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Analytical Methods in Engineering (EE300) Major -1 9 Oct 2004 25-8-1425

DO NOT USE CALCULATOR

Any type of calculator is not allowed.

1) The following matrices are given

$$A=\begin{bmatrix}2&4&0&6&3\\5&1&0&4&0\\3&0&0&0&0\\4&0&1&0&0\end{bmatrix}\quad B=\begin{bmatrix}0&1&0&0&1\\0&0&2&0&0\\1&0&0&1&1\\1&1&0&1&1\end{bmatrix},$$

Calculate a)AB

2)a)Write the following linear equations in matrix form.

b)reduce the system into echolen form

c)Show that the system has unique solution

d)Solve the system and obtain x,y,z

$x+2y+z=4$

$2y+2z=4$

$-2x+4y+z=3$

3)a)Write the following linear equations in matrix form.

b)reduce the system into echolen form

c)Show that the system has multiple solution

d)How many free variables does this system have.

(Do not solve the equations)

$x+2y+3z+4w=5$

$2x-y+2z-3w=6$

$4x+3y+8z+5w=16$

$5y+4z+11w=4$

4)It is known that the vectors X,Y,Z are linearly dependent. Calculate q.

$$X=\begin{bmatrix}1\\2\\3\end{bmatrix},\quad Y=\begin{bmatrix}0\\5\\7\end{bmatrix},\quad Z=\begin{bmatrix}1\\12\\q\end{bmatrix}$$

5)Examine the following linear homogenous system.

$$\begin{bmatrix}2&3&-5\\4&10&7\\2&7&12\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

a)Find two nontrivial solution sets for this system

b)Are your solutions linearly dependent or independent.

c)How many independent nontrivial solutions does this system have.

6)The following matrix is given

$$A=\begin{bmatrix}a&b&c\\x&y&z\\p&q&r\end{bmatrix}$$

It is known that $\det(A)=\Delta A=20$. Calculate the determinant of the following matrices.

$$B=\begin{bmatrix}a&b&c\\2a+x&2b+y&2c+z\\p&q&r\end{bmatrix},\quad C=\begin{bmatrix}a&b&c\\2a+2x&2b+2y&2c+2z\\p&q&r\end{bmatrix}$$

$$D=\begin{bmatrix}-a&b&-c\\x&-y&z\\-p&q&-r\end{bmatrix},\quad E=\begin{bmatrix}p&q&r\\a&b&c\\x&y&z\end{bmatrix},$$

7)Calculate the inverse of the following matrix using Gauss elimination technique.

$$A=\begin{bmatrix}1&2&3\\0&1&4\\0&2&3\end{bmatrix}$$

8) A,B,C,D,X *are all matrices in the following* equations. A,B,C,D are known matrices and X is unknown matrix. Obtain X from each matrix equation.

a) $XAB+XC+CA=A+XB$

b) $AX+BAX=CX+DA$

c) $XA+XA^{-1}=B$

9)Calculate the rank of the following matrices

$$A=\begin{bmatrix}2&6&4&2\\0&0&1&2\\1&3&2&1\\3&3&3&3\end{bmatrix},\quad B=\begin{bmatrix}1&2\\3&4\\5&6\\7&8\\9&0\end{bmatrix},\quad C=\begin{bmatrix}0&0&0\\1&1&1\\2&2&2\\3&3&3\\4&4&4\end{bmatrix},$$

$$D=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\end{bmatrix}$$

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2) $\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 2 & 2 & 4 \\ -2 & 4 & 1 & 3 \end{bmatrix} \xrightarrow{2R_1+R_3\to R_3} \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & 8 & 3 & 11 \end{bmatrix} \xrightarrow{-4R_2+R_3\to R_3} \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & -5 & -5 \end{bmatrix}$

$\text{rank } A = \text{rank } \bar{A} = 3 = n$ Unique solution

$-5z = -5 \Rightarrow z = 1$

$2y + 2z = 4 \Rightarrow y = 1$

$x + 2y + z = 4 \Rightarrow x = 1$

3) $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & -1 & 2 & -3 & 6 \\ 4 & 3 & 8 & 5 & 16 \\ 0 & 5 & 4 & 11 & 4 \end{bmatrix} \xrightarrow[-4R_1+R_3\to R_3]{-2R_1+R_2\to R_2} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -4 & -11 & -4 \\ 0 & -5 & -4 & -11 & -4 \\ 0 & 5 & 4 & 11 & 4 \end{bmatrix}$

