M1-If you multiply a matrix by a vector from the right, it is equivalent to multiplying each column by the each element of the vector and adding all the results.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} = 10\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} + 20\begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} + 30\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 10a_{11} + 20a_{12} + 30a_{13} \\ 10a_{21} + 20a_{22} + 30a_{23} \\ 10a_{31} + 20a_{32} + 30a_{33} \end{bmatrix}$$

M2-Multiplying by a diagonal matrix from the right.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{bmatrix} = \begin{bmatrix} 10a_{11} & 20a_{12} & 30a_{13} \\ 10a_{21} & 20a_{22} & 30a_{23} \\ 10a_{31} & 20a_{32} & 30a_{33} \end{bmatrix}$$

M3-Multiplying by a diagonal matrix from the left.

$$\begin{bmatrix} 10 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 30 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 10a_{11} & 10a_{12} & 10a_{13} \\ 20a_{21} & 20a_{22} & 20a_{23} \\ 30a_{31} & 30a_{32} & 30a_{33} \end{bmatrix}$$

M4-Inverse of a diagonal matrix is a diagonal matrix whose values are the inverse of each element

$$\begin{bmatrix} a_{11} & 0.... & 0 \\ 0 & a_{22}.... & 0 \\ .... & ..... & ..... \\ 0 & 0.... & a_{nn} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & .... & 0 \\ 0 & \frac{1}{a_{22}} & .... & 0 \\ .... & ..... & & ..... \\ 0 & 0.... & & \frac{1}{a_{nn}} \end{bmatrix}$$

Proof: Multiply two matrices, you will get unit matrix.

M5-Inverse of two matrix product

$[AC]^{-1} = C^{-1}A^{-1}$

Proof: Replace $C^{-1}A^{-1}$ for $[AC]^{-1}$ in the inversion formula.

$[AC]\,[AC]^{-1} = I$

$[AC]\ [C^{-1}A^{-1}] = I$

$AC\ C^{-1}A^{-1} = I$

$A\,I\,A^{-1} = I$

$A\ A^{-1} = I$

$I = I$

M6 -Problem: Calculate the inverse of T=[ABC]

M7-Theorem: if A^{-1} exists and AB=AC => B=C

Proof: Multiply both sides by A^{-1}

$A^{-1}[AB] = A^{-1}[AC]$

$A^{-1}AB = A^{-1}AC$

$I\,B = I\,C$

$B=C$

M8-Comment: if A^{-1} does not exist and AB=AC in general B≠C

M9-Theorem: if AB=0and A^{-1} exists => B=0

Proof: Multiply both sides by A^{-1}

$A^{-1}[AB] = A^{-1}0$

$A^{-1}AB = 0$

$I\,B = 0$

$B=0$

M10-Theorem: if AB=0 and A≠0, B≠0 then A should be singular and B should be singular

Proof: Assume A is multiplied by a vector

$$\begin{bmatrix} C_1 & C_2 & ... & C_n \\ . & & & \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3. \end{bmatrix} = a_1C_1 + a_2C_2 + a_3C_3 = 0$$

This means columns of A are linearly dependent.

M11-Theorem: if A singular then AB is singular and BA is singular

Proof:

M12-Theorem: $(A^{-1})^T = (A^T)^{-1}$

Proof:

MATRIX EQUATIONS

M13-Problem if AX=B Calculate X.

Solution: multiply both sides by A^{-1}

$AX=B$

$A^{-1}AX = A^{-1}B$

$X == A^{-1}B$

M14-Problem if AX+CX=B Calculate X.

Solution:

$AX+CX=B$

$[A+C]X=B$

Multiply both sides by $A+C]^{-1}$

$[A+C]^{-1}\ [A+C]\,X = [A+C]^{-1}B$

$I\,X = [A+C]^{-1}B$

$X = [A+C]^{-1}B$

M15-Comment $[A+B]^{-1} \neq A^{-1} + B^{-1}$

M16-Problem if AX+CX=B Calculate X.

Solution

$AX+CX=B$

$[A+C]X=B$

Multiply both sides by $A+C]^{-1}$

$[A+C]^{-1}\ [A+C]\,X = [A+C]^{-1}B$

$I\,X = [A+C]^{-1}B$

$X = [A+C]^{-1}B$

M18-Problem: AX+XC=B

Solution No direct solution exists