Temel Formuller

$\sin^2 x+\cos^2 x=1$

cos(x+y)=cosx cosy - sinx siny (1)

sin(x+y)=sinx cosy + siny cosx (2)

sin(-x)=- sinx (3)

cos(-x)=cos(x) (4)

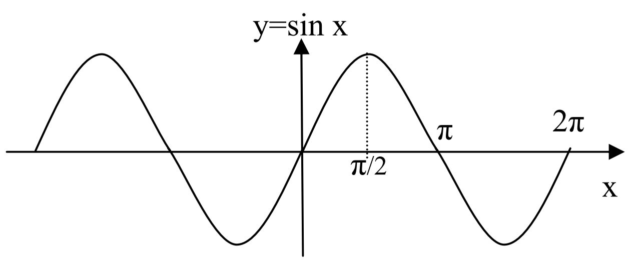
tan(-x)=-tan(x) (4.a)

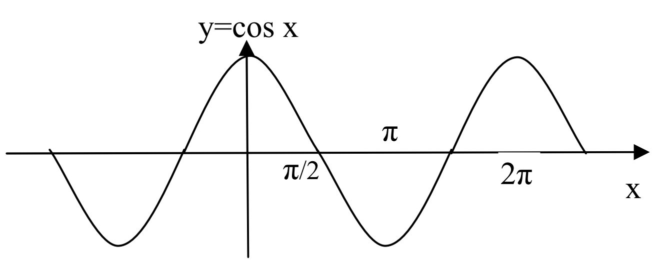
x	0	30	45	60	90
sinx	0	0.5	0.707	0.8	1
cosx	1	0.8	0.707	0.5	0

1)

x	0	30	45	60	90
sin x	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
	0	0.5	0.707	0.866	1

x	0	30	45	60	90
cos x	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
	1	0.866	0.707	0.5	0





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(1) de y yerine –y yaz ve 3,4 bagintilarini kullan

cos(x-y)=cosx cos(-y) - sinx sin(-y)

cos(x-y)=cosx cos y + sinx sin y (5)

(2) de y yerine –y yaz ve 3,4 bagintilarini kullan

sin(x-y)=sinx cos(-y) - sinx cos(-y)

sin(x-y)=sinx cosy - siny cos x (6)

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(1) ve (5) i topla

cos(x+y)=cosx cosy - sinx siny

cos(x-y)=cosx cos y + sinx sin y

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cos(x+y)+cos(x-y)=2 cosx cosy

cosx cosy= $\frac{1}{2}$ **[cos(x+y) +cos(x-y)] (7)**

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(5) den (1) i cikar

cos(x-y)=cosx cos y + sinx sin y

cos(x+y)=cosx cosy - sinx siny

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cos(x-y)-cos(x+y)=2 sinx siny

sinx siny = $\frac{1}{2}$ **[cos(x-y)- cos(x+y)] (8)**

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(2) ve (6) yi topla

sin(x+y)=sinx cosy +siny cosx (2)

sin(x-y)=sinx cosy - siny cos x (6)

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sin(x+y) + sin(x-y) =2 sinx cosy

sinx cosy = $\frac{1}{2}$ **[sin(x+y)+ sin(x-y)] (9)**

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(1) de x=90 koy

cos(x+90)=cosx cos90 - sinx sin90

cos(x+90)=– sinx (10)

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(1) de x=-90 koy

cos(x-90)=cosx cos(-90) - sinx sin(-90)

cos(x- 90)= sinx (11)

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(2) de x=90 koy

sin(x+90)=sinx cos90 + sin90 cosx

sin(x+ 90)= cosx (12)

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(2) de x=-90 koy

sin(x-90)=sinx cos(-90) + sin(-90) cosx

sin(x- 90)= -cosx (13)

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(1) de x=90, y=-q koy

cos(90-q)=cos90 cos(-q) – sin90 sin(-q)

cos(90-q)= sinq (14)

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(2) de x=90, y=-q koy

sin(90-q)=sin90 cos(-q) + cos90 sin(-q)

sin(90-q)= cosq (15)

