Properties of Determinant

1.If two rows are exchanged then the value of determinant is multiplied by -1.

$A=\begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \\ 15 & 20 & 30 \end{bmatrix} \rightarrow \det A = -15 \quad R_2 \leftrightarrow R_3 \quad B=\begin{bmatrix} 2 & 5 & 8 \\ 15 & 20 & 30 \\ 3 & 6 & 9 \end{bmatrix} \det B = -(-15)=15$

$R_1 \leftrightarrow R_3 \quad C=\begin{bmatrix} 15 & 20 & 30 \\ 3 & 6 & 9 \\ 2 & 5 & 8 \end{bmatrix} \rightarrow \det C = 15, \quad D=\begin{bmatrix} 3 & 6 & 9 \\ 15 & 20 & 30 \\ 2 & 5 & 8 \end{bmatrix} \rightarrow \det D = -15$

2.If one row is multiplied by a scalar and added to another row, the value of determinant is unchanged.

$Q=\begin{bmatrix} 1 & 1 & 2 \\ 10 & 20 & 30 \\ 3 & 4 & 12 \end{bmatrix} \quad -10R_1+R_2 \rightarrow R_2 \quad T=\begin{bmatrix} 1 & 1 & 2 \\ 0 & 10 & 10 \\ 3 & 4 & 12 \end{bmatrix} \quad -3R_1+R_3 \rightarrow R_3 \quad V=\begin{bmatrix} 1 & 1 & 2 \\ 0 & 10 & 10 \\ 0 & 1 & 6` \end{bmatrix}$

$-0.1R_2+R_3 \rightarrow R_3 \quad Z=\begin{bmatrix} 1 & 1 & 2 \\ 0 & 10 & 10 \\ 0 & 0 & 5` \end{bmatrix}$ det Q=50, det T=50, det V=50, det Z=50

3. If one row is multiple of another row the determinant is zero.

$A=\begin{bmatrix} 1 & 3 & 7 \\ 10 & 30 & 70 \\ 2 & 7 & 6 \end{bmatrix}$ $R_2=10R_1$ Det A =0 $\quad B=\begin{bmatrix} 4 & 7 & 3 \\ 5 & 0 & 1 \\ 8 & 14 & 6 \end{bmatrix}$ $R_3=2R_1$ Det B =0 $\quad C=\begin{bmatrix} 3 & 8 & 2 \\ 6 & 9 & 0 \\ -3 & -8 & -2 \end{bmatrix}$ $R_3=-R_1$ Det C =0

4. If rows are linearly dependent the determinant is zero.

$P=\begin{bmatrix} 1 & 7 & 5 \\ 10 & 20 & 30 \\ 12 & 27 & 36 \end{bmatrix}$ $R_3= R_1 +R_2$ $-R_3+ R_1 +R_2=0$ Det P =0 $\quad Q=\begin{bmatrix} 1 & 2 & 4 \\ 10 & 0 & 30 \\ 12 & 4 & 38 \end{bmatrix}$ $R_3= 2R_1 +R_2$ $-R_3+ 2R_1 +R_2=0$ Det Q =0

5. If one row is multiplied by a scalar then the determinant value is also multiplied by the same scalar.

$M=\begin{bmatrix} 1 & 7 & 5 \\ 3 & 2 & 8 \\ 4 & 6 & 9 \end{bmatrix}$ Det M=30. $\quad N=\begin{bmatrix} 1 & 7 & 5 \\ 30 & 20 & 80 \\ 4 & 6 & 9 \end{bmatrix}$ Det N=10Det M Det N=300 $\quad R=\begin{bmatrix} 1 & 7 & 5 \\ 30 & 20 & 80 \\ 400 & 600 & 900 \end{bmatrix}$ Det R=100Det N Det R=30 000

6. If there is a zero row the determinant is zero.

$A=\begin{bmatrix} 1 & 7 & 5 \\ 0 & 0 & 0 \\ 4 & 6 & 9 \end{bmatrix}$ Det A=0. $\quad B=\begin{bmatrix} 1 & 7 & 5 \\ 4 & 6 & 9 \\ 0 & 0 & 0 \end{bmatrix}$ Det B=0.

6. Column operations have the same effect as row operations.

.6.1. If two columns are exchanged then the value of determinant is multiplied by -1.

$X=\begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \\ 15 & 20 & 30 \end{bmatrix} \rightarrow \det X = -15 \quad C_1 \leftrightarrow C_3 \quad W=\begin{bmatrix} 8 & 5 & 2 \\ 9 & 6 & 3 \\ 30 & 20 & 15 \end{bmatrix} \det W = -(-15)=15$

6.2 .If one column is multiplied by a scalar and added to another column, the value of determinant is unchanged.

6.3. If one column is multiple of another column the determinant is zero.

6.4. If columns are linearly dependent the determinant is zero.

6.5 If one column is multiplied by a scalar then the determinant value is also multiplied by the same scalar.

$K=\begin{bmatrix} 1 & 7 & 5 \\ 3 & 2 & 8 \\ 4 & 6 & 9 \end{bmatrix}$ Det K=30, $\quad E=\begin{bmatrix} 1 & 7 & 50 \\ 3 & 2 & 80 \\ 4 & 6 & 90 \end{bmatrix}$ Det E=300. $\quad F=\begin{bmatrix} 1 & 70 & 50 \\ 3 & 20 & 80 \\ 4 & 60 & 90 \end{bmatrix}$ Det F=3000.