KISMI INTEGRASYON.

$$\int u\,dv = uv - \int v\,du$$

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61) $\int x\, e^{3x}\, dx = ?$

$u = e^{3x}$, ==> $du = 3\, e^{3x}\, dx$

$dv = x\, dx$, ==> $v = \int x\, dx = \frac{1}{2}x^2$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\, e^{3x}\, dx = e^{3x}\,\frac{1}{2}x^2 - \int \frac{1}{2}x^2\, 3e^{3x}\, dx$$

$$\int x\, e^{3x}\, dx = \frac{1}{2}e^{3x}x^2 - \frac{3}{2}\int x^2\, e^{3x}\, dx$$

integral icinde ilk integralde x e^{3x} var idi. Simdi $x^2\, e^{3x}$ var. Daha zor bir integral elde ettik. Bu donusum faydasiz bir donusumdur.

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62) $\int x\, e^{3x}\, dx = ?$

$u = x$, ==> $du = dx$

$dv = e^{3x}\, dx$, ==> $v = \int e^{3x}\, dx = \frac{1}{3}e^{3x}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x\, e^{3x}\, dx = x\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}\, dx$$

$$= x\frac{1}{3}e^{3x} - \frac{1}{3}\frac{e^{3x}}{3}$$

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63) $\int x^2\, e^{3x}\, dx = ?$

$u = x^2$, ==> $du = 2x\, dx$

$dv = e^{3x}\, dx$, ==> $v = \int e^{3x}\, dx = \frac{1}{3}e^{3x}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2\, e^{3x}\, dx = x^2\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}\, 2x\, dx$$

$$= x^2\frac{1}{3}e^{3x} - \frac{2}{3}\int e^{3x}\, x\, dx$$

Sag taraftaki integrali yukarida hesaplamistik. Onu yerine koyalim.

$$= x^2\frac{1}{3}e^{3x} - \frac{2}{3}\left(x\frac{1}{3}e^{3x} - \frac{1}{3}\frac{e^{3x}}{3}\right)$$

$$= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{e^{3x}}{27}$$

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65) $\int x^3\, e^{3x}\, dx = ?$

$u = x^3$, ==> $du = 3x^2\, dx$

$dv = e^{3x}\, dx$, ==> $v = \int e^{3x}\, dx = \frac{1}{3}e^{3x}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2\, e^{3x}\, dx = x^3\frac{1}{3}e^{3x} - \int \frac{1}{3}e^{3x}\, 3x^2\, dx$$

$$= \frac{1}{3}x^3e^{3x} - \int e^{3x}\, x^2\, dx$$

$$= \frac{1}{3}x^3e^{3x} - \left(\frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{e^{3x}}{27}\right)$$

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71) $\int x \cos 3x\, dx$

$u = x$, ==> $du = dx$

$dv = \cos 3x\, dx$, ==> $v = \int \cos 3x\, dx = \frac{1}{3}\sin 3x$

$$\int u\,dv = uv - \int v\,du$$

$$\int x \cos 3x\, dx = x\,\frac{1}{3}\sin 3x - \int \frac{1}{3}\sin 3x\, dx$$

$$= x\,\frac{1}{3}\sin 3x + \frac{1}{9}\cos 3x\, dx$$

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72) $\int x \sin 3x\, dx$

$u = x$, ==> $du = dx$

$dv = \sin 3x\, dx$, ==> $v = \int \sin 3x\, dx = -\frac{1}{3}\cos 3x$

$$\int u\,dv = uv - \int v\,du$$

$$\int x \sin 3x\, dx = -x\,\frac{1}{3}\cos 3x - \int -\frac{1}{3}\cos 3x\, dx$$

$$= -x\,\frac{1}{3}\cos 3x + \frac{1}{9}\sin 3x\, dx$$

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74) $I=\int x^2 \cos 3x\, dx$

$u=x^2$, $\Longrightarrow du=2x\, dx$

$dv=\cos 3x\, dx$, $\Longrightarrow v=\int \cos 3x\, dx = \frac{1}{3}\sin 3x$

$$\int u\, dv = uv - \int v\, du$$

$I = \int x^2 \cos 3x\, dx = x^2 \frac{1}{3}\sin 3x - \int \frac{1}{3}\, 2x \sin 3x\, dx$

$I = x^2 \frac{1}{3}\sin 3x + \frac{2}{3}\int x \sin 3x\, dx$

(72) den esitligin sag tarafindaki integral yerine yazilirsa.

$I = x^2 \frac{1}{3}\sin 3x + \frac{2}{3}\left(- x \frac{1}{3}\cos 3x + \frac{1}{9}\sin 3x\, dx\right)$

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81) $I=\int x^n \cos bx\, dx$ icin bir indirgeme formulu elde edin.

