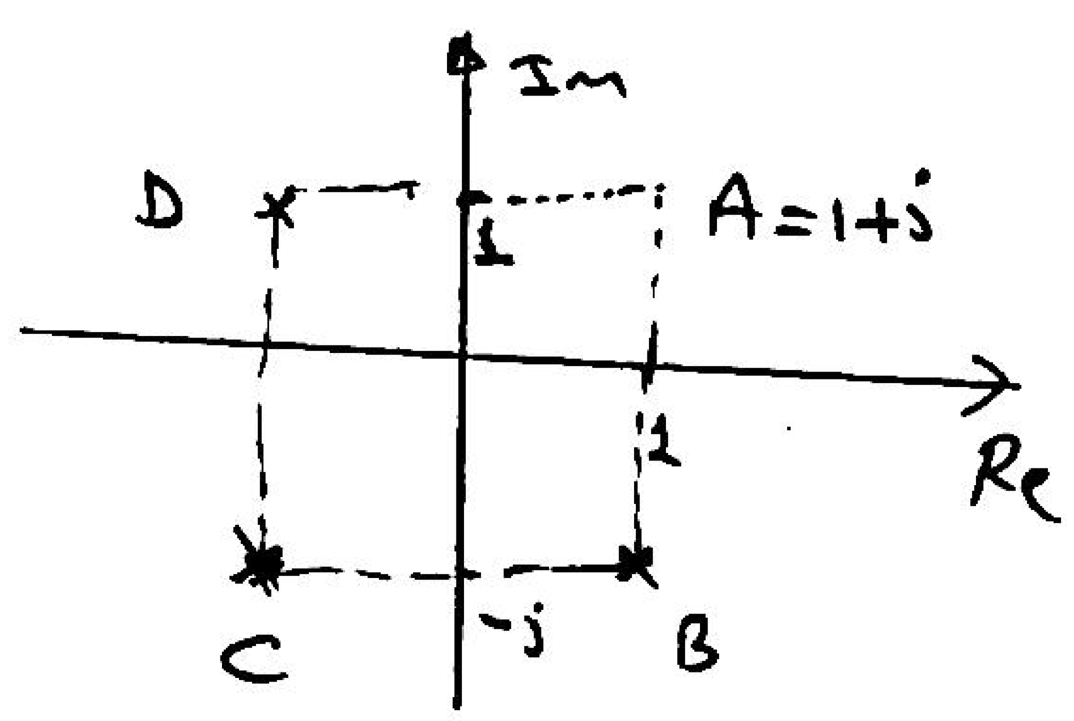
Example $F(s) = 2s + 1$ $\quad A = 1+j \quad B = 1-j \quad C = -1-j \quad D = -1+j$

calculate A', B', C', D' and Draw the contour.

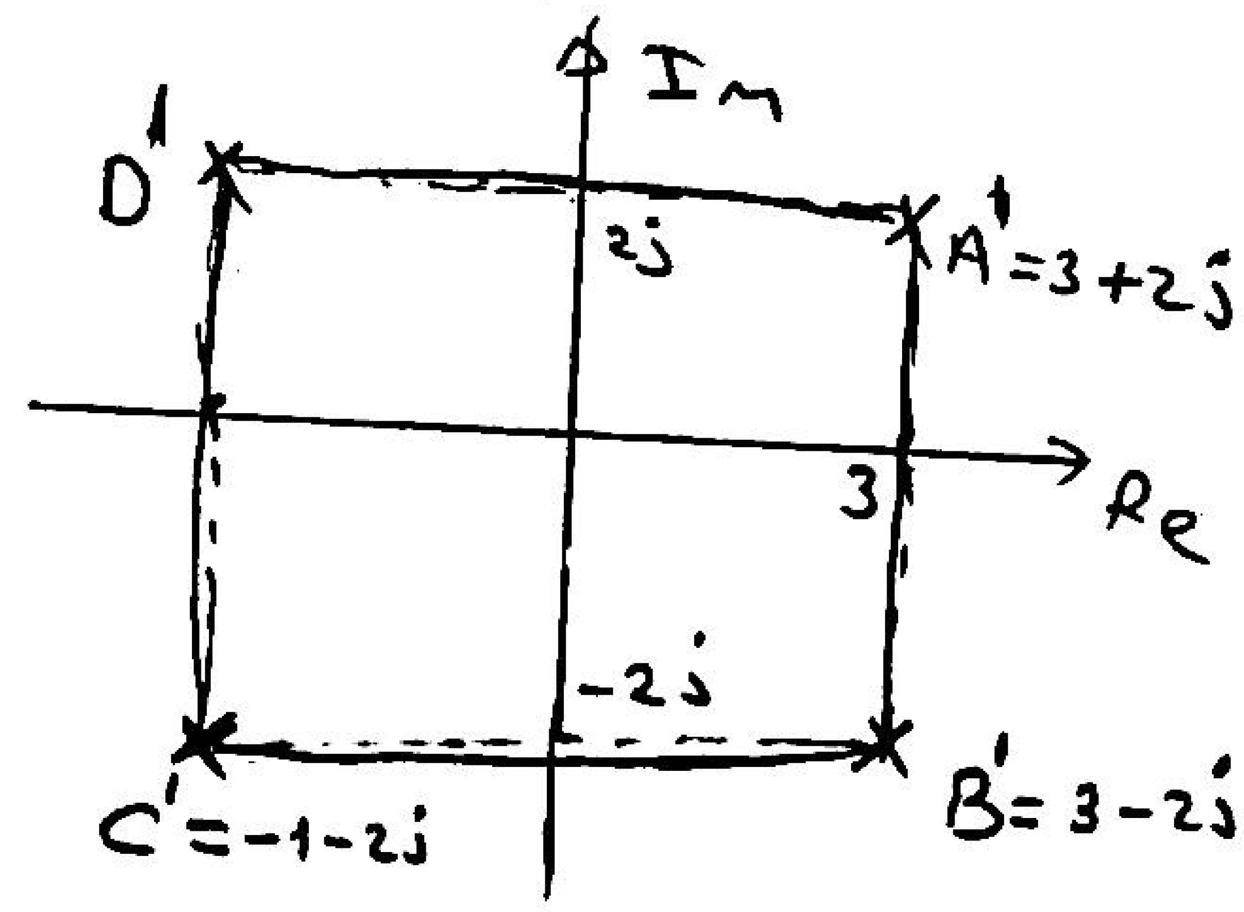
$$A' = F(s)\Big|_{s=1+j} = (2s+1)\Big|_{s=1+j} = 2(1+j)+1 = 2+2j+1 = 3+2j$$

$$B' = F(s)\Big|_{s=1-j} = (2s+1)\Big|_{s=1-j} = 2(1-j)+1 = 3-2j$$

$C' = 2(-1-j)+1 = -1-2j$ $\qquad D' = 2(-1+j)+1 = -1+2j$



S domain.
(s plane)



$F(s) = (2s+1)$ domain
(F(s) plane)

Example $f(s) = \dfrac{s}{s+2}$. $A = 1+j$ $\quad B = 1-j$ $\quad C = -1-j$ $\quad D = -1+j$

$P = 1$ $\quad \varphi = -j$ $\quad R = -1$ $\quad T = j$

$$A' = \left.\frac{s}{s+2}\right|_{s=1+j} = \frac{1+j}{1+j+2} = \frac{1+j}{3+j} = \frac{(1+j)(3-j)}{(3+j)(3-j)} = \frac{3-j+3j-j^2}{3^2+1^2} = \frac{4+2j}{10} = 0{,}4+0{,}2j$$

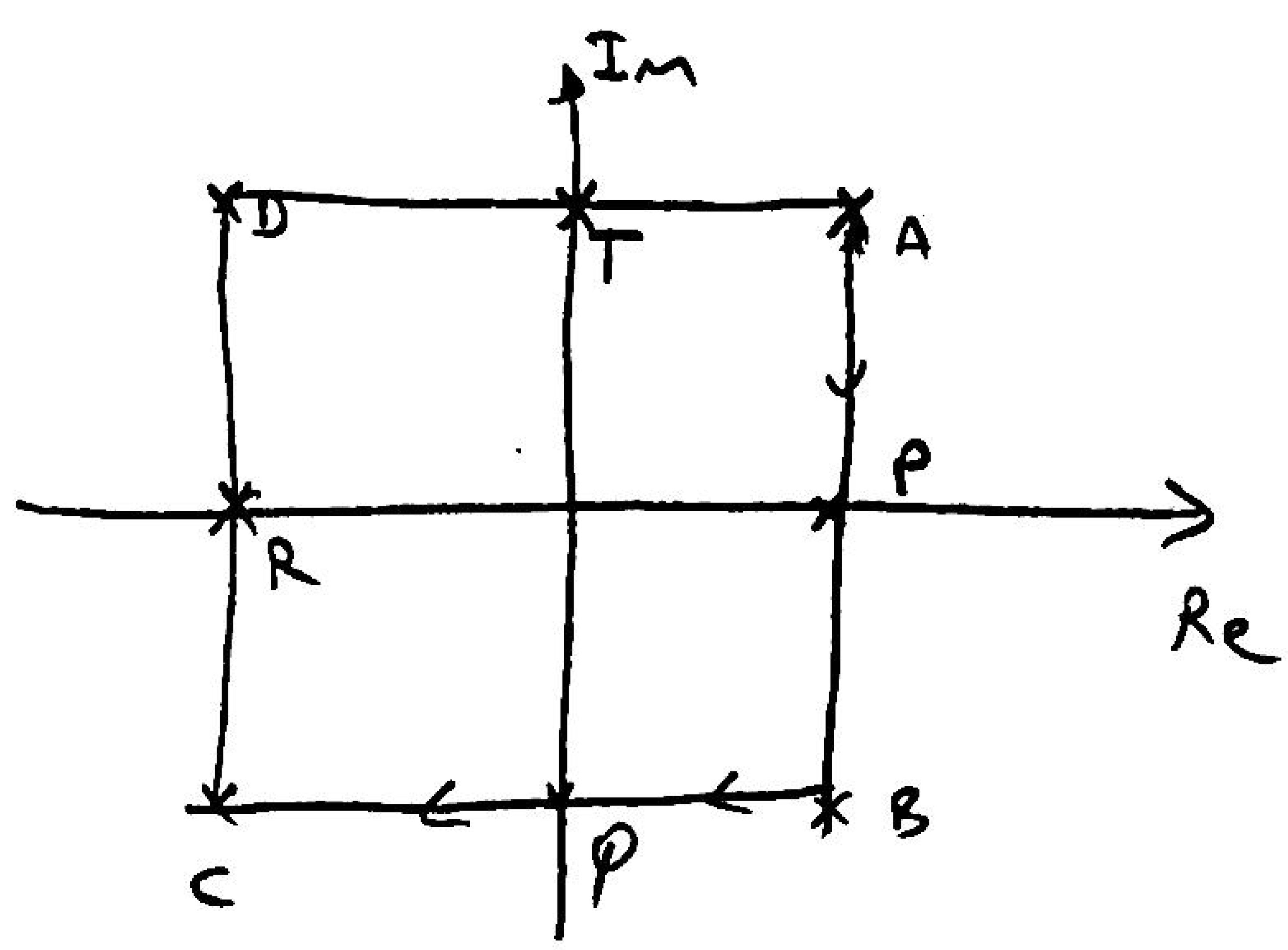
$$B' = \left.\frac{s}{s+2}\right|_{s=1-j} = \frac{1-j}{1-j+2} = \frac{4-2j}{10} = 0{,}4 - 0{,}2j$$

$$C' = \left.\frac{s}{s+2}\right|_{s=-1-j} = \frac{-1-j}{-1-j+2} = -j \qquad D' = \frac{-1+j}{-1+j+2} = j$$

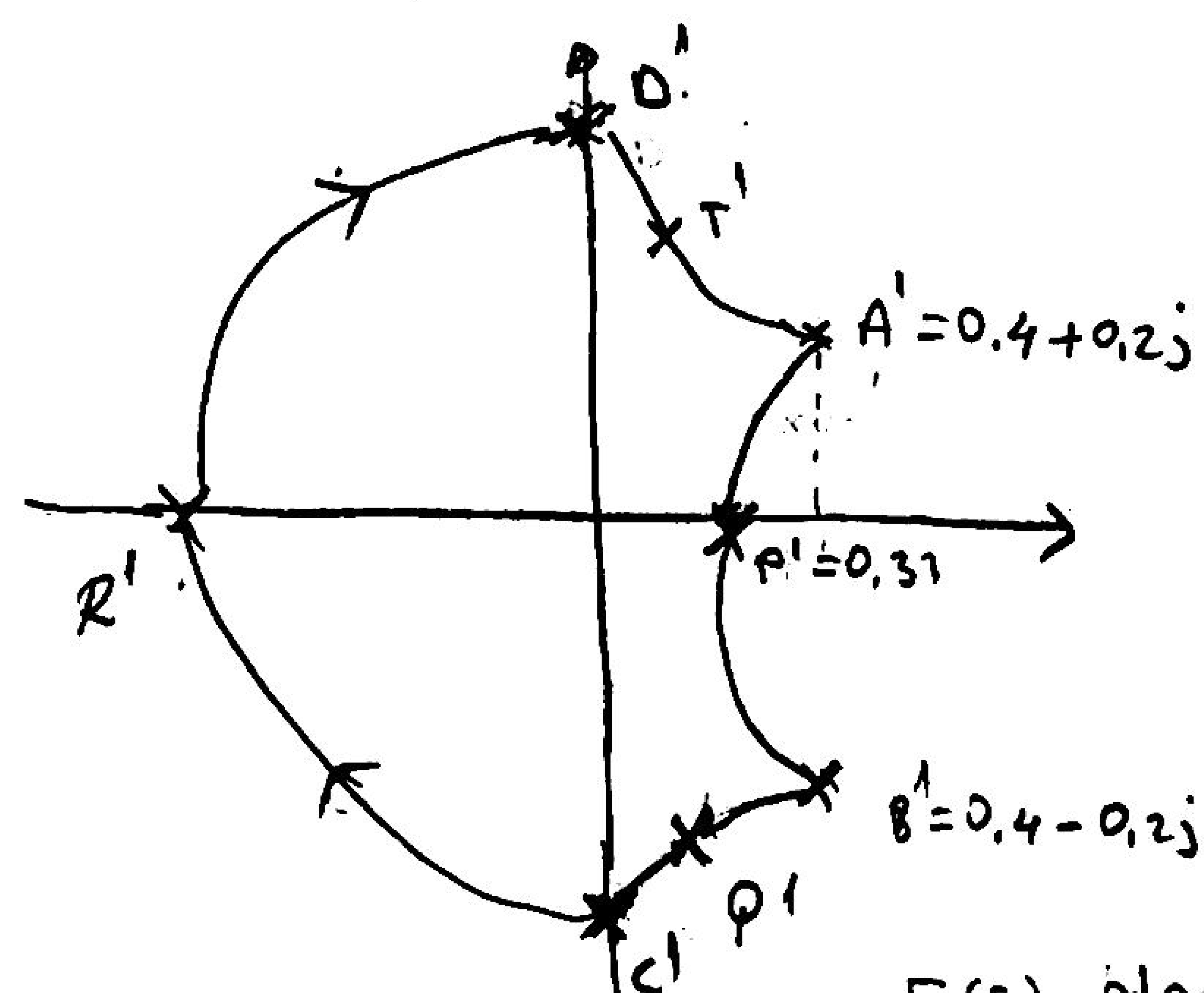
$$P' = \left.\frac{s}{s+2}\right|_{s=1} = \frac{1}{1+2} = 0{,}33 \qquad \varphi' = \frac{-j}{-j+2} = \frac{1-2j}{5} = 0{,}2-0{,}4j \qquad R' = \frac{-1}{-1+2} = -1$$

$$T' = \frac{j}{j+2} = \frac{1+2j}{5} = 0{,}2+0{,}4j$$

s	$A=1+j$	$P=1$	$B=1-j$	$\varphi=-j$	$C=-1-j$	$R=-1$	$D=-1+j$	$T=j$
$F(s)$	$A'=0{,}4+0{,}2j$	$P'=0{,}33$	$B'=0{,}4-0{,}2j$	$\varphi'=0{,}2-0{,}4j$	$C'=-j$	$R'=-1$	$D'=j$	$T'=0{,}2+0\ldots$



s plane

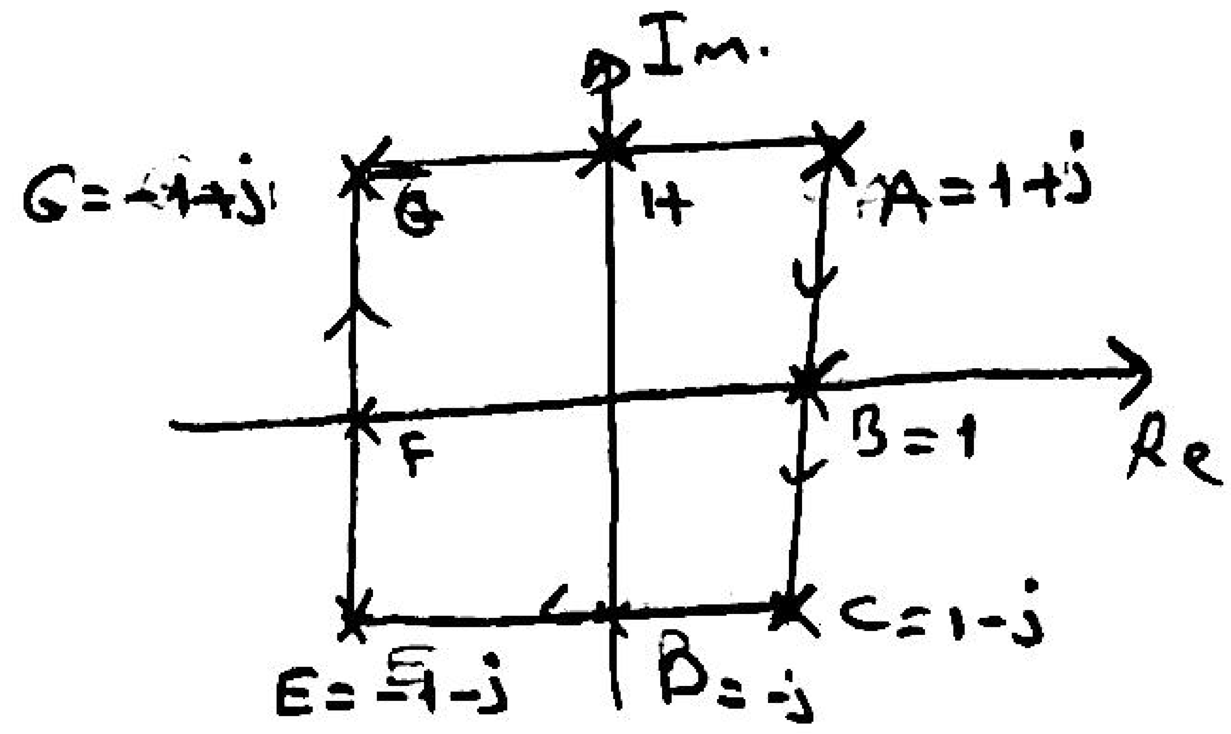


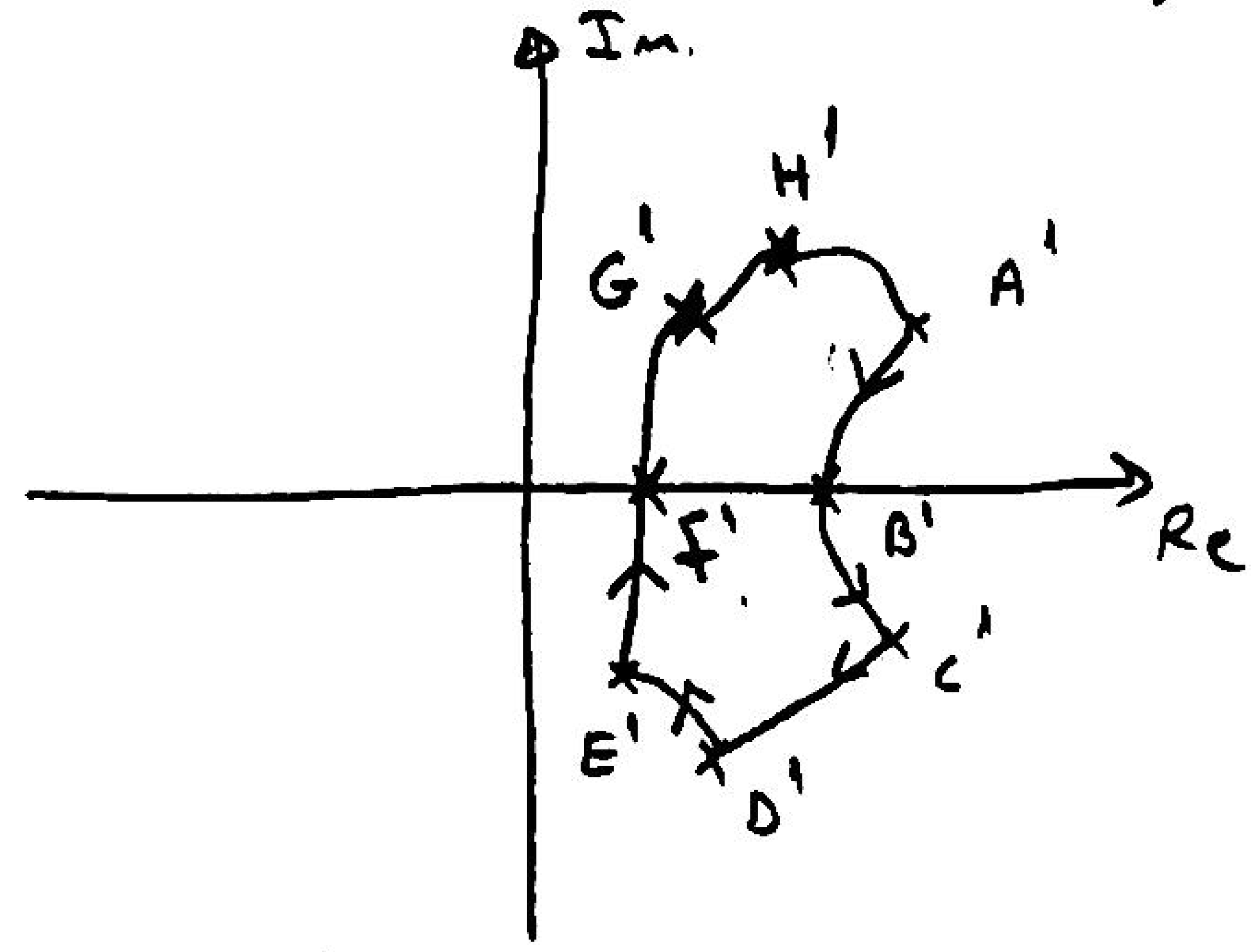
F(s) plane

$A\,P\,B\,\varphi\,C\,R\,D\,T\,A \longrightarrow A'\,P'\,B'\,\varphi'\,C'\,R'\,D'\,T'\,A'$

Example c21 $f(s) = \dfrac{s+2}{s+4}$

a)



