$$\int \frac{P_n(x)}{\sqrt{ax^2+bx+c}}\,\mathrm{dx} = Q_{n-1}(x)\sqrt{ax^2+bx+c} + \lambda\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\int \frac{P_n(x)}{\sqrt{ax^2+bx+c}}\,\mathrm{dx} = Q(x)\sqrt{ax^2+bx+c} + \lambda\int \frac{dx}{\sqrt{ax^2+bx+c}}$$

iki tarafin turevini alalim

$$\frac{P_n(x)}{\sqrt{ax^2+bx+c}} = \frac{dQ(x)}{dx}\sqrt{ax^2+bx+c} + Q(x)\frac{2ax+b}{2\sqrt{ax^2+bx+c}} + \frac{\lambda}{\sqrt{ax^2+bx+c}}$$

iki tarafi karekoklu ifade ile carpalim

$$P_n(x) = \frac{dQ(x)}{dx}(ax^2+bx+c) + \frac{1}{2}Q(x)(2ax+b) + \lambda$$

$$\frac{x^2}{\sqrt{1-x^2}}\,dx$$

P(x)=x^2, a=-1, b=0; c=1; Q(x)=Ax+B, Q '(x)=A,

(Q(x) P(x) den bir derece dusukolmalidir. P(x) ikinci dereceden o halde Q(x) birincidereceden olmali)

$$P_n(x) = \frac{dQ(x)}{dx}(ax^2+bx+c) + \frac{1}{2}Q(x)(2ax+b) + \lambda$$

x^2 = A(ax^2+bx+c)+0.5(Ax+B)(2ax+0)+λ

x^2 = A(-x^2+1)+(Ax+B)(-1)x+λ

x^2 = -Ax^2+A - Ax^2 -B x +λ

x^2 + 0x + 0= (-A-A) x^2 - B x +A+ λ

1= -A-A → A=-1/2

0= -2B → B=0

0=A + λ → λ=1/2

Q(x)=Ax+B=-1/2x, λ=1/2

$$\int \frac{x^2}{\sqrt{-x^2+1}}\,\mathrm{dx} = -0.5x^2\sqrt{-x^2+1} + 0.5\int \frac{dx}{\sqrt{1-x^2}}$$

$$= -0.5x^2\sqrt{-x^2+1} + 0.5\arcsin x$$

5. $\int \frac{dx}{(x-p)^n \sqrt{ax^2+bx+c}}$ integralinin hesabı:

Bu tip integrallerde

$$\frac{1}{x-p} = t \Rightarrow x = p + \frac{1}{t}$$