$$\sqrt{a^2-x^2}=a\sqrt{1-\left(\frac{x}{a}\right)^2}$$

$$\sqrt{a^2+x^2}=a\sqrt{1+\left(\frac{x}{a}\right)^2}$$

$$\frac{1}{\sqrt{a^2-x^2}}=\frac{1}{a\sqrt{1-\left(\frac{x}{a}\right)^2}}$$

$$\frac{1}{\sqrt{a^2+x^2}}=\frac{1}{a\sqrt{1+\left(\frac{x}{a}\right)^2}}$$

$$\frac{1}{a^2+x^2}=\frac{1}{a\left(1+\left(\frac{x}{a}\right)^2\right)}$$

$ax^2+bx+c \rightarrow (px+q)^2+M$
$(px+q)^2-M$

$$3x^2+5x+16=\left(\sqrt{3}x+\frac{5}{2\sqrt{3}}\right)^2+\frac{167}{12}$$

$$3x^2+5x-1=\left(\sqrt{3}x+\frac{5}{2\sqrt{3}}\right)^2-\frac{37}{12}$$

$$3x^2-5x+6=\left(\sqrt{3}x-\frac{5}{2\sqrt{3}}\right)^2+\frac{47}{12}$$

$$3x^2-5x-2=\left(\sqrt{3}x-\frac{5}{2\sqrt{3}}\right)^2-\frac{49}{12}$$

$-ax^2+bx+c \rightarrow M-(px+q)^2$
$-M-(px+q)^2$

$$-3x^2+5x+10=\frac{145}{12}-\left(\sqrt{3}x-\frac{5}{2\sqrt{3}}\right)^2$$

$$-3x^2-5x+1=\frac{37}{12}-\left(\sqrt{3}x+\frac{5}{2\sqrt{3}}\right)^2$$

Pr471)

$$\int\frac{1}{\sqrt{1-(ax)^2}}\,dx=?$$

u=ax, du=a dx, dx=du/a

$$\int\frac{dx}{\sqrt{1-(ax)^2}}=\int\frac{du/a}{\sqrt{1-u^2}}=\frac{1}{a}arc\sin ax$$

Pr472) $\int\frac{1}{\sqrt{b^2-x^2}}\,dx=\arcsin\frac{x}{b}$ oldugunu gosterin

x=bt, dx=b dt,

$$\int\frac{dx}{\sqrt{b^2-x^2}}=\int\frac{bdt}{\sqrt{b^2-(bt)^2}}=\int\frac{bdt}{b\sqrt{1-t^2}}$$

$$=\arcsin t=\arcsin\frac{x}{b}$$

$$\int\frac{1}{1+(ax)^2}\,dx=\frac{1}{a}arc\tan ax$$

$$\int\frac{1}{a^2+x^2}\,dx=\frac{1}{a}arc\tan\frac{x}{a}$$

$$\int\frac{1}{\sqrt{(ax)^2-1}}\,dx=\frac{1}{a}arc\cosh ax$$

$$\int\frac{1}{\sqrt{x^2-a^2}}\,dx=arc\cosh\frac{x}{a}$$

$$\int\frac{1}{\sqrt{(ax)^2+1}}\,dx=\frac{1}{a}arc\sinh ax$$

$$\int\frac{1}{\sqrt{x^2+a^2}}\,dx=arc\sinh\frac{x}{a}$$

----------------- ----------------

A) $\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx = ?$

a,b,c nin degerlerine gore integral asagidaki formlardan birine getirilebilir. (ozel durumlar haric. $\int\sqrt{-x^2}\,dx$, $\int\sqrt{-(x+1)^2}\,dx$ cozumsuzdur)

$$\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx = \int \frac{1}{\sqrt{(px+q)^2+M}}\,dx$$

$$= \int \frac{1}{\sqrt{(px+q)^2-M}}\,dx$$

$$= \int \frac{1}{\sqrt{M-(px+q)^2}}\,dx$$

$u=px+q \quad \rightarrow \quad du=pdx \quad \rightarrow \quad dx=\frac{du}{p}$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}\,dx = \frac{1}{p}\int \frac{1}{\sqrt{u^2+M}}\,du$$

$$= \frac{1}{p}\int \frac{1}{\sqrt{u^2-M}}\,du$$

$$= \frac{1}{p}\int \frac{1}{\sqrt{M-u^2}}\,dx$$

Bu hallere gelen integraller asagidaki formullere gore integre edilir.

$$\int \frac{1}{\sqrt{b^2-x^2}}\,dx = \arcsin\frac{x}{b}$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = arc\cosh\frac{x}{a}$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = arc\sinh\frac{x}{a}$$