$u=x^n$, $\Longrightarrow du=n x^{n-1} dx$

$dv=\cos bx\, dx$, $\Longrightarrow v=\int \cos bx\, dx = \frac{1}{b}\sin bx$

$$\int u\, dv = uv - \int v\, du$$

$I = \int x^n \cos bx\, dx = x^n \frac{1}{b}\sin bx - \int \frac{1}{b}\, n x^{n-1} \sin bx\, dx$

$I = \frac{1}{b} x^n \sin bx + \frac{n}{b}\int x^{n-1} \sin bx\, dx$

$$\boldsymbol{\int x^n \cos bx\, dx = \frac{1}{b} x^n \sin bx + \frac{n}{b}\int x^{n-1} \sin bx\, dx}$$

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85) $I_n = \int \cos^n x\, dx$ icin bir indirgeme formulu elde edin.

Cozum:

$I_n=\int \cos^{n-1} x \cos x\, dx$ seklinde yazalim.

$u= \cos^{n-1} x$, $\Longrightarrow du=(n-1) \cos^{n-2}x\, (-\sin x)\, dx$

$dv=\cos x\, dx$, $\Longrightarrow v=\int \cos x\, dx = \sin x$

$$\int u\, dv = uv - \int v\, du$$

$I_n = \int \cos^{n-1} x \cos x\, dx = \cos^{n-1}x \sin x - I_x$ (85A)

$I_x=\int \sin x\, (n-1)\cos^{n-2}x\, (-\sin x)\, dx$

$I_x= -\int (n-1)\cos^{n-2}x \sin^2 x\, dx$

ikinci taraftaki integrali duzenleyelim.

$I_x = -\int \sin x\, (n-1)\cos^{n-2}x \sin x\, dx = -(n-1)\int \cos^{n-2}x \sin^2 x\, dx$

$= -(n-1)\int \cos^{n-2}x\, (1-\cos^2 x)\, dx$

$= -(n-1)\int (\cos^{n-2}x - \cos^n x)\, dx$

$= -(n-1)\int \cos^{n-2}x - (n-1)\int \cos^n x\, dx$

$I_x = -(n-1)\int \cos^{n-2}x - (n-1)\, I_n$

Buldugumuz bu degeri 85A daki yerine koyalim.

$I_n = \cos^{n-1}x \sin x - I_x$

$I_n = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x - (n-1)\, I_n$

$I_n + (n-1)\, I_n = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x$

$n\, I_n = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x$

$I_n = \frac{1}{n}\cos^{n-1}x \sin x + \frac{n-1}{n}\int \cos^{n-2}x$

$$\boldsymbol{\int \cos^n x\, dx = \frac{1}{n}\cos^{n-1}x \sin x + \frac{n-1}{n}\int \cos^{n-2}x}$$

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121) $I=\int e^{ax} \cos bx\, dx$ integralini hesaplayin.

$u= e^{ax}$, $\Longrightarrow du=a e^{ax} dx$,

$dv=\cos bx\, dx \Longrightarrow v=\int \cos bx\, dx= \frac{1}{b}\sin bx$

$$\int u\, dv = uv - \int v\, du$$

$I = \int e^{ax} \cos bx\, dx = e^{ax} \frac{1}{b}\sin bx - \int \frac{1}{b}\sin bx\; a e^{ax} dx$

$= e^{ax} \frac{1}{b}\sin bx - \frac{a}{b}\int \sin bx\, e^{ax} dx$

$= e^{ax} \frac{1}{b}\sin bx - \frac{a}{b}\, I_2$ (121.A)

Simdi de kalan I_2 integraline yeniden kismi integrasyon uygulayalim.

$I_2=\int \sin bx\, e^{ax} dx=?$

$u= e^{ax}$, $\Longrightarrow du=a e^{ax} dx$,

$dv=\sin bx\, dx \Longrightarrow v=\int \sin bx\, dx= -\frac{1}{b}\cos bx$

$$\int u\, dv = uv - \int v\, du$$

$I_2 = \int e^{ax} \sin bx\, dx = -e^{ax} \frac{1}{b}\cos bx - \int -\frac{1}{b}\cos bx\; a e^{ax} dx$

$= -e^{ax} \frac{1}{b}\cos bx + \frac{a}{b}\int \cos bx\, e^{ax} dx$

elde ettigimiz bu I_2 degerini 121.A) da yerine koyalim.

$I= e^{ax} \frac{1}{b}\sin bx - \frac{a}{b}\, I_2$

$$I = e^{ax} \frac{1}{b} \sin bx - \frac{a}{b} \left(- e^{ax} \frac{1}{b} \cos bx + \frac{a}{b} \int \cos bx \, e^{ax} \, dx \right)$$

$$I = \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int \cos bx \, e^{ax} \, dx$$

Dikkat edilirse kalan integral ilk olarak tanimladigimiz integraldir onun yerine de I yazalim.

$$I = \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} I$$

$$I + \frac{a^2}{b^2} I = \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx$$

$$I\left(1 + \frac{a^2}{b^2} \right) = \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx$$

$$I\left(\frac{b^2 + a^2}{b^2} \right) = \frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx$$

$$I = \frac{b^2}{a^2 + b^2} \left(\frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx \right)$$

Sonuc:

$$\int e^{ax} \cos bx = \frac{b^2}{a^2 + b^2} \left(\frac{1}{b} e^{ax} \sin bx + \frac{a}{b^2} e^{ax} \cos bx \right)$$