Solutions of linear equations by CRAMMER Method (n equations, n unknowns)

$a_{11}x_1+ a_{12}x_2+\ldots\ldots a_{1n}x_n=b_1$
$a_{21}x_1+ a_{22}x_2+\ldots\ldots A_{2n}x_n=b_2$
$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots \rightarrow$
$a_{n1}x_1+ a_{n2}x_2+\ldots\ldots A_{nn}x_n=b_n$

$$\begin{bmatrix} a_{11} & a_{12}\ldots. & a_{1n} \\ a_{21} & a_{22}\ldots. & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2}\ldots. & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \ldots\ldots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots\ldots \\ b_n \end{bmatrix}$$

$$A=\begin{bmatrix} a_{11} & a_{12}\ldots. & a_{1n} \\ a_{21} & a_{22}\ldots. & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2}\ldots. & a_{nn} \end{bmatrix}, \quad B_1=\begin{bmatrix} b_1 & a_{12}\ldots. & a_{1n} \\ b_2 & a_{22}\ldots. & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ b_n & a_{n2}\ldots. & a_{nn} \end{bmatrix}, \quad B_2=\begin{bmatrix} a_{11} & b_1\ldots. & a_{1n} \\ a_{21} & b_2\ldots. & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & b_n\ldots. & a_{nn} \end{bmatrix}\ldots\ldots \quad .B_n=\begin{bmatrix} a_{11} & a_{12}\ldots. & b_1 \\ a_{21} & a_{22}\ldots. & b_2 \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2}\ldots. & b_n \end{bmatrix}$$

$$x_1=\frac{\Delta B_1}{\Delta A}, \quad x_2=\frac{\Delta B_2}{\Delta A}, \quad x_3=\frac{\Delta B_1}{\Delta A}, \quad \ldots\ldots\ldots \quad x_n=\frac{\Delta B_n}{\Delta A},$$

2x+6y+8z=4,
4x+17y+14z=15,
x+13y+14z=30

$$A=\begin{bmatrix} 2 & 6 & 8 \\ 4 & 17 & 14 \\ 1 & 13 & 14 \end{bmatrix} \quad B_1=\begin{bmatrix} 4 & 6 & 8 \\ 15 & 17 & 14 \\ 30 & 13 & 14 \end{bmatrix} \quad B_2=\begin{bmatrix} 2 & 4 & 8 \\ 4 & 15 & 14 \\ 1 & 30 & 14 \end{bmatrix} \quad B_3=\begin{bmatrix} 2 & 6 & 4 \\ 4 & 17 & 15 \\ 1 & 13 & 30 \end{bmatrix}$$

$$\Delta A=+2\begin{vmatrix} 17 & 14 \\ 13 & 14 \end{vmatrix}-4\begin{vmatrix} 6 & 8 \\ 13 & 14 \end{vmatrix}+1\begin{vmatrix} 6 & 8 \\ 17 & 14 \end{vmatrix}=2\ (56)-4\,(-20)+1\ (-52)=140$$

$$\Delta B_1=+4\begin{vmatrix} 17 & 14 \\ 13 & 14 \end{vmatrix}-15\begin{vmatrix} 6 & 8 \\ 13 & 14 \end{vmatrix}+30\begin{vmatrix} 6 & 8 \\ 17 & 14 \end{vmatrix}=4\ (56)-15\,(-20)+30\ (-52)=-1036$$

$\Delta B_2 = 252$ $\Delta B_3 = 140$

$$x_1=\frac{\Delta B_1}{\Delta A}=\frac{-1036}{140}=-7.4, \qquad x_2=\frac{\Delta B_2}{\Delta A}=\frac{252}{140}=1.8, \qquad x_3=\frac{\Delta B_1}{\Delta A}=\frac{140}{140}=1$$

Solutions of linear equations by Matrix inversion Method (n equations, n unknowns)

$$\begin{bmatrix} a_{11} & a_{12}\ldots. & a_{1n} \\ a_{21} & a_{22}\ldots. & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2}\ldots. & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \ldots\ldots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots\ldots \\ b_n \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ \ldots\ldots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12}\ldots. & a_{1n} \\ a_{21} & a_{22}\ldots. & a_{2n} \\ \ldots\ldots & \ldots\ldots & \ldots\ldots \\ a_{n1} & a_{n2}\ldots. & a_{nn} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ \ldots\ldots \\ b_n \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & 8 \\ 4 & 17 & 14 \\ 1 & 13 & 14 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 30 \end{bmatrix} \rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 6 & 8 \\ 4 & 17 & 14 \\ 1 & 13 & 14 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 15 \\ 30 \end{bmatrix} = \begin{bmatrix} 0.4 & 0.143 & -0.37 \\ -0.3 & 0.143 & 0.028 \\ 0.25 & -0.143 & 0.071 \end{bmatrix} \begin{bmatrix} 4 \\ 15 \\ 30 \end{bmatrix} = \begin{bmatrix} -7.4 \\ 1.8 \\ 1 \end{bmatrix}$$

x=-7.4 y=1.8 z=1