EE 300 Dr. Ramazan

Analytical Methods in Engineering **(EE300)** Major -1 6 Oct 2003 10-8-1424

1)Given the following matrices, calculate
a)A+B, b)B-10A, c)A+B^T, d) B^T-A^T

$$A=\begin{bmatrix}1 & 7\\3 & 6\end{bmatrix},\quad B=\begin{bmatrix}10 & 70\\30 & 60\end{bmatrix}$$

2)Calculate AB, if A and B matrices are as follows

$$A=\begin{bmatrix}1 & 0 & a & a\\b & 0 & 0 & 1\\2 & 1 & 1 & 0\end{bmatrix},\quad B=\begin{bmatrix}0 & 1\\c & 2\\a & 0\\a & 0\end{bmatrix}$$

3)Find values of *p* and *q* such that the following vectors *x,y, z* are linearly **dependent**.

$$x=\begin{bmatrix}2\\1\\3\end{bmatrix},\quad y=\begin{bmatrix}0\\1\\1\end{bmatrix},\quad z=\begin{bmatrix}6\\p\\q\end{bmatrix}$$

4)Try to bring the following matrices into upper triangular form.

$$A=\begin{bmatrix}1 & 2 & 3 & 4\\3 & 7 & 0 & 14\\2 & 4 & 7 & 0\\-4 & -8 & 5 & 7\end{bmatrix},\quad B=\begin{bmatrix}1 & 2 & 9\\3 & 7 & 11\\2 & 5 & 2\end{bmatrix}$$

5)Calculate determinant of the following matrices.

$$A=\begin{bmatrix}1 & 3 & 2 & 8\\1.5 & 1 & 7 & 0\\2 & 6 & 9 & 11\\4 & 12 & 8 & -5\end{bmatrix},\quad B=\begin{bmatrix}3 & 0 & 0 & 0\\5 & 2 & 4 & 6\\9 & 3 & 1 & 4\\5 & 0 & 0 & 2\end{bmatrix}$$

6)Examine the following linear equations
x+2y=5, *2x+y=4,* *3x+5y=q*
a)Determine q such that this system has unique solution.
b)Determine q such that this system has no solution.

7)Examine the following homogenous system
x+y+z=0, x+2y+3z=0, x+3y+5z=0
a)Does this system have a nontrivial solution,
b)Calculate the solution space for this system

8)The following matrix is given

$$A=\begin{bmatrix}a & x & p\\b & y & q\\c & z & r\end{bmatrix}$$

It is known that det(A)=ΔA=20

Calculate the determinant of the following matrices.

$$B=\begin{bmatrix}b & y & q\\a & x & p\\c & z & r\end{bmatrix},\quad C=\begin{bmatrix}10b & 10y & 10q\\a & x & p\\c & z & r\end{bmatrix}$$

$$D=\begin{bmatrix}10a & 10x & 10p\\b & y & q\\c & z & r\end{bmatrix},\quad E=\begin{bmatrix}9a & 9x & 9p\\9b & 9y & 9q\\9c & 9z & 9r\end{bmatrix}$$

$$F=\begin{bmatrix}a & x & p\\b & y & q\\a+10b+c & x+10y+z & p+10q+r\end{bmatrix}$$

9)*A* and *B* matrices are as follows

$$A=\begin{bmatrix}1 & 0 & 0\\4 & 0 & 1\\6 & 1 & 0\end{bmatrix},\quad B=\begin{bmatrix}1 & 0 & 0\\x & 0 & 1\\y & 1 & 0\end{bmatrix},$$

It is known that $A^{-1}=B$. Calculate *x* and *y*.

10) Calculate the inverse of the following matrices.

$$A=\begin{bmatrix}2 & 3 & 4.5\\0 & 2 & 8\\0 & 0 & 1\end{bmatrix},\quad B=\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 2\end{bmatrix}$$

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4)

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 7 & 0 & 14 \\ 2 & 4 & 7 & 0 \\ -4 & -8 & 5 & 7 \end{bmatrix} \Rightarrow \quad \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -9 & 2 \\ 0 & 0 & 1 & -8 \\ 0 & 0 & 0 & 159 \end{bmatrix}$$

$$\begin{pmatrix} 1 & 2 & 9 \\ 3 & 7 & 11 \\ 2 & 5 & 2 \end{pmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 9 \\ 0 & 1 & -16 \\ 0 & 1 & -16 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 9 \\ 0 & 1 & -16 \\ 0 & 0 & 0 \end{bmatrix}$$

5)

$$A = \begin{bmatrix} 1 & 3 & 2 & 8 \\ 1.5 & 1 & 7 & 0 \\ 2 & 6 & 9 & 11 \\ 4 & 12 & 8 & -5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & 2 & 8 \\ 0 & -3.5 & 4 & -12 \\ 0 & 0 & 5 & -5 \\ 0 & 0 & 0 & -77 \end{bmatrix}$$