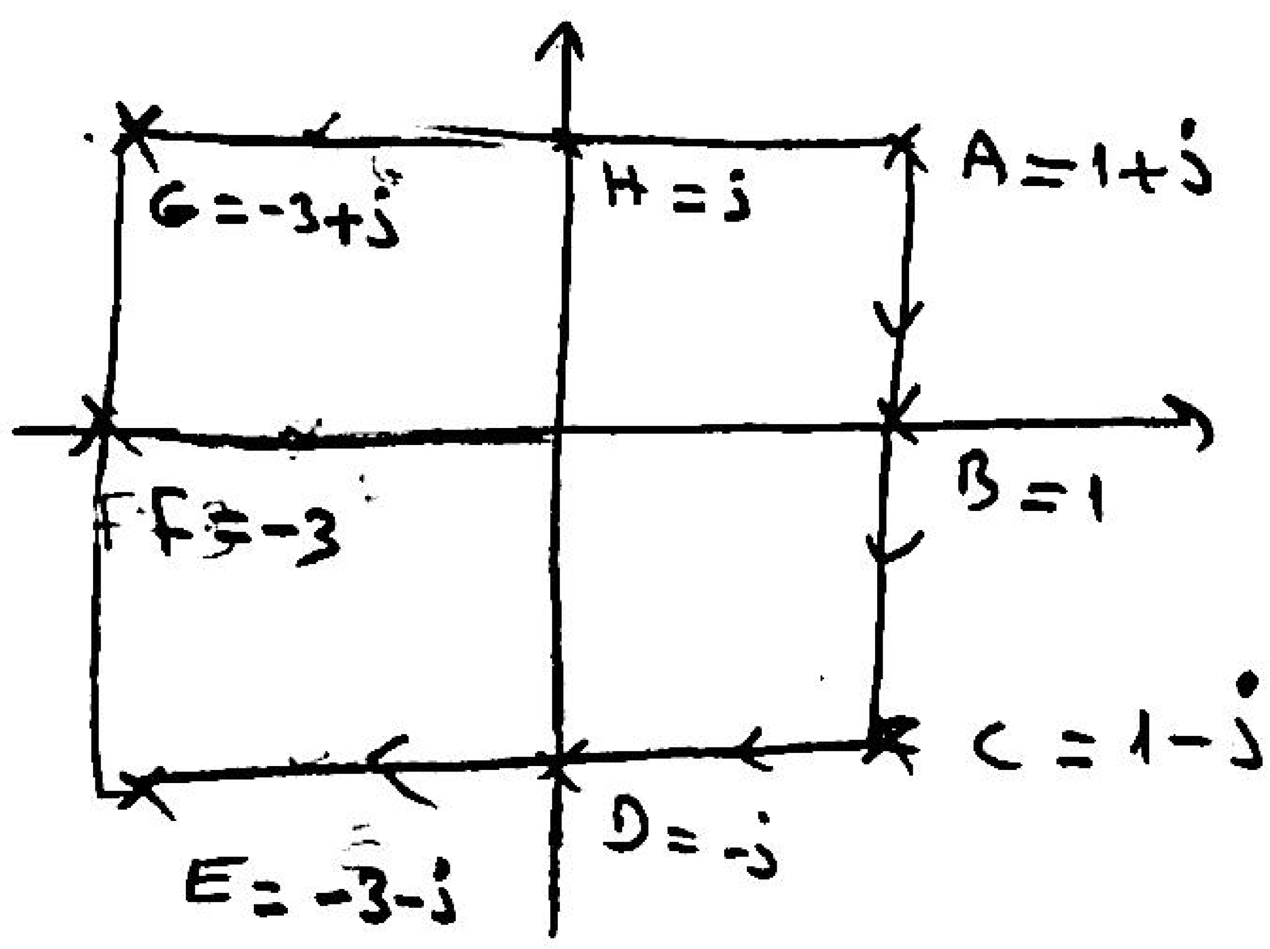


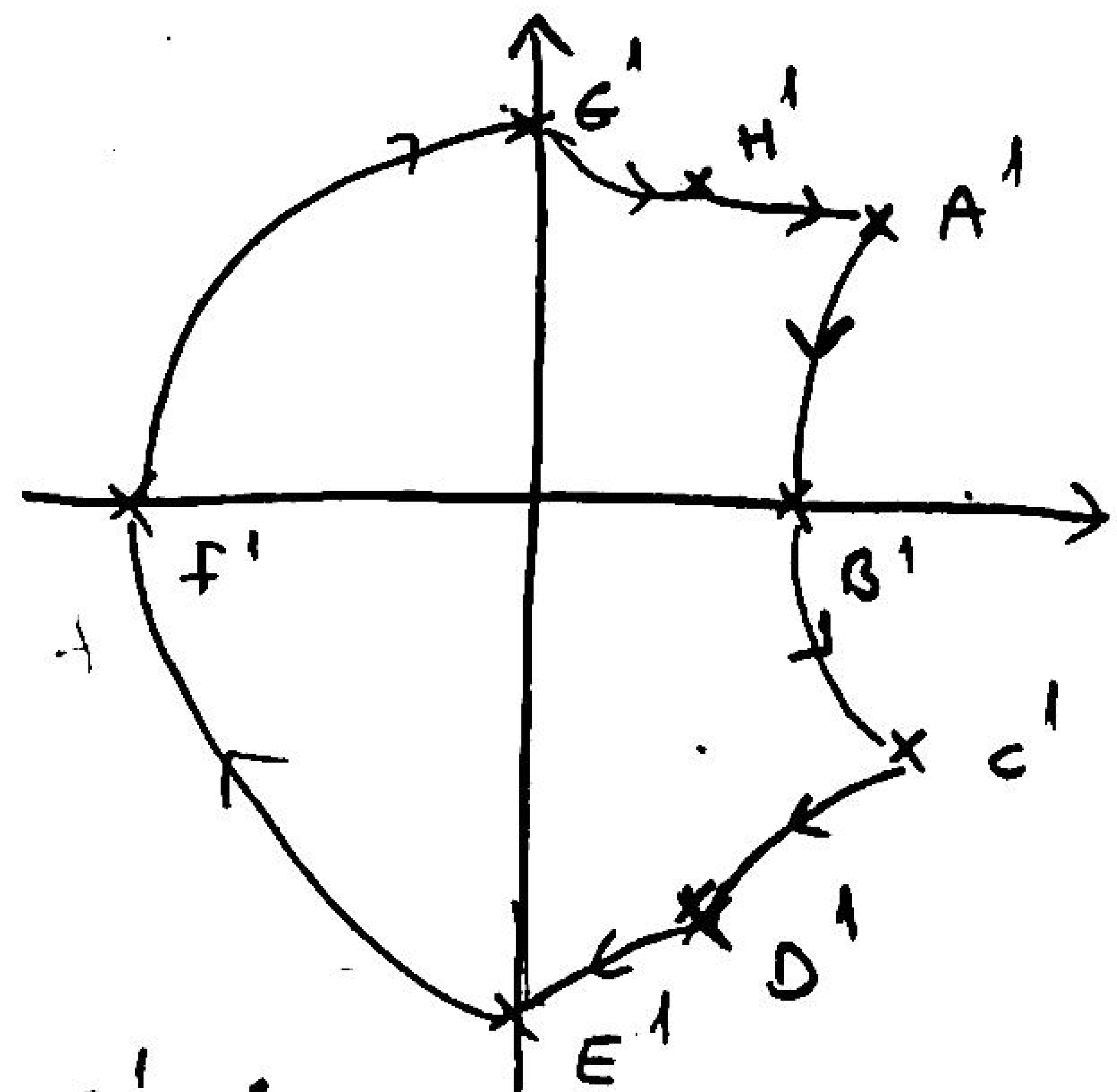
$A' = 0{,}61 + 0{,}07j$ $B' = 0{,}6$ $C' = 0{,}61 - 0{,}07$ $D' = 0{,}52 - 0{,}12j$ $E' = 0{,}4 - 0{,}2$ $F' = 0{,}33$

$G' = 0{,}4 + 0{,}2j$ $H' = 0{,}52 + 0{,}12j$

---

b)

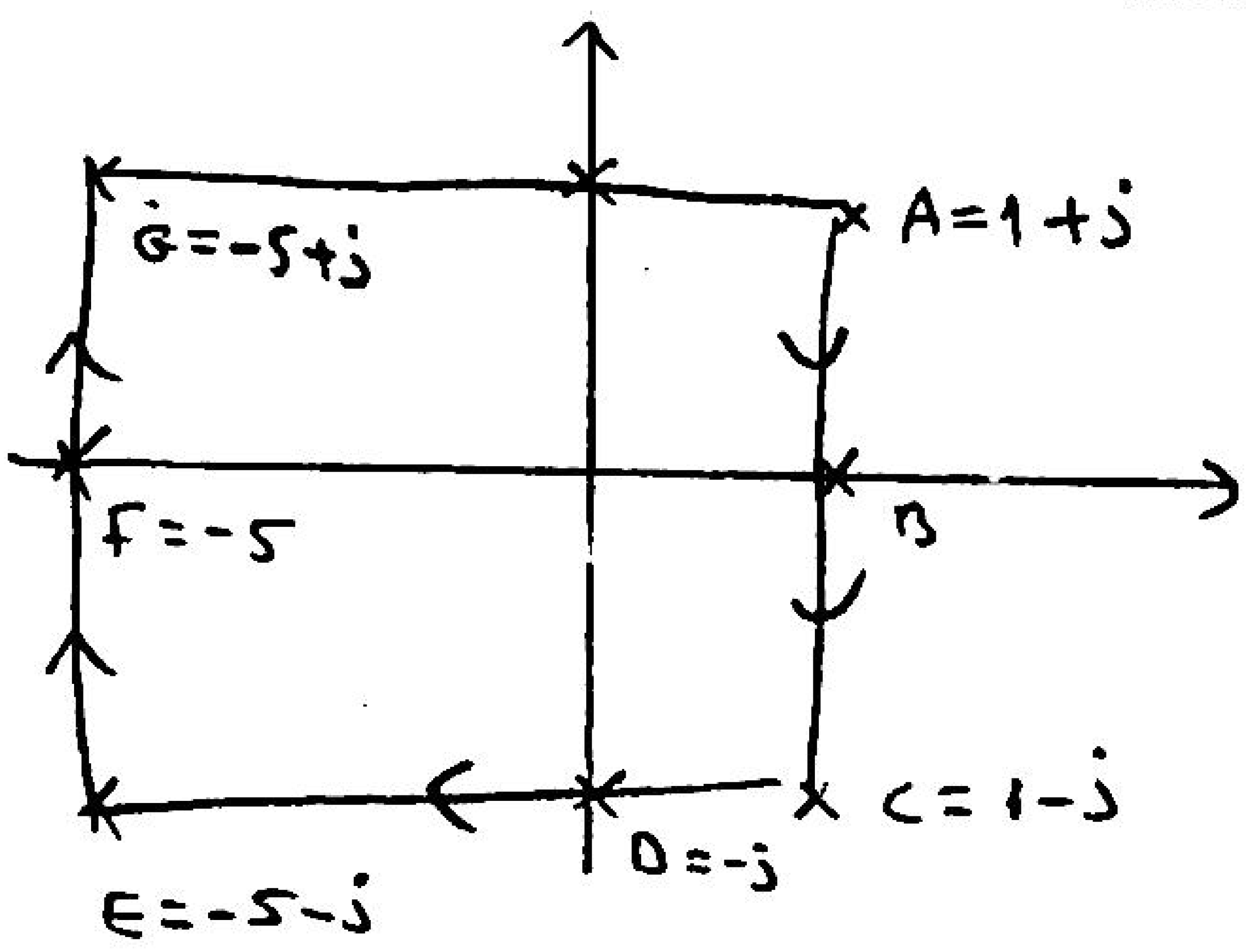


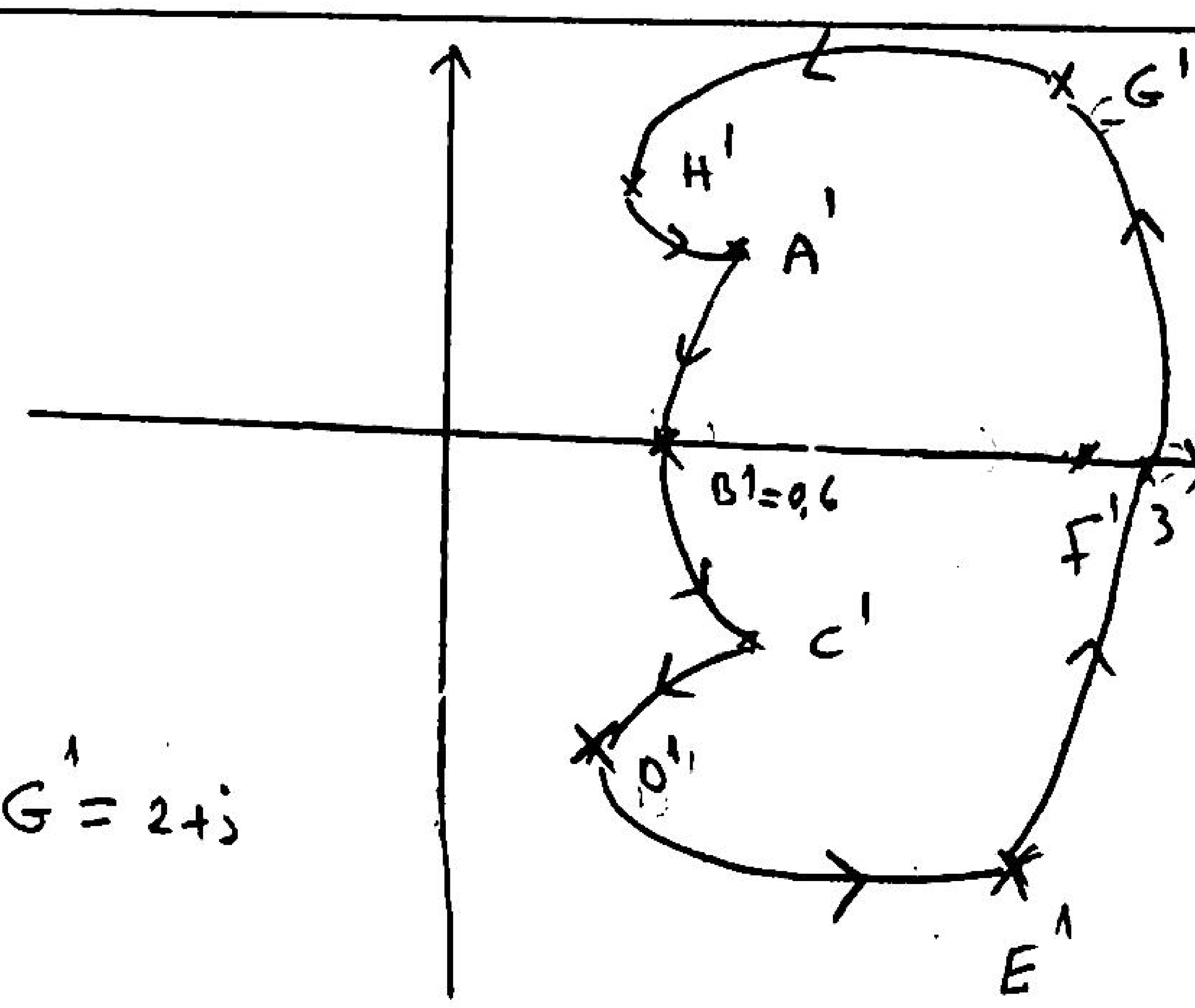


A', B', C', D', H' are same $E' = -j$ $F' = -1$ $G' = j$

---

c)

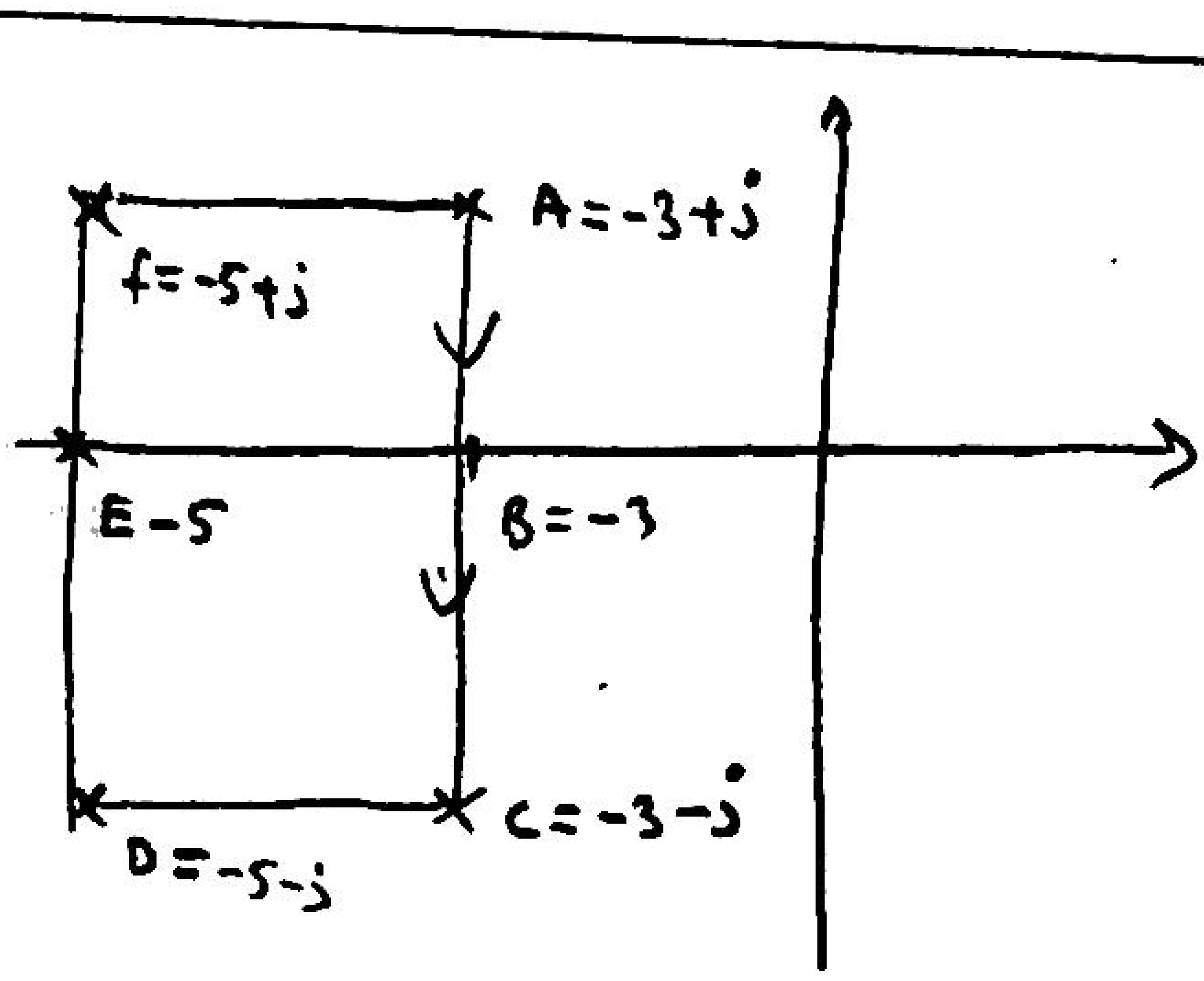




A', B', C', D', H' are same $E' = 2-j$ $F' = 3$ $G' = 2+j$

---

d)



$A' = j$

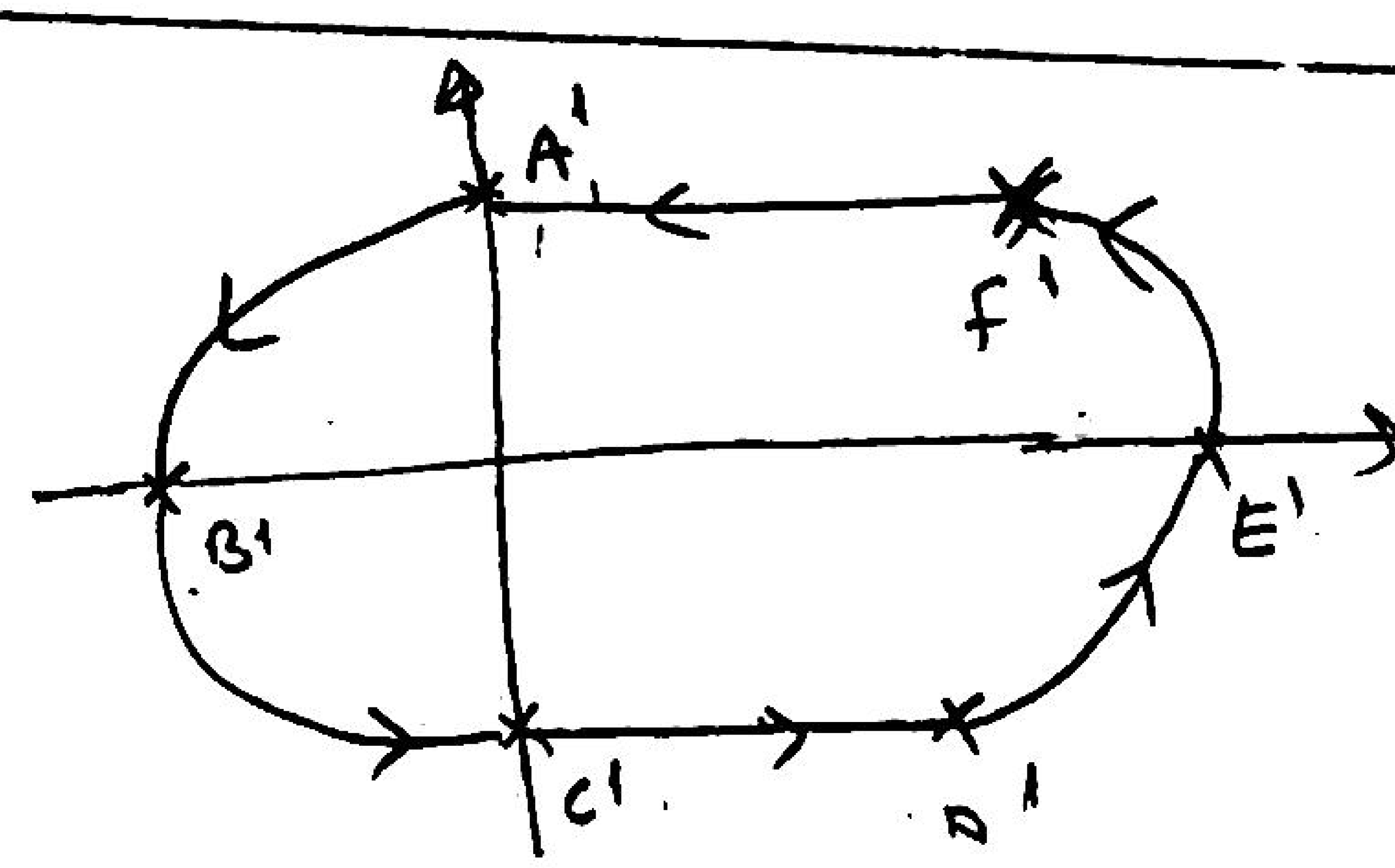
$B' = -1$

$C' = -j$

$D' = 2-j$

$E' = 3$

$F' = 2+j$



Cauchy theorem: Γ_s contour in s plane

Γ_F contour in F(s) plane

if Γ_s does not encircle any pole and zero
then Γ_F " " the origin. (case a)

if Γ_s encircles a zero in clockwise direction
then Γ_F " the origin " " (case b)