$-R_2+R_3 \to R_3$
$+R_2+R_4 \to R_4$
$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & -5 & -4 & -11 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\text{rank } A = \text{rank } \bar{A} = 2 \qquad n = 4$

rank $A < n$ multiple solution

4) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 1 & 12 & 9 \end{bmatrix} \xrightarrow{-R_1+R_3\to R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 0 & 10 & 9-3 \end{bmatrix} \xrightarrow{-2R_2+R_3\to R_3} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 7 \\ 0 & 0 & 9-17 \end{bmatrix}$

in order that x, y, z to be linearly dependent the rank of the matrix $\begin{bmatrix} x^T \\ y^T \\ z^T \end{bmatrix}$ must be less then three. Dr Ram

so the last line must be zero. 3
(otherwise rank will be three)

$$9-17=0 \Rightarrow 9=17$$

5)
$$\begin{bmatrix} 2 & 3 & -5 \\ 4 & 10 & 7 \\ 2 & 7 & 12 \end{bmatrix} \begin{matrix} -2R_1+R_2 \to R_2 \\ -R_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 2 & 3 & -5 \\ 0 & 4 & 17 \\ 0 & 4 & 17 \end{bmatrix} \to \begin{bmatrix} 2 & 3 & -5 \\ 0 & 4 & 17 \\ 0 & 0 & 0 \end{bmatrix}$$

rank $A = 2 = r$ $n=3$ There is $n-r=3-2=1$

independent non trivial Solution.

To find the solution. Use the equations

$$2x+3y-5z=0$$
$$4y+17z=0$$

Set $z=\frac{4}{17} \Rightarrow y=-1$ $x=\frac{-3y+5z}{2}=\frac{3+5\frac{4}{17}}{2}$

$$=\frac{3}{2}+\frac{20}{34}=\frac{51+20}{34}=\frac{71}{34}$$

Set $z=1 \Rightarrow y=-\frac{17}{4}$ $x=\frac{-3y+5z}{2}=\frac{-3\cdot\frac{-17}{4}+5}{2}=\frac{51}{8}+\frac{5}{2}$

$$=\frac{51+20}{8}=\frac{71}{8}$$

$$V_1=\begin{bmatrix} \frac{71}{34} \\ -1 \\ \frac{4}{17} \end{bmatrix} \qquad V_2=\begin{bmatrix} \frac{71}{8} \\ -\frac{17}{4} \\ 1 \end{bmatrix}$$

$$1\times\frac{4}{17}=\frac{4}{17} \qquad -\frac{17}{4}\times\frac{4}{17}=-1 \qquad \frac{71}{8}\times\frac{4}{17}=\frac{71}{34}$$

$V_2 \times \frac{4}{17} = V_1$

Solutions are Linearly dependent

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$$A = \begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}$$

B: $2R_1 + R_2 \rightarrow R_2$

$$B = \begin{bmatrix} a & b & c \\ 2a+x & 2b+y & 2c+z \\ p & q & r \end{bmatrix}$$

$\Delta A = \Delta B$ row operations does not change det

C: $2R_2 \rightarrow R_2$

$$C_1 = \begin{bmatrix} a & b & c \\ 2x & 2y & 2z \\ p & q & r \end{bmatrix} \quad 2R_1 + R_2 \rightarrow R_2 \quad C_2 = \begin{bmatrix} a & b & c \\ 2a+2x & 2b+2y & - \\ p & q & r \end{bmatrix}$$

$\Delta C_1 = 2 \cdot \Delta A = 40$ (row multiplied by 2 increases determinant value by 2 times)

$\Delta C_2 = \Delta C_1 = 20$ (row operations)

$\Delta C_2 = \Delta C = 40$

D:

$$D_1 = \begin{bmatrix} -a & -b & -c \\ x & y & z \\ p & q & r \end{bmatrix} \quad D_2 = \begin{bmatrix} -a & -b & -c \\ x & y & z \\ -p & -q & -r \end{bmatrix} \quad D_3 = \begin{bmatrix} -a & -(-b) & -c \\ x & -y & z \\ -p & -(-q) & -r \end{bmatrix}$$

