

Solution of Linear Equations by Gauss Elimination (diagonal form)
(Number of equations = Number of unknowns)

The aim is
$$\begin{bmatrix} X & X & X & X & X & X \\ X & X & X & X & X & X \\ X & X & X & X & X & X \\ X & X & X & X & X & X \\ X & X & X & X & X & X \end{bmatrix} \Rightarrow \begin{bmatrix} X & X & X & X & X & X \\ 0 & X & X & X & X & X \\ 0 & 0 & X & X & X & X \\ 0 & 0 & 0 & X & X & X \\ 0 & 0 & 0 & 0 & X & X \end{bmatrix} \Rightarrow \begin{bmatrix} X & 0 & 0 & 0 & 0 & X \\ 0 & X & 0 & 0 & 0 & X \\ 0 & 0 & X & 0 & 0 & X \\ 0 & 0 & 0 & X & 0 & X \\ 0 & 0 & 0 & 0 & X & X \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 8 \\ 4 & 17 & 14 \\ 1 & 13 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 30 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 4 & 17 & 14 & 15 \\ 1 & 13 & 14 & 30 \end{bmatrix} \begin{array}{l} -2R_1 + R_2 \rightarrow R_2 \\ -0.5R_1 + R_3 \rightarrow R_3 \end{array} \Rightarrow \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 10 & 10 & 28 \end{bmatrix} \begin{array}{l} -2R_2 + R_3 \rightarrow R_3 \end{array} \Rightarrow \begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 8 & 4 \\ 0 & 5 & -2 & 7 \\ 0 & 0 & 14 & 14 \end{bmatrix} \begin{array}{l} -\left(\frac{-2}{14}\right)R_3 + R_2 \rightarrow R_2 \\ -\left(\frac{8}{14}\right)R_3 + R_1 \rightarrow R_1 \end{array} \Rightarrow \begin{array}{l} \frac{1}{7}R_3 + R_2 \rightarrow R_2 \\ -\frac{6}{7}R_3 + R_1 \rightarrow R_1 \end{array}$$

$$\begin{bmatrix} \frac{1}{7}x0+2 & \frac{1}{7}x0+6 & \frac{1}{7}x14+8 & \frac{1}{7}x14+4 \\ -\frac{6}{7}x0+0 & -\frac{6}{7}x0+5 & -\frac{6}{7}x14-2 & -\frac{6}{7}x14+7 \\ 0 & 0 & 14 & 14 \end{bmatrix} = \begin{bmatrix} 2 & 6 & 0 & -4 \\ 0 & 5 & 0 & 9 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

$$-\frac{6}{5}R_2 + R_1 \rightarrow R_1 \Rightarrow \begin{bmatrix} -\frac{6}{5}x0+2 & -\frac{6}{5}x5+6 & -\frac{6}{5}x0+0 & -\frac{6}{5}x9-4 \\ 0 & 5 & 0 & 9 \\ 0 & 0 & 14 & 14 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & -14.8 \\ 0 & 5 & 0 & 9 \\ 0 & 0 & 14 & 14 \end{bmatrix}$$

$$2x = -14.8 \Rightarrow x = -7.4,$$

$$5y = 9 \Rightarrow y = 1.8,$$

$$14z = 14 \Rightarrow z = 1$$