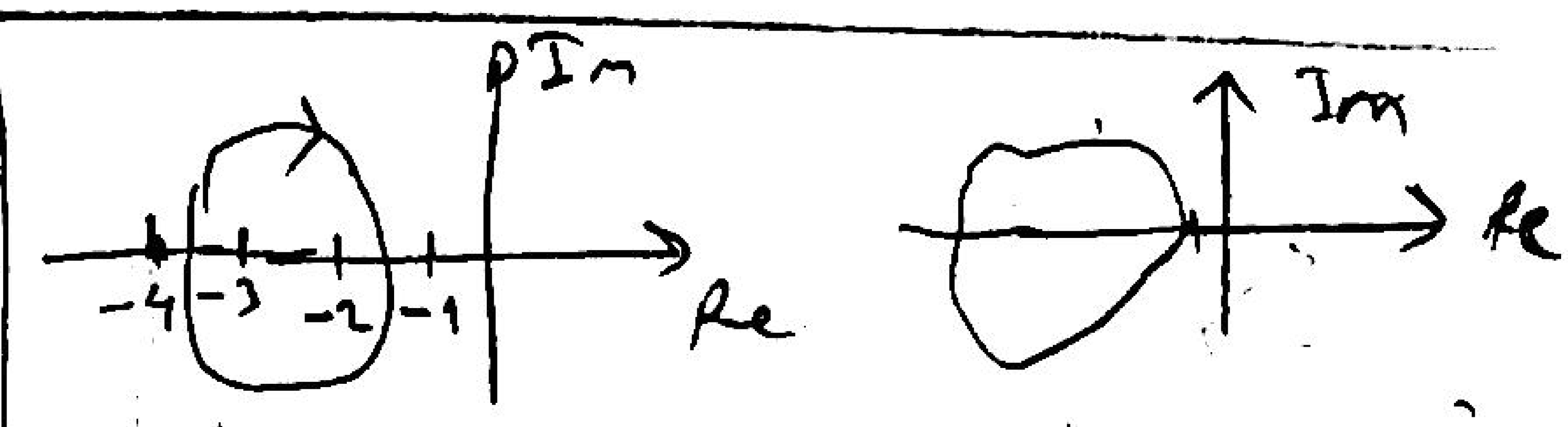
if Γ_s encircles a pole in clockwise direction
then Γ_F " the origin in anticlockwise " (case d)

---

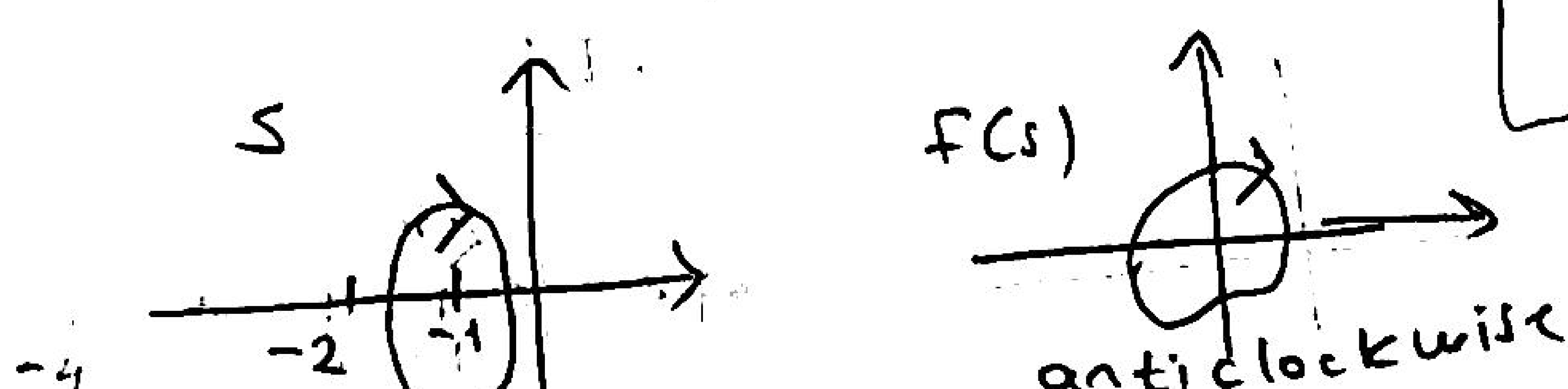
if Γ_s encircles Z zeros and P poles in clockwise direction
then Γ_F " the origin $N = Z - P$ times

if N is positive encirclement is clockwise
if N is negative " is anticlockwise
if $N = 0$ no encirclement (case c)

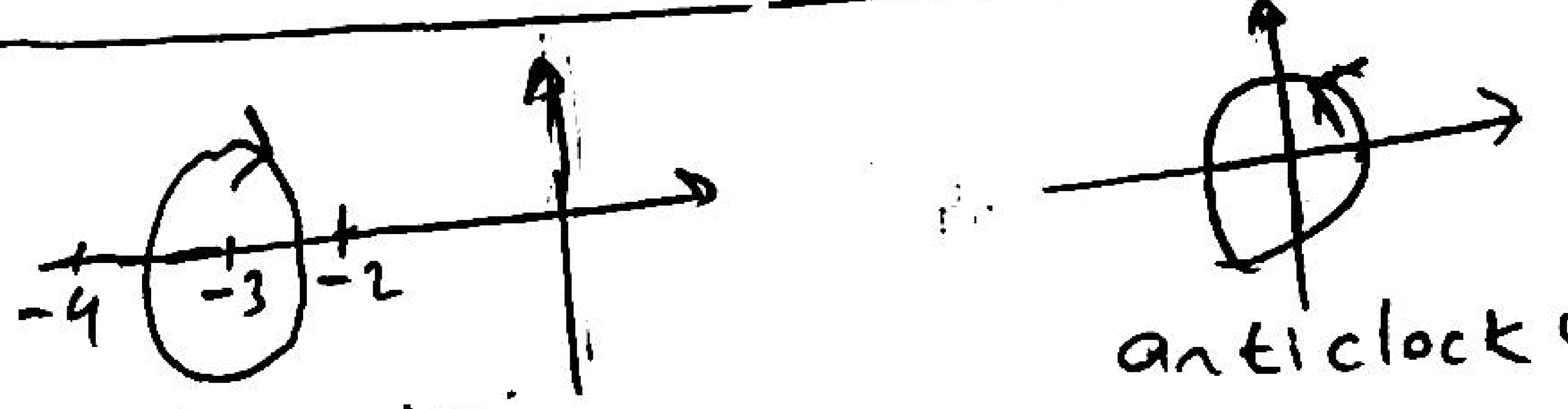
$$F(s) = \frac{(s+1)(s+2)}{(s+3)(s+4)}$$



$N = 1 - 1 = 0$

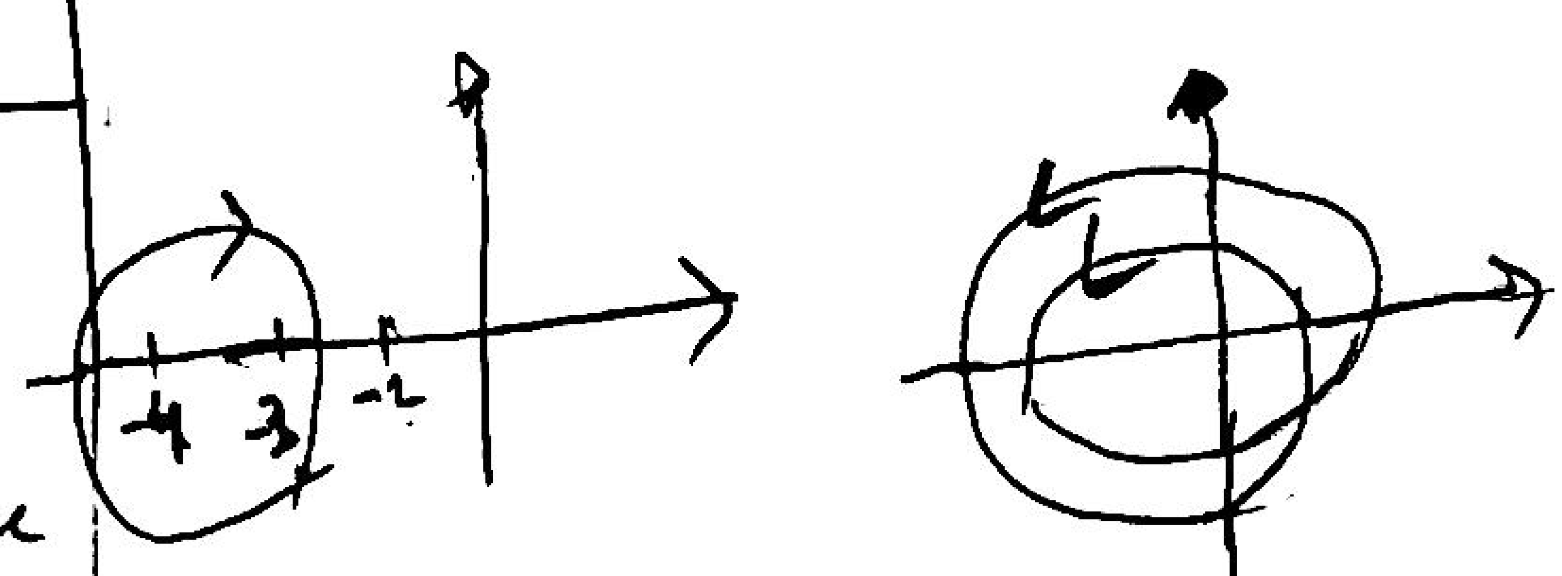


clockwise $N = 1 - 0 = 1$ anticlockwise



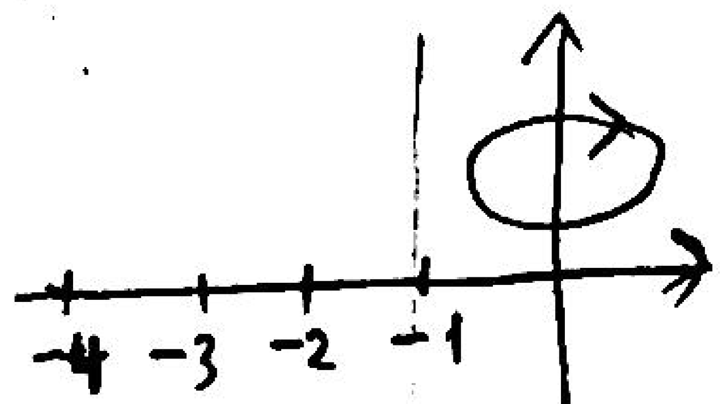
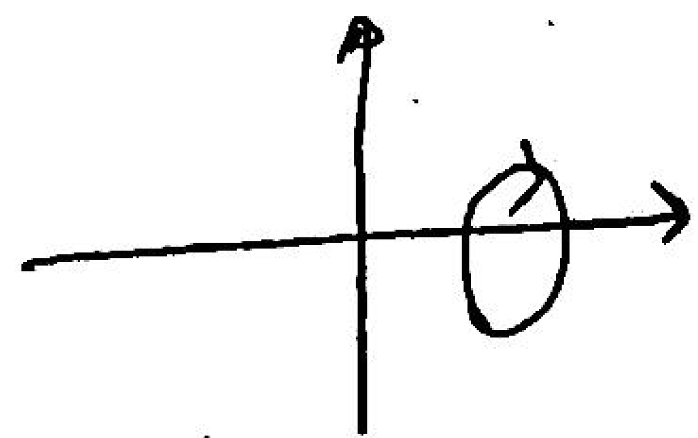
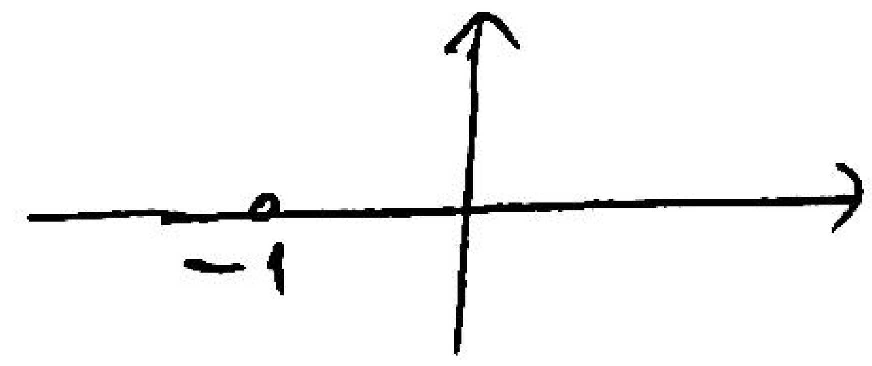
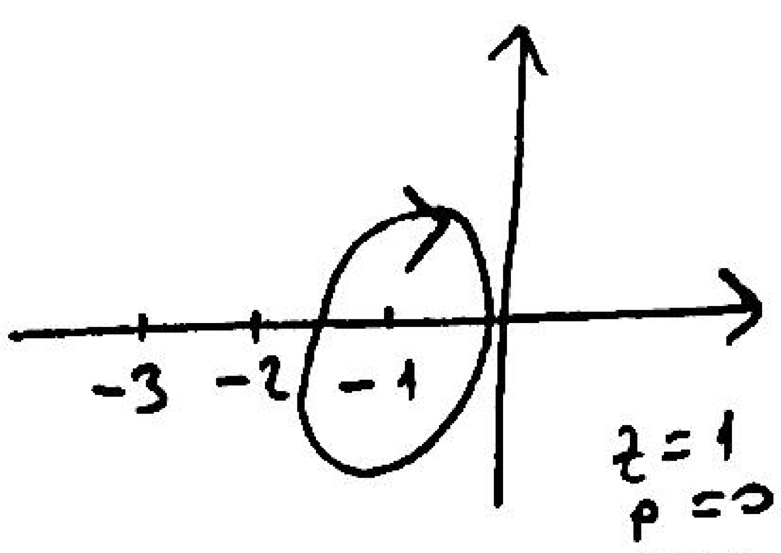
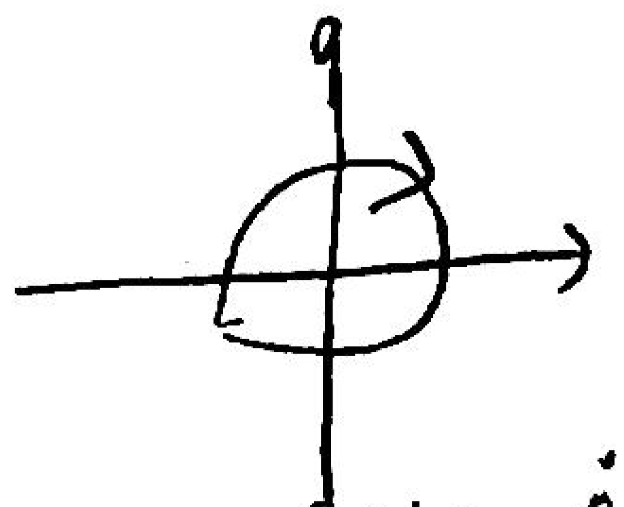
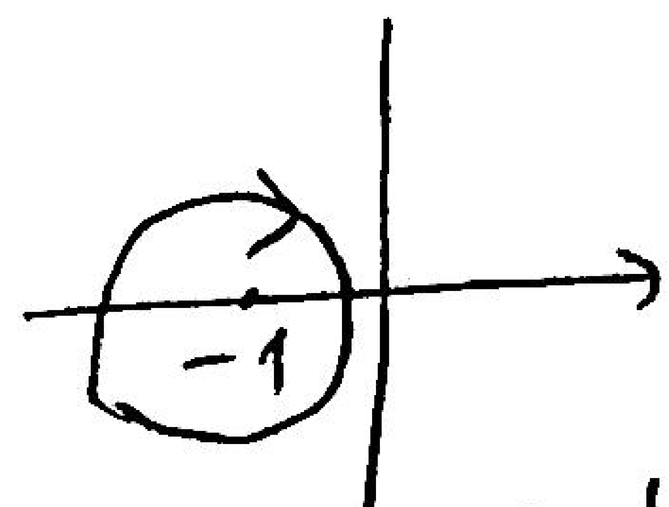
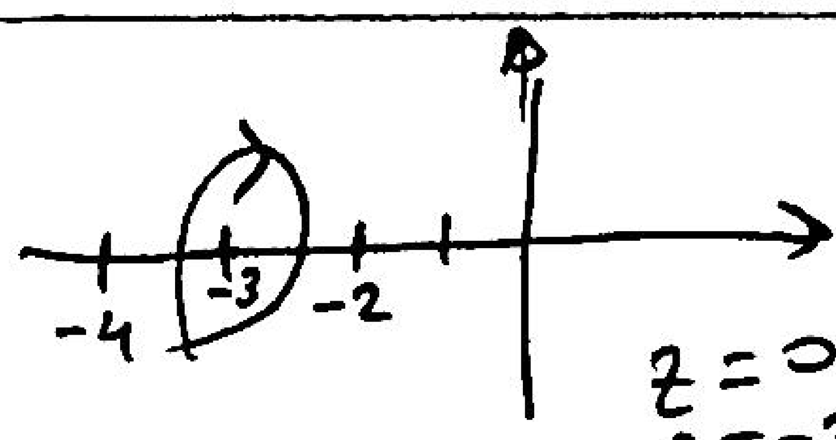
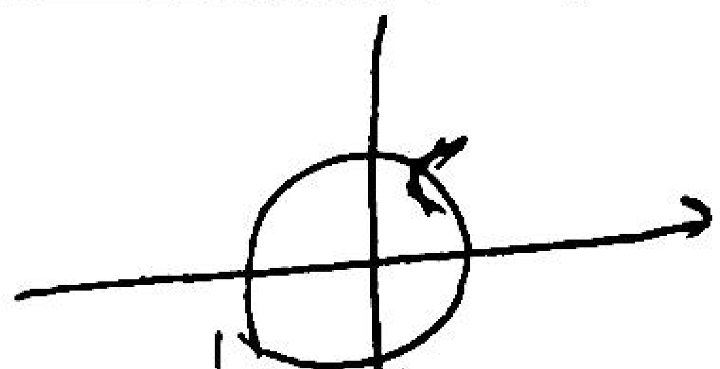
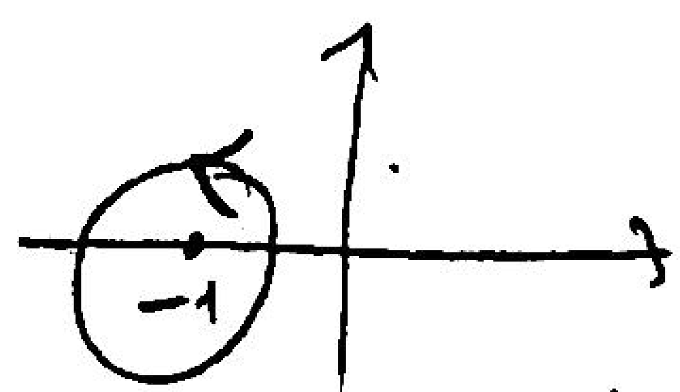
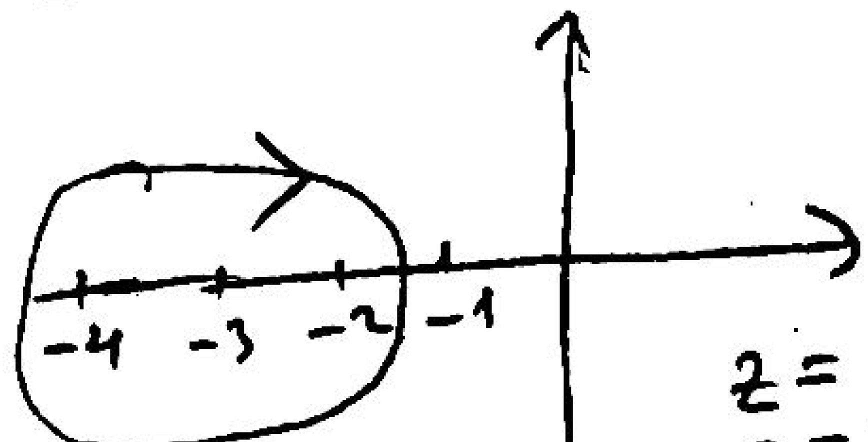
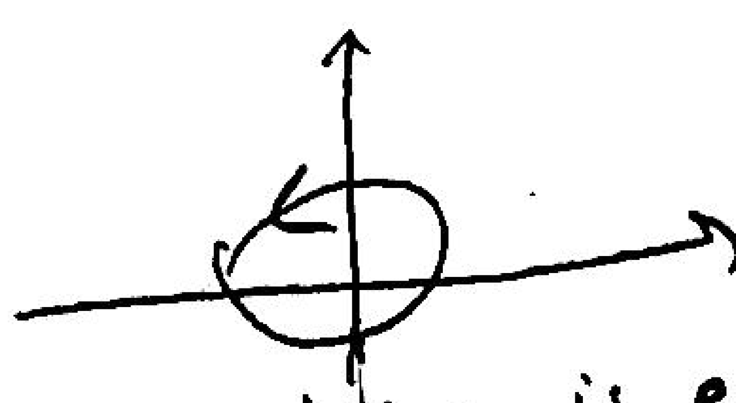
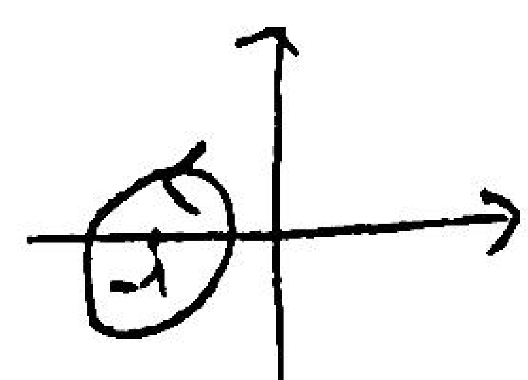
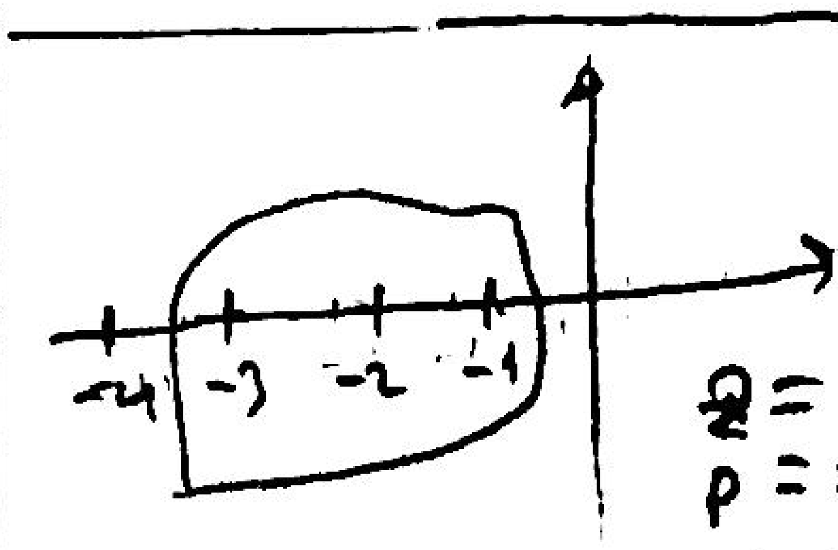
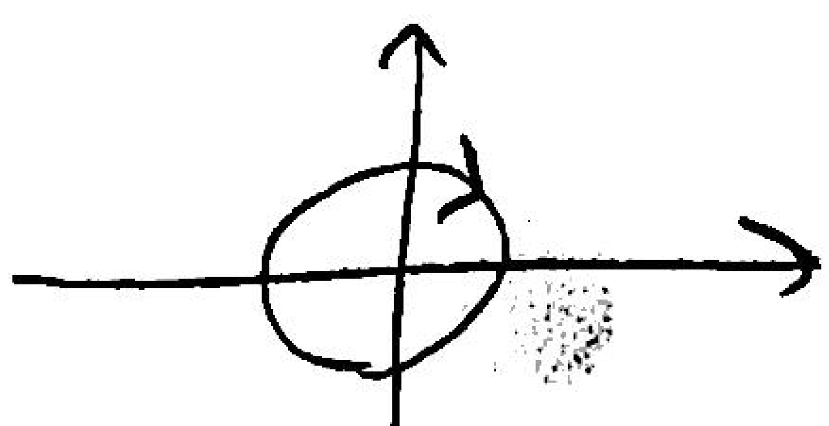
clockwise anticlockwise

