An easy procedure to determine linear independence of vectors

1)Put the vectors into rows of a matix

2)Reduce the matrix into echolen form by row operations

3)If there is a zero row then the vectors are linearly dependent. If there is no zero row then the vectors are linearly independent.

Example L1

$$X=\begin{bmatrix}1\\2\\5\end{bmatrix},\quad Y=\begin{bmatrix}2\\-3\\3\end{bmatrix},\quad Z=\begin{bmatrix}3\\-1\\8\end{bmatrix}$$

$$A=\begin{bmatrix}X^T\\Y^T\\Z^T\end{bmatrix}=\begin{bmatrix}1&2&5\\2&-3&3\\3&-1&8\end{bmatrix},\quad \begin{matrix}R_2-2R_1\rightarrow R_2\\R_3-2R_1\rightarrow R_3\\R_3-3R_1\rightarrow R_4\end{matrix}$$

$$\begin{bmatrix}1&2&5\\0&-7&-7\\0&-7&-7\end{bmatrix}\ R_3\text{-}R_2\rightarrow R_3\ \begin{bmatrix}1&2&5\\0&-7&-7\\0&0&0\end{bmatrix}$$

The vectors X,Y,Z are linearly dependent

The vectors X,Y,Z have two independent vectors,

(because echolen form matrix has two nonzero rows.

The matrix A has rank 2.

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Example L2

$$X=\begin{bmatrix}1\\2\\5\\6\\2\end{bmatrix},\quad Y=\begin{bmatrix}2\\-3\\3\\0\\6\end{bmatrix},\quad Z=\begin{bmatrix}1\\-5\\-2\\-6\\4\end{bmatrix},\quad W=\begin{bmatrix}0\\7\\7\\12\\-2\end{bmatrix},$$

$$B=\begin{bmatrix}1&2&5&6&2\\2&-3&3&0&6\\1&-5&-2&-6&4\\0&7&7&12&-2\end{bmatrix}\quad \begin{matrix}R_2-2R_1\rightarrow R_2\\R_3-R_1\rightarrow R_3\end{matrix}\quad \begin{bmatrix}1&2&5&6&2\\0&-7&-7&-12&2\\0&-7&-7&-12&2\\0&7&7&12&-2\end{bmatrix}$$

$$\begin{bmatrix}1&2&5&6&2\\0&-7&-7&-12&2\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}$$

The vectors X,Y,Z,W are linearly dependent

The vectors X,Y,Z,W have two independent vectors. (The echolen form has two nonzero rows)

The matrix B has rank 2

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Example L3

$$C=\begin{bmatrix}1&1&2&3&1\\1&1&3&5&4\\2&5&8&11&6\\-1&2&4&4&4\end{bmatrix}\ \xrightarrow{\text{Row operation}}\ \begin{bmatrix}1&1&2&3&1\\0&0&-1&-1&2\\0&0&0&-1&13\\0&0&0&0&17\end{bmatrix}$$

The matrix C has 4 independent rows.

The matrix C has rank 4.

Example L4

$$D=\begin{bmatrix}1&2&2&3&1\\2&4&3&5&4\\1&2&6&7&-7\\3&6&8&10&12\end{bmatrix}\qquad \begin{bmatrix}1&1&2&3&1\\0&0&-1&-1&2\\0&0&0&-1&-13\\0&0&0&0&0\end{bmatrix}$$

The matrix D has 3 independent rows

The matrix D has rank 3

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Example L5

$$\begin{bmatrix}1&1&2&3&1\\1&1&3&5&4\\2&5&8&11&6\\-1&2&4&4&4\end{bmatrix}\ \xrightarrow{\text{Row operation}}\ \begin{bmatrix}1&1&2&3&1\\0&3&4&5&4\\0&0&1&2&3\\0&0&0&-2&-5\end{bmatrix}$$

The matrix has 4 independent rows

The matrix has rank 4.

Example L5

$$\begin{bmatrix}1&2&2&3&1\\2&4&3&5&4\\1&2&6&7&10\\3&6&8&10&12\end{bmatrix}\ \xrightarrow{\text{Row operation}}\ \begin{bmatrix}1&1&2&3&1\\0&0&-1&-1&2\\0&0&0&0&17\\0&0&0&-1&13\end{bmatrix}$$

The matrix has 4 independent rows

The matrix has rank 4.
