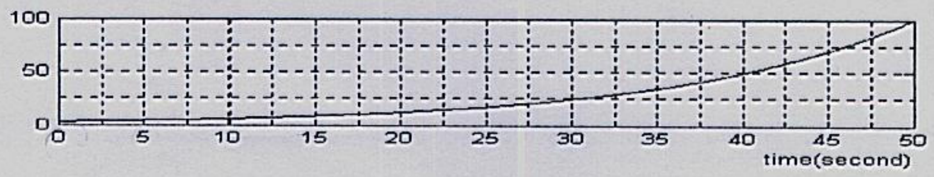
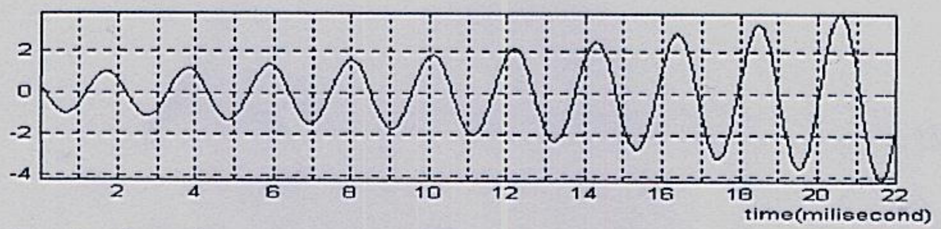
EE300 Quiz No:5

1)Figure 1) represents the function $x(t)=c\,e^{\alpha t}$, time in seconds. Calculate the coefficients c and α. (Approximate values are required)

2)Figure 2) represents the function $x(t)=e^{\alpha t}\cos(wt)$ time in miliseconds. Calculate the coefficients α and w. (Approximate values are required)





3)Eigenvales of a dynamic system are $\lambda_1=2+3i$ and $\lambda_2=2-3i$. This dynamic system has the following differential equations. $\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \begin{bmatrix} a & -8 \\ 2 & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$, Calculate a,d

---

Solutions 1) from the plot

t	30	40
$x(t)$	25	50

$$x(t) = ce^{\alpha t}$$

$$x(30) = ce^{\alpha 30} \qquad x(40) = ce^{\alpha 40}$$

$$\boxed{25 = ce^{\alpha 30}} \; (1) \qquad \boxed{50 = ce^{\alpha 40}} \; (2)$$

dived equation 2 by equation 1

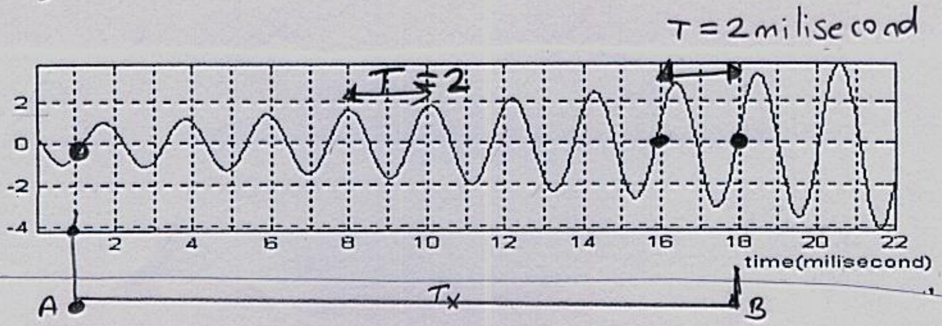
$$\frac{50}{25} = \frac{ce^{\alpha 40}}{ce^{\alpha 30}} \Rightarrow 2 = e^{\alpha(40-30)} \Rightarrow 2 = e^{\alpha 10}$$

$$Ln\,2 = Ln\,e^{\alpha 10} \Rightarrow Ln\,2 = \alpha \times 10 \times \underbrace{Ln\,e}_{1} \Rightarrow \alpha = \frac{Ln\,2}{10}$$

$$\boxed{\alpha = 0.069}$$

$$25 = ce^{\alpha 30} \Rightarrow 25 = ce^{0.069 \times 30} \Rightarrow \boxed{c = 3.15}$$

(2) we can measure the period either from peak to peak or from zero-crossing to zero crossing.



it is more accurate if we measure the period over a long time interval

from A to B, time interval is 18 − 1 = 17 milisecond

from A to B there are 8 zero crossings.

