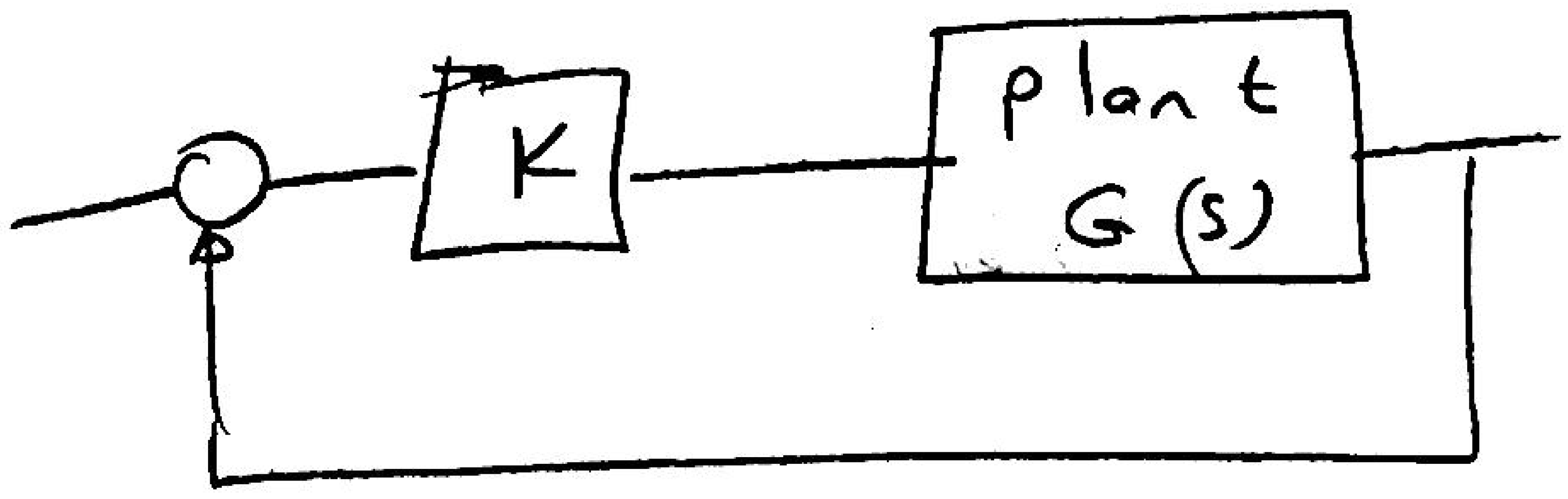
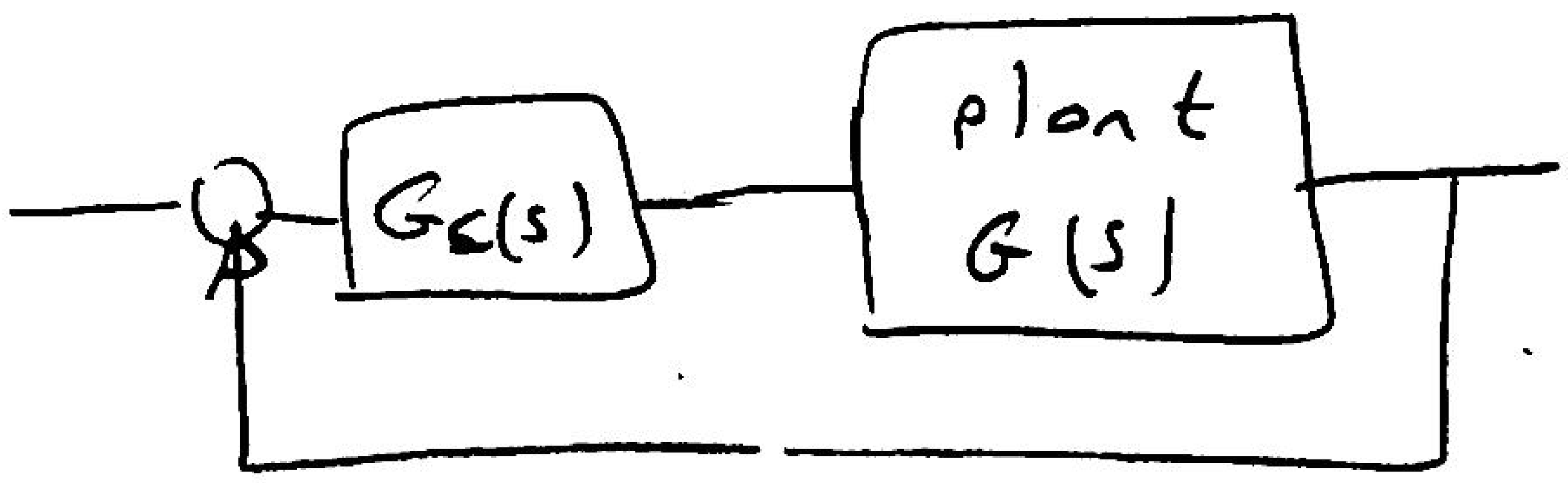
we need to use dynamic controller



