

Solution of Linear Equations by Gaus Elimination

$$\begin{bmatrix} 2 & 6 & 8 \\ 4 & 17 & 14 \\ 1 & 13 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 30 \end{bmatrix} \quad \text{Augmented Matrix} \quad \Rightarrow \quad \begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix}$$

Multiply the first row by -2 and add the result to the second row.

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix} \quad R_2 - 2R_1 \rightarrow R_2$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 4-2*2 & 17-2*6 & 14-2*8 & 15-2*4 \\ 1 & 13 & 14 & 30 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 1 & 13 & 14 & 30 \end{bmatrix}$$

Multiply the first row by -0.5 and add the result to the third row.

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 1 & 13 & 14 & 30 \end{bmatrix} \quad R_3 - 0.5R_1 \rightarrow R_3$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 1-0.5*2 & 13-0.5*6 & 14-0.5*8 & 3-0.5*4 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 10 & 10 & 30 \end{bmatrix}$$

Multiply the second row by -2 and add the result to the third row.

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 10 & 10 & 28 \end{bmatrix} \quad R_3 - 2R_2 \rightarrow R_3$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 10-2*5 & 10-2*(-2) & 28-2*7 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

Write the resulting equations in separate form.

$$2x + 6y + 8z = 4$$

$$5y - 2z = 17$$

$$14z = 14$$

Now solve the equations starting from the bottom.

$$14z = 14 \Rightarrow z = \frac{14}{14} = 1$$

$$5y - 2z = 7 \Rightarrow 5y - 2*1 = 7 \Rightarrow 5y = 9 \Rightarrow y = \frac{9}{5} = 1.8$$

$$2x + 6y + 8z = 4 \Rightarrow 2x + 6*1.8 + 8*1 = 4 \Rightarrow 2x + 10.8 + 8 = 4 \Rightarrow x = -7.4$$

During the process we converted the coefficient matrix into **upper triangular Form**

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 & 6 & 4 \\ 0 & 5 & -7 & 1 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

Explanation

$$2x + 6y + 8z = 4 \quad (E1)$$

$$4x + 17y + 14z = 15 \quad (E2)$$

$$x + 13y + 14z = 30 \quad (E3)$$

Multiply both side of equation (E1) by -2 .

$$2x + 6y + 8z = 4 \Rightarrow -4x - 12y - 16z = -8$$

Add this modified equation (E1) to equation (E2).

$$\begin{array}{r} -4x - 12y - 16z = -8 \\ + \quad 4x + 17y + 14z = 15 \\ \hline 5y - 2z = 7 \end{array} \quad (E4)$$

Multiply both side of equation (E1) by -0.5 .

$$2x + 6y + 8z = 4 \Rightarrow -x - 3y - 4z = -2$$

Add this modified equation (E1) to equation (E3).

$$\begin{array}{r} -x - 3y - 4z = -2 \\ + \quad x + 13y + 14z = 30 \\ \hline 10y + 10z = 28 \end{array} \quad (E5)$$

Now Solve (E4) and (E5)

$$5y - 2z = 7 \quad (E4)$$

$$10y + 10z = 28 \quad (E5)$$

Multiply equation (E4) by -2 and add to equation (E5)

$$\begin{array}{r} 5y - 2z = 7 \Rightarrow -10y + 4z = -14 \\ + \quad 10y + 10z = 28 \\ \hline 14z = 14 \end{array}$$

Thus $z = 1$. Substitute $z = 1$ into equation (E4) and obtain y .

$$5y - 2z = 7 \Rightarrow 5y - 2 \cdot 1 = 7$$

$$5y = 9 \Rightarrow y = \frac{9}{5} = 1.8$$

Substitute $z = 1$ and $y = 1.8$ into the first equation of (E1) and solve for x .

$$2x + 6y + 8z = 4 \Rightarrow 2x + 6 \cdot 1.8 + 8 \cdot 1 = 4$$

$$\Rightarrow 2x + 10.8 + 8 = 4 \Rightarrow x = -7.4$$

Result $x = -7.4$ $y = 1.8$ $z = 1$