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(1) de y=x koy

$\cos(x+x)=\cos x \cos x - \sin x \sin x$

$\cos(2x) = \cos^2 x-\sin^2 x$

$\sin^2 x=1-\cos^2 x$ koy

$\cos^2 x -\sin^2 x= \cos^2 x -(1- \cos^2 x)= 2\cos^2 x -1$

$$\mathbf{\cos(2x) = 2\cos^2 x -1} \qquad \textbf{(16)}$$

veya

$$\mathbf{\cos^2 x = \frac{1}{2} [1+\cos(2x)]} \qquad \textbf{(17)}$$

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(1) de y=x koy

$\cos(x+x)=\cos x \cos x - \sin x \sin x$

$\cos(2x) = \cos^2 x-\sin^2 x$

$\cos^2 x=1-\sin^2 x$ koy

$\cos^2 x -\sin^2 x= 1- \sin^2 x - \sin^2 x = 1-2\sin^2 x$

$$\mathbf{\cos(2x) =1- 2\sin^2 x} \qquad \textbf{(18)}$$

veya

$$\mathbf{\sin^2 x = \frac{1}{2} [1-\cos(2x)]} \qquad \textbf{(19)}$$

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(2) de y=x koy

$\sin(x+x)=\sin x \cos x + \sin x \cos x=2 \sin x \cos x$

$$\mathbf{\sin 2x =2 \sin x \cos x} \qquad \textbf{(20)}$$

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(16) ve(17) de da q=2x koy

$$\mathbf{\cos q = 2\cos^2 \frac{q}{2} -1} \qquad \textbf{(21)}$$

$$\mathbf{\cos q = 1-2\sin^2 \frac{q}{2}} \qquad \textbf{(22)}$$

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(20) de q=2x koy

$$\mathbf{\sin q =2 \sin \frac{q}{2} \cos \frac{q}{2}} \qquad \textbf{(23)}$$

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	90	180	270	-90	-180
sinx	1	0	-1	-1	0
cosx	0	-1	0	0	-1

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$\sin^3 q=\sin q \sin^2 q$

(19) bagintisinda verilen $\sin^2 q$ yerine koy

$\sin^3 q= \sin q \ \frac{1}{2} [1-\cos(2q)] = \frac{1}{2} \mathbf{[} \sin q- \sin q \cos(2q)\mathbf{]}$

(9) bagintisinda x=q, y=2q koy

$\sin q \cos(2q)= \frac{1}{2} [\sin(q+2q)+ \sin(q-2q)]$

$= \frac{1}{2} [\sin(3q)- \sin(q)]$

$\sin^3 q= \frac{1}{2} \mathbf{[} \sin q- \frac{1}{2} [\sin(3q)- \sin q] \mathbf{]}$

$= \frac{1}{2} \sin q - \frac{1}{4} \sin(3q) +\frac{1}{4} \sin q$

$$\mathbf{\sin^3 q= = - \frac{1}{4} \sin(3q) +\frac{3}{4} \sin q} \qquad \textbf{(34)}$$

veya

$$\mathbf{\sin(3q)= -4\sin^3 q +3\sin q} \qquad \textbf{(35)}$$

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$\cos^3 q=\cos q \cos^2 q$

(17) bagintisinda verilen $\cos^2 q$ yerine degerini koy

$\cos^3 q= \cos q \ \frac{1}{2} [1+\cos(2q)] = \frac{1}{2} \mathbf{[} \cos q+ \cos q \cos(2q)\mathbf{]}$

(7 bagintisinda x=q, y=2q koy

$\cos q \cos 2q=\frac{1}{2} [\cos(q+2q) +\cos(q-2q)]$

$= \frac{1}{2} [\cos(3q)+ \cos q]$

$\cos^3 q=\frac{1}{2} \mathbf{[} \cos q+\frac{1}{2} [\cos(3q)+ \cos q]\mathbf{]}$