$N = 0 - 1 = -1$



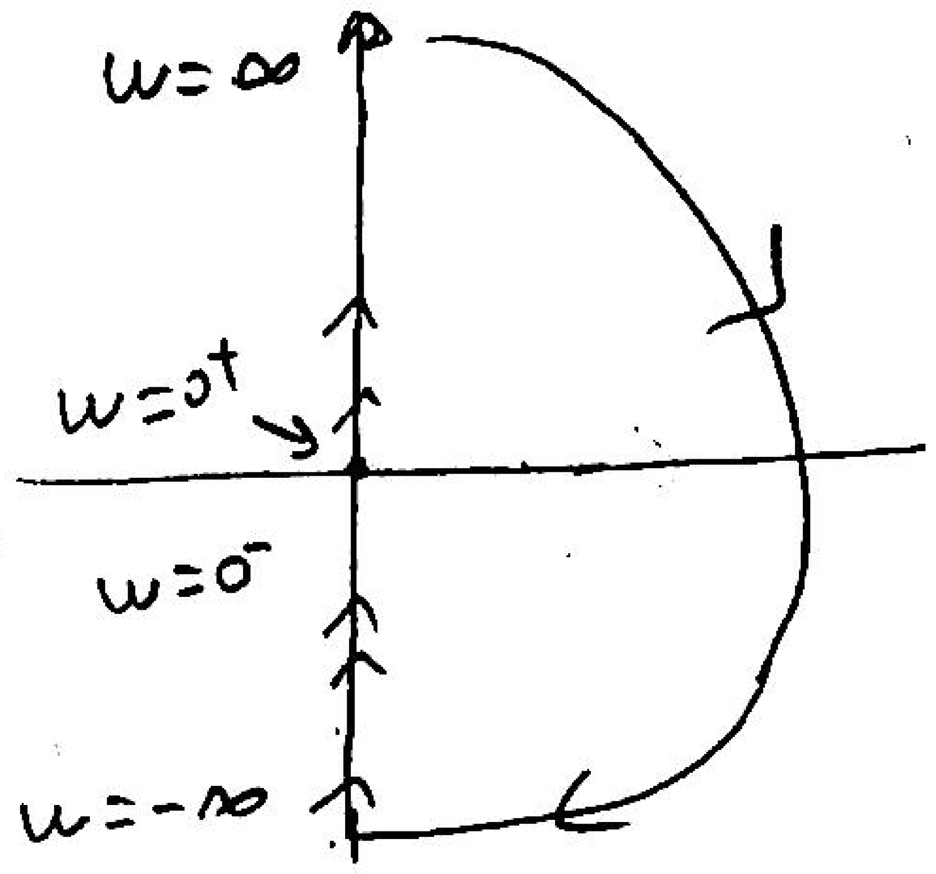
$N = 0 - 2 = -2$

$$F(s) = \frac{(s+1)(s+2)}{(s+3)(s+4)} \qquad T(s) = 1 + F(s) = 1 + \frac{(s+1)(s+2)}{(s+3)(s+4)}$$

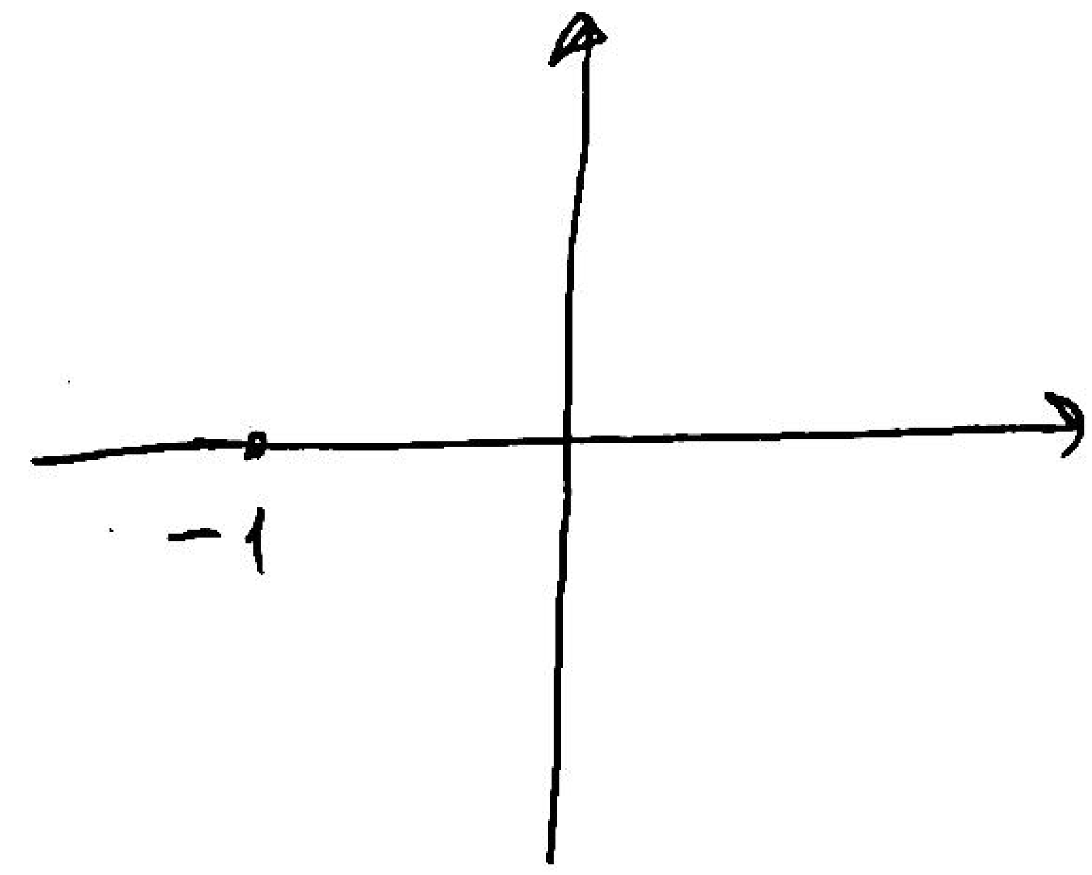
s plane	F(s) plane	T(s) plane
 $z=0$, $p=0$, $N=z-p=0$	 origin is not encircled	 point -1 is not encircled
 $z=1$, $p=0$, $N=z-p=1$	 origin is encircled once	 -1 is encircled once
 $z=0$, $p=-2$, $N=-2$	 origine is encircled once (anticlockwise)	 -1 is encircled once (anticlockwise)
 $z=1$, $p=2$, $N=1-2=-1$	 origine is encircled once (anticlockwise	 -1 is encircled once (anticlockwise)
 $z=2$, $p=1$, $N=2-1=1$	 origine is encircled once (clock wise)	 -1 is encircled once (clockwise)

Point 0 in F(s) plane $\equiv$ Point -1 in T(s) plain

NYQUIST STABILITY CRITERIA



splane



$1+f(s)=1+KG(s)$
plane

if there is no pole or zero in the right half plane then no encirclement of the point -1.

$$\frac{Y}{R}=\frac{KG(s)}{1+KG(s)} \qquad G(s)=\frac{A(s)}{B(s)}$$

$G(s)$ = open loop system

open loop zeros = roots of $A(s)$

open loop poles = roots of $B(s)$

$$\frac{KG(s)}{1+KG(s)}=\frac{KA(s)/B(s)}{1+KA(s)/B(s)}=\frac{KA(s)}{B(s)+KA(s)}$$

$$1+KG(s)=1+KA(s)/B(s)=\frac{B(s)+KA(s)}{B(s)}$$

closed loop poles = roots of $B(s)+KA(s)$

= zeros of $1+KG(s)$

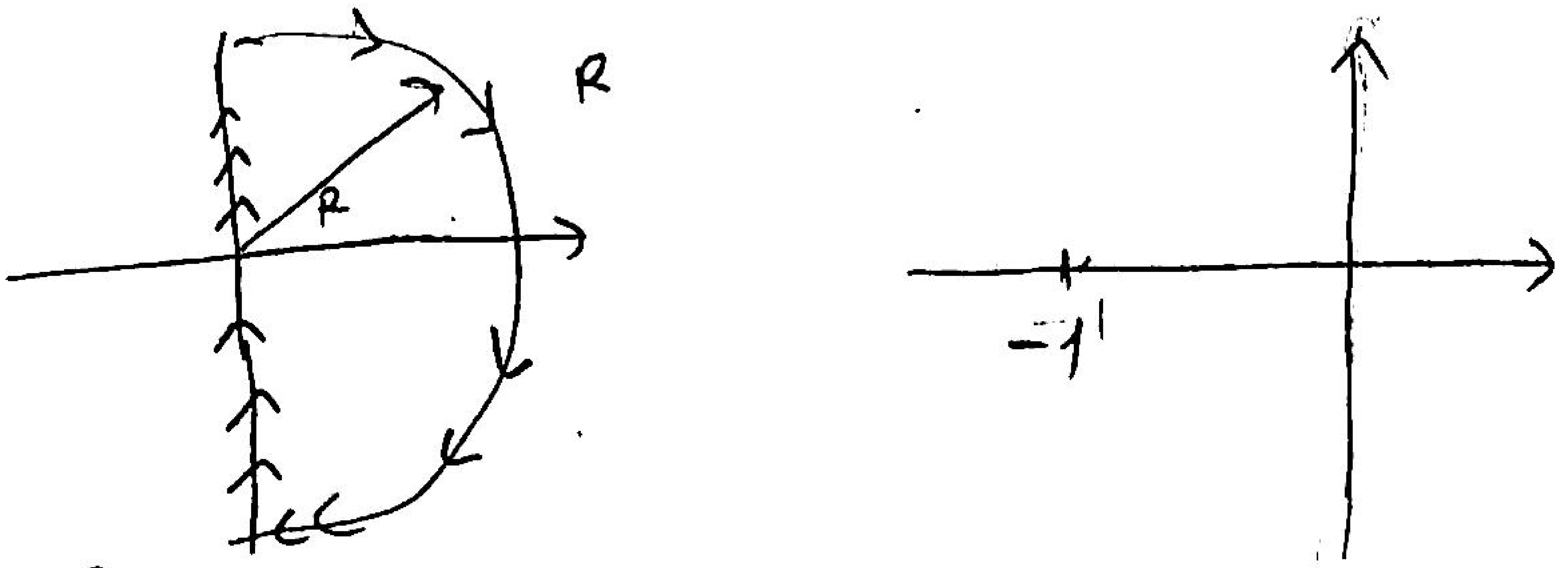
Important – closed loop poles = zeros of $1+KG(s)$

Z_c = Number of closed loop poles in the right half plane.

= Number of unstable closed loop poles)

P_c = Number of unstable roots of $B(s)$

we draw s to 1+F(s) contour mapping



R is very large

N = number of encirclement of the point -1

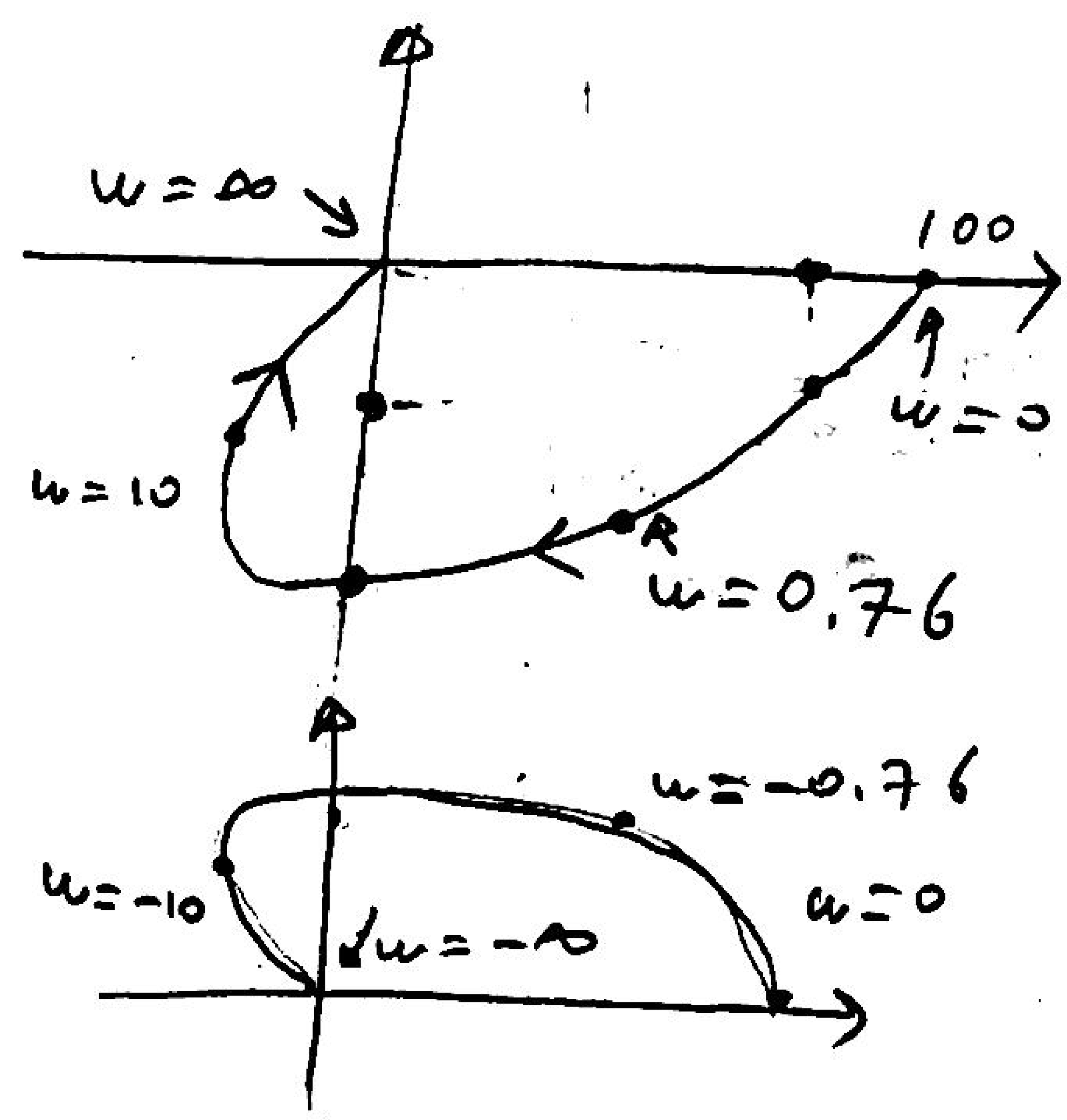
$$N = Z_c - P_c$$

$$Z_c = N + P_c$$

if $Z_c = 0$ the closed loop system is stable

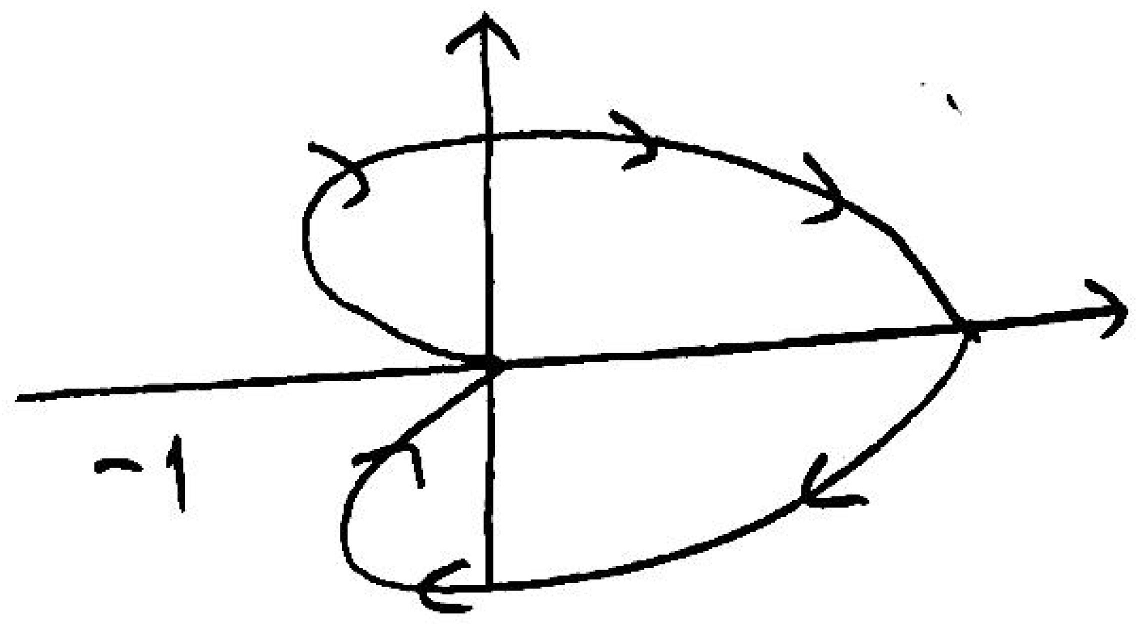
Example: $G(s) = \dfrac{1000}{(s+1)(s+10)}$ $G(j\omega) = \dfrac{1000}{(j\omega+1)(j\omega+10)}$

ω	$G(j\omega)$	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$	Re $G(j\omega)$	Im $G(j\omega)$
0		100	0		
0.1		96	-5.7		
0.76		79.6	-41.5		
1		70.7	-50.7		
2		50.2	-74.7		
10		6.8	-129		
20		2.24	-150		
100		0.1	-173		
∞		0	-180		
$-\infty$		0	+180		
-100		0.1	+173		
-20		2.24	+150		
-10		6.8	+129		
-2		50.2	+74		
-1		70.7	+50.7		
-0.76		79.6	+41.5		
-0.1		96	+5.7		
-0		100	0		



Note $\omega = 0$ to $\omega = \infty$ is symmetrical to $\omega = 0$ to $\omega = -\infty$

$$G(s) = \frac{1000}{(s+1)(s+10)}$$



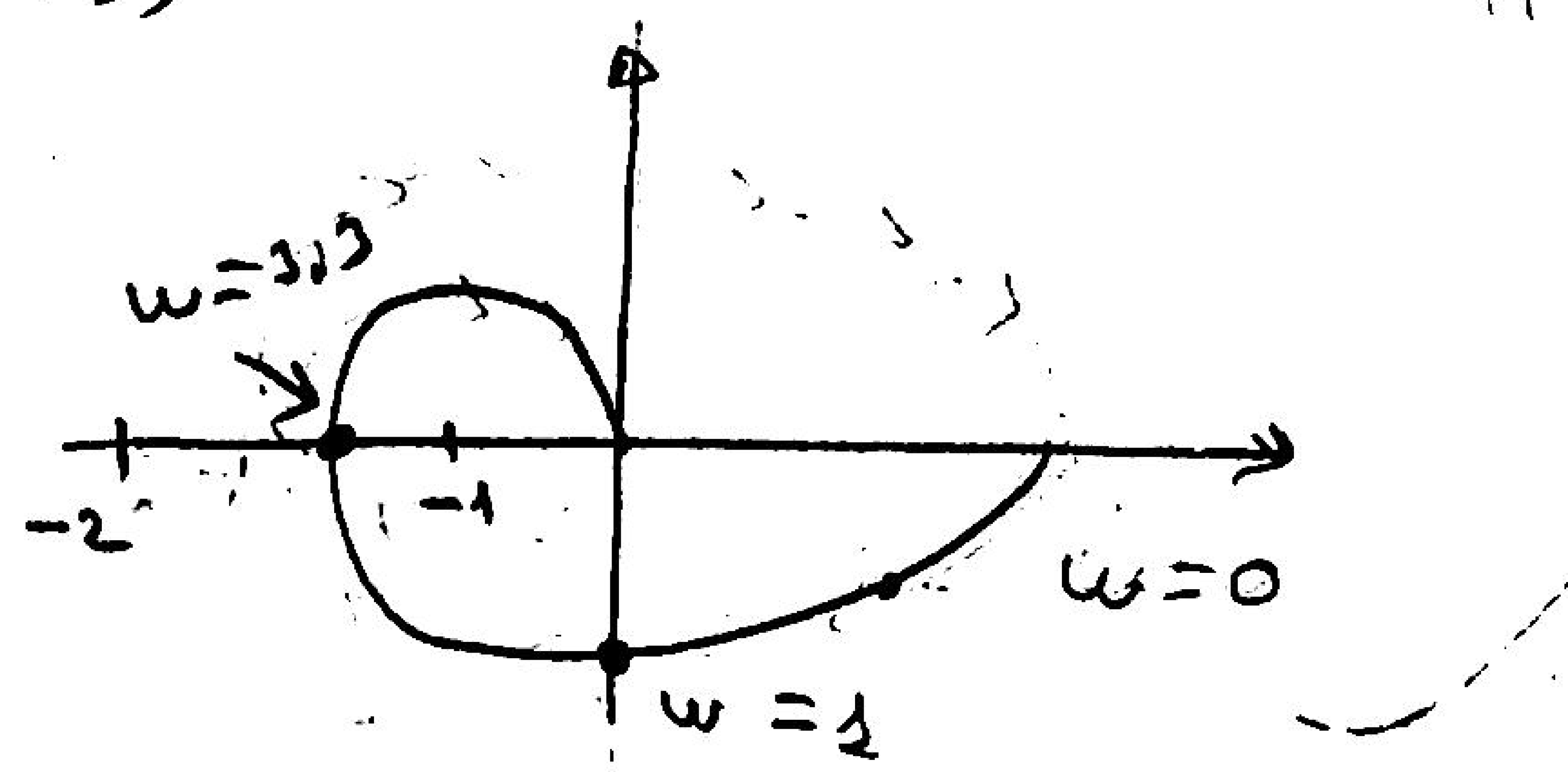
$P_c = 0$ (no unsteble poles of $G(s)$)

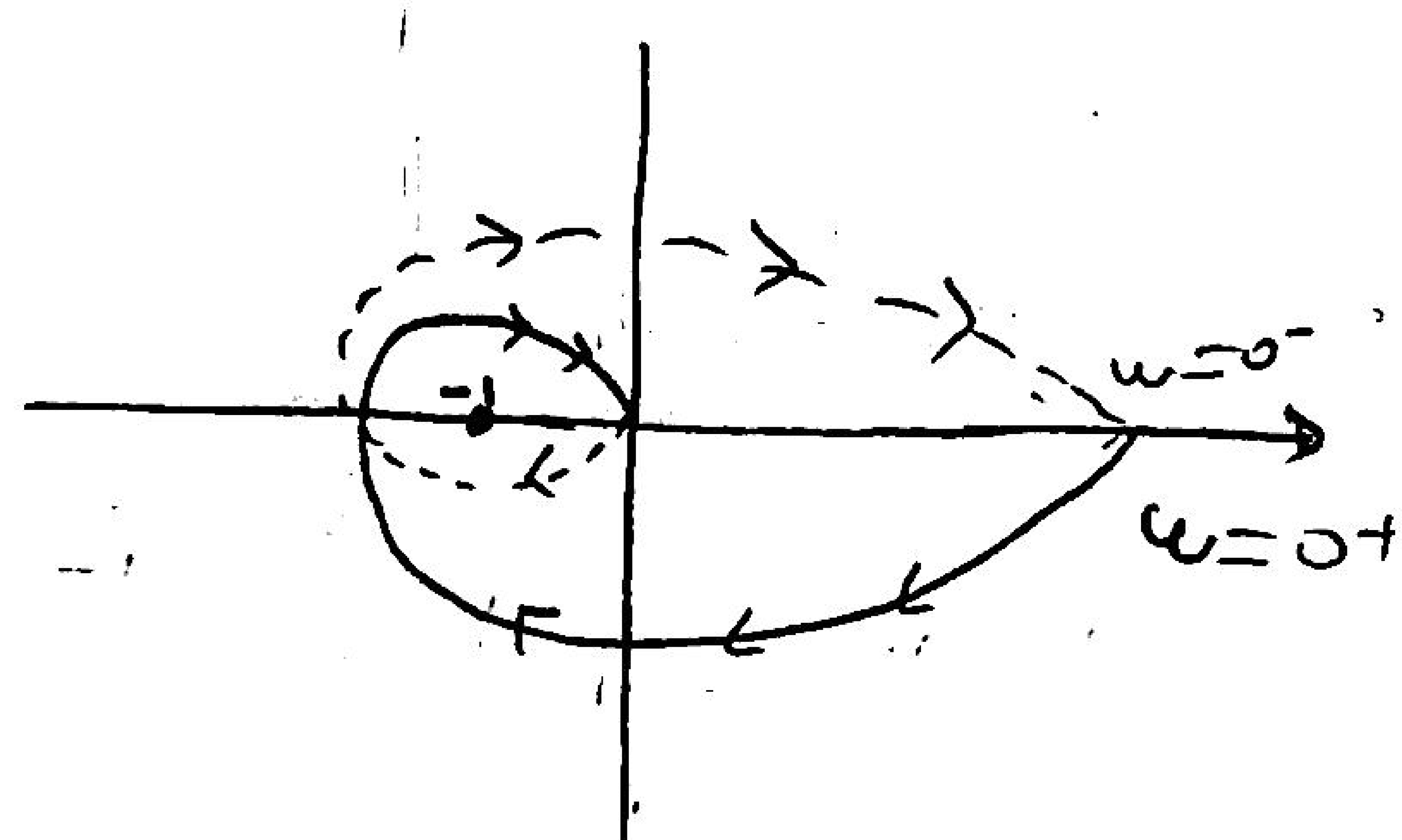
$N = 0$ The point -1 is not encircled

$Z_c = N + P_c = 0 + 0$ The closed loop system does not have poles in the right half plane
The system is stable.

