

Mat II		Ödev No		3	
Adı Soyadı:					
Öğrenci No:					
	0	0		0	0
	1	1		1	1
	2			2	2
	3			3	3
	4			4	4
	5			5	5
	6			6	6
	7			7	7
	8			8	8
	9			9	9

Uygun degisken donusumu ile integrallerii cozun.

$$\int \frac{1}{\sqrt{1-(ax)^2}} dx = \frac{1}{a} \arcsin ax \text{ seklindeki hazır formulu}$$

kullanmayın, formulu kendiniz turetin. Bazi integraller

$$\int \frac{dx}{\sin^n x}, \int \frac{dx}{\cos^n x} \text{ haline gelir. Bu halde birakabilirsiniz}$$

Sorular	degisken x=	Cevaplar
1) $\int \sqrt{36-x^2} dx$	$x=6\cos t$ $dx=-6\sin 6t$	$18 \arcsin (x/6) + 3x \sqrt{1-(x/6)^2}$
2) $\int \sqrt{7-2x^2} dx$		
3) $\int \frac{1}{\sqrt{36-x^2}} dx$		
4) $\int \frac{1}{\sqrt{7-2x^2}} dx$		
5) $\int \frac{1}{\sqrt{36+x^2}} dx$		
6) $\int \sqrt{36+x^2} dx$		
7) $\int \frac{2}{(x+1)^4} dx$		
8) $\int \frac{2}{4+(x+2)^2} dx$		
9) $\int \frac{\sqrt{(x+2)^3}}{5+\sqrt{(x+2)^5}} dx$		

$\sqrt{a^2-x^2}$	$x=a \sin t$	$dx=a \cos t dt$
$\sqrt{x^2-a^2}$	$x = \frac{a}{\cos t}$	$dx = \frac{a \sin t}{\cos^2 t} dt$
$\sqrt{x^2+a^2}$	$x=a \tan t$	$dx=(1+\tan^2 t)dt$
$\sqrt{ax+b}$	$t=(ax+b)^n$ $ax+b=t^{1/n}$	$dx=t$

$$1) I = \int \sqrt{36-x^2} dx \quad x=6 \sin t, \quad dx=6 \cos t$$

$$\begin{aligned} \int \sqrt{36-x^2} dx &= \int \sqrt{36-36\sin^2 t} \cdot 6\cos t dt = \int 6\cos t \cdot 6\cos t dt \\ &= 36 \int \cos^2 t dt = 36 \int \frac{1}{2} (1+\cos(2t)) dt = \frac{36}{2} \int (1+\cos(2t)) dt \\ &= 18 \int dt + 18 \int \cos(2t) dt = 18t + \frac{18}{2} (\sin 2t) + C \\ &= 18t + 9 \sin 2t + C \end{aligned}$$

$$I = 18 \arcsin (x/6) + 9 \sin(2 \arcsin(x/6))$$

$$\begin{aligned} \sin(2q) &= 2 \sin q \cos q = 2 \sin q \sqrt{1-\sin^2 q} \\ \sin(2 \arcsin(x/6)) &= 2 \sin (\arcsin(x/6)) \sqrt{1-\sin^2(\arcsin(x/6))} \\ &= 2(x/6) \sqrt{1-(x/6)^2} \end{aligned}$$

$$I = 18 \arcsin (x/6) + 9 (2(x/6) \sqrt{1-(x/6)^2})$$

$$I = 18 \arcsin (x/6) + 3x \sqrt{1-(x/6)^2}$$