$= \frac{1}{4} \cos 3q +\frac{3}{4} \cos q$

$$\mathbf{\cos^3 q= = \frac{1}{4} \cos 3q +\frac{3}{4} \cos q} \qquad \textbf{(36)}$$

$$\mathbf{\cos 3q=4 \cos^3 q -3 \cos q} \qquad \textbf{(37)}$$

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$$\tan(x+y)= \frac{\sin(x+y)}{\cos(x+y)}=\frac{\sin x\cos y+\sin y\cos x}{\cos x\cos y-\sin\sin y}$$

payi ve paydayi cosx cosy ifadesine bolelim

$$=\frac{\dfrac{\sin x\cos y+\sin y\cos x}{\cos x\cos y}}{\dfrac{\cos x\cos y-\sin\sin y}{\cos x\cos y}}=\frac{\dfrac{\sin x\cos y}{\cos x\cos y}+\dfrac{\sin y\cos x}{\cos x\cos y}}{\dfrac{\cos x\cos y}{\cos x\cos y}-\dfrac{\sin\sin y}{\cos x\cos y}}$$

$$=\frac{\dfrac{\sin x}{\cos x}+\dfrac{\sin y}{\cos y}}{1-\dfrac{\sin x}{\cos x}\dfrac{\sin y}{\cos y}}=\frac{\tan x+\tan y}{1-\tan x\tan y}$$

$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x\tan y} \qquad \textbf{(41)}$$

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tan(-x)=-tan(x) oldugu dikkate alinirsa

$$\tan(x-y)=\frac{\tan x-\tan y}{1+\tan x\tan y} \qquad \textbf{(42)}$$

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(41) de y=x koy

$$\tan(2x)=\frac{2\tan x}{1-\tan^2 x}$$ **(43)**

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(43) de q=2x koy

$$\tan(q)=\frac{2\tan\frac{q}{2}}{1-\tan^2\frac{q}{2}}$$ **(44)**

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$$\tan x=\frac{\sin x}{\cos x}=\frac{\sin x}{\sqrt{1-\sin^2 x}}=\frac{\sqrt{1-\cos^2 x}}{\cos x}$$ **(51)**

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$$\tan x=\frac{\sin x}{\sqrt{1-\sin^2 x}} \quad\Rightarrow\quad \tan^2 x=\frac{\sin^2 x}{1-\sin^2 x}$$

$\tan^2 x(1-\sin^2 x) = \sin^2 x$

$\tan^2 x - \tan^2 x\sin^2 x = \sin^2 x$

$\tan^2 x - \tan^2 x\sin^2 x - \sin^2 x=0$

$-\tan^2 x\sin^2 x - \sin^2 x=-\tan^2 x$

$-\sin^2 x(\tan^2 x+1)=-\tan^2 x$

$$\sin^2 x=\frac{\tan^2 x}{1+\tan^2 x} \quad\Rightarrow \sin x=\frac{\tan x}{\sqrt{1+\tan^2 x}}$$ **(52)**

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$$\tan x=\frac{\sqrt{1-\cos^2 x}}{\cos x} \quad\Rightarrow\quad \tan^2 x=\frac{1-\cos^2 x}{\cos^2 x}$$

$\tan^2 x\cos^2 x= 1-\cos^2 x$

$\cos^2 x(1+\tan^2 x)=1$

$$\cos^2 x=\frac{1}{1+\tan^2 x} \quad\Rightarrow \cos x=\frac{1}{\sqrt{1+\tan^2 x}}$$ **(53)**

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(23) den

$\sin q = 2\sin\frac{q}{2}\cos\frac{q}{2}$

(52)ve (53) den

$$\sin\frac{x}{2}=\frac{\tan\frac{x}{2}}{\sqrt{1+\tan^2\frac{x}{2}}} \qquad \cos\frac{x}{2}=\frac{1}{\sqrt{1+\tan^2\frac{x}{2}}}$$

Ucu birlestirilirse

$$\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}=2\frac{\tan\frac{x}{2}}{\sqrt{1+\tan^2\frac{x}{2}}}\frac{1}{\sqrt{1+\tan^2\frac{x}{2}}}$$

$$\sin x=\frac{\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$ **(54)**

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(21) ve (53) den

$$\cos x = 2\cos^2\frac{x}{2}-1=2\left(\frac{1}{\sqrt{1+\tan^2\frac{x}{2}}}\right)^2-1$$

$$\cos x=\frac{1-\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$ **(55)**

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(54) ve (55) icin baska bir yol.