Example $G(s) = \dfrac{100}{(s+1)(s+2)(s+3)}$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$	Re	Im
0	16.6	0	16.6	0
0.1	16.5	-10	16.2	-3
0.9	10.8	-82	1.3	-10
1	10	-90	0	-10
3.3	1.68	-179	-1.6	-0.01
3.4	1.57	+178	-1.57	0.04
5	0.62	154	-0.50	0.27
30	0.032	101	-0.0007	0.03
100	0.0000001	93		
1000	0	90°	0	0.00001
∞	0	90°	0	0





$P_c = 0$ G(s) has no unstable roots (Open loop system is stable)

$N = 2$

-1 is encircled twice (clockwise direction)

$Z_c = N - P_c = 2$ there are two unstable roots for the closed loop system.

$$G(s) = \frac{100}{(s+1)(s+2)(s+3)} = \frac{100}{s^3+6s^2+11s+6}$$

$$\frac{G(s)}{1+G(s)} = \frac{100}{s^3+6s^2+11s+106}$$

$$s^3+6s^2+11s+106=0$$

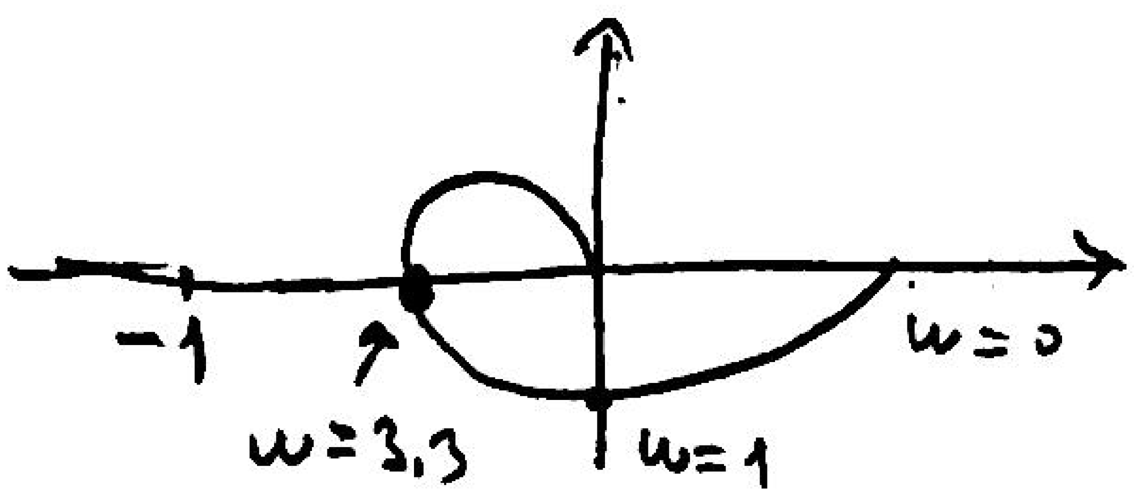
$s_1 = -6.71$

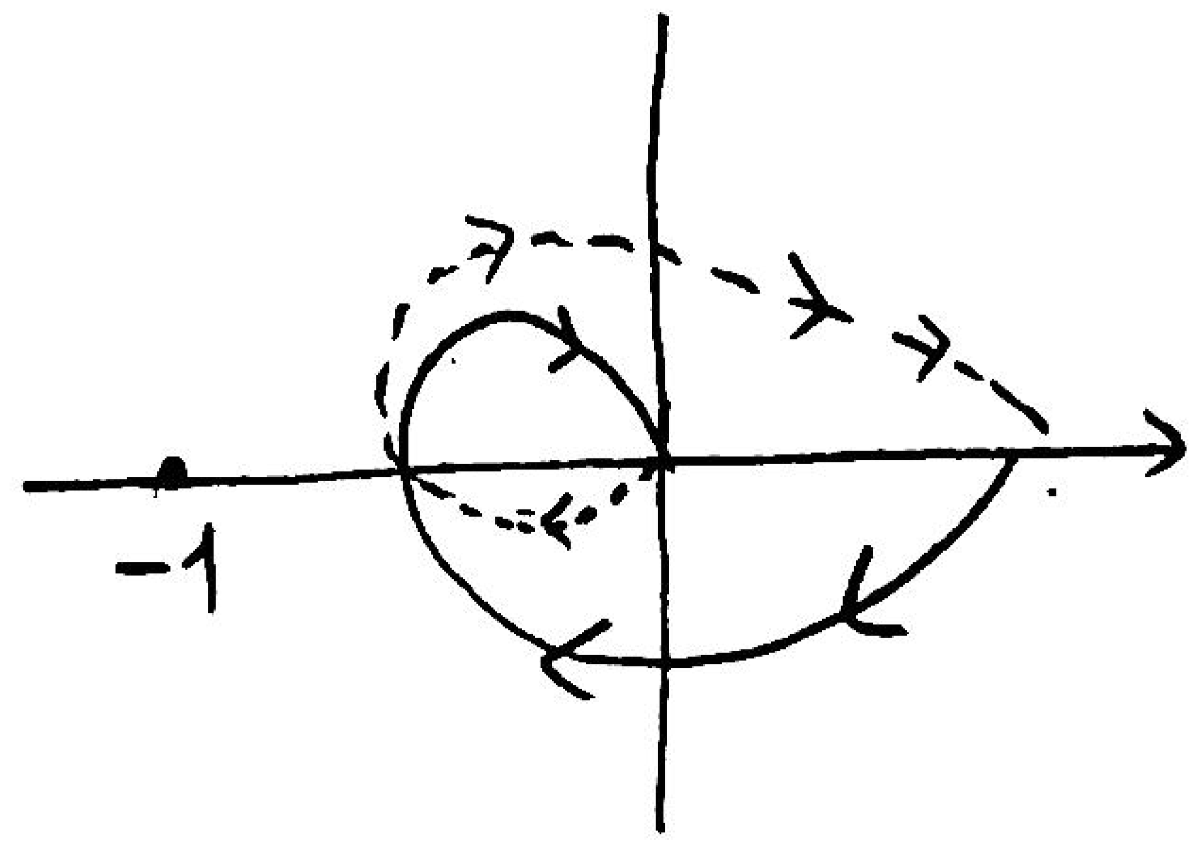
$s_2 = 0,35 + 3.95j$

$s_3 = 0,35 - 3.95j$

unstable roots

Exomple $G(s) = \dfrac{10}{(s+1)(s+2)(s+3)}$



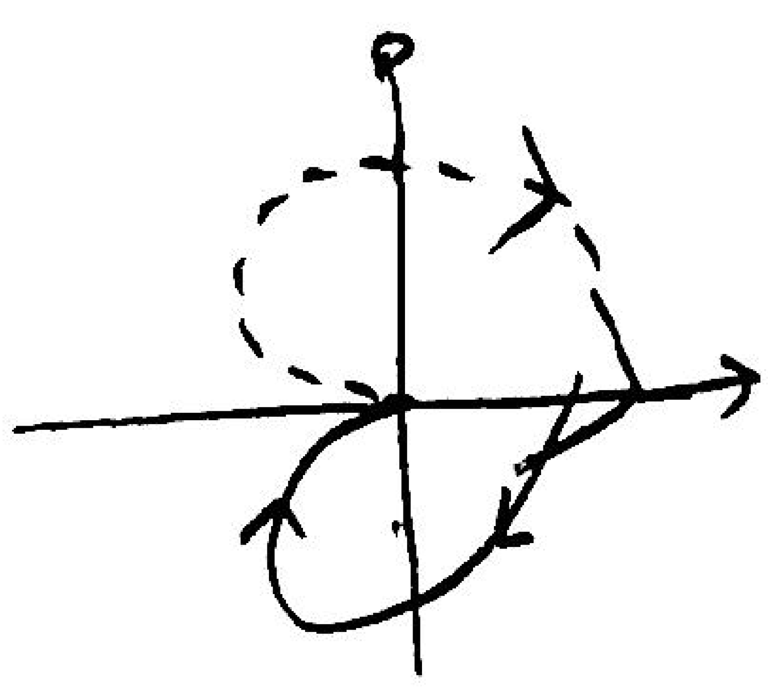


$P_c = 0$

$N = 0$

$Z_c = N + P_c = 0$ stable

Example $G(s) = \dfrac{10(s+4)}{(s+1)(s+2)\ s+3}$

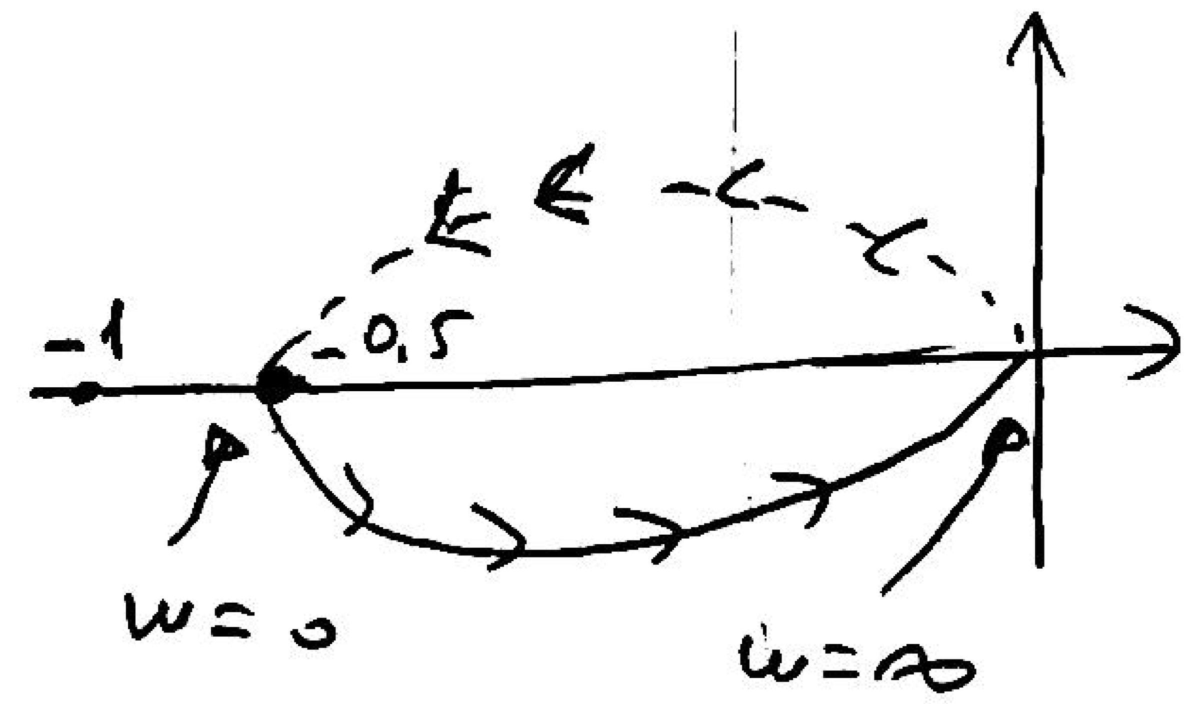


stable

$$G(s) = \frac{1}{(s-1)(s+2)}$$

35²

w	$\|G(jw)\|$	$\angle G(jw)$	
0	0.5	-180	-0.5
0.1	0.496	-177	-0.496 + 0.02j
0.2	0.48	-174	-0.48 - 0.076j
1	0.31	-164	-0.3 - 0.1j
2	0.15	-161	-0.15 - 0.05
10	0.0098	-174	-0.097 + 0.001
1000	10^{-7}	-179	$10^{-7} + 10^{-8}j$



$Z_c = N + P_c$

$P_c = 1$ (1 unstable open loop pole)

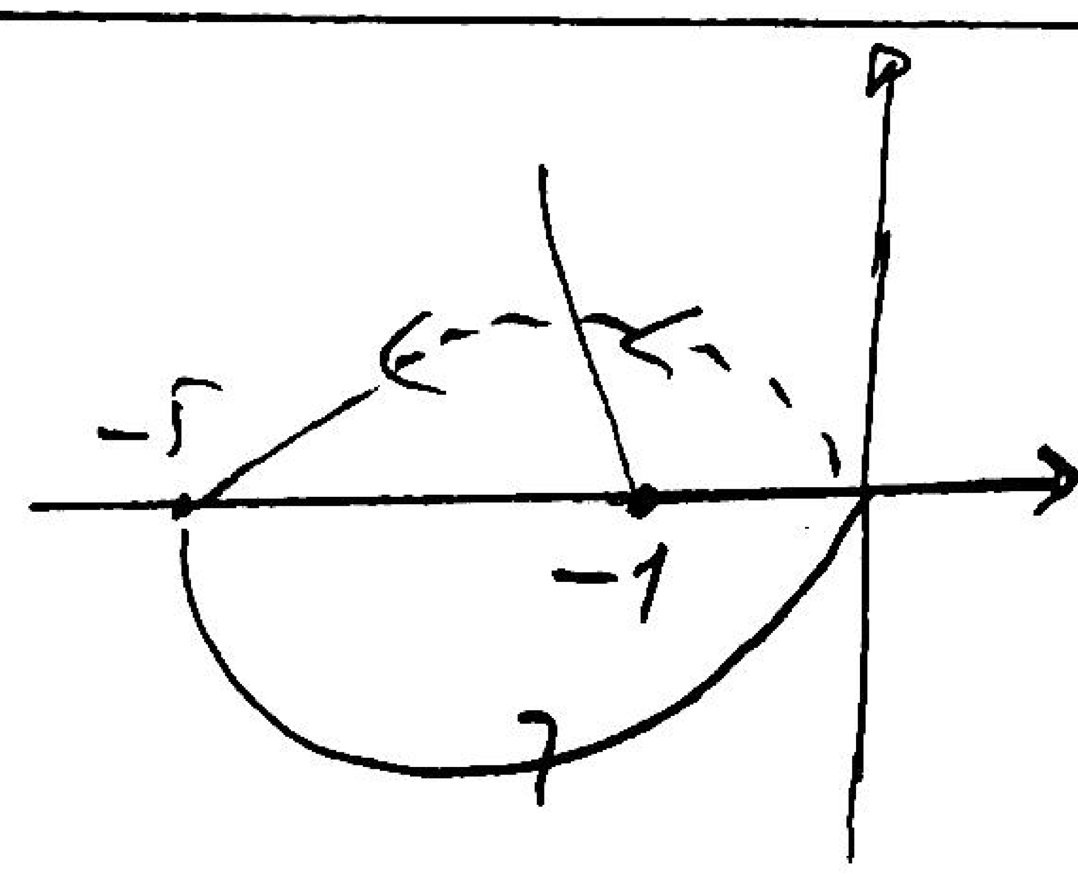
N = 0 (no encircelement)

$Z_c = N + P_c = 0 + 1$ 1 unsteble root

<u>Unstable</u>

$$G(s) = \frac{10}{(s-1)(s+2)}$$

w	$\|G(jw)\|$	$\angle G(jw)$	Re	Im
0	5	-1.8	-5	0
0.1	4.96	-177	-4.9	-0.2
0.2	4.87	-174	-4.8	0.4
1	3.16	-161	-3	1
2	1.58	-161	-1.5	0.5
10	0.09	-174	-0.0	0.0085



