$\operatorname{arccos} x = \ln(x + \sqrt{x^2 - 1}),$ sorusunun cozumu.

arc cosh x=y,

cosh y=x

$(e^y + e^{-y})/2 = x,$

$Q=e^y$ tanimi yapalim.

$Q^{-1}=e^{-y}$ olur.

cosh $y=(e^y + e^{-y})/2=x$, → $(Q+Q^{-1})/2=x$ → $Q + \frac{1}{Q} = 2x$

$\frac{Q^2 + 1}{Q} = 2x$, $Q^2-2xQ+1=0,$

$$Q_{1,2} = \frac{2x \pm \sqrt{(2x)^2 - 4}}{2} = x \pm \sqrt{x^2 - 1}$$

$Q=e^y$ → $e^y = x \pm \sqrt{x^2 - 1}$

$$\text{Ln}(e^y) = \text{Ln}(x \pm \sqrt{x^2 - 1})$$

$$y = \text{Ln}(x \pm \sqrt{x^2 - 1}) = \text{arc cosh } x$$

± ifadesinde hem (+) hem (–) her ikisi de cozumdur. Pratikte
+ kullanilir.

Turev tanimi: $\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

111) **f(x)=ax^2 nin turevinin 2ax oldugunu turev tanimi kullanarak gosterin.**

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{a(x+\Delta x)^2 - ax^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{a(x^2+2x\Delta x+\Delta x^2) - ax^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{ax^2 + a\ 2x\Delta x + 2\Delta x^2\ - ax^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{a\ 2x\Delta x + 2\Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} (a\ 2x + \Delta x) = a\ 2x + 0 = 2ax$$

121) **f(x)=sin(x) nin turevinin cos(x) oldugunu turev tanimi kullanarak gosterin.**

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) = f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin(x)\cos(\Delta x) + \sin(\Delta x)\cos(x) - \sin(x)}{\Delta x}$$

$\Delta x \to 0$ icin cos(Δx)=1 olur ve sin(x)cos(Δx)-sin(x)=0 olur.

$$\lim_{\Delta x \to 0} \frac{\sin(\Delta x)\cos(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin(\Delta x)}{\Delta x} \cos(x) = 1 \cos(x) = \cos(x)$$

122) $f = e^{ax} \rightarrow u=ax,\ f= e^{U},\ \frac{df}{du} = e^{U},\quad \frac{du}{dx} = a,\qquad \frac{df}{dx} = \frac{df}{du}\frac{du}{dx} = e^{U}\ a = e^{ax}\ a$

123) $f=Ln(ax) \rightarrow u=ax,\ f=Ln(u),\ \frac{df}{du} = \frac{1}{u},\quad \frac{du}{dx} = a,\qquad \frac{df}{dx} = \frac{df}{du}\frac{du}{dx} = \frac{1}{u}a = \frac{1}{ax}a = \frac{1}{x}$

$\int \frac{1}{x} dx = Ln(x) + C = Ln(x) + Ln(a) = Ln(ax)$

Ters fonk turevleri.

$$\frac{df^{-1}(x)}{dx}=\frac{1}{\left.\frac{df}{dx}\right|_{x\to f^{-1}(x)}}$$

P711): $f(x)=x^3$, $f^{-1}(x)=x^{1/3}$, $(f^{-1})'=?$

$$\frac{d(x^{1/3})}{dx}=\frac{1}{\left.\frac{d(x^3)}{dx}\right|_{x\to x^{1/3}}}=\frac{1}{\left.3x^2\right|_{x\to x^{1/3}}}=\frac{1}{3(x^{1/3})^2}$$

$(f^{-1})' =(1/3)x^{-2/3}$

Bildigimiz yontemle cozersek.

$[x^{1/3}]' =1/3\ x^{1/3-1}=1/3\ x^{-2/3}$

bulunur.

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p772) $f(x)=x^2$, $(f^{-1})' =?$

$$\frac{df^{-1}(x)}{dx}=\frac{1}{\left.\frac{df}{dx}\right|_{x\to f^{-1}(x)}}$$

$$(\sqrt{x})'=\frac{1}{\left.(x^2)'\right|_{x\to\sqrt{x}}}=\frac{1}{\left.2x\right|_{x\to\sqrt{x}}}=\frac{1}{2\sqrt{x}}$$

Bildigimiz yolla cozersek.