(53) ve (44) birlestirilirse

$$\cos x=\frac{1}{\sqrt{1+\tan^2 x}}=\frac{1}{\sqrt{1+\left(\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}\right)^2}}$$

basitlestirmek icin q=tan(x/2) tanimi yapalim.

$$=\frac{1}{\sqrt{1+\left(\frac{2q}{1-q^2}\right)^2}}=\frac{1}{\sqrt{1+\frac{4q^2}{1-2q^2+q^4}}}=\frac{1}{\sqrt{\frac{1-2q^2+q^4}{1-2q^2+q^4}+\frac{4q^2}{1-2q^2+q^4}}}$$

$$=\frac{1}{\sqrt{\frac{1+2q^2+q^4}{1-2q^2+q^4}}}=\frac{1}{\sqrt{\frac{(1+q^2)^2}{(1-q^2)^2}}}=\frac{1-q^2}{1+q^2}=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\cos x=\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$ **(56)**

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(52) ve (44) birlestirilirse

$$\sin x = \frac{\tan x}{\sqrt{1+\tan^2 x}} = \frac{\dfrac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}}{\sqrt{1+\left(\dfrac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}\right)^2}}$$

q=tan(x/2) tanimi yapalim.

$$= \frac{\dfrac{2q}{1-q^2}}{\sqrt{1+\left(\dfrac{2q}{1-q^2}\right)^2}} = \frac{\dfrac{2q}{1-q^2}}{\sqrt{1+\dfrac{4q^2}{1-2q^2+q^4}}} = \frac{\dfrac{2q}{1-q^2}}{\sqrt{\dfrac{1-2q^2+q^4}{1-2q^2+q^4}+\dfrac{4q^2}{1-2q^2+}}}$$

$$= \frac{\dfrac{2q}{1-q^2}}{\sqrt{\dfrac{1-2q^2+q^4}{1-2q^2+q^4}+\dfrac{4q^2}{1-2q^2+q^4}}}$$

$$= \frac{\dfrac{2q}{1-q^2}}{\sqrt{\dfrac{1+2q^2+q^4}{1-2q^2+q^4}}} = \frac{\dfrac{2q}{1-q^2}}{\sqrt{\dfrac{(1+q^2)^2}{(1-q^2)^2}}}$$

$$= \frac{2q}{1-q^2}\frac{1-q^2}{1+q^2} = \frac{2q}{1+q^2} = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} \qquad \textbf{(57)}$$

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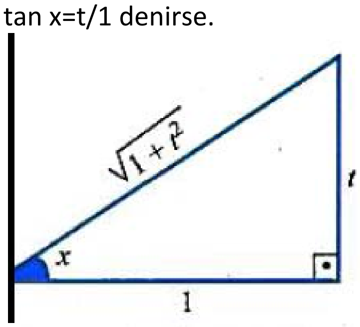
tan x=t/1 denirse.



$$\sin x = \frac{t}{\sqrt{1+t^2}}, \quad \cos x = \frac{1}{\sqrt{1+t^2}}, \quad dx = \frac{1}{1+t^2}dt$$

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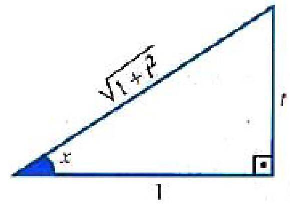

tan x/2= t,

$$\sin\frac{x}{2} = \frac{t}{\sqrt{1+t^2}}, \quad \cos\frac{x}{2} = \frac{1}{\sqrt{1+t^2}}$$

$$\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2} = 2\frac{t}{\sqrt{1+t^2}}\frac{1}{\sqrt{1+t^2}} = \frac{2t}{1+t^2},$$

$$\cos x = 2\cos^2\frac{x}{2} - 1 = 2\frac{1}{1+t^2} - 1 = \frac{1-t^2}{1+t^2}$$

$$\frac{x}{2} = \arctan t \Rightarrow dx = \frac{2\,dt}{1+t^2}$$

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Ters Trigonometrik Fonksiyonlar