$P_c = 1$ (1 unstable root

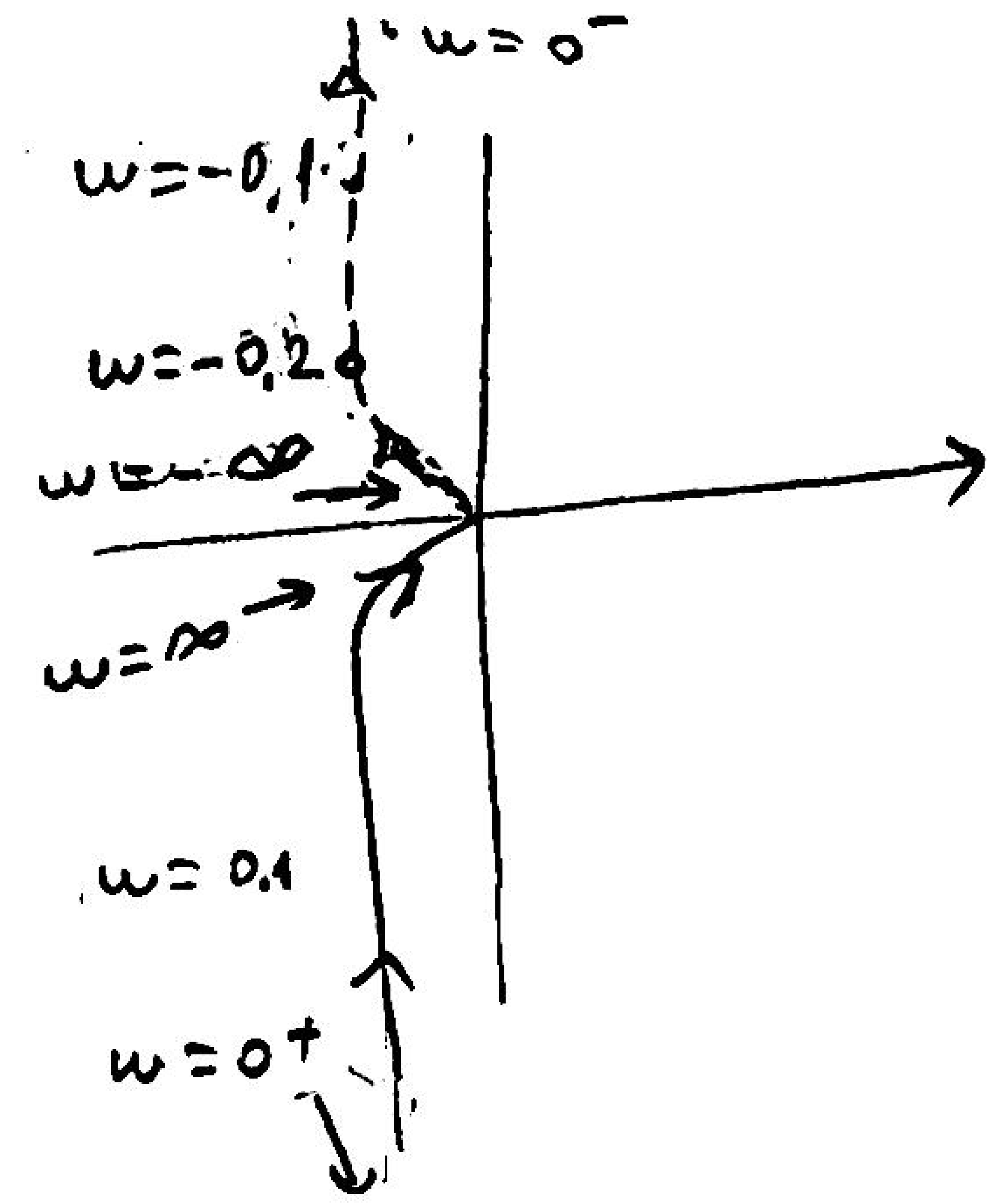
N = -1 (counterclockwise encirclement

$Z_c = N - P = 1 - 1 = 0$

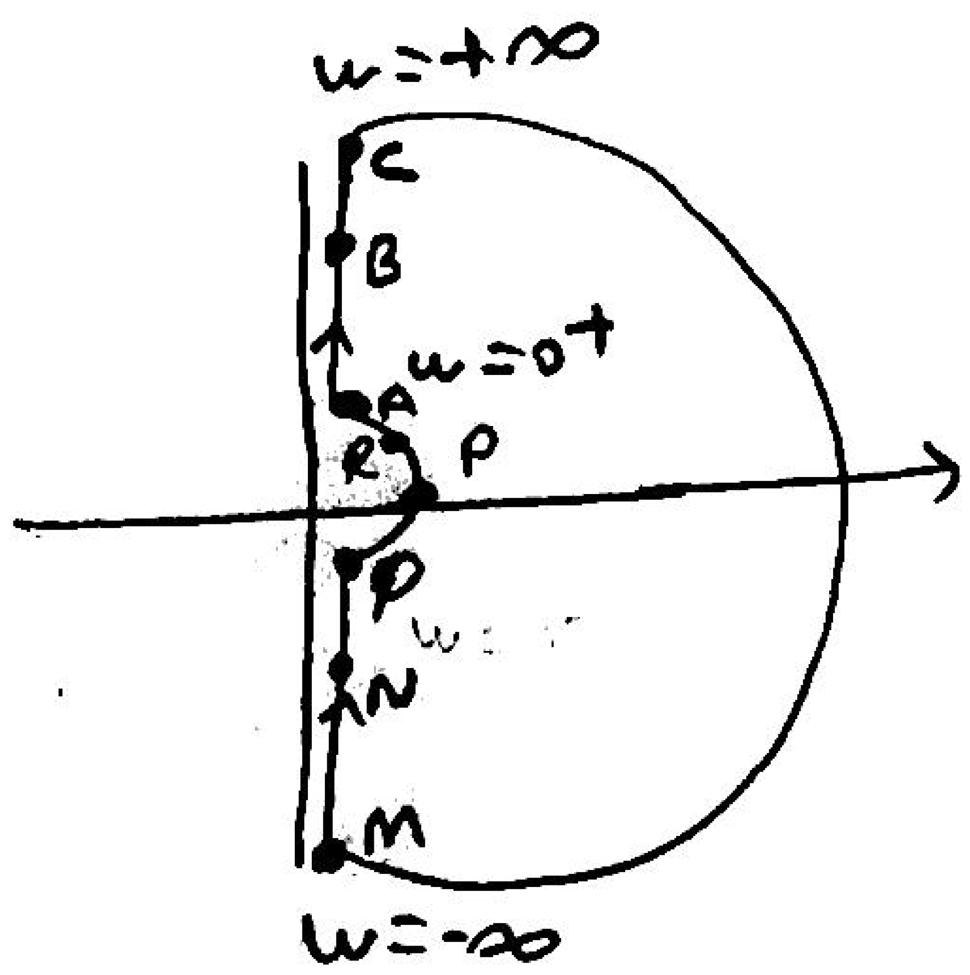
Stable

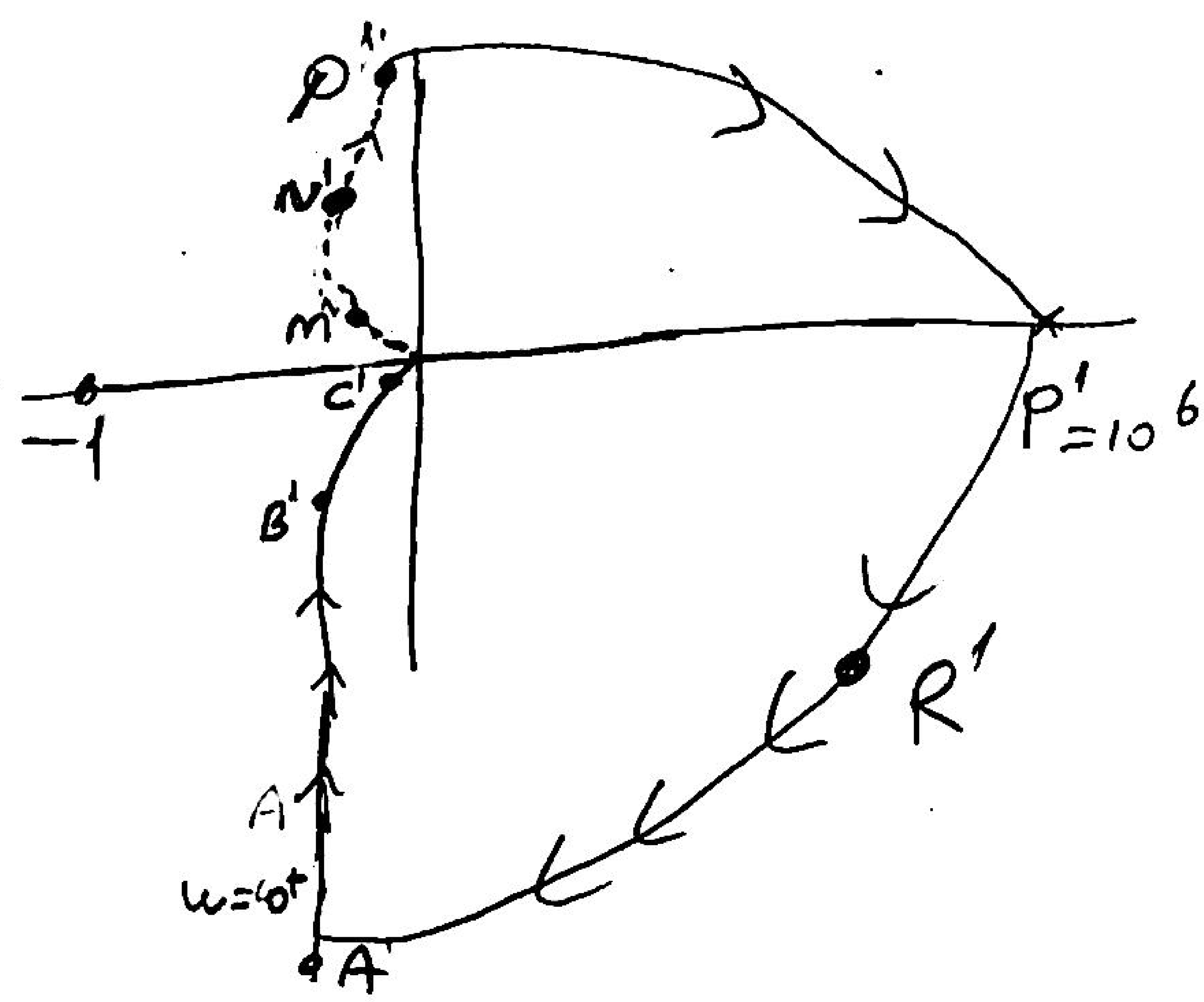
$G(s) = \frac{10}{s(s+10)}$ $\quad |G(j\omega)| = \frac{10}{\omega\sqrt{\omega^2+10^2}}$ $\quad \angle G(j\omega) = -90 - \tan^{-1}\frac{\omega}{10}$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
0	∞	−90
0,1	9,9	−90,5
0,2	5	−91
1	0,99	−95
10	0,07	−135
1000	0,001	−174
∞	0	−180



Question How can we join $\omega = 0^+$ to $\omega = 0^-$





Take point $P = 0,00001$

$$G(s) = \frac{10}{s(s+10)} = \frac{10}{0,00001(0,00001+1)}$$

$$= 10^6$$

Take point $R = 0,0001 + 0.0001j$

$$G(s) = \frac{10}{(0,0001 + 0,0001j)(0,00001 + 0.0001 + 1)}$$

$$R' = 50000 - 50000j$$

$Z_c = N + P_c$

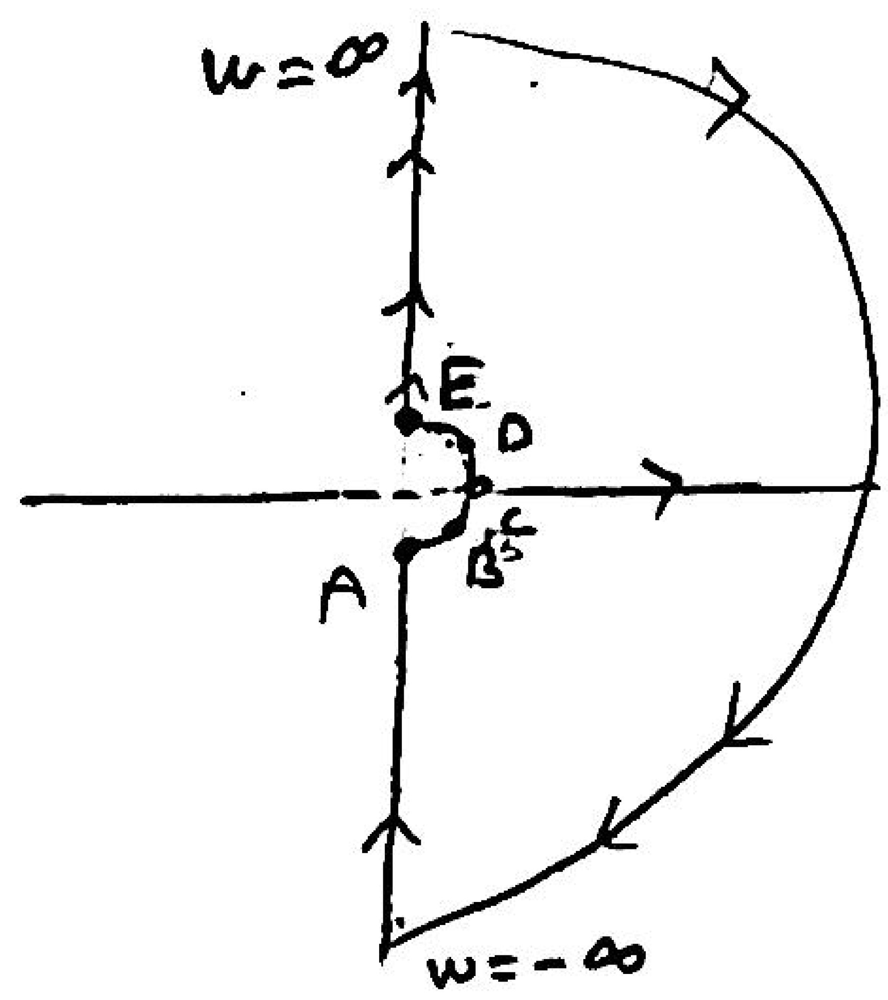
$P_c = 0$

$N = 0$

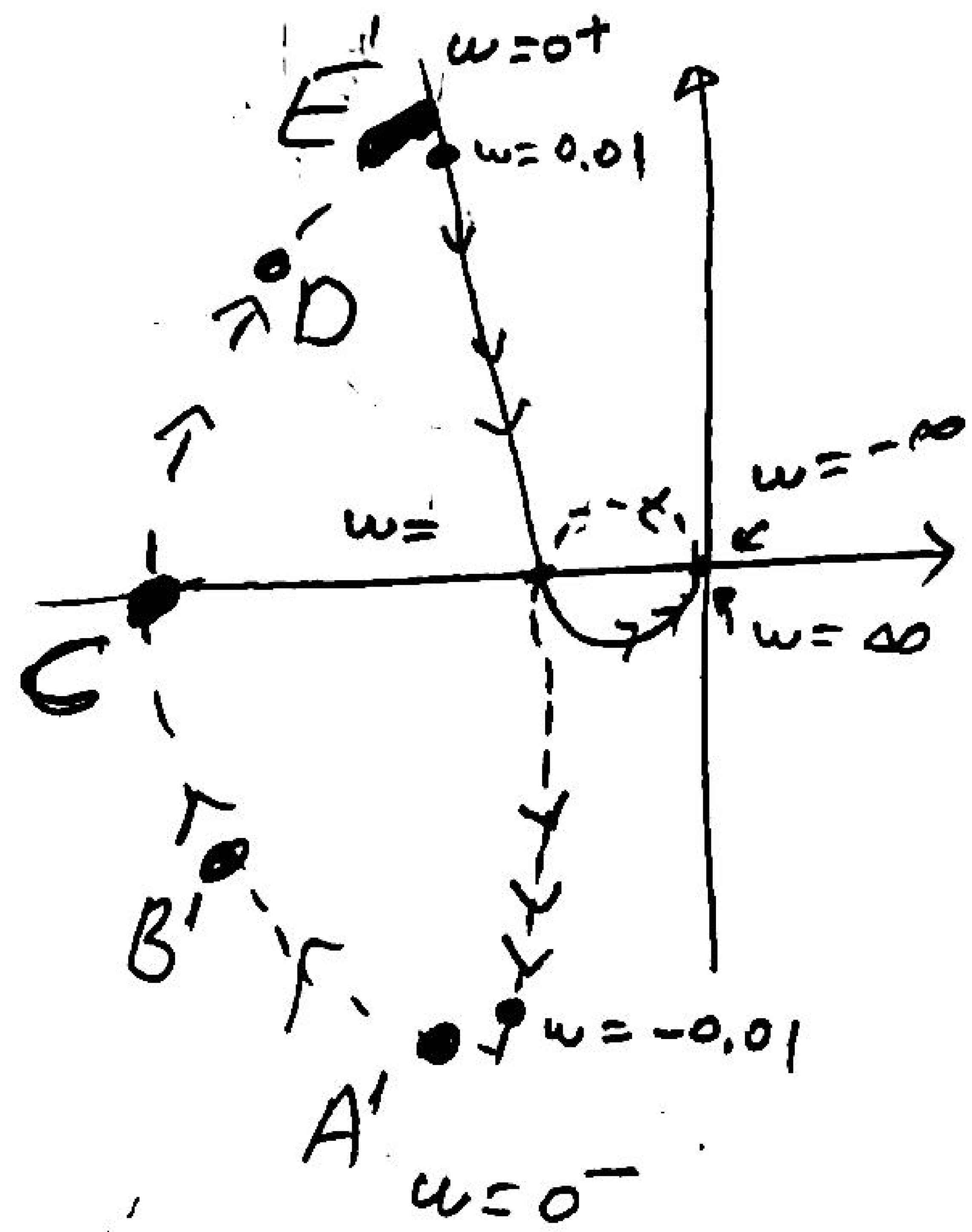
$Z_c = 0 + 0 = 0$

Stable

Example: $G(s) = \dfrac{k(s+2)}{s(s-1)}$



s plane



$A \to s = -0.0001j \qquad G(s) = \dfrac{k(-0.0001j)}{(-0.0001j)(-0.0001j+1)} = -3 - 20000$

$B \to s = 0.0001j - 0.0001j \qquad G(s) = -10000 - 10000j$

$C \to s = 0.0001 \qquad G(s) = -20000$

$D \to s = 0.0001 + 0.0001j \qquad G(s) = -10000 + 10000j$

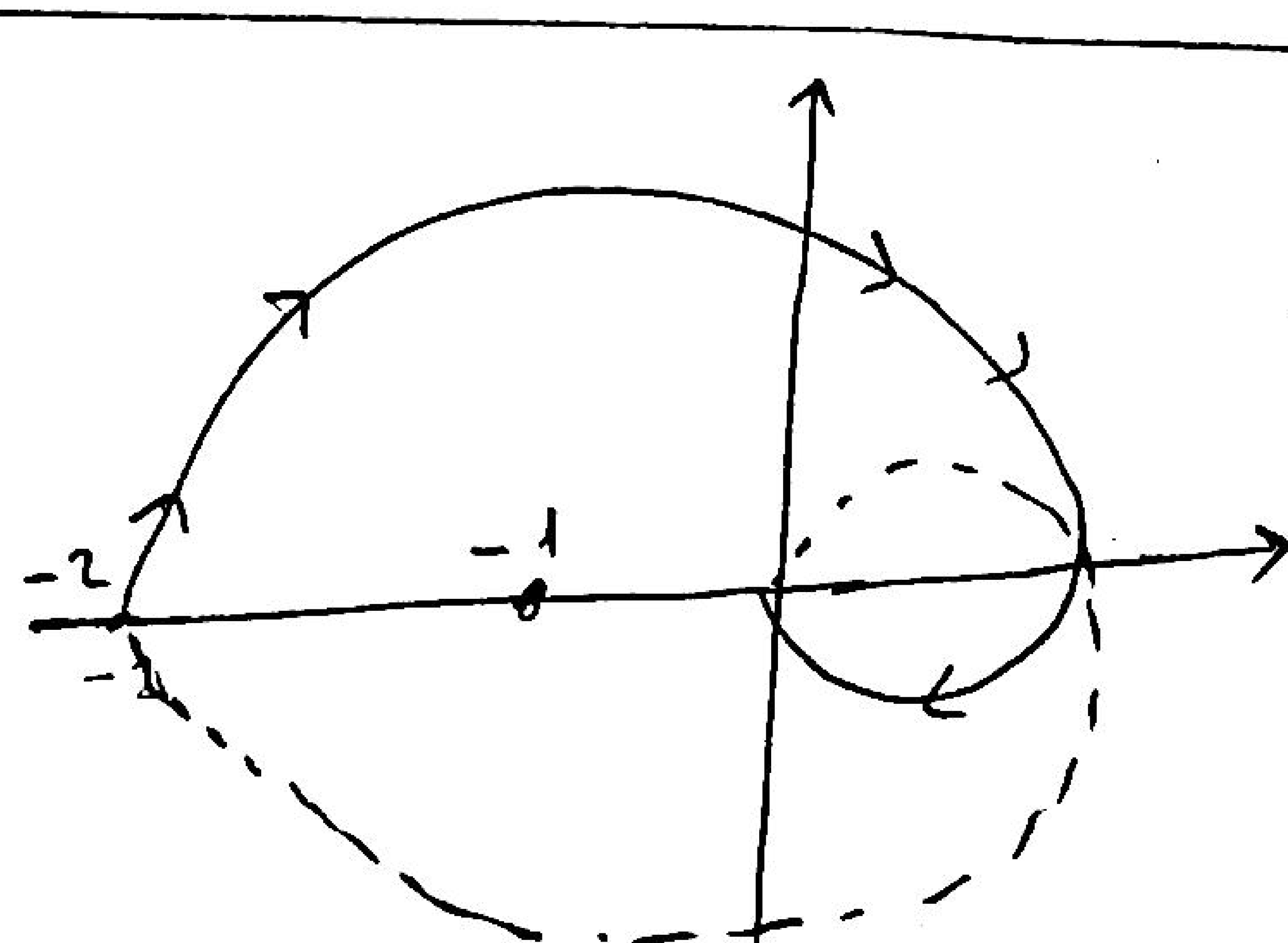
$E \to s = 0.0001j \qquad G(s) = -3 + 20000$

$P_c = 1$

Example

$G(s) = \dfrac{k(s-2)}{(s+1)^2}$

w	$\lvert G(jw)\rvert$	$\angle G(jw)$
0	2	−180
0.1	1.98	+165°
0.5	1.64	112°
0.7	1.42	90°
1	1.11	63°
2.236	0.5	0°
100	0.01	−87°
∞	0	−90°