$\cancel{\det(A) = 1 \times (-3.5) \times 5 (-77) = 647.5}$

$$\det(A) = 1\,(-3.5)\,(5)\,(-77) = 647.5$$

$$B = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 5 & 2 & 4 & 6 \\ 9 & 3 & 1 & 4 \\ 9 & 0 & 0 & 2 \end{bmatrix} \qquad \det B = 3 \begin{vmatrix} 2 & 4 & 6 \\ 3 & 1 & 4 \\ 0 & 0 & 2 \end{vmatrix} = 3 \times 2 \begin{vmatrix} 2 & 4 \\ 3 & 1 \end{vmatrix} = -60$$

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6)

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 1 & 4 \\ 3 & 5 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 0 & -3 & -6 \\ 0 & -1 & -15+9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 5 \\ 0 & -3 & -6 \\ 0 & 0 & -13+9 \end{bmatrix}$$

$-13 + 9 = 0 \qquad 9 = 13$ unique solution

$9 \neq 13$ No solution

1) $A = \begin{bmatrix} 1 & 7 \\ 3 & 6 \end{bmatrix}$ $\quad B = \begin{bmatrix} 10 & 70 \\ 30 & 60 \end{bmatrix}$

$A + B = \begin{bmatrix} 11 & 77 \\ 33 & 66 \end{bmatrix}$ $\quad B - 10A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$A + B^T = \begin{bmatrix} 1 & 7 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 10 & 30 \\ 70 & 60 \end{bmatrix} = \begin{bmatrix} 11 & 37 \\ 73 & 66 \end{bmatrix}$$

$$B^T - A^T = \begin{bmatrix} 10 & 30 \\ 70 & 60 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 27 \\ 63 & 54 \end{bmatrix}$$

2)
$$\begin{bmatrix} 1 & 0 & a & a \\ b & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ c & 2 \\ a & 0 \\ a & 0 \end{bmatrix} = \begin{bmatrix} 2a^2 & 1 \\ a & b \\ c+a & 4 \end{bmatrix}$$

B.A is impossible

3)
$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \\ 6 & p & q \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & p-3 & q-9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & \cancel{q-10} \end{bmatrix}$$

$q-9-(p-3)$

$p = 4$
$q = 10$

$q-9-p+3=0$

$\boxed{q-p=6}$

or

$$\begin{bmatrix} 2 & 0 & 6 \\ 1 & 1 & p \\ 3 & 1 & q \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 6 \\ 0 & 1 & p-3 \\ 0 & 1 & q-9 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 0 & 6 \\ 0 & 1 & p-3 \\ 0 & 0 & q-9-(p-3) \end{bmatrix}$$

$q-p-6=0$ $\quad \boxed{q-p=6}$ is the solution

any values which satisfies $q-p=6$

7) (3)

$$\begin{bmatrix}1&1&1\\1&2&3\\1&3&5\end{bmatrix} \to \begin{bmatrix}1&1&1\\0&1&2\\0&2&4\end{bmatrix} \to \begin{bmatrix}1&1&1\\0&1&2\\0&0&0\end{bmatrix}$$

$$\left(\begin{array}{l} x+y+z=0 \\ y+2z=0 \end{array}\right) \quad \text{One variable arbitrary}$$

Set $\underline{z=1}$ $\quad \left(\begin{array}{l} x+y+1=0 \\ y+2=0 \end{array}\right) \Rightarrow \begin{array}{l} y=-2 \\ x=1 \end{array} \qquad v=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-2\\1\end{pmatrix}$

One arbitrary variable: solution space contains only one vector

8)

$$A=\begin{bmatrix}a&x&p\\b&y&q\\c&z&r\end{bmatrix}$$

$\Delta A = 20$ $\qquad \Delta D = 200$

$\Delta B = -20$ $\qquad \Delta F = 20$

$\Delta C = -200$ $\qquad \Delta E = 729 \times 20 = 14580$

9)

$$\begin{bmatrix}1&0&0\\4&0&1\\6&1&0\end{bmatrix}\begin{bmatrix}1&0&0\\x&0&1\\y&1&0\end{bmatrix} = \begin{bmatrix}1&0&0\\4+y&1&0\\6+x&0&1\end{bmatrix} \Rightarrow \begin{array}{l} 4+y=0 \quad y=-4 \\ 6+x=0 \quad x=-6 \end{array}$$

10)

$$A=\left[\begin{array}{ccc|ccc}2&3&4.5&1&0&0\\0&2&8&0&1&0\\0&0&1&0&0&1\end{array}\right] \to \left[\begin{array}{ccc|ccc}2&3&4.5&1&0&0\\0&2&0&0&1&-8\\0&0&1&0&0&1\end{array}\right] \to$$

$$\left[\begin{array}{ccc|ccc}2&3&0&1&0&-4.5\\0&2&0&0&1&-8\\0&0&1&0&0&1\end{array}\right] \to \left[\begin{array}{ccc|ccc}2&0&0&1&-1.5&-4.5\\0&2&0&0&1&-8\\0&0&1&0&0&4\end{array}\right] \to \left[\begin{array}{ccc|ccc}1&0&0&0.5&-0.75&-2.25\\0&1&0&0&0.5&-4\\0&0&1&0&0&4\end{array}\right]$$

$$\begin{pmatrix}2&0&0\\0&2&0\\0&0&2\end{pmatrix}^{-1} = \begin{pmatrix}0.5&0&0\\0&0.5&0\\0&0&0.5\end{pmatrix}$$

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