Linear independence of vectors

1) p=[1 5 3], p is a 3 dimensional **row** vector

$q=\begin{bmatrix}3\\1\end{bmatrix}$, q is a 2 dimensional **column** vector

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Definition: $X_1, X_2, X_3, \ldots, X_n$, are vectors

If there are numbers $a_1, a_2, a_3, \ldots, a_n$, such that

$$a_1X_1+a_2X_2+a_3X_3+\ldots+a_nX_n=0$$

then vectors $X_1, X_2, X_3, \ldots, X_n$ are linearly dependent.

For two vectors,

$$a_1X_1+a_2X_2=0$$

or

$$a_1X_1=-a_2X_2$$

$$X_1=-\frac{a_2}{a_1}X_2=\alpha X_2$$

i.e. if the two vectors are multiples of each other then the two vectors are linearly dependent

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2) $m=\begin{bmatrix}1\\3\\2\end{bmatrix}$, $n=\begin{bmatrix}10\\30\\20\end{bmatrix}$

It is clear that n=10m or m=0.1n. Thus m and n are linearly dependent

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3) $m=\begin{bmatrix}1\\3\end{bmatrix}$, $n=\begin{bmatrix}2\\1\end{bmatrix}$

There is no number α that satisfies the equation n= α m. Thus m and n are linearly independent

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4) $m=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$, $n=\begin{bmatrix}0\\0\\0\\4\end{bmatrix}$

It is clear that n=4m. m and n are linearly dependent

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5) $m=\begin{bmatrix}1\\3\\-1\\10\end{bmatrix}$, $n=\begin{bmatrix}-10\\-30\\10\\-100\end{bmatrix}$

n=-10m. m and n are linearly dependent

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6) $m=\begin{bmatrix}0\\2\\1\end{bmatrix}$, $n=\begin{bmatrix}0\\0\\3\end{bmatrix}$

There is no number α that satisfies the eqation n= αm m and n are linearly independent.

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6) $m=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $n=\begin{bmatrix}0\\5\\0\end{bmatrix}$, $p=\begin{bmatrix}8\\0\\0\end{bmatrix}$

am+bn+cp=0

$$a\begin{bmatrix}0\\0\\1\end{bmatrix}+b\begin{bmatrix}0\\5\\0\end{bmatrix}+c\begin{bmatrix}8\\0\\0\end{bmatrix}=0$$

a 0 +b 0+c 8=0 → c 8=0 → c=0

a 0 +b 5+c 0=0 → b 5=0 → b=0

a 1 +b 0+c 0=0 → a 1=0 → a=0

There is no numbers (except zero) that satisfies the equation am+bn+cp=0. Thus the vectors m,n,p are linearly independent.

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7) $m=\begin{bmatrix}0\\8\\2\end{bmatrix}$, $n=\begin{bmatrix}0\\0\\3\end{bmatrix}$, $p=\begin{bmatrix}4\\5\\6\end{bmatrix}$

am+bn+cp=0

$$a\begin{bmatrix}0\\8\\2\end{bmatrix}+b\begin{bmatrix}0\\0\\3\end{bmatrix}+c\begin{bmatrix}4\\5\\6\end{bmatrix}=0$$

a 0 +b 0+c 4=0 → c 4=0 → c=0

a 8 +b 0+c 5=0 → 8a + 5c=0 → 8a + 5 0=0 →a =0

a 2 +b 3+c 6=0 → 2a+0+0=0 → a=0

There are no numbers (except a=0, b=0, c=0) that satisfies the equation am+bn+cp=0. Thus the vectors m,n,p are linearly independent.

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7) $m=\begin{bmatrix}1\\3\\2\end{bmatrix}$, $n=\begin{bmatrix}2\\6\\4\end{bmatrix}$, $p=\begin{bmatrix}0\\0\\1\end{bmatrix}$

$$a\begin{bmatrix}1\\3\\2\end{bmatrix}+b\begin{bmatrix}2\\6\\4\end{bmatrix}+c\begin{bmatrix}0\\0\\1\end{bmatrix}=0$$

a 1 +b 2+c 0=0 → a+2b=0 → a=-2b

a 3 +b 6+c 0=0 → 3a + 6b=0 → a=-2b

a 2 +b 4+c 1=0 → 2a+4b+c=0 → replace a=-2b

2(-2b)+ 4b+c=0 → -4b+4b +c=0 → c=0

The values c=0, a=-2b satisfies the equation.

Example c=0, b=1, a=-2 or c=0, b=10, a=-20 etc satisfies the equation am+bn+cp=0

Vectors are linearly dependent.