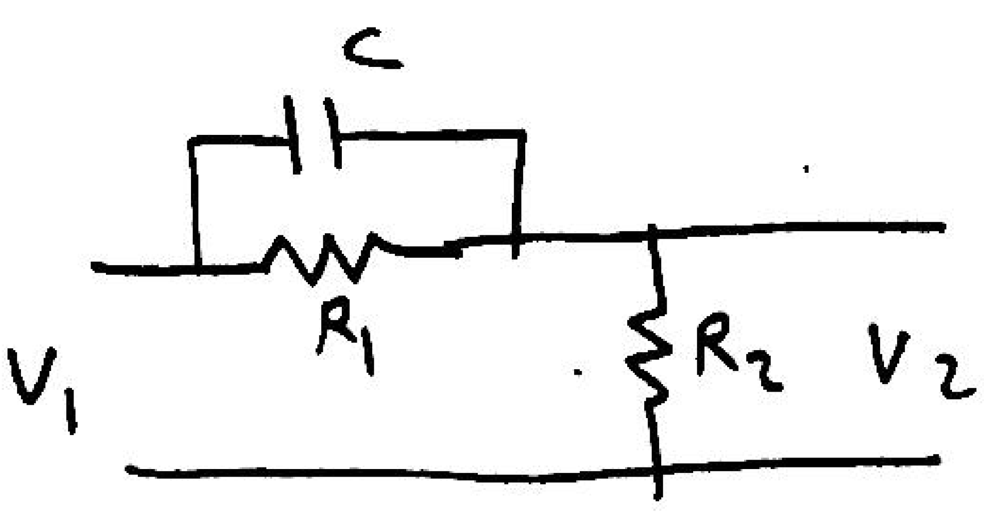
A constant gain controller



$G_c(s)$ dynamic controller

Possible forms of $G_c(s)$ used in industry

$$G_c(s) = K\frac{s+a}{s+b}, \quad G_c(s) = K_p + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_p s + K_I}{s}$$

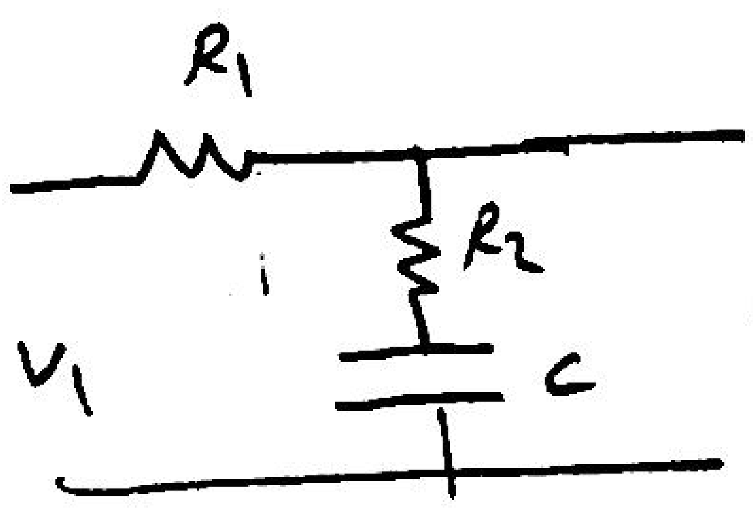


$$G_c(s) = \frac{V_2(s)}{V_1(s)} = \alpha\frac{s+a}{s+b}$$

$$a = \frac{1}{R_1 C}$$

$$b = \frac{1}{\frac{R_1 R_2}{R_1+R_2}C}$$

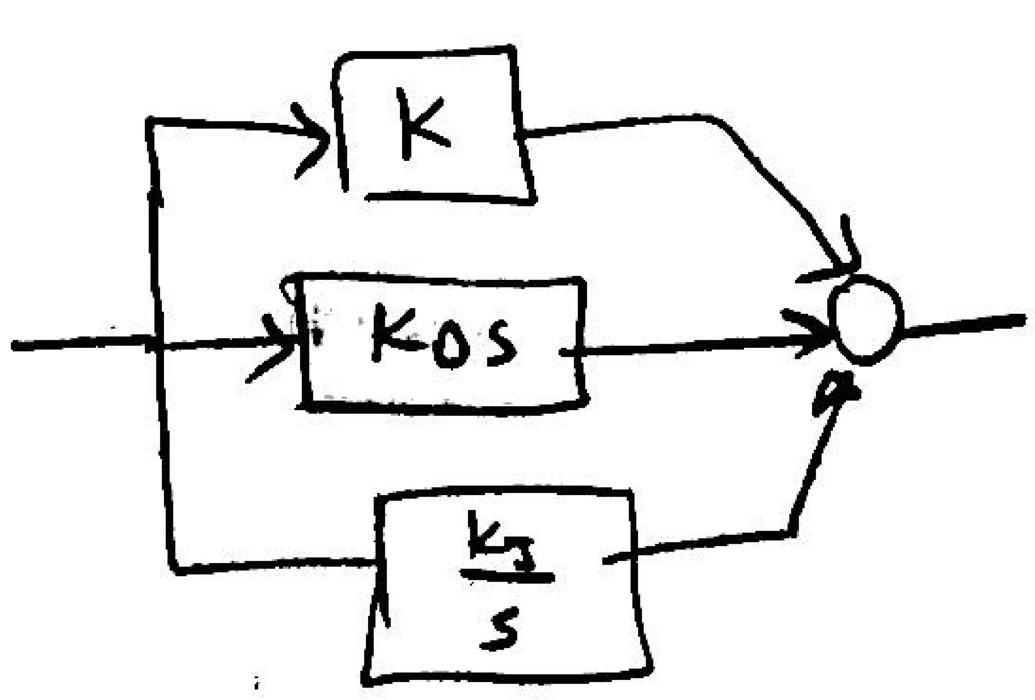
$$\alpha = \frac{R_2}{R_1+R_2}\,\frac{b}{a}$$



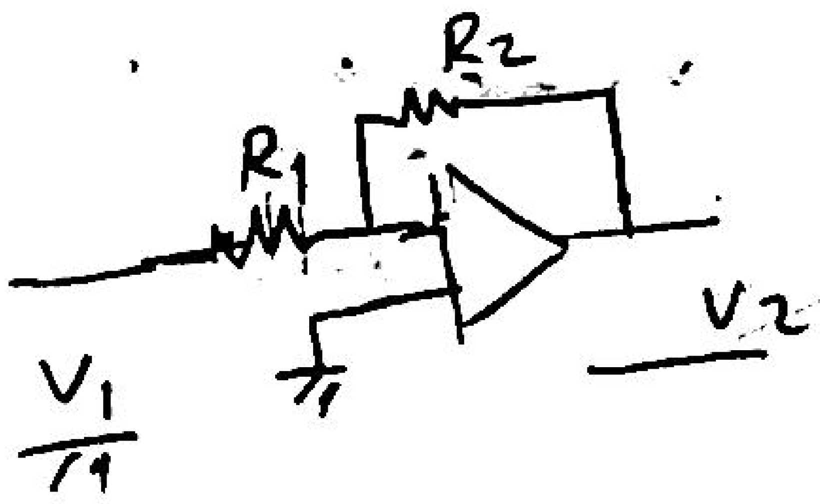
$$G_c(s) = \frac{V_2(s)}{V_1(s)} = \alpha\frac{s+a}{s+b}$$