Not:

$$\text{arc}\sinh x = \text{Ln}(x \pm \sqrt{x^2+1})$$

$$\text{arc}\cosh x = \text{Ln}(x \pm \sqrt{x^2-1})$$

------------------------- ---------------

$$\text{Pr } 431) = \int \frac{1}{\sqrt{-x^2+2x+3}}\,dx$$

$-x^2+2x+3= - (px+q)^2+M$

$-x^2+2x+3= - (p^2x^2+2pqx+q^2) +M$

$-x^2+2x+3= - p^2x^2-2pqx-q^2+M$

$-1=-p^2 \;\rightarrow p=1$

$2 = -2pq \;\rightarrow pq=-1,\; q=-1$

$-q^2+M=3 \;\rightarrow M=4$

$-x^2+2x+3= - (x-1)^2+4$

$$= \int \frac{1}{\sqrt{4-(x-1)^2}}\,dx$$

u=x-1, du=dx

$$= \int \frac{1}{\sqrt{4-u^2}}\,du = \arcsin\frac{u}{2} = \arcsin\frac{x-1}{2}$$

------------------- --------------

B) $\int \frac{mx+n}{\sqrt{ax^2+bx+c}}\,dx = ?$

$$\int \frac{\frac{2a}{m}(mx+n)}{\frac{2a}{m}\sqrt{ax^2+bx+c}}\,dx = \frac{m}{2a}\int \frac{\frac{2a}{m}(mx+n)}{\sqrt{ax^2+bx+c}}\,dx$$

$$= \frac{m}{2a}\int \frac{2ax+\frac{2an}{m}}{\sqrt{ax^2+bx+c}}\,dx$$

$$= \frac{m}{2a}\int \frac{2ax+b-b+\frac{2an}{m}}{\sqrt{ax^2+bx+c}}\,dx$$

$$= \frac{m}{2a}\int \frac{2ax+b}{\sqrt{ax^2+bx+c}}\,dx + \frac{m}{2a}\int \frac{-b+\frac{2an}{m}}{\sqrt{ax^2+bx+c}}\,dx$$

Birinci integral icin

$u=ax^2+bx+c$, $du=2ax+b$

$$\frac{m}{2a}\int\frac{2ax+b}{\sqrt{ax^2+bx+c}}\,\mathrm{dx}=\frac{m}{2a}\int\frac{du}{\sqrt{u}}=\frac{m}{2a}\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

$$\frac{m}{2a}\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}=\frac{m}{2a}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}=\frac{m}{a}\sqrt{u}=\frac{m}{a}\sqrt{ax^2+bx+c}$$

ikinci integral icin.

$$\frac{m}{2a}\int\frac{-b+\frac{2an}{m}}{\sqrt{ax^2+bx+c}}\,\mathrm{dx}$$

$$=\frac{m}{2a}\left(-b+\frac{2an}{m}\right)\int\frac{1}{\sqrt{ax^2+bx+c}}\,\mathrm{dx}$$

a,b,c nin degerlerine gore integra degeri hesaplanir lin degeri hesaplanir.

------------------- ---------------

C) $Q=\int\frac{1}{(px+q)\sqrt{ax^2+bx+c}}\,\mathrm{dx}=?$

$$t=\frac{1}{px+q}\quad\rightarrow\quad px+q=\frac{1}{t}\quad\rightarrow\quad x=\frac{1}{pt}-\frac{q}{p}$$

$$x=\frac{1-qt}{pt}\ ,\qquad dx=-\frac{1}{pt^2}$$

$$ax^2+bx+c=a\left(\frac{1-qt}{pt}\right)^2+b\left(\frac{1-qt}{pt}\right)+c$$

$$=a\frac{1-2qt+q^2t^2}{p^2t^2}+b\frac{(1-qt)pt}{p^2t^2}+c\frac{p^2t^2}{p^2t^2}$$

$$=\frac{a-2aqt+aq^2t^2}{p^2t^2}+\frac{bpt-bpqt^2}{p^2t^2}+\frac{cp^2t^2}{p^2t^2}$$

$$=\frac{\left(aq^2-bpq+cp^2\right)t^2+(-2aq+b)t+a}{p^2t^2}$$

$$\sqrt{ax^2+bx+c}=$$

$$=\sqrt{\frac{\left(aq^2-bpq+cp^2\right)t^2+(-2aq+b)t+a}{p^2t^2}}$$

$$=\frac{\sqrt{\left(aq^2-bpq+cp^2\right)t^2+(-2aq+b)t+a}}{pt}$$

$$Q=\int\frac{1}{px+q}\frac{dx}{\sqrt{ax^2+bx+c}}$$

$$Q=\int t\frac{-\frac{1}{pt^2}dt}{\frac{\sqrt{\left(aq^2-bpq+cp^2\right)t^2+(-2aq+b)t+a}}{pt}}$$

$$Q=-\int\frac{dt}{\sqrt{\left(aq^2-bpq+cp^2\right)t^2+(-2aq+b)t+a}}$$

$$\int\frac{dx}{(x-1)\sqrt{x^2+3}}\quad:-\int\frac{dt}{\sqrt{4t^2+2t+1}}$$
,

a=1; b=0; c=3; p=1; q=-1;

[a*q^2-b*p*q+c*p^2, -2*a*q+b, a]