

Mat II

Ödev No 4

Adı Soyadı:

Öğrenci No:

		0	5	0				
0	0			0	0	0		
1	1			1	1	1		
2				2	2	2		
3				3	3	3		
4				4	4	4		
5				5	5	5		
6				6	6	6		
7				7	7	7		
8				8	8	8		
9				9	9	9		

Sorular

Cevaplar

1) $\int \frac{1-\sin x}{1+\cos x} dx$

$$\tan(x/2) - \ln(1 + \tan^2(x/2)) + C$$

2) $\int \frac{2-\sin x}{1+2\cos x} dx$

3) $\int \frac{\sin x + \cos^2 x}{4+2\tan x} dx$

$$\tan(x/2) = t,$$

$$\tan(x) = \frac{2 \tan(x/2)}{1 - \tan^2(x/2)} = \frac{2t}{1 - t^2},$$

$$\sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{2t}{1 + t^2}$$

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} = \frac{1 - t^2}{1 + t^2}$$

$$\tan(x/2) = t, \quad x = \arctan 2t, \quad dx = \frac{2}{1+t^2} dt$$

$$I = \int \frac{1 - \sin x}{1 + \cos x} dx = \int \frac{1 - \frac{2t}{1+t^2}}{1 + \frac{1-t^2}{1+t^2}} \frac{2dt}{1+t^2} = \int \frac{\frac{1+t^2-2t}{1+t^2}}{\frac{1+t^2+1-t^2}{1+t^2}} \frac{2dt}{1+t^2}$$

$$= \int \frac{1+t^2-2t}{2} \frac{2dt}{1+t^2} = \int \frac{1+t^2-2t}{1+t^2} dt$$

$$= \int \frac{1+t^2}{1+t^2} dt - \int \frac{2t}{1+t^2} dt = \int 1 dt - \ln(1+t^2) = t - \ln(1+t^2)$$

$$I = t - \ln(1+t^2) = \tan(x/2) - \ln(1 + \tan^2(x/2))$$

Not: Kesirlerin integralinde paydanın derecesi küçükse (veya eşitse) bolme yapılır. Örnek: $\int \frac{3t^2+2}{1+t^2} dt = ?$

$$\begin{array}{r} 3t^2+2 \\ - 3t^2+3 \\ \hline -1 \end{array}$$

$$\int \frac{3t^2+2}{1+t^2} dt = \int \left(3 + \frac{-1}{1+t^2} \right) dt = \int 3 dt - \int \frac{1}{1+t^2} dt$$

$$= 3t + \arctan t$$