$$a = \frac{1}{R_2 C} \qquad b = \frac{1}{(R_1+R_2)C}$$

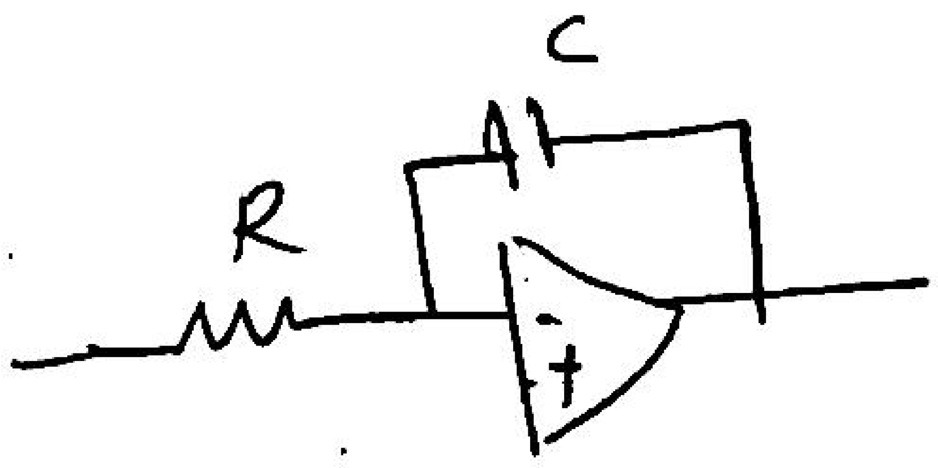
$$\alpha = \frac{b}{a}$$



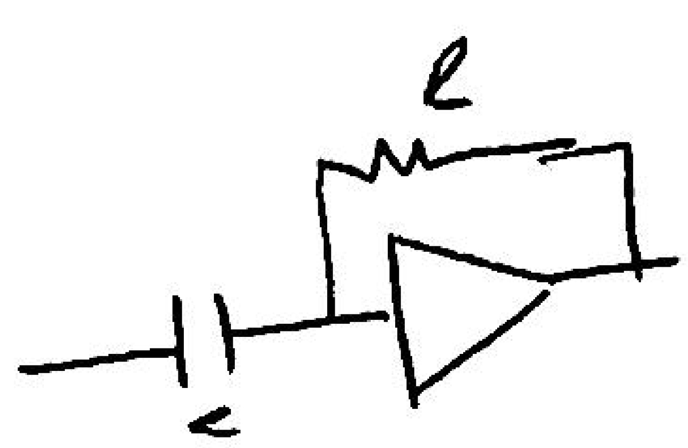
$$G_c(s) = K + K_D s + \frac{K_I}{s}$$



$$G_1(s) = -\frac{R_2}{R_1} = K \quad \text{Constant gain}$$



$$G(s) = \frac{-\frac{1}{sC}}{R} = -\frac{1/RC}{s} = \frac{K_I}{s} \quad \text{(integrator)}$$



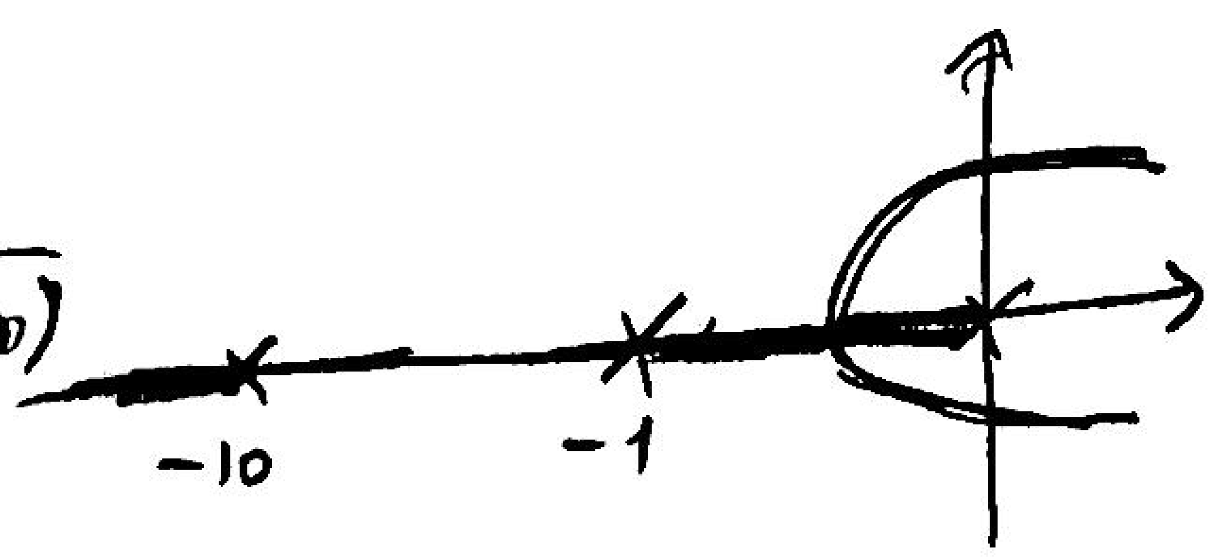
$$G(s) = -\frac{R}{1/sC} = -RCs = K_D s \quad \text{(Derivative)}$$

