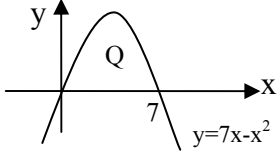


ALAN HESABI ORNEKLER

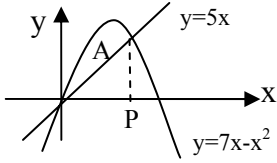
115) Sekilde gosterilen q bolgesinin Alanini hesaplayin



COZUM:

$$Q = \int_{x=0}^{x=7} (7x - x^2) dx = \left(7 \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_{x=0}^{x=7} = \left(7 \frac{7^2}{2} - \frac{7^3}{3} \right) - 0 = 57.16$$

121) Sekilde gosterilen A bolgesinin Alanini hesaplayin



COZUM:

$$A = \int_{x=0}^{x=P} [f(x) - g(x)] dx = \int_{x=0}^{x=P} [(7x - x^2) - 5x] dx$$

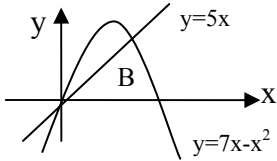
Burada P $y = 7x - x^2$ ve $y = 5x$ egrilerinin kesim noktasidir.

$$7x - x^2 = 5x, \quad 2x - x^2 = 0, \quad x^2 - 2x = 0, \quad x = 0, \quad x = 2,$$

$$y = 7x - x^2 = 7 \cdot 2 - 2^2 = 10, \quad \text{veya } y = 5x = 5 \cdot 2 = 10.$$

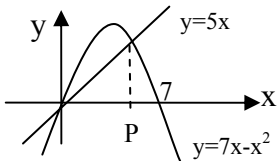
$$A = \int_{x=0}^{x=2} (2x - x^2) dx = \left(2 \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_{x=0}^{x=2} = \left(2 \frac{2^2}{2} - \frac{2^3}{3} \right) - (0) = \frac{4}{3}$$

122) Sekilde gosterilen B bolgesinin Alanini hesaplayin



COZUM: Bolgeyi iki parca halinde dusunmemiz gerekir.

Burada P $y = 7x - x^2$ ve $y = 5x$ egrilerinin kesim noktasidir. $x = 2$



$$B = \int_{x=0}^{x=P} 5x dx + \int_{x=P}^{x=7} (7x - x^2) dx = B_1 + B_2$$

Yukarida P $x = 2$ ve $y = 10$ bulundu

$$B = \int_{x=0}^{x=2} 5x dx + \int_{x=2}^{x=7} (7x - x^2) dx = \left(5 \frac{x^2}{2} \right) \Big|_{x=0}^{x=2} + \left(7 \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_{x=2}^{x=7}$$

ayri ayri hesaplayalim.

$$B_1 = \left(5 \frac{x^2}{2} \right) \Big|_{x=0}^{x=2} = \left(5 \frac{2^2}{2} \right) - 0 = \left(5 \frac{2^2}{2} \right) = 10$$

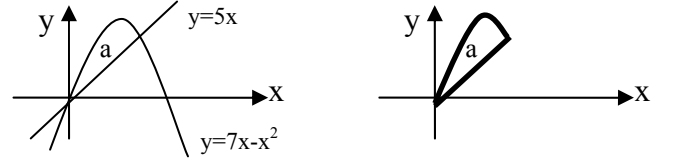
$$B_2 = \left(7 \frac{x^2}{2} - \frac{x^3}{3} \right) \Big|_{x=2}^{x=7} = \left(7 \frac{7^2}{2} - \frac{7^3}{3} \right) - \left(7 \frac{2^2}{2} - \frac{2^3}{3} \right) = 45.83$$

$$B = B_1 + B_2 = 10 + 45.83 = 55.83$$

Not: $B = Q - A = 57.16 - (4/3) = 55.83$ olarak da hesaplannabilir. Sonuclar ayni olacaktır.

EGRI UZUNLUGU

141) Sekilde gosterilen a bolgesini kaplayan telin uzunlugunu bulun.



COZUM: Teli iki parca halinde dusunup ayri ayri hesaplariz.