8. Aşağıdaki bağıntıların doğruluğunu gösteriniz.

a) $\sin(\arctan x) = \frac{x}{\sqrt{1+x^2}}$

b) $\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}}$

c) $\tan(\arccos x) = \frac{\sqrt{1-x^2}}{x}$

ç) $\tan(\arcsin x) = \frac{x}{\sqrt{1-x^2}}$

d) $\arcsin(\cos x) = \frac{\pi}{2} - x$

e) $\cos(a + c\sin x) = \sqrt{1-x^2}$

f) $\arcsin x + \arcsin t = \arcsin(x\sqrt{1-t^2} + t\sqrt{1-x^2})$

g) $\arccos x + \arccos t = \arccos\left(xt - \sqrt{(1-x^2)(1-t^2)}\right)$

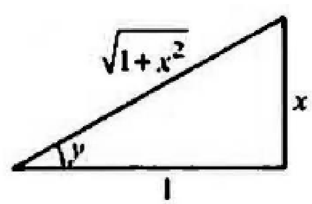
h) $\arctan x - \arctan t = \arctan\frac{x-t}{1+xt}$

ı) $\sin(2\arcsin x) = 2x\sqrt{1-x^2}$

a) $\sin(\arctan x) = \dfrac{x}{\sqrt{1+x^2}}$

$$\arctan x = y \Rightarrow \tan y = x \Rightarrow \frac{\sin y}{\cos y} = x \Rightarrow \frac{\sin y}{\sqrt{1-\sin^2 y}} = x \Rightarrow \sin^2 y = x^2 - x^2\sin^2 y$$

$$\Rightarrow \sin^2 y = \frac{x^2}{1+x^2} \Rightarrow \sin y = \frac{x}{\sqrt{1+x^2}} \Rightarrow \sin(\arctan x) = \frac{x}{\sqrt{1+x^2}}$$



b) $\cos(\arctan x) = \dfrac{1}{\sqrt{1+x^2}}$

$$y = \arctan x \Rightarrow x = \tan y \Rightarrow \cos y = \frac{1}{\sqrt{1+x^2}} \Rightarrow$$

$$\cos(\arctan x) = \frac{1}{\sqrt{1+x^2}} \quad \textit{olur.}$$

c) $\tan(\arccos x) = \dfrac{\sqrt{1-x^2}}{x}$

$$y = \arccos x \Rightarrow x = \cos y \Rightarrow \sqrt{1-x^2} = \sin y \Rightarrow \tan y = \frac{\sqrt{1-x^2}}{x}$$

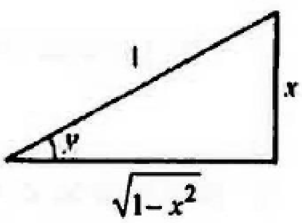
$$\Rightarrow \tan(\arccos x) = \frac{\sqrt{1-x^2}}{x}$$

ç) $\tan(\arcsin x) = \dfrac{x}{\sqrt{1-x^2}}$

$$y = \arcsin x \Leftrightarrow \sin y = x \Rightarrow \tan y = \frac{x}{\sqrt{1-x^2}}$$

olur. Buradan

$\tan y = \tan(\arcsin x) = \dfrac{x}{\sqrt{1-x^2}}$ bulunur.



d) $\arcsin(\cos x) = \dfrac{\pi}{2} - x$

$$\cos x = y \Rightarrow \sin\left(\frac{\pi}{2} - x\right) = y \Rightarrow \frac{\pi}{2} - x = \arcsin y \Rightarrow \frac{\pi}{2} - x = \arcsin(\cos x)$$