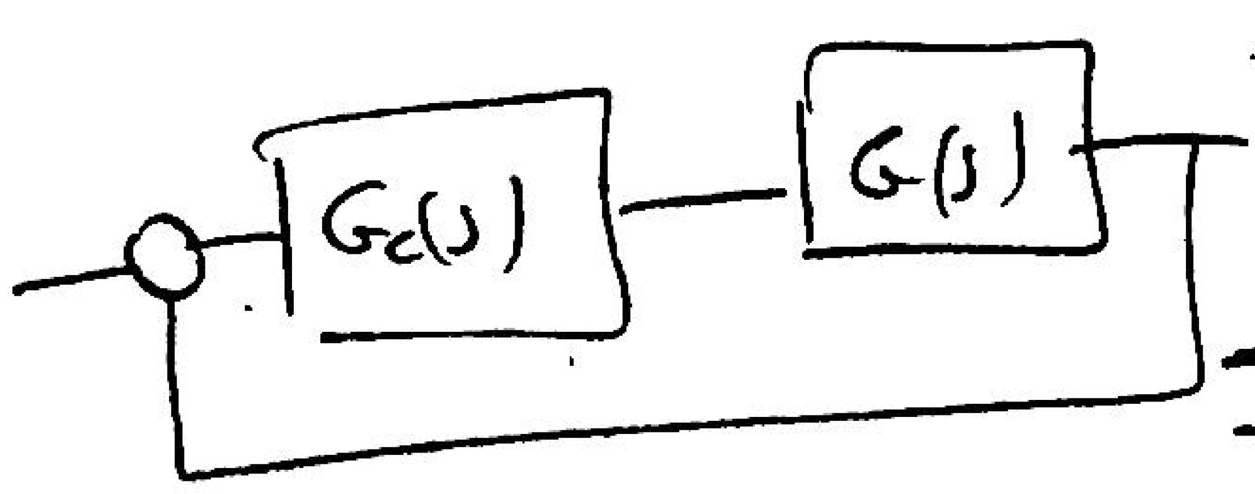
PID control $\quad G_c(s) = K + \frac{K_I}{s} + K_D s$

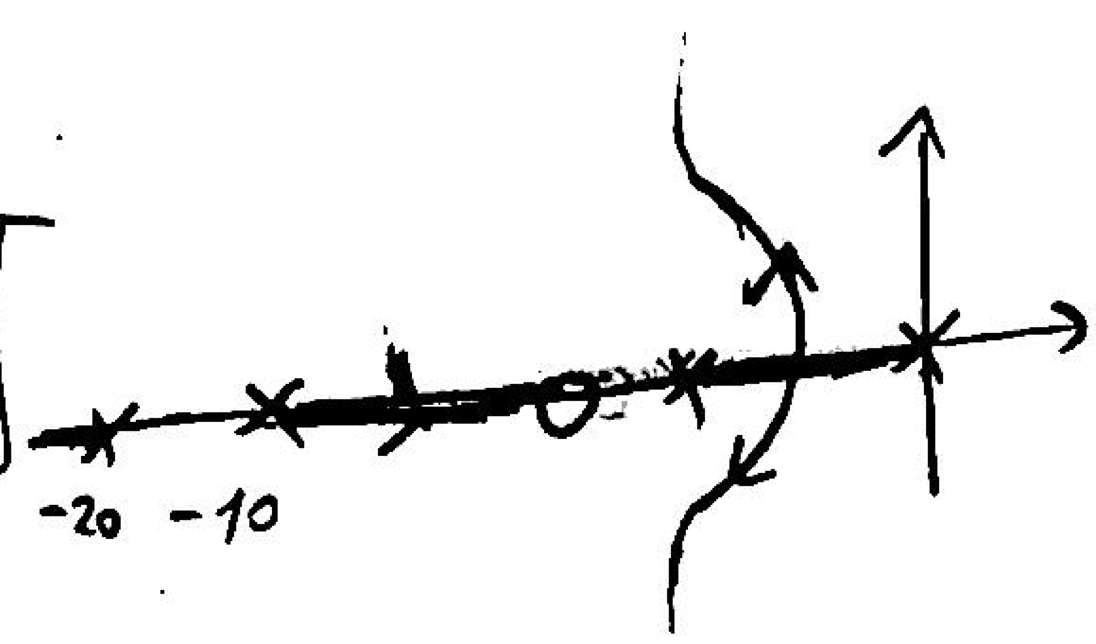
Proportional, Integration, Derivative
P, I, D.