$P_c = 0$

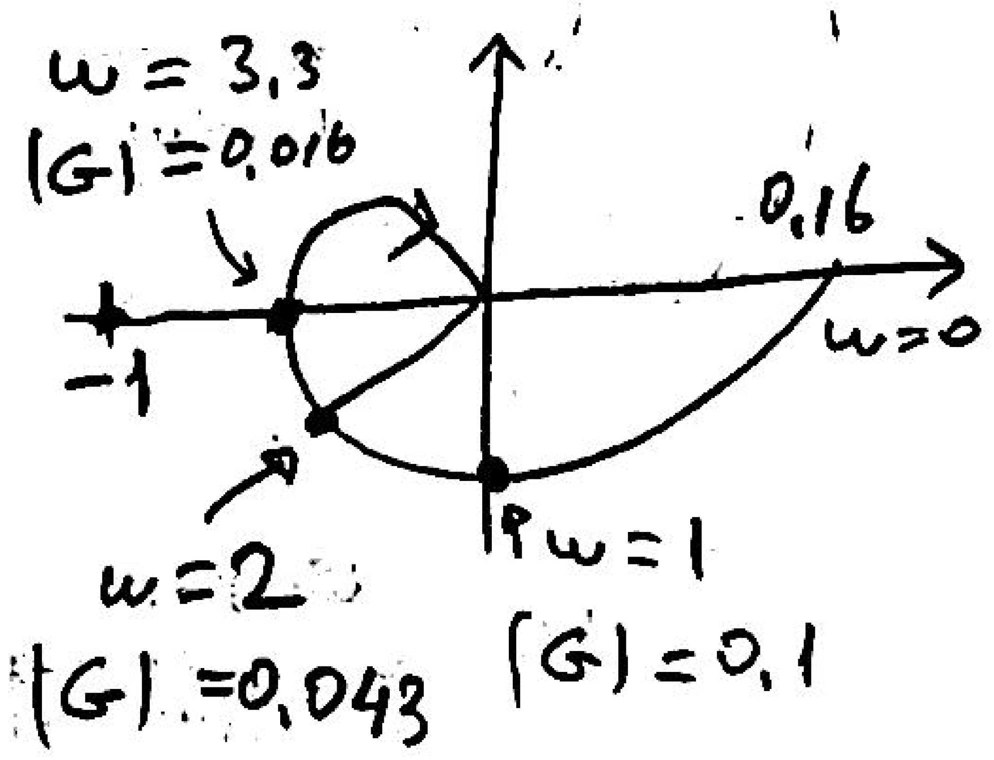
$N = 1$

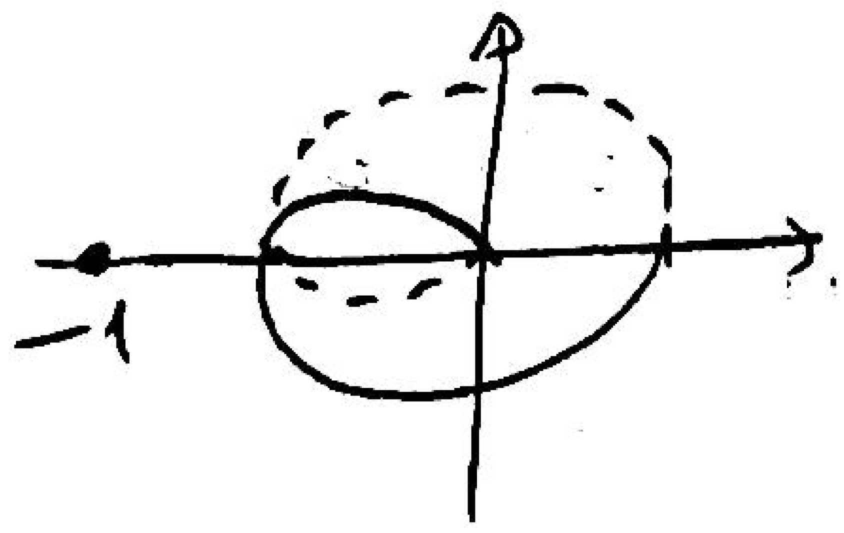
$Z_c = N + P_c = 1$

Unstable

Relative stability

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$





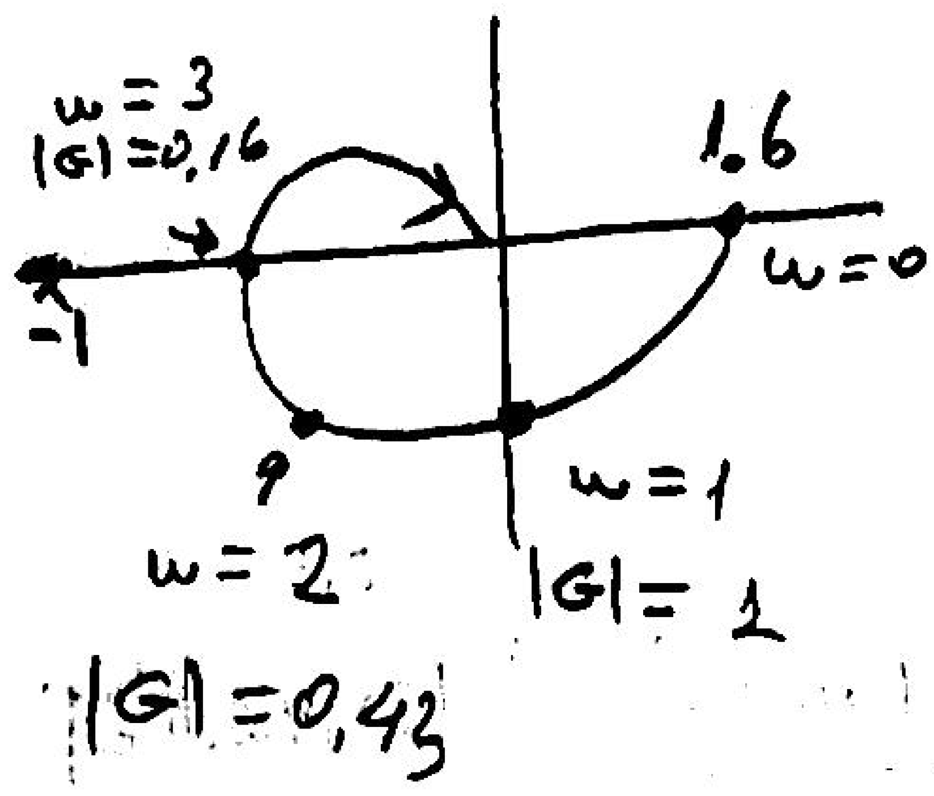
$P_c = 0$

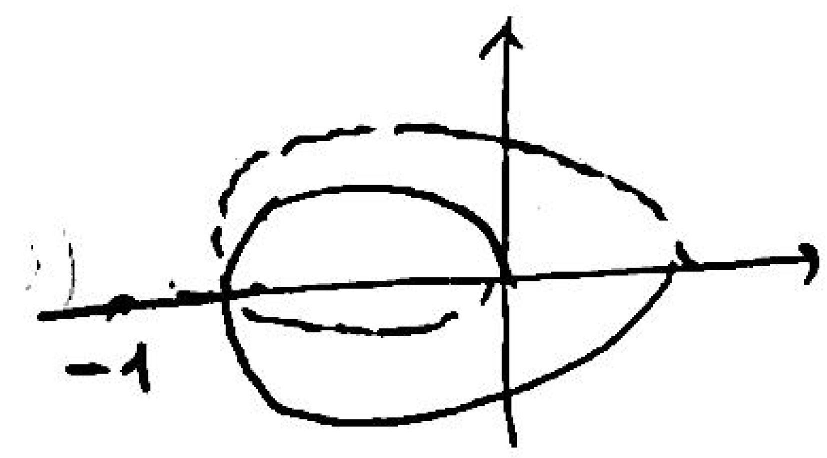
$N = 0$

$Z_c = P_c + N = 0$

stable

$$G(s) = \frac{10}{(s+1)(s+2)(s+3)}$$





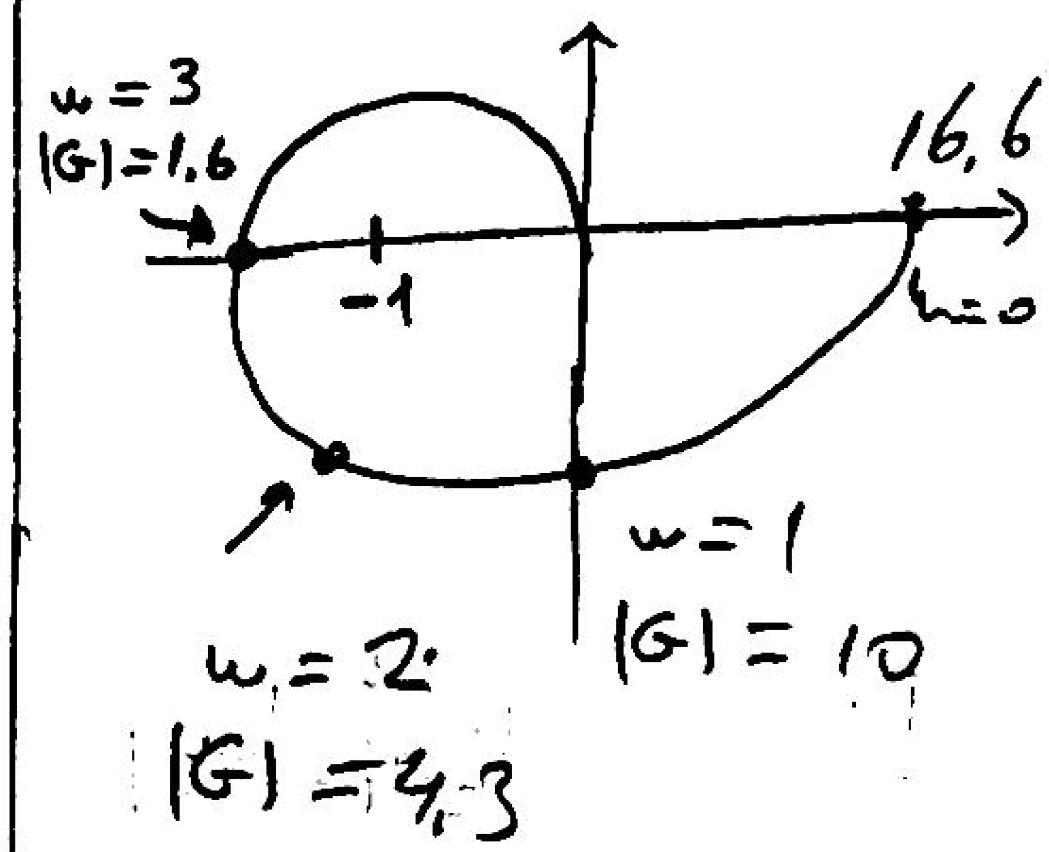
$P_c = 0$

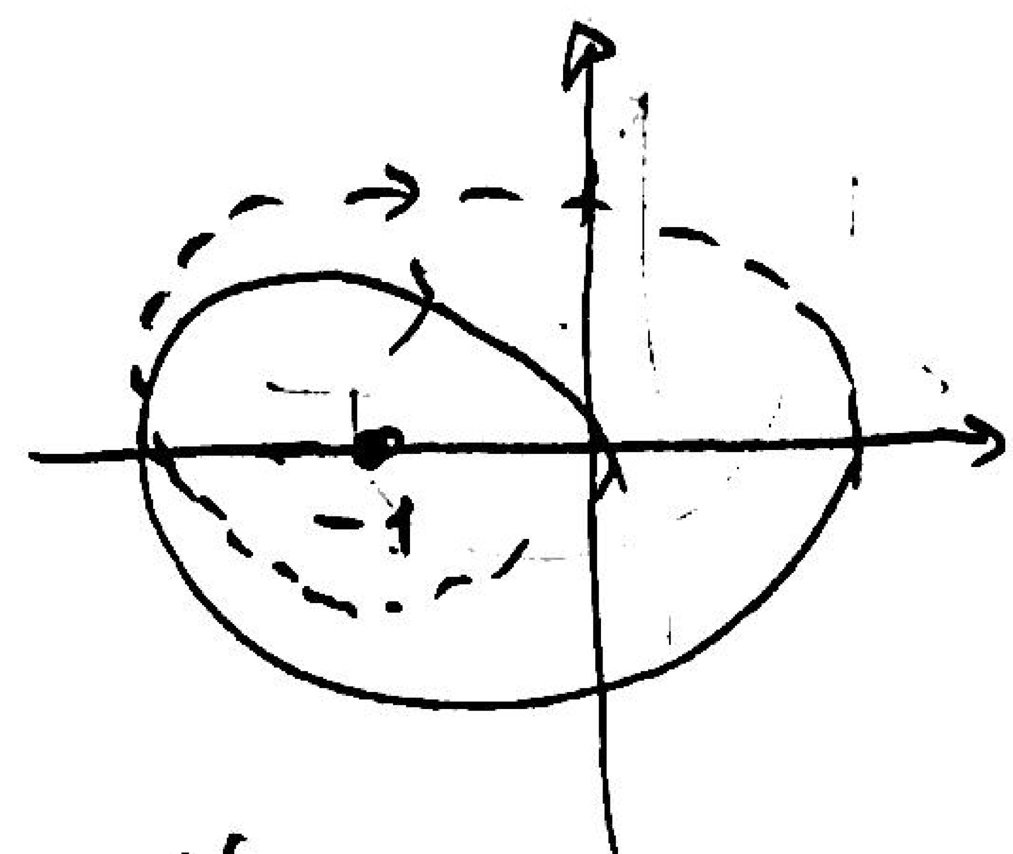
$N = 0$

$Z_c = P + N = 0$

stable

$$G(s) = \frac{100}{(s+1)(s+2)(s+3)}$$





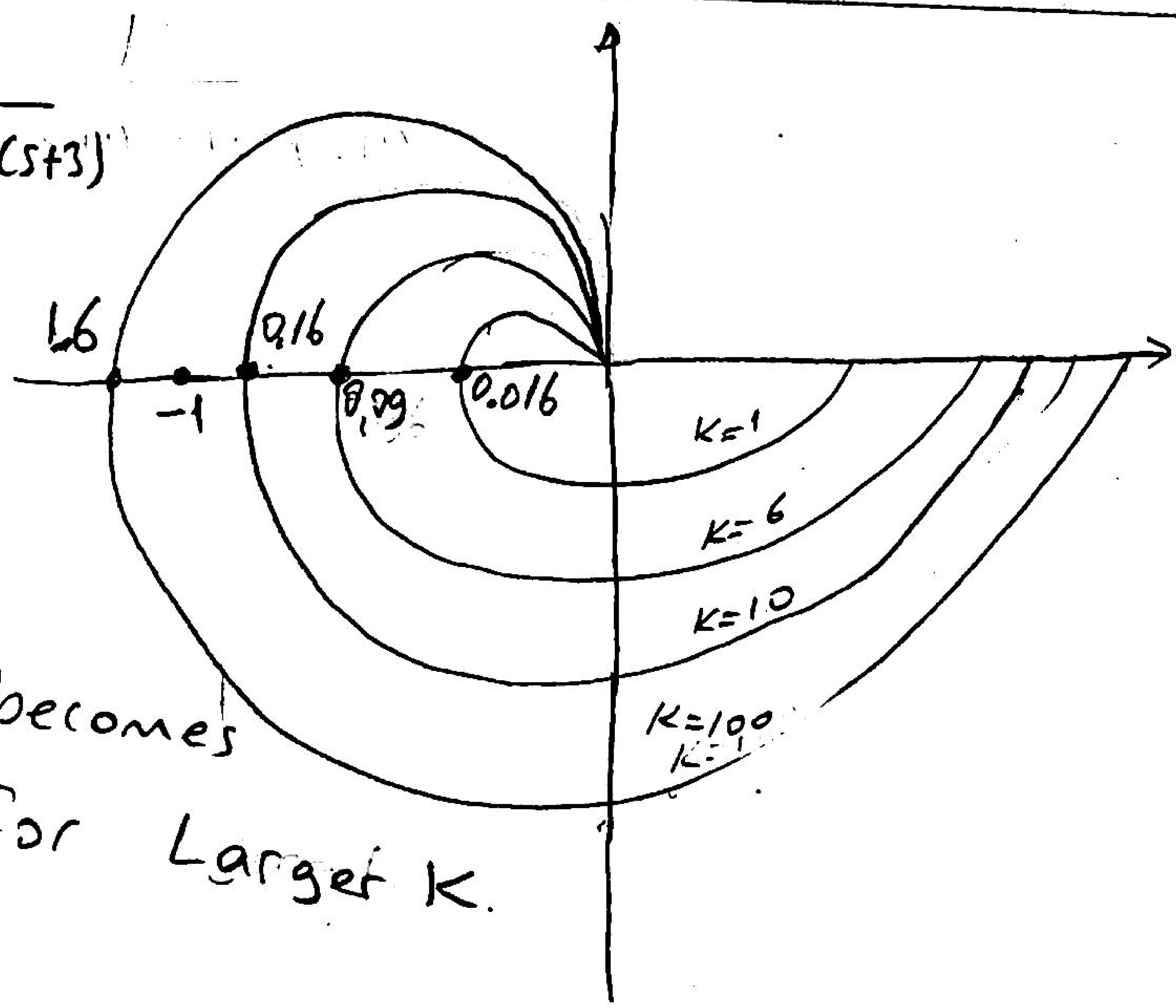
$N = 2$

$P_c = 0$

$Z_c = N + P_c = 2$

Unstable

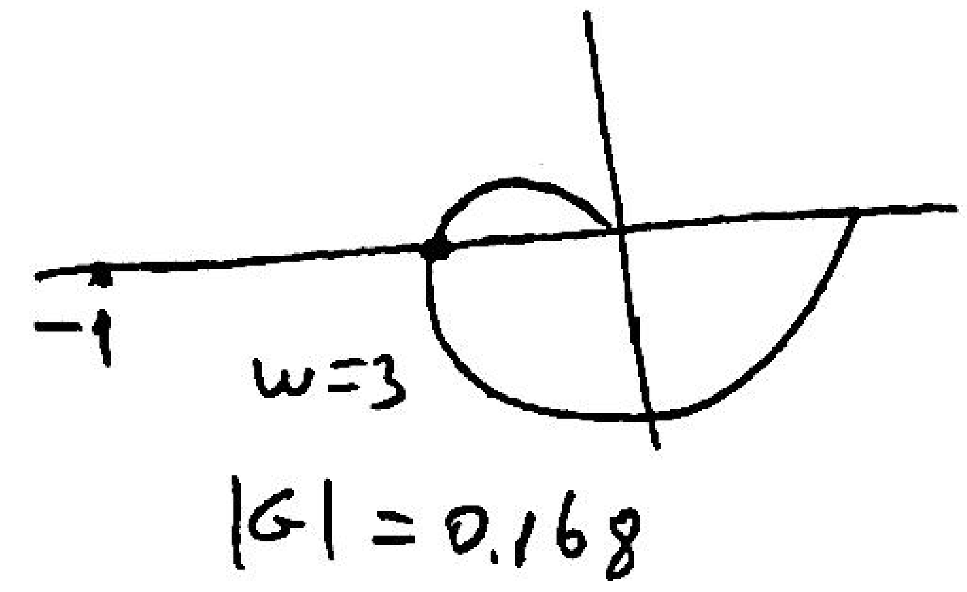
$$G(s) = \frac{K}{(s+1)(s+2)(s+3)}$$



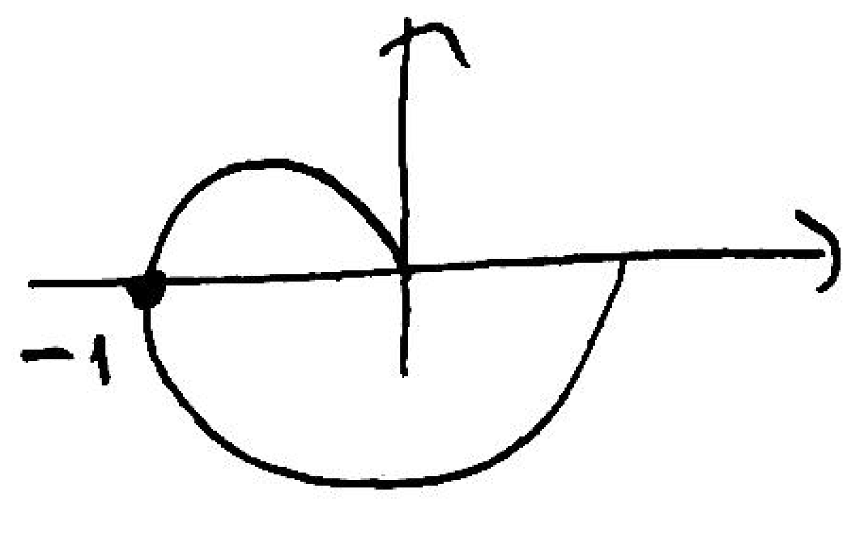
The System becomes Unstable for Larger K.

Gain margin Phase Margin

$$G(s) = \frac{1}{(s+1)(s+2)(s+3)}$$



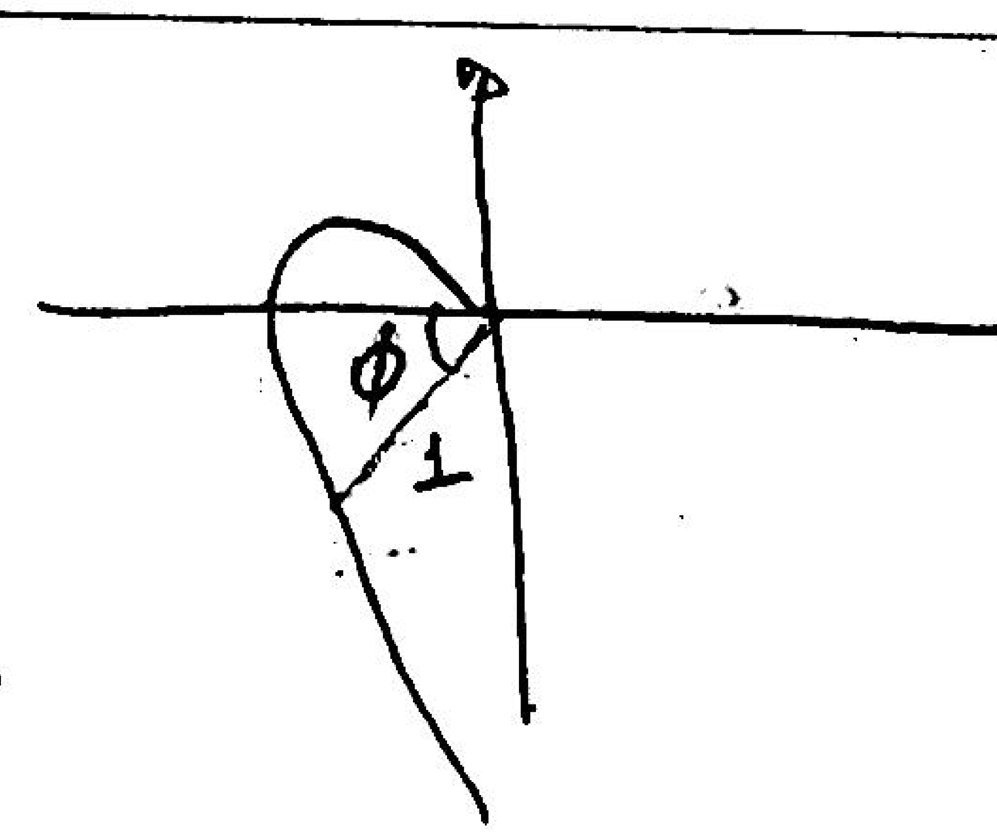
$$G(s) = \frac{59}{(s+1)(s+2)(s+3)}$$



Gain margin is $\frac{1}{0.168} = 59.5$

498

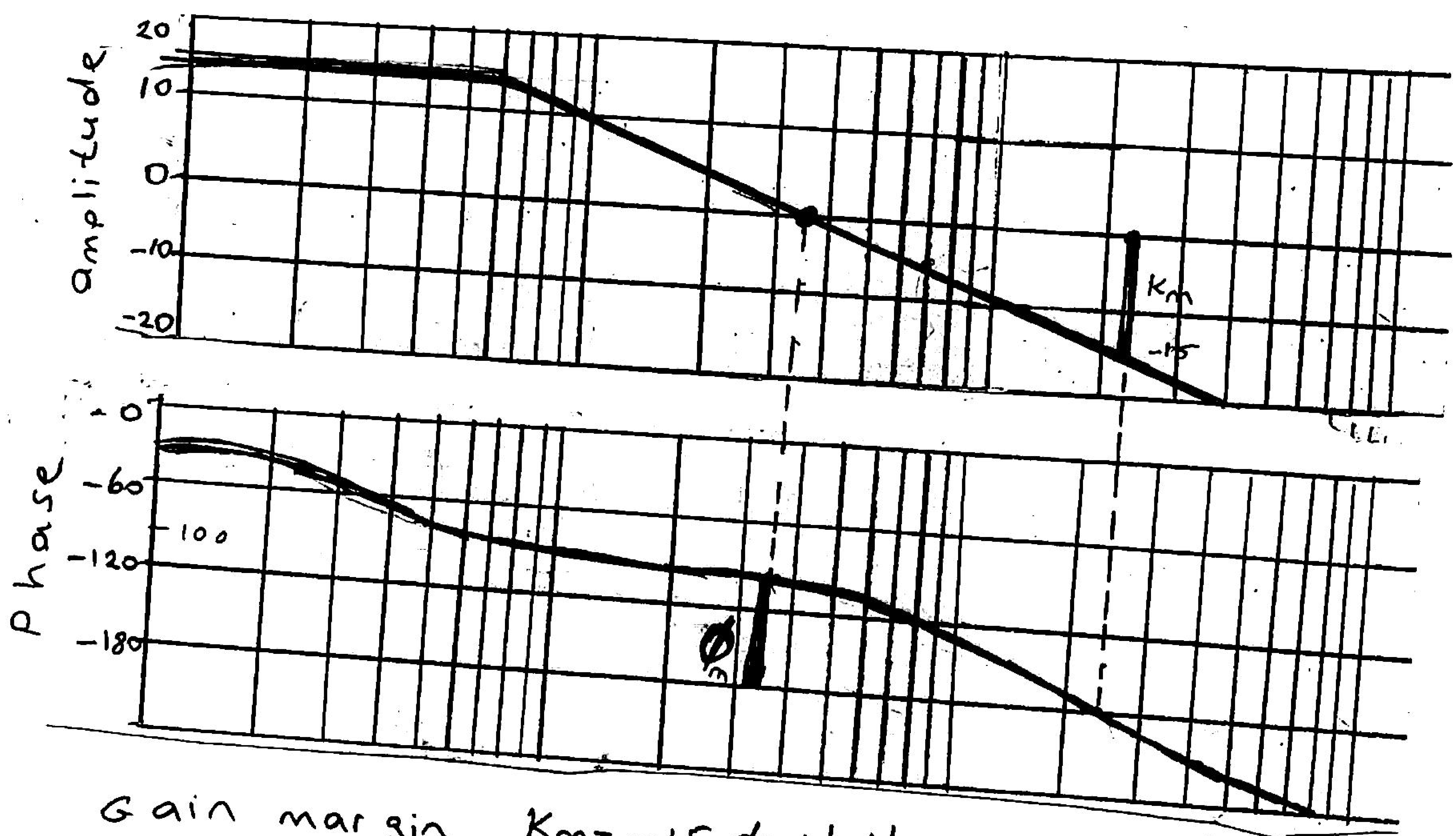
$$G(s) = \frac{1}{s(s+1)(s+2)}$$



ϕ is phase margin

458

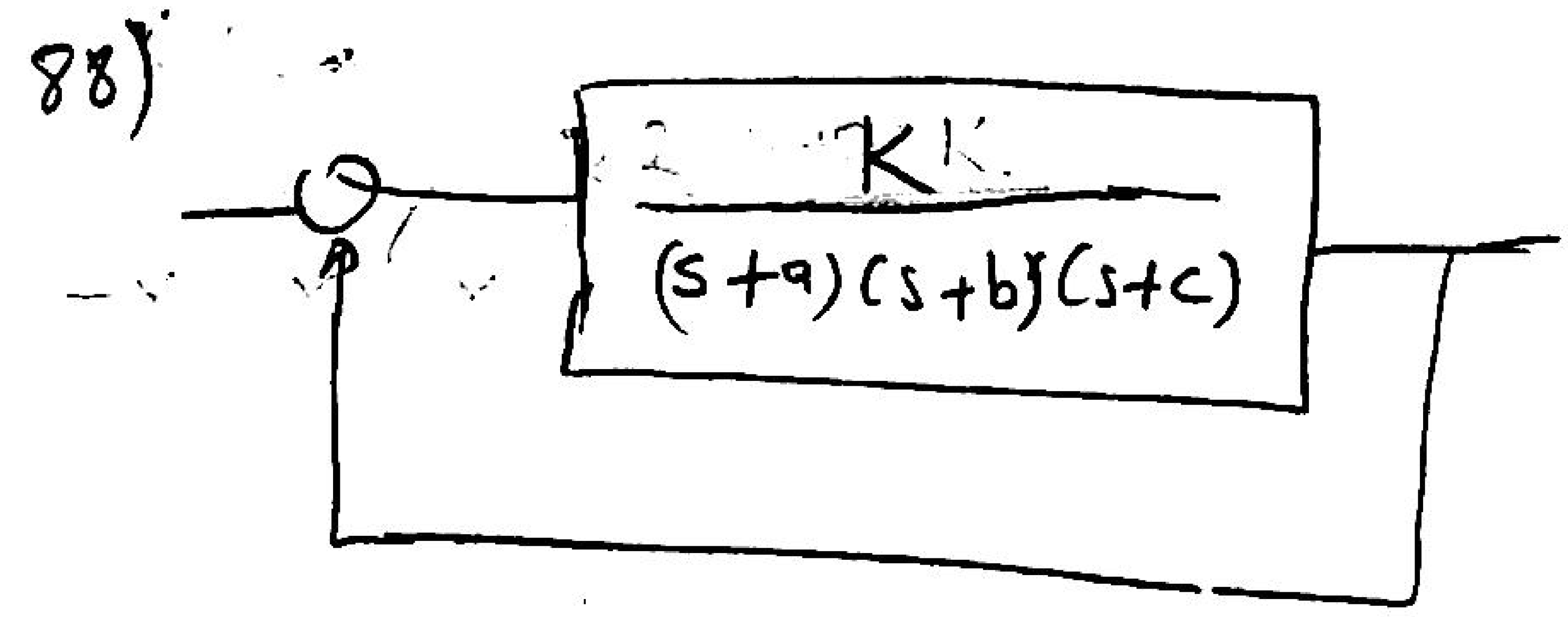
Gain and phase margin in Bode Diagram



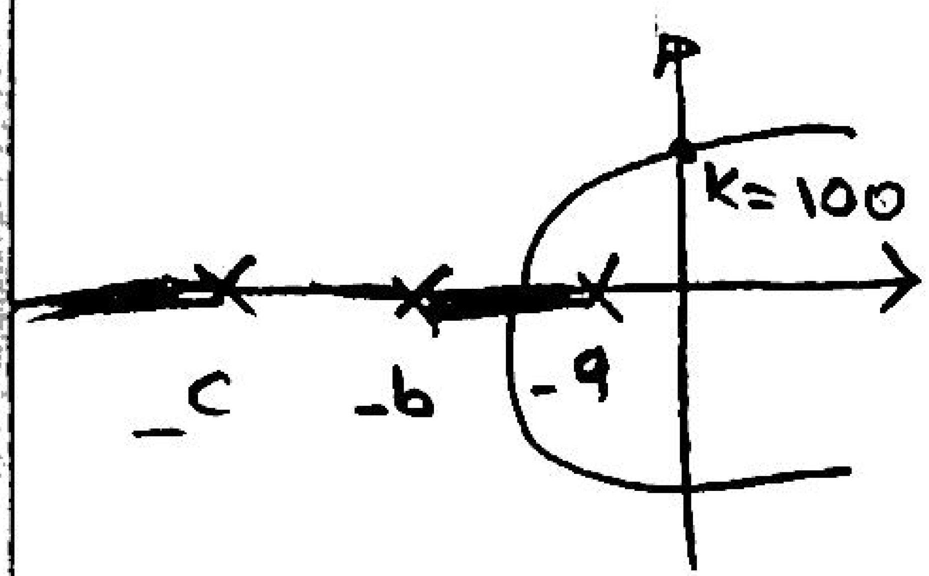
Gain margin $K_m = -15$ decibell

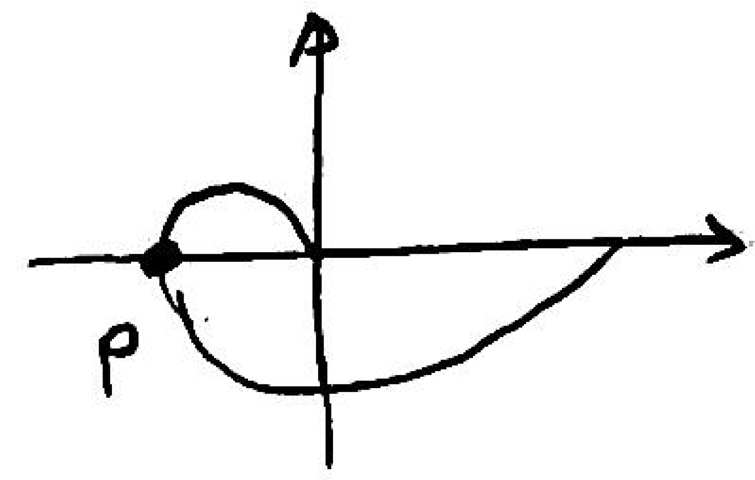
phase margin $\phi_m = 180 - 100 = 80^\circ$