$(-1)R_1 \rightarrow R_1 \quad \Delta D_1 = \Delta A(-1) = -20$

$-1(R_3) \rightarrow R_3 \quad \Delta D_2 = (-1)\Delta D_1 = -(-20) = 20$

$(-1 C_2) \rightarrow C_2 \quad \Delta D_3 = (-1)\ \Delta D_2 = -(-(-20)) = -20$

Second column is multiplied by (-1)

E) $\begin{matrix} a & b & c \\ x & y & z \\ p & q & r \end{matrix}$ $R_1 \leftrightarrow R_2$ $E1 = \begin{bmatrix} x & y & z \\ a & b & c \\ p & q & r \end{bmatrix}$ $R_1 \leftrightarrow R_3$ $E_2 = \begin{bmatrix} p & q & r \\ a & b & c \\ x & y & z \end{bmatrix}$

$$\Delta E_1 = -\Delta A = -20$$

$$\Delta E_2 = -\Delta E_1 = -(-20) = 20$$

7) $\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 2 & 3 & 0 & 0 & 1 \end{bmatrix}$ $-2R_2+R_3 \to R_3$ $\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 0 & 1 & 0 \\ 0 & 0 & -5 & 0 & -2 & -1 \end{bmatrix}$ $\frac{-R_3}{-5} \to R_3$

$-3R_3 + R_1 \to R_1$
$-4R_3 + R_2 \to R_2$
$\begin{bmatrix} 1 & 2 & 0 & 1 & -3(\frac{2}{5}) & -3(-\frac{1}{5}) \\ 0 & 1 & 0 & 0 & -4(\frac{2}{5})+1 & -4(\frac{1}{-5}) \\ 0 & 0 & 1 & 0 & \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$

$$= \left[\begin{array}{ccc|ccc} 1 & 2 & 0 & 1 & -1.2 & 0.6 \\ 0 & 1 & 2 & 0 & -0.6 & 0.8 \\ 0 & 0 & 1 & 0 & 0.4 & -0.2 \end{array}\right] -2R_2+R_1 \to R_1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 & -1 \\ 0 & -0.6 & 0.8 \\ 0 & 0.4 & -0.2 \end{bmatrix}}_{A^{-1}}$$

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(8)

$$XAB + XC - XB = A - CA$$

$$X[AB + C - B] = A - CA$$

$$X\underbrace{[AB+C-B][AB+C-B]^{-1}}_{I} = \{A-CA\}[AB+C-B]^{-1}$$

$$XI = X = [A-CA][AB+C-B]$$

b)

$$AX + BAX - CX = DA$$

$$[A + BA - C]X = DA$$

$$\underbrace{[A+BA-C]^{-1}[A+BA-C]}_{I}X = [A+BA-C]^{-1}DA$$

$$IX = X = [A+BA-C]^{-1}DA$$

c) $XA + XA^{-1} = B$

$$X[A + A^{-1}] = B$$

$$X\underbrace{[A+A^{-1}][A+A^{-1}]^{-1}}_{I} = B[A+A^{-1}]$$

$$\boxed{XI = X = B[A+A^{-1}]}$$

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$$A=\begin{bmatrix} 2 & 6 & 4 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 3 & 3 & 3 & 3 \end{bmatrix} \begin{matrix} -0.5R_1+R_3\to R_3 \\ -1.5R_1+R_4\to R_4 \end{matrix} \begin{bmatrix} 2 & 6 & 4 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -6 & -3 & 0 \end{bmatrix}$$

order the rows

$$A=\begin{pmatrix} 2 & 6 & 4 & 2 \\ 0 & -6 & -3 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Echolen form

rank $A=3$

$$B=\begin{bmatrix} 1 & 3 & 5 & 7 & 9 \\ 2 & 4 & 6 & 8 & 0 \end{bmatrix} -2R_1+R_2\to R_2 \begin{pmatrix} 1 & 3 & 5 & 7 & 9 \\ 0 & -2 & -4 & -16 & -17 \end{pmatrix}$$

Echolen form

rank $B=2$

$$C=\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix} \begin{matrix} -2R_2+R_3\to R_3 \\ -3R_2+R_4\to R_4 \\ -4R_2+R_5\to R_5 \end{matrix} \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Echolen form

$\boxed{\text{rank } C=1}$

$$D=\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{rank } D=0$$

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