Example

$$G(s) = \frac{1}{s(s+1)(s+10)}$$

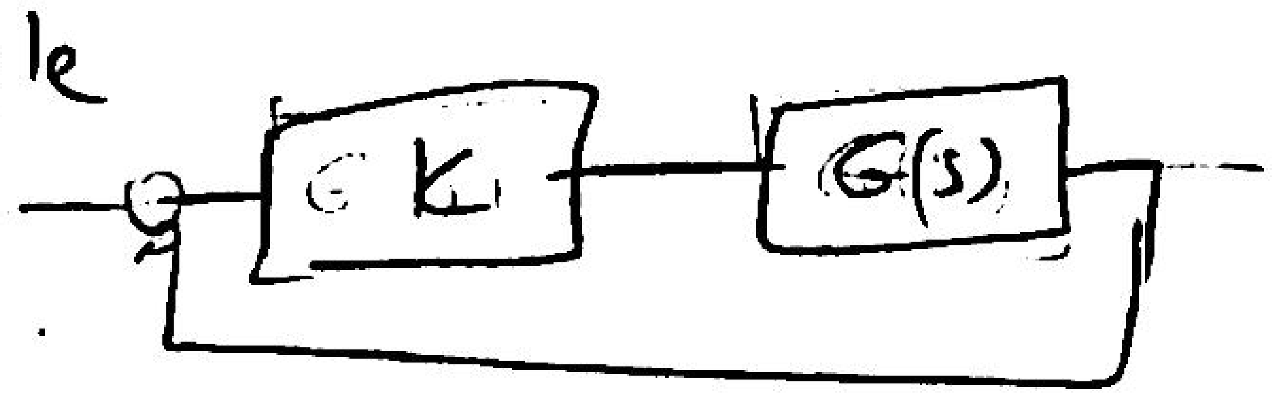




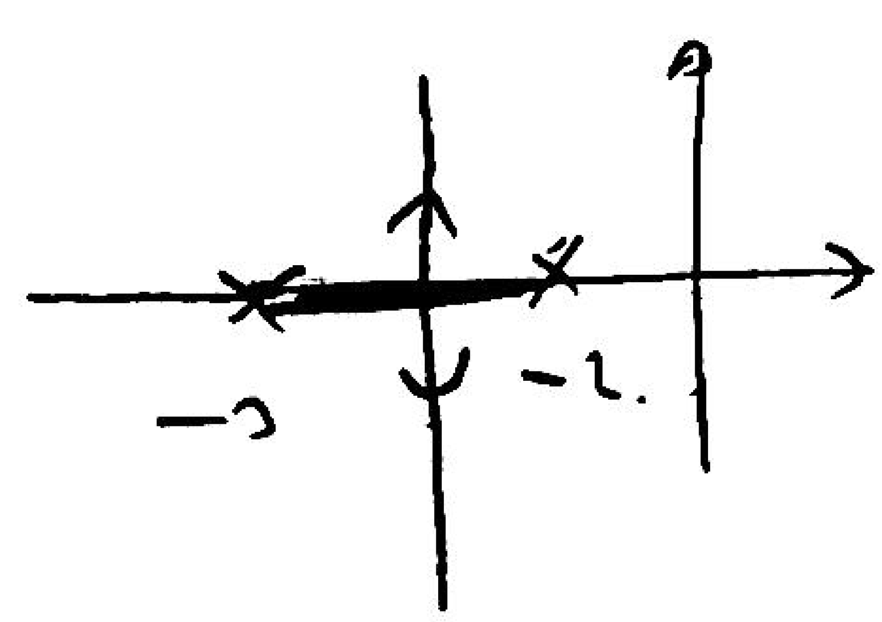


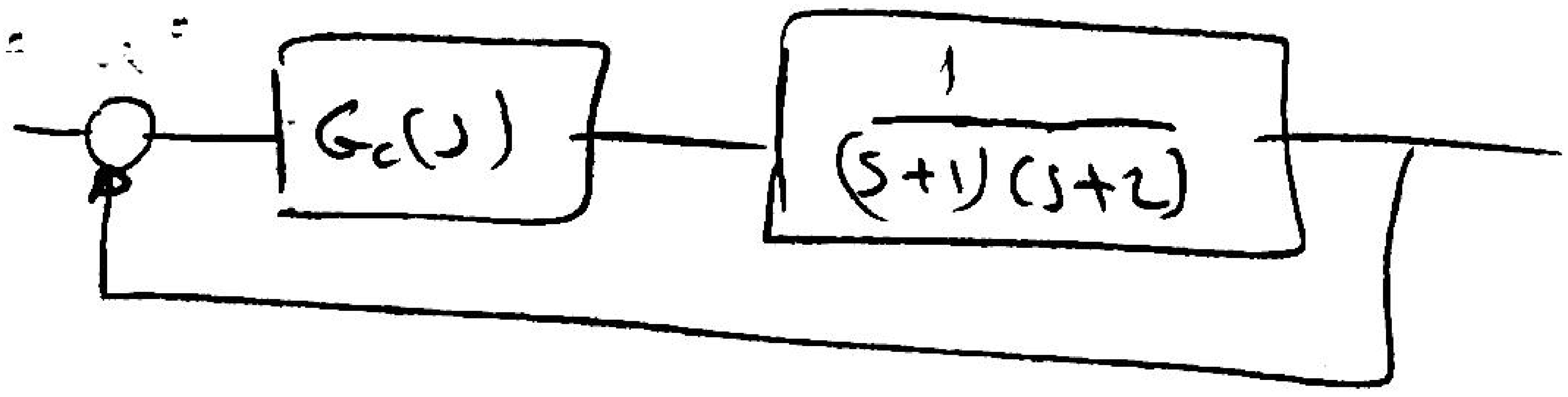
$$G_c(s) = K\frac{s+2}{s+20}$$

Example



$$G(s) = \frac{1}{(s+2)(s+3)}$$