$[x^{1/2}]' =1/2\ x^{1/2-1}=1/2\ x^{-1/2}$

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p772) $f(x)=x^5+x$, $(f^{-1})'(2)=?$

$f(x)=x^5+x$, $f^{-1}(x)=?$

$f^{-1}(x)$bilmiyoruz. Hesaplamasi da kolay degil. Fakat soruda sadece $f^{-1}(x)$ in turevinin x=2 deki degeri soruluyor.

$$\frac{df^{-1}(x)}{dx}=\frac{1}{\left.\frac{df}{dx}\right|_{x\to f^{-1}(x)}}$$

$$(f^{-1}(x))'=\frac{1}{\left.(x^5+x)'\right|_{x\to f^{-1}(x)}}=\frac{1}{\left.(5x^4+1)\right|_{x\to f^{-1}(x)}}=\frac{1}{5[f^{-1}(x)]^4+1},$$

$f^{-1}(2)=?$

$x^5+x=2$, x=?

x=1 koklerden birisidir. o halde $f^{-1}(2)=1$ alabiliriz.

$$(f^{-1}(2))'=\frac{1}{5[1]^4+1}=\frac{1}{6},$$

$$\frac{df^{-1}(x)}{dx}=\frac{1}{\left.\frac{df}{dx}\right|_{x\to f^{-1}(x)}}$$

Ters trigonometrik turevler.

tan(arc tan x)=x
sin(arc sin x)=x
cos(arc cos x)=x

arc tan(tan x)=x
arc sin(sin x)=x
arc cos(cos x)=x

$\tan^2 x =(\tan x)^2$
$\tan x^2 \neq \tan^2 x$

$\tan^2(\text{arc tan } x) =[\tan(\text{arc tan } x)]^2 =x^2$
$\tan^3(\text{arc tan } x) =x^3$

$\sin^2(\text{arc sin } x) =x^2$
$\cos^2(\text{arc cos } x) =x^2$

sin (arc cos x) =?
sin (arc tan x) =?
tan (arc sin x) =?

Bu durumda ic ve disi ayni yapmaya calisiriz.
Once temel trigonometrik bagintilari hatirlamamiz lazim.

	sin x	**cos x**	**tan x**
sin x		$\sqrt{1-\cos^2 x}$	$\frac{\tan x}{\sqrt{1+\tan^2 x}}$
cos x	$\sqrt{1-\sin^2 x}$		$\frac{1}{\sqrt{1+\tan^2 x}}$
tan x	$\frac{\sin x}{\sqrt{1-\sin^2 x}}$	$\frac{\sqrt{1-\cos^2 x}}{\cos x}$	

p33) sin (arc cos x) =?

sin (arc cos x) = $\sqrt{1-\cos^2(\text{arc cos x})}$ = $\sqrt{1-x^2}$

benzer sekilde

p34) cos (arc sin x) = $\sqrt{1-\sin^2(\text{arc sin x})}$ = $\sqrt{1-x^2}$

p35) sin (arc tan x) =?

$\sin x=\frac{\tan x}{\sqrt{1+\tan^2 x}}$ oldugundan

sin (arc tan x) $=\frac{\tan(\arctan x)}{\sqrt{1+\tan^2(\arctan x)}}$

sin (arc tan x) $=\frac{x}{\sqrt{1+x^2}}$

olarak bulunur.

p37) cos (arc tan x)=?

$\cos x=\frac{1}{\sqrt{1+\tan^2 x}}$ oldugundan

$$\cos(\text{arc tan x}) = \frac{1}{\sqrt{1+\tan^2(\arctan x)}}$$

$$\cos(\text{arc tan x}) = \frac{1}{\sqrt{1+x^2}}$$

421)f(x)=arc sin x ise f '(x)=?

$$\frac{f^{-1}(x)}{dx} = \frac{1}{\left.\frac{df}{dx}\right|_{x\to f^{-1}(x)}}$$

f(x)=sin x, f $^{-1}$(x)=arc sin x

$$(\arcsin x)' = \frac{1}{(\sin x)'|_{x\to\arcsin x}}$$

$$= \frac{1}{\cos x|_{x\to\arcsin x}}$$

$$= \frac{1}{\cos(\arcsin x)}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

422)f(x)=arc cos x ise f '(x)=?

$$(\arccos x)' = \frac{1}{(\cos x)'|_{x\to\arccos x}}$$

$$= \frac{1}{-\sin x|_{x\to\arccos x}}$$

$$= \frac{1}{-\sin(\arccos x)}$$

$$= \frac{1}{-\sqrt{1-x^2}}$$

423)f(x)=arc tan x ise f '(x)=?

$$(\arctan x)' = \frac{1}{(\tan x)'|_{x\to\arctan x}}$$

$$= \frac{1}{(1+\tan^2 x)|_{x\to\arctan x}}$$

$$= \frac{1}{(1+\tan^2(\text{arc tanx})}$$

$$= \frac{1}{1+x^2}$$