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e) $\arcsin x + \arcsin t = \arcsin\left(x\sqrt{1-t^2} + t\sqrt{1-x^2}\right)$

$\arcsin x = u$, $\arcsin t = v$, denirse $x = \sin u$, $t = \sin v$ olur.

$$\sin(u+v) = \sin u \cos v + \sin v \cos u = \sin u\sqrt{1-\sin^2 v} + \sin v\sqrt{1-\sin^2 u}$$
$$= x\sqrt{1-t^2} + t\sqrt{1-x^2} \Rightarrow \sin(u+v) = x\sqrt{1-t^2} + t\sqrt{1-x^2}$$

$$\arcsin x + \arcsin t = \arcsin\left(x\sqrt{1-t^2} + t\sqrt{1-x^2}\right)$$

f) $\arccos x + \arccos t = \arccos\left(xt - \sqrt{(1-x^2)(1-t^2)}\right)$

$u = \arccos x$, $v = \arccos t$, denirse $x = \cos u$, $t = \cos v$ olur.

$$\cos(u+v) = \cos u \cos v - \sin u \sin v = \cos u.\cos v - \sqrt{1-\cos^2 u}\,\sqrt{1-\cos^2 v}$$
$$= x.t - \sqrt{1-x^2}\,\sqrt{1-t^2} \Rightarrow u + v = \arccos\left(xt - \sqrt{1-x^2}\,\sqrt{1-t^2}\right)$$

$$\arccos x + \arccos t = \arccos\left(xt - \sqrt{1-x^2}\,\sqrt{1-t^2}\right)$$

g) $\arctan x - \arctan t = \arctan\left(\dfrac{x-t}{1+xt}\right)$

$u = \arctan x$, $v = \arctan t \Rightarrow x = \tan u$, $t = \tan v$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u.\tan v} = \frac{x-t}{1+xt} \Rightarrow u - v = \arctan\left(\frac{x-t}{1+xt}\right) \Rightarrow$$

$$\arctan x - \arctan v = \arctan\left(\frac{x-t}{1+xt}\right)$$

h) $\sin(2\arcsin x) = 2x\sqrt{1-x^2}$

$\arcsin x = u \Leftrightarrow x = \sin u$ *olur.* $\sin(2u) = 2\sin u \cos u = 2\sin u\sqrt{1-\sin^2 u} = 2x\sqrt{1-x^2}$

$\Rightarrow \sin(2\arcsin x) = 2x\sqrt{1-x^2}$

11. Aşağıdaki ifadeleri hesaplayınız.

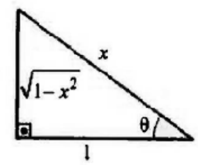
a) cot(arcsecx) **b)** sin(arccotx)

d) sin(arccosx) **e)** sin(arcsecx)

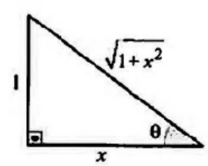
c) cos(arccotx)

f) cos(arccscx)

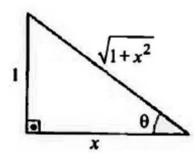
a) $\cot(\text{arcsec}x) = \dfrac{1}{\sqrt{1-x^2}}$



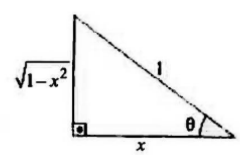
c) $\cos(\text{arccot}x) = \dfrac{x}{\sqrt{1+x^2}}$



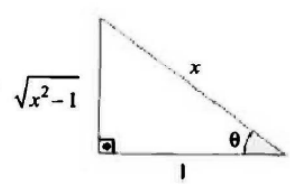
b) $\sin(\text{arccot}x) = \dfrac{1}{\sqrt{1+x^2}}$



d) $\sin(\arccos x) = \sqrt{1-x^2}$



e) $\sin(\text{arcsec}x) = \dfrac{\sqrt{x^2-1}}{x}$



f) $\cos(\text{arccsc}x) = \dfrac{\sqrt{x^2-1}}{x}$