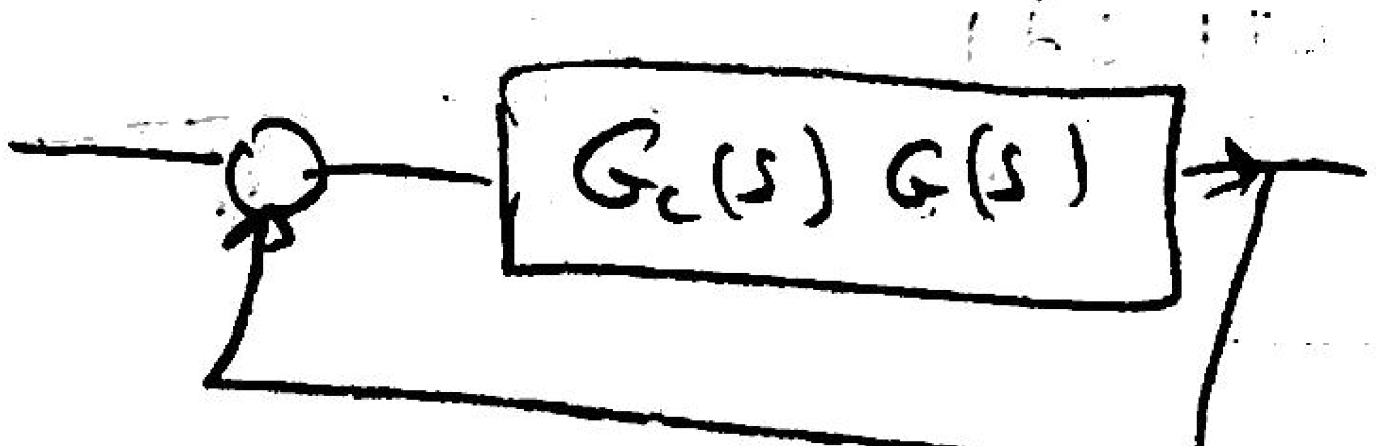


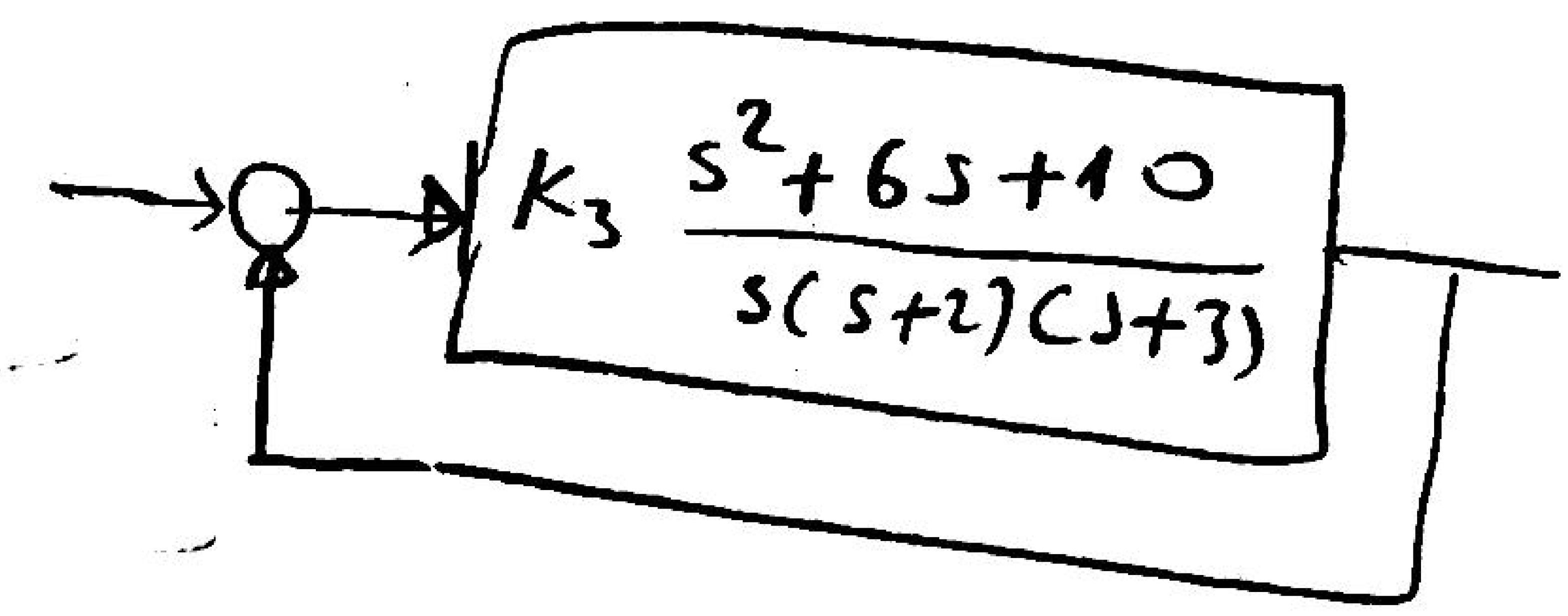
$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3 s^2 + K_1 s + K_2}{s}$$

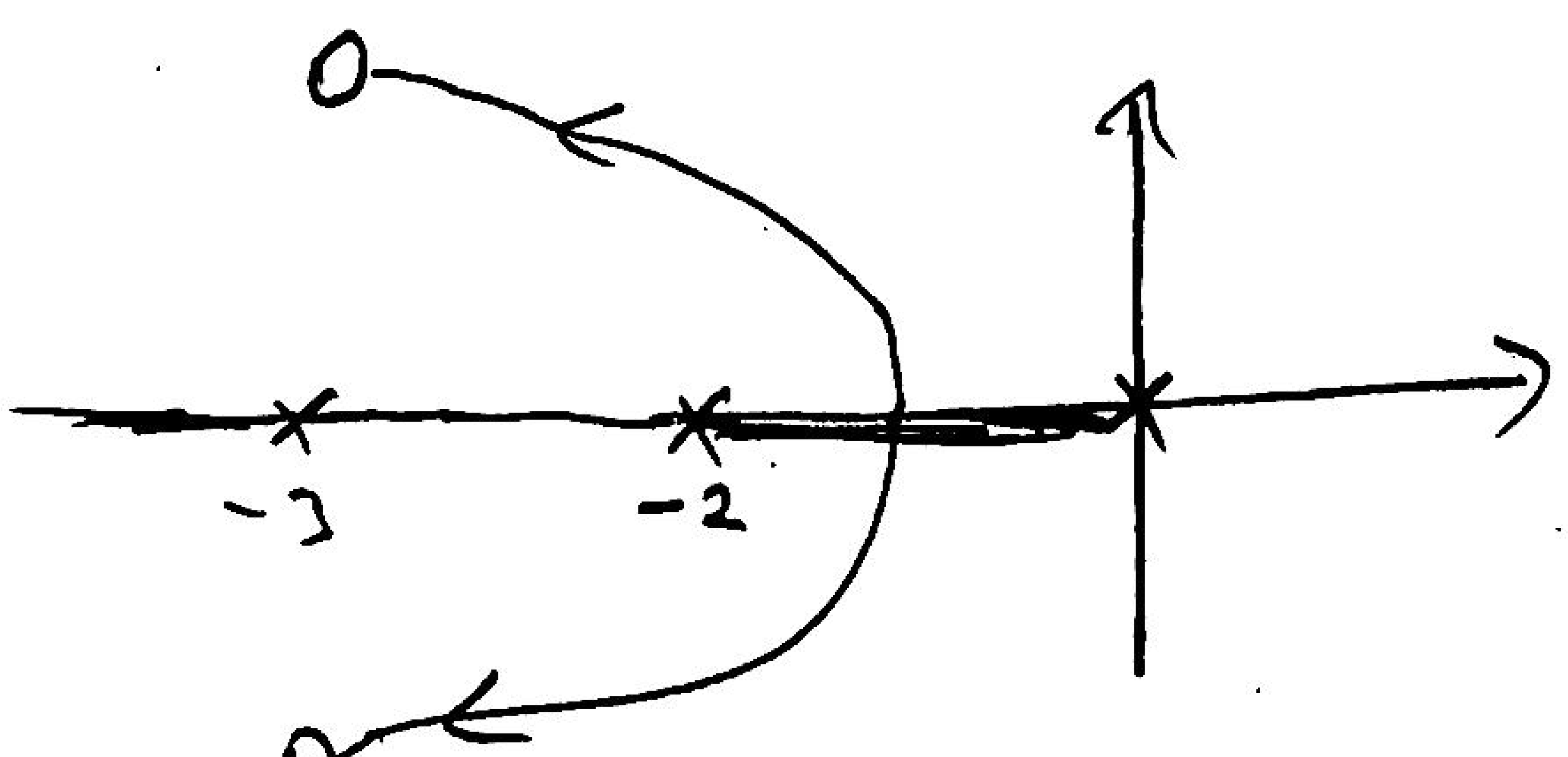
$$= \frac{K_3\left(s^2 + \frac{K_1}{K_3} s + \frac{K_2}{K_3}\right)}{s}$$

