

$$A = \begin{bmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ \dots & \dots & \dots \\ a_{n1} & a_{n2} \dots & a_{nn} \end{bmatrix} \rightarrow A^{-1} = \frac{1}{\Delta A} \begin{bmatrix} C_{11} & C_{12} \dots & C_{1n} \\ C_{21} & C_{22} \dots & C_{2n} \\ \dots & \dots & \dots \\ C_{n1} & C_{n2} \dots & C_{nn} \end{bmatrix}^T = \frac{1}{\Delta A} \begin{bmatrix} +M_{11} & -M_{12} & \dots & +M_{1n} \\ -M_{21} & +M_{22} & \dots & -M_{2n} \\ \dots & \dots & \dots & \dots \\ (-1)^{n+1}M_{n1} & (-1)^{n+1}M_{n2} \dots & (-1)^{2n}M_{nn} \end{bmatrix}^T$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 8 \end{bmatrix}$$

$$S_{11} = \begin{bmatrix} 5 & 6 \\ 8 & 8 \end{bmatrix},$$

$$S_{12} = \begin{bmatrix} 4 & 6 \\ 7 & 8 \end{bmatrix},$$

$$S_{13} = \begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix},$$

$$S_{21} = \begin{bmatrix} 2 & 3 \\ 8 & 8 \end{bmatrix},$$

$$S_{22} = \begin{bmatrix} 1 & 3 \\ 7 & 8 \end{bmatrix},$$

$$S_{23} = \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix},$$

$$S_{31} = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix},$$

$$S_{32} = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix},$$

$$S_{33} = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix},$$

$$M_{11} = \Delta S_{11} = -8, \quad M_{12} = \Delta S_{12} = -10, \quad M_{13} = \Delta S_{13} = -3, \quad ,$$

$$M_{21} = \Delta S_{21} = -8, \quad M_{22} = \Delta S_{22} = -13, \quad M_{23} = \Delta S_{23} = -6, \quad ,$$

$$M_{31} = \Delta S_{31} = -3, \quad M_{32} = \Delta S_{32} = -6, \quad M_{33} = \Delta S_{33} = -3, \quad ,$$

$$Ac = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} = \begin{bmatrix} +M_{11} & -M_{12} & +M_{13} \\ -M_{21} & +M_{22} & -M_{23} \\ +M_{31} & -M_{32} & +M_{33} \end{bmatrix} = \begin{bmatrix} -8 & -(-10) & -3 \\ -(-8) & -13 & -(-6) \\ -3 & -(-6) & -3 \end{bmatrix} = \begin{bmatrix} -8 & 10 & -3 \\ 8 & -13 & 6 \\ -3 & 6 & -3 \end{bmatrix}$$

Using the first row $\Delta A = 1 \times M_{11} - 2 \times M_{12} + 3 \times M_{13} = 1(-8) - 2(-10) + 3(-3) = -8 + 20 - 9 = 3$

Or using the second row $\Delta A = -4 \times M_{21} + 5 \times M_{22} - 6 \times M_{23} = -4(-8) + 5(-13) - 6(-6) = 32 - 65 + 36 = 3$

$$A^{-1} = \begin{bmatrix} M_{11} & -M_{12} & M_{13} \\ -M_{21} & M_{22} & -M_{23} \\ M_{31} & -M_{32} & M_{33} \end{bmatrix}^T = \frac{1}{3} \begin{bmatrix} -8 & 10 & -3 \\ 8 & -13 & 6 \\ -3 & 6 & -3 \end{bmatrix}^T = \frac{1}{3} \begin{bmatrix} -8 & 8 & -3 \\ 10 & -13 & 6 \\ -3 & 6 & -3 \end{bmatrix} = \begin{bmatrix} -8/3 & 8/3 & -3/3 \\ 10/3 & -13/3 & 6/3 \\ -3/3 & 6/3 & -3/3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -2.667 & 2.667 & -1 \\ 3.33 & -4.33 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow \begin{matrix} S_{11} = \begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}, & S_{12} = \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix}, & S_{13} = \begin{bmatrix} 4 & 5 \\ 7 & 8 \end{bmatrix}, \\ S_{21} = \begin{bmatrix} 2 & 3 \\ 8 & 9 \end{bmatrix}, & S_{22} = \begin{bmatrix} 1 & 3 \\ 7 & 9 \end{bmatrix}, & S_{23} = \begin{bmatrix} 1 & 2 \\ 7 & 8 \end{bmatrix}, \\ S_{31} = \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}, & S_{32} = \begin{bmatrix} 1 & 3 \\ 4 & 6 \end{bmatrix}, & S_{33} = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} \end{matrix}$$

$$M_{11} = \Delta S_{11} = -3, \quad M_{12} = \Delta S_{12} = -6, \quad M_{13} = \Delta S_{13} = -3, \quad ,$$

Using the first row $\Delta A = 1 \times M_{11} - 2 \times M_{12} + 3 \times M_{13} = 1(-3) - 2(-6) + 3(-3) = -3 + 12 - 9 = 0$

Since $\Delta A = 0$ A^{-1} does not exist.