L_1 : ust kisimdaki egri parcasinin uzunlugu.

$$L_1 = \int_{x=0}^{x=P} dl = \int_{x=0}^{x=P} \sqrt{1 + (y')^2} dx = \int_{x=0}^{x=P} \sqrt{1 + (y')^2} dx$$

$$y = 7x - x^2, \quad y' = 7 - 2x$$

$$= \int_{x=0}^{x=2} \sqrt{1 + (7 - 2x)^2} dx$$

L_2 : alt kisimdaki dogru parcasinin uzunlugu.

$$L_2 = \int_{x=0}^{x=2} dl = \int_{x=0}^{x=2} \sqrt{1 + (y')^2} dx = \int_{x=0}^{x=2} \sqrt{1 + (5)^2} dx = \sqrt{1 + (5)^2} \int_{x=0}^{x=2} dx = 2\sqrt{26}$$

$$\text{Toplam uzunluk} = L_1 + L_2,$$

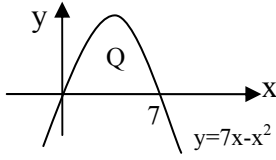
KUTLE HESABI

Cizgisel kutlede: Kutle = yogunluk x uzunluk

Alanlarin kutlesi Kutle = yogunluk x Alan

Cisimlerin kutlesi Kutle = yogunluk x Hacim

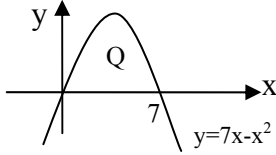
122) Sekilde gosterilen Q bolgesinin yogunlugu $\sigma(x) = 10$ olarak verilmektedir. Q bolgesinin kutlesini hesaplayin



COZUM:

Burada yoğunluk sabittir. Bolgenin alani daha once $Q=57.16$ olarak bulunmustu. O halde kutle $m=10 \cdot 57.16=571.6$ birim olarak bulunur.

124) Sekilde gosterilen Q bolgesinin yogunlugu $\sigma(x)=(x^3+3x)$ olarak verilmektedir. Q bolgesinin kutlesini hesaplayin

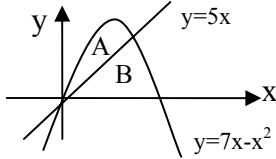


COZUM:

$$m = \int_{x=0}^{x=7} \sigma(x) f(x) dx = \int_{x=0}^{x=7} (x^3 + 3x)(7x - x^2) dx$$

$$m = \int_{x=0}^{x=7} (-x^5 + 7x^4 - 3x^3 + 21x^2) dx = 4521.9$$

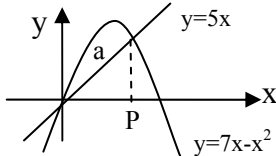
135) Sekilde gosterilen A bolgesinin yogunlugu $\sigma(x)=10$, B bolgesinin yogunlugu $\sigma(x)=20$, seklinde degismektedir. Bolgenin toplam agirligini bulun.



COZUM:

A bolgesinin alani $4/3$,
A bolgesinin agirligi $(4/3)10=40/3=13.3$
B bolgesinin alani 55.83 ,
B bolgesinin agirligi $55.83 \cdot 20=1116.6$
Toplam agirlik $1116.6+13.3=1129.9$

137) Sekilde gosterilen a bolgesinin yogunlugu $\sigma(x)=(x^3+3x)$ seklinde degismektedir. Bolgenin agirligini bulmak icin gerekli integralleri yazin. Kutleyi hesaplayin.



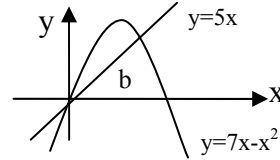
COZUM:

$$m = \int_{x=0}^{x=P} \sigma(x) dA = \int_{x=0}^{x=P} \sigma(x)[f(x)-g(x)]dx = \int_{x=0}^{x=P} (x^3 + 3x)[(7x - x^2) - 5x]dx$$

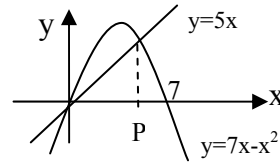
Burada P egrilerin kesim noktasi, $x=2$, $y=10$ noktasidir.

$$\int_{x=0}^{x=2} (x^3 + 3x)[(7x - x^2) - 5x]dx = \int_{x=0}^{x=2} (x^3 + 3x)(2x - x^2)dx$$

136) Sekilde gosterilen a bolgesinin yogunlugu $\sigma(x)=e^{2x}$ seklinde degismektedir. Bolgenin agirligini bulmak icin gerekli integralleri yazin. .