Select $\frac{K_1}{K_3} = 6$ $\quad \frac{K_2}{K_3} = 10$

$$G_c(s) = \frac{K_3\,(s^2 + 6s + 10)}{s}$$







$s^2 + 6s + 10 = 0$

$s_1 = -3 + j$

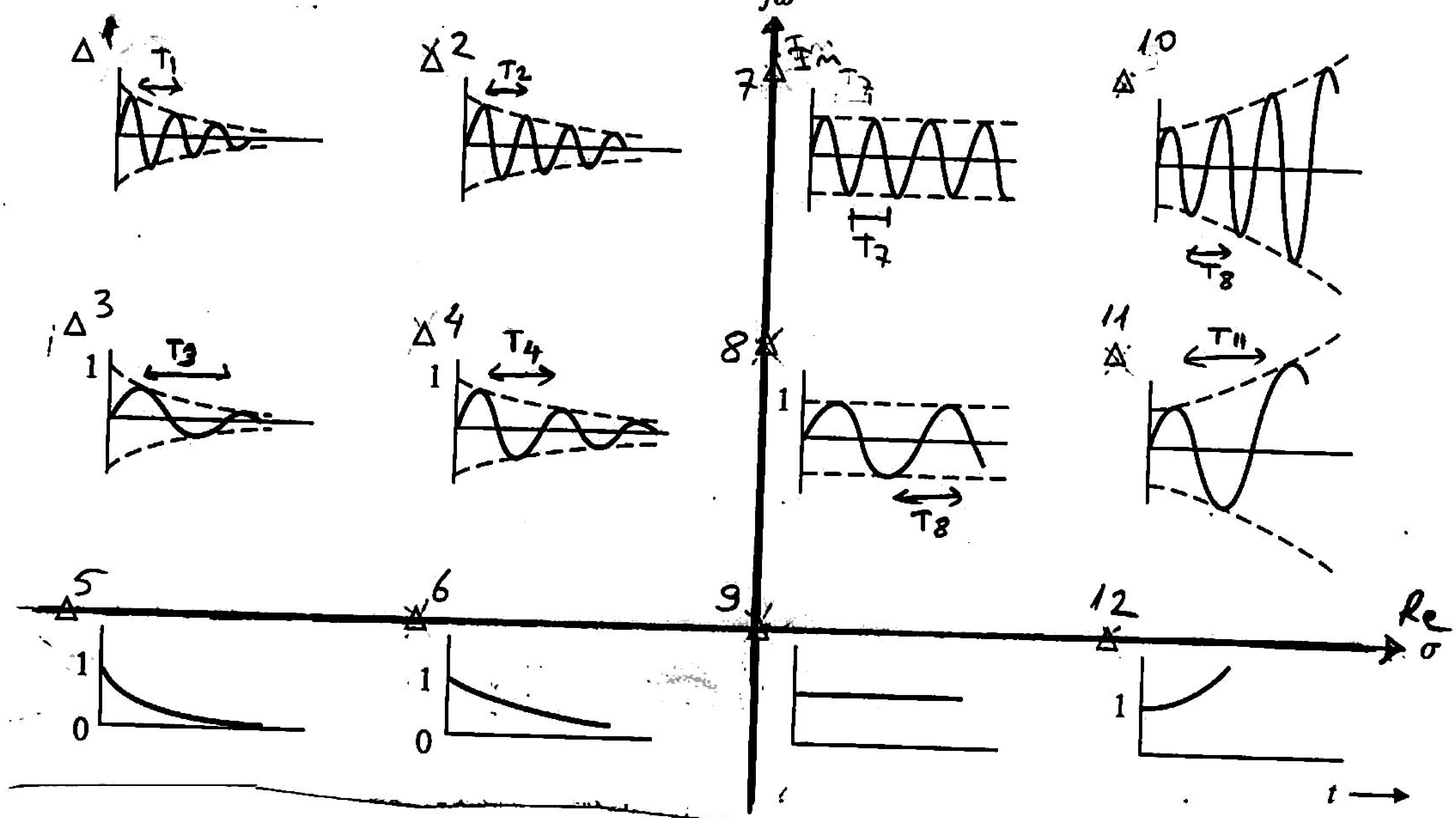
$s_2 = -3 - j$

We changed the shape of root locus

Location of poles and system Response

(All complex roots are conjugate pairs)

Impulse Responses



1, 3 fast decay (dies very fast)