88)



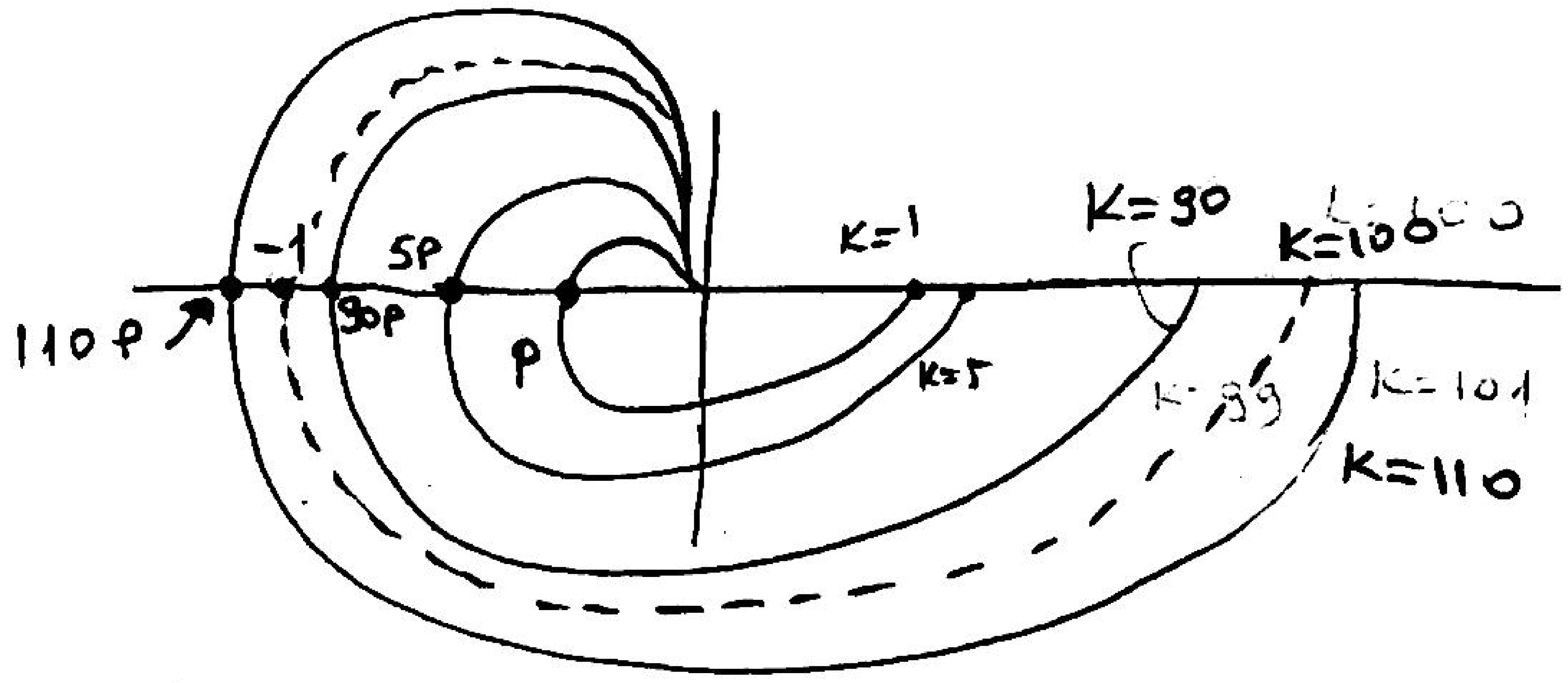
Root Locus and Polar plot of this System are shown below. Calculate the point P. what is gain margin





Polar plot for K=1

Solution the system is Unstable for K>100.
Polar plot is drawn for different K below.

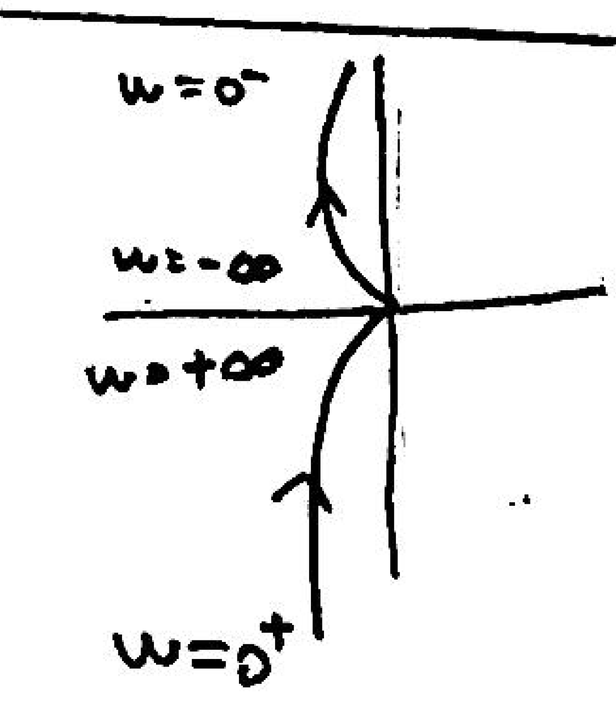


for K=100 the curve passes through the point -1

$100P = -1 \qquad P = \frac{-1}{100} = -0.01$

Gain margin is 100.

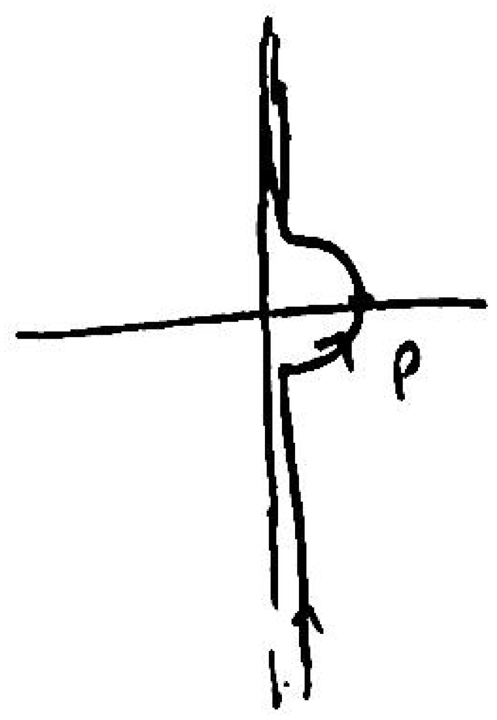
89) $G(s) = \dfrac{10}{s(s+3)}$

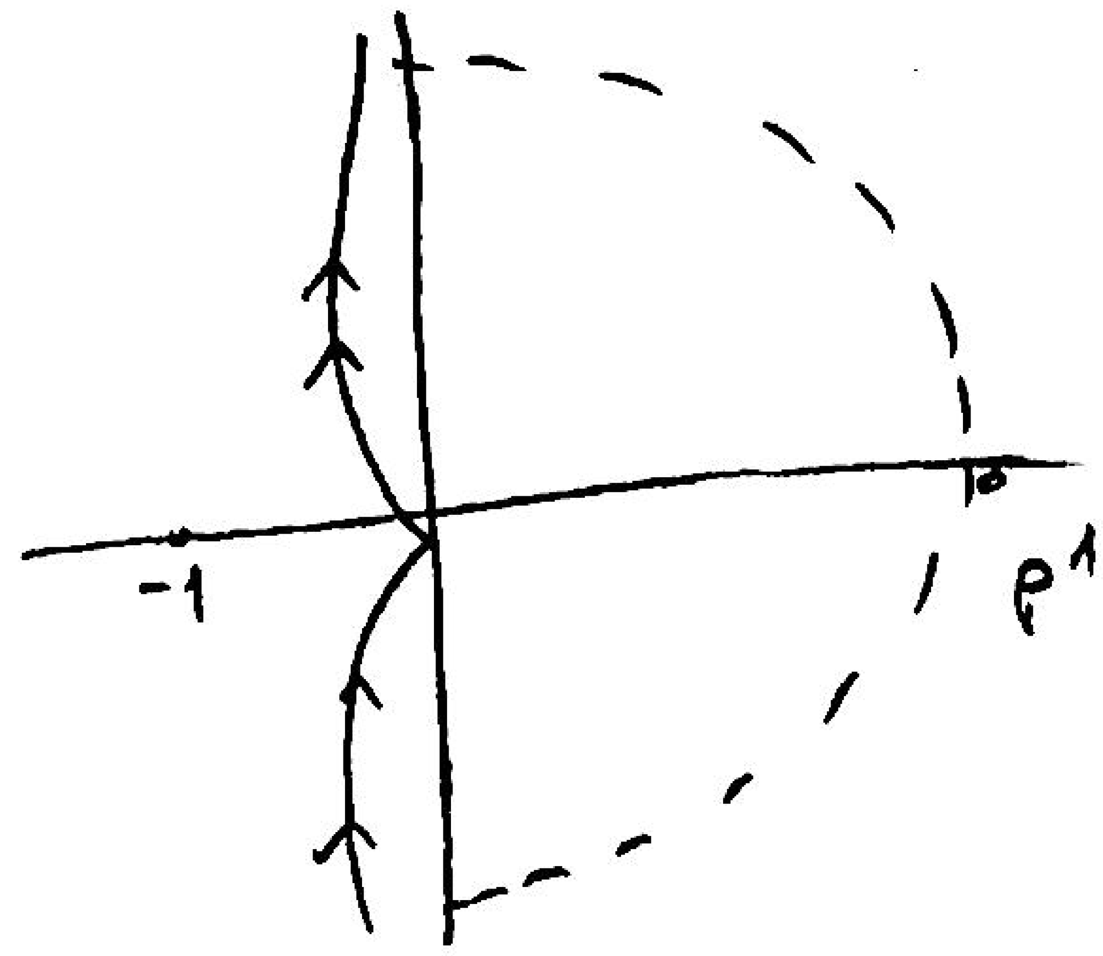


Complete the Nyquist contour.

89) solution.

s plane



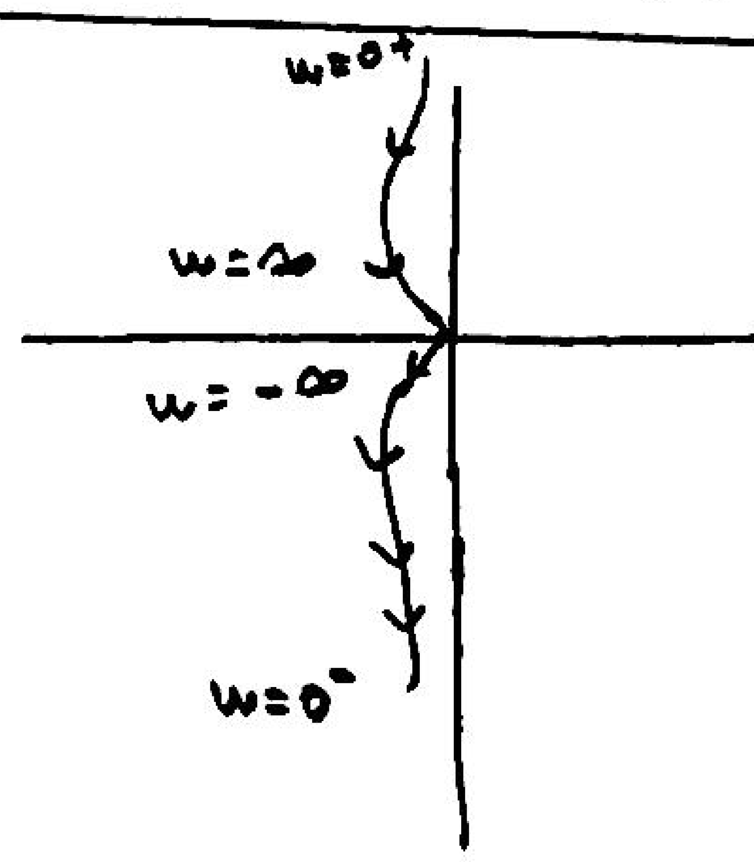


G(S) plane

replace $s = p = 0{,}0001$

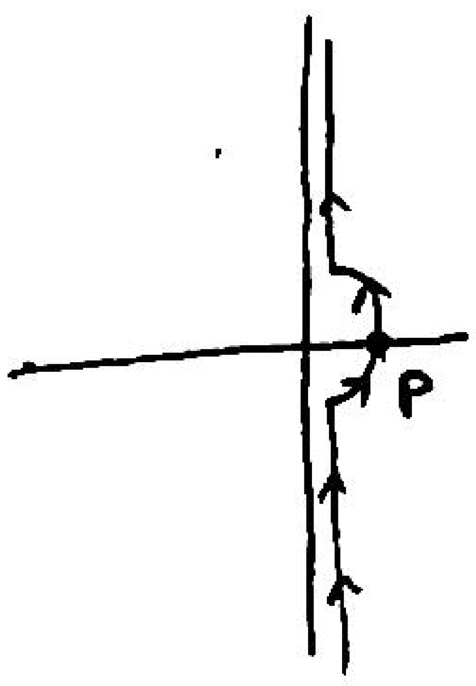
$$G(s) = \frac{10}{0.0001\,(0.0001+3)} \cong \frac{10}{0{,}0001 \times 3} = 33\,000 = p'$$

90) $G(s) = \dfrac{10}{s(s-3)}$



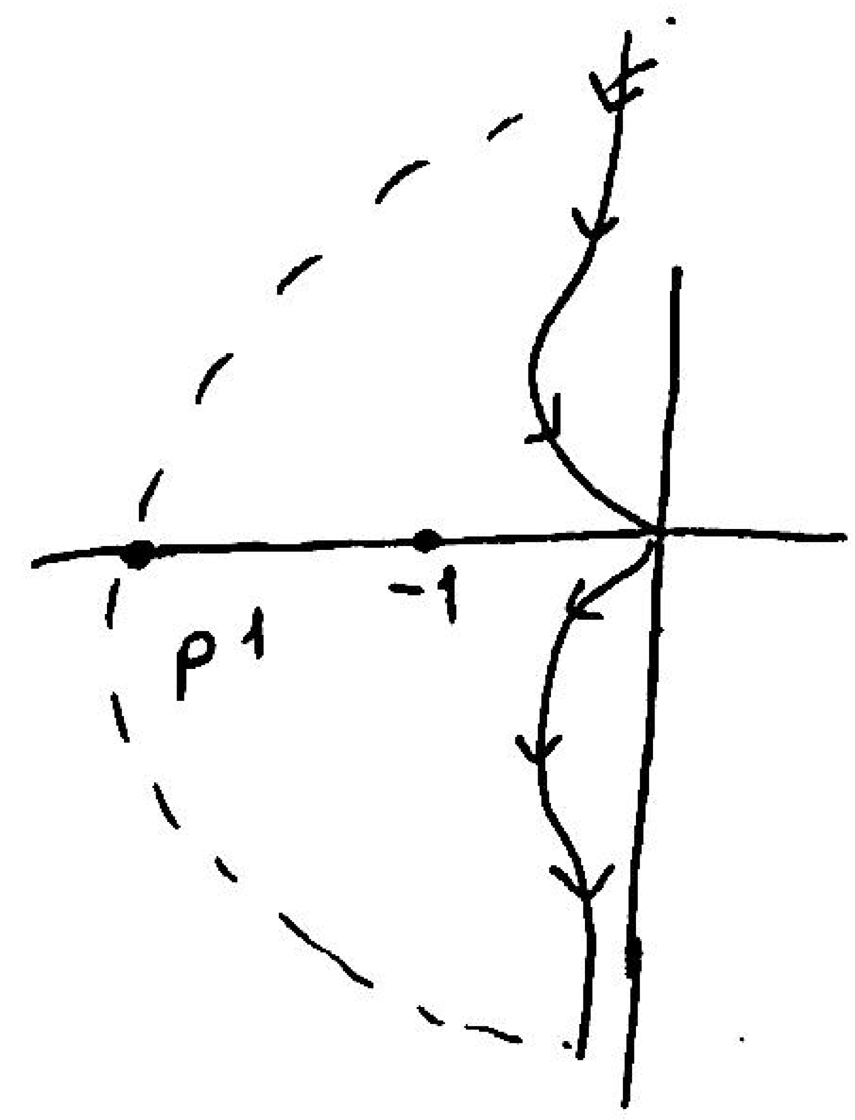
complete the Nyquist contour.

Solution

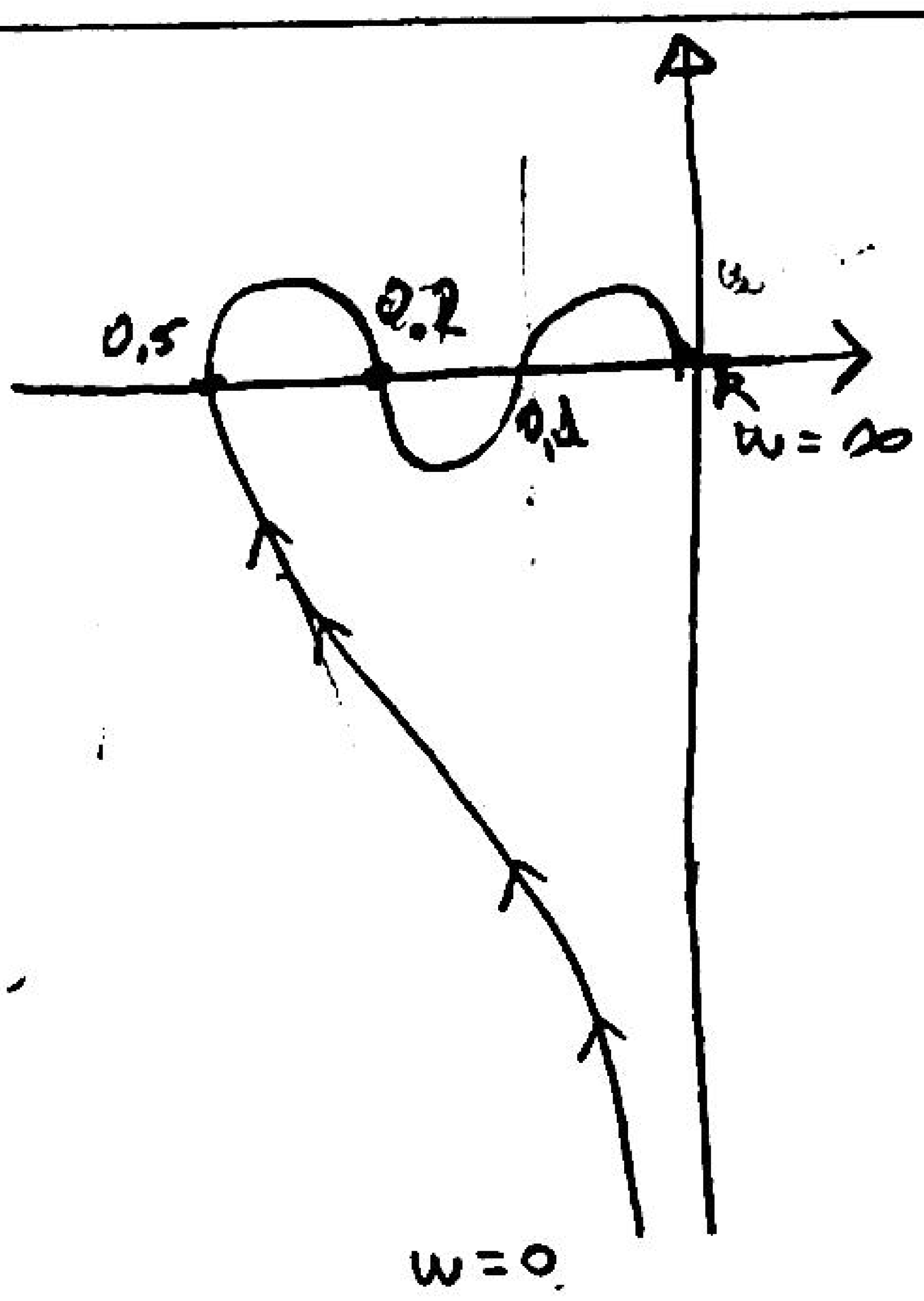


$s = p = 0{,}0001$

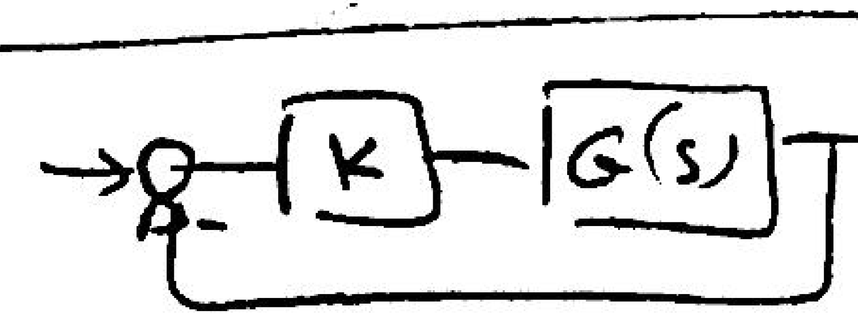
$$G(s) = \frac{10}{0.0001\,(j0.0001-3)} = -33\,000$$



91)



$$G(s) = \frac{(s+a)(s+b)}{s(s+1)(s+2)(s+5)(s+6)}$$



a) complete the contour

b) for which values of K the system is stable?