COZUM: Bolgeyi iki parca halinde dusunmemiz gerekir. Burada P $y=7x-x^2$ ve $y=5x$ egrilerinin kesim noktasidir. $x=2$

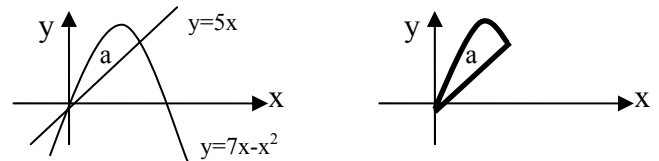


$$\int_{x=0}^{x=P} \sigma(x)[5x - 0] dx + \int_{x=P}^{x=7} \sigma(x)(7x - x^2) dx$$

$$= \int_{x=0}^{x=P} e^{2x}[5x - 0] dx + \int_{x=P}^{x=7} e^{2x}(7x - x^2) dx$$

$$= \int_{x=0}^{x=2} e^{2x}[5x - 0] dx + \int_{x=2}^{x=7} e^{2x}(7x - x^2) dx$$

141) Sekilde gosterilen a bolgesini kaplayan telin yogunlugu $\sigma(x)=\sin(x)$ seklinde degismektedir. Telin kutlesini hesaplamak icin gerekli integralleri yazin.



COZUM: Teli iki parca halinde dusunup ayri ayri hesaplariz.

m_1 : ust kisimdaki egri parcasinin uzunlugu.

$$m_1 = \int_{x=0}^{x=P} \sigma(x) dl = \int_{x=0}^{x=P} \sigma(x) \sqrt{1+(y')^2} dx = \int_{x=0}^{x=P} \sin(x) \sqrt{1+(y')^2} dx$$

$$y=7x - x^2, \quad y'=7-2x$$

$$= \int_{x=0}^{x=2} \sin(x) \sqrt{1+(7-2x)^2} dx$$

m_2 : alt kisimdaki dogru parcasinin uzunlugu.

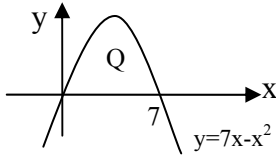
$$m_2 = \int_{x=0}^{x=2} \sigma(x) dl = \int_{x=0}^{x=2} \sigma(x) \sqrt{1+(y')^2} dx = \int_{x=0}^{x=2} \sin(x) \sqrt{1+(5)^2} dx$$

Toplam kutle= m_1+m_2 ,

AGIRLIK MERKEZI

$$x_M = \frac{\int x \sigma \, dm}{\int \sigma \, dm} = \frac{\int x \sigma \, dm}{M}, \quad y_M = \frac{\int y \sigma \, dm}{\int \sigma \, dm} = \frac{\int y \sigma \, dm}{M},$$

124) Sekilde gosterilen Q bolgesinin yogunlugu $\sigma(x)=1$ olarak verilmektedir. Q bolgesinin agirlik merkezininin x_M koordinatini hesaplayin.



COZUM:

$$x_M = \frac{\int x \sigma \, dm}{M} = \frac{\int x \sigma \, dm}{57.16} = \frac{\int x(7x-x^2) dx}{57.16} = \frac{1}{57.16} \int_0^7 x(7x-x^2) dx$$

$$x_M = \frac{1}{57.16} \int_0^7 (7x^2 - x^3) dx = \frac{1}{57.16} \left(7 \frac{x^3}{3} - \frac{x^4}{4} \right) \Big|_{x=0}^{x=7} = \frac{200.08}{57.16} = 3.5$$

y_M koordinatinin hesabi genelde iki katli inegral ile hesaplanir.