$T_1 = T_2$ $\quad$ $T_3 = T_4$ $\quad$ $T_2 < T_4 \Rightarrow f_2 > f_4$

$T_1 < T_3 \Rightarrow f_1 > f_3$

1, 2 are more oscillatory than 3, 3

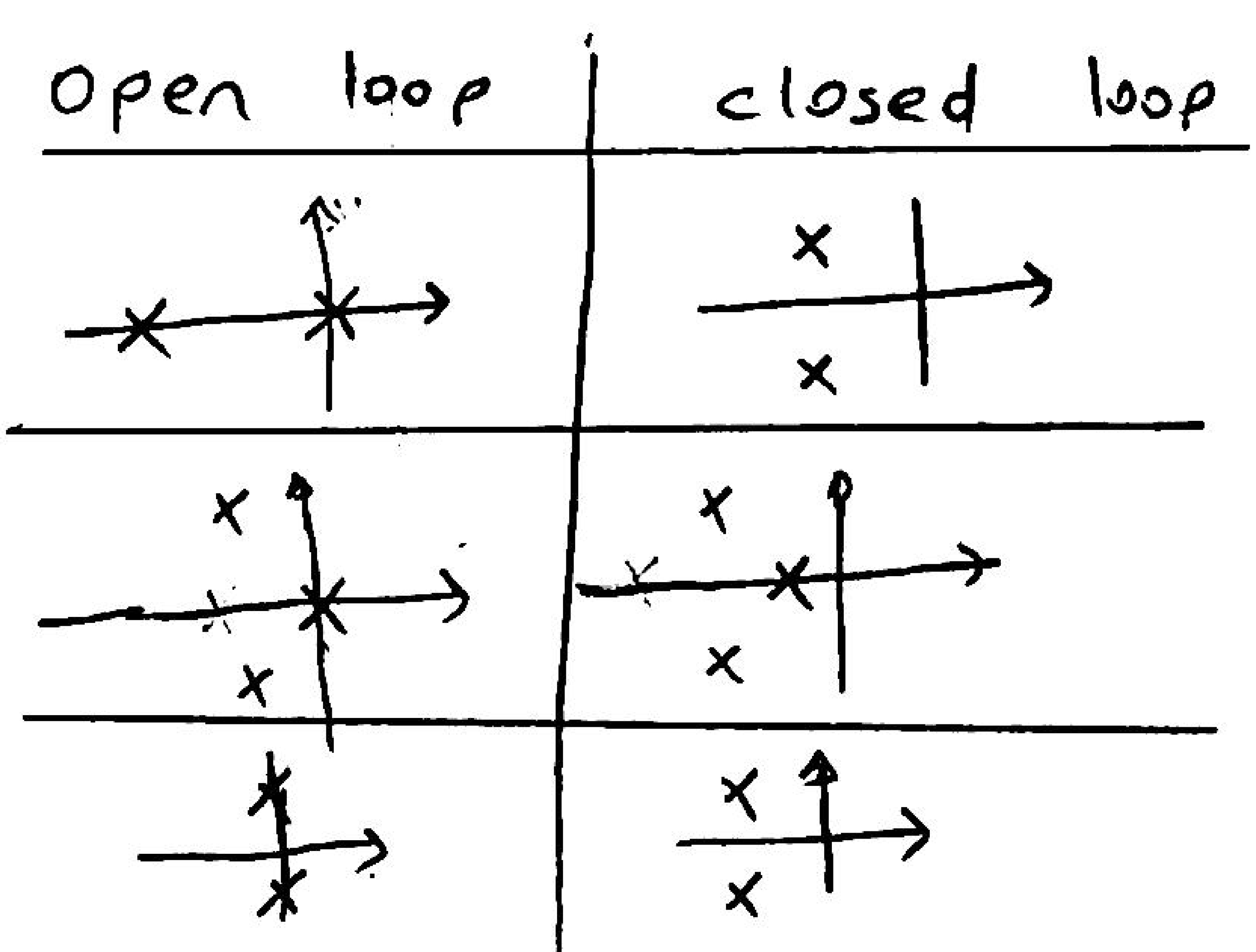
$T_1 = T_2 = T_7 = T_8$ $\quad$ $T_3 = T_4 = T_8 = T_{11}$

7-8 oscillation. $\quad$ 10, 11 12 Unstable

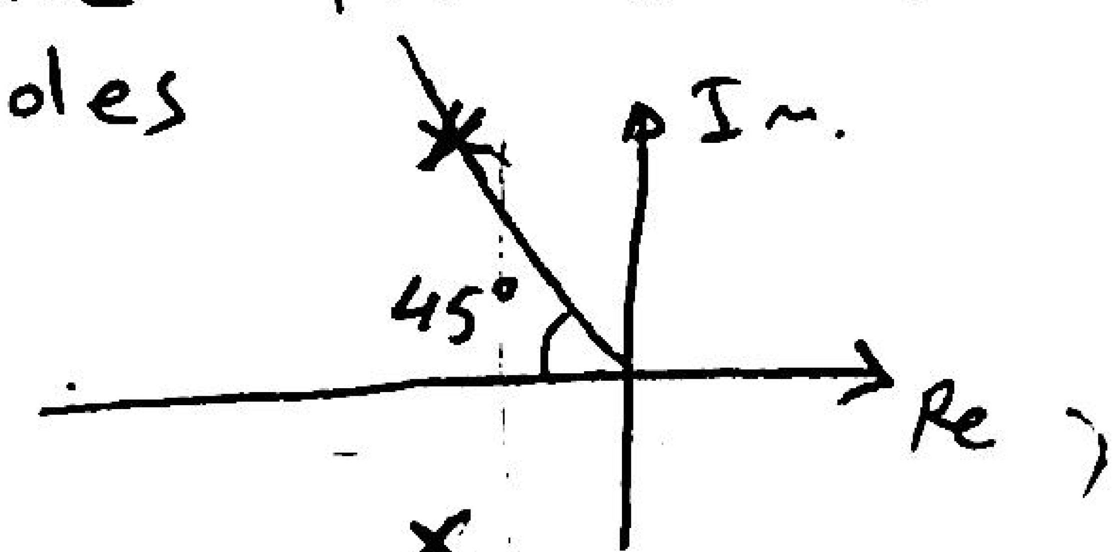
7-8-9 conditionaly stable

1-2-3-4-5-6 stable

Design with Root Locus



What is the best place for closed loop poles



1) try to push towards Left
2) try to put on 45° line