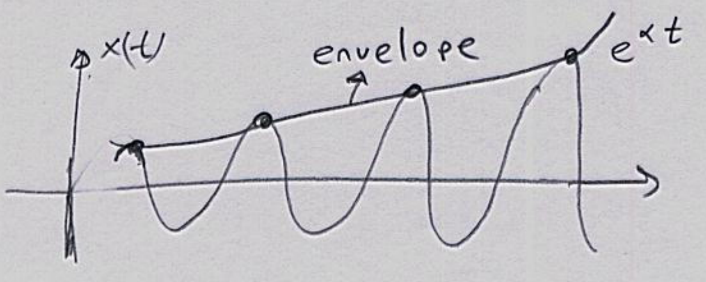
$$\text{Period} = \frac{17}{8} = 2.125 \text{ miliseconds} = 2.125\ 10^{-3} \text{ seconds}$$

$$w = \frac{2\pi}{T} = \frac{2 \times 3.14}{2.125\ 10^{-3}} = 2955 \ \frac{\text{radian}}{\text{second}}$$

Note: $f = \frac{1}{T} = 470$ hertz

The graph is $x(t) = e^{\alpha t} \cos wt$.

The envelope of graph is $e^{\alpha t}$



we may take any values of the peak from the graph

$t = 8$ milisecond $\quad x(t) \cong 1.5$

$t = 20.5$ ms $\quad x(t) \cong 4$

solve it as in question 1.

$$x(t) = e^{\alpha t}$$

$$x(8 \times 10^{-3}) = e^{\alpha \, 8 \, 10^{-3}}$$

$$1.5 = e^{\alpha \, 0,008}$$

$$\alpha = \frac{\ln 1.5}{0.008} = 50.68$$

$$x(t) = e^{\alpha t}$$

$$x(20.5 \; 10^{-3}) = e^{\alpha \, 20.5 \; 10^{-3}}$$

$$4 = e^{\alpha \, 0.0205}$$

$$\alpha = \frac{\ln 4}{0.0205} = 67.62$$

The difference is because of reading the grephics less accuratels.

$$\boxed{x(t) = e^{67.6 t} \cos(2955 \, t)}$$

or

$$\boxed{x(t) = e^{50.68 t} \cos(2955 t)}$$

(3)

$$A = \begin{bmatrix} a & -8 \\ 2 & d \end{bmatrix} \qquad \lambda_1 = 2+3i \qquad \lambda_2 = 2-3i$$

$$A - \lambda I = \begin{bmatrix} a-\lambda & -8 \\ 2 & d-\lambda \end{bmatrix}$$

$$\det[A - \lambda I] = (a-\lambda)(d-\lambda) - 2\times(-8)$$

$$= ad - a\lambda - \lambda d + \lambda^2 + 16$$

$$= \lambda^2 - (a+d)\lambda + ad + 16$$

The eigenvalues are given by λ_1 and λ_2

$$(\lambda-\lambda_1)(\lambda-\lambda_2) = \lambda^2 - (\lambda_1+\lambda_2)\lambda + \lambda_1\lambda_2$$

equate two terms

$$\lambda^2 - (a+d)\lambda + ad + 16 = \lambda^2 - (\lambda_1+\lambda_2)\lambda + \lambda_1\lambda_2$$

$$-(a+d) = -(\lambda_1+\lambda_2) = -4$$

$$ad + 16 = \lambda_1\lambda_2 = +13$$

Note $\lambda_1 + \lambda_2 = 2+3i + 2-3i = 4$

$\lambda_1\lambda_2 = (2+3i)(2-3i) = 2^2+3^2 = 13$

$ad = 13 - 16 = -3 \qquad d = -\frac{3}{a}$

$a + d = 4 \qquad a + (-\frac{3}{a}) = 4 \Rightarrow a^2 - 4a - 3 = 0 \nearrow a_1 = 4.645 \quad \searrow a_2 = -0.645$

$d_1 = -\frac{3}{4.6458} = -0.645 \qquad d_2 = -\frac{3}{-0.645} = 4.645$

possible solutions

$$\begin{bmatrix} 4.645 & -8 \\ 2 & -0.645 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} -0.645 & -8 \\ 2 & 4.645 \end{bmatrix}$$

Second method

$$\det[A-\lambda I] = \lambda^2 - (a+d)\lambda + ad + 16$$

$$\lambda_{1,2} = \frac{a+d \mp \sqrt{(a+d)^2 - 4\times(ad+16)}}{2}$$

$$\frac{a+d+\sqrt{}}{2} = 2+3i \qquad \frac{a+d-\sqrt{}}{2} = 2-3i$$

or

$$\frac{a+d}{2} + \frac{\sqrt{}}{2} = 2 + 3i$$

$$\frac{a+d}{2} = 2 \Rightarrow \boxed{a+d=4}$$

$$\frac{\sqrt{}}{2} = 3i \Rightarrow \sqrt{(m)} = 6i \Rightarrow m = (6i)^2 = -36$$

$$(a+d)^2 - 4(ad+16) = -36$$

$$\downarrow$$

$$4^2 - 4(ad+16) = -36$$

$$16 - 4ad - 64 = -36$$

$$-4ad = 12 \qquad \boxed{ad=-3}$$

Solve these two equations as before