Dr. Ramazan

Analytical Methods in Engineering (EE300) Major 1 **Computer ID**

1)Examine the following price list

Apple (kg)	Orange (kg)	Pear (kg)	Total Price (SR)
2	2	0	4
0	5	6	15
2	7	6	19

a) Write the necessary equations in matrix form.
b) Examine the existence of solution. (unique solution, multiple solution, no solution)

$$\begin{bmatrix} 2 & 2 & 0 \\ 0 & 5 & 6 \\ 2 & 7 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 19 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 2 & 0 & 4 \\ 0 & 5 & 6 & 15 \\ 2 & 7 & 6 & 19 \end{array}\right] \quad -R_1+R_3 \to R_3 \quad \left[\begin{array}{cccc} 2 & 2 & 0 & 4 \\ 0 & 5 & 6 & 15 \\ 0 & 5 & 6 & 15 \end{array}\right]$$

$$-R_2+R_3 \to R_3 \quad \begin{bmatrix} 2 & 2 & 0 & 4 \\ 0 & 5 & 6 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\text{rank } A = 2$ $\quad$ $\text{rank } \tilde{A} = 2$ $\quad$ $n = 3$

multiple solution

2) It is known that the vectors X,Y are linearly dependent. Calculate p, q.

$$X = \begin{bmatrix} p \\ 0.7 \\ 3 \end{bmatrix}, \quad Y = \begin{bmatrix} 2.5 \\ 7 \\ q \end{bmatrix},$$

$y = \alpha x$

$$2.5 = \alpha p$$
$$7 = \alpha\, 0.7 \Rightarrow \alpha = 10$$
$$q = \alpha\, 3$$

$\alpha p = 2.5 \quad p = \frac{2.5}{\alpha} = 0.25$

$q = 3\alpha = 3 \times 10 = 30$

3) Examine the following equation systems

$$\begin{bmatrix} 3 & 2 & 4 \\ 9 & 6 & 12 \\ 1.5 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

a)Find two nontrivial solutions for this system
b)How many independent nontrivial solutions does this system have.

$$\begin{bmatrix} 3 & 2 & 4 \\ 9 & 6 & 12 \\ 1.5 & 1 & 2 \end{bmatrix} \quad \begin{matrix} -3R_1+R_2 \to R_2 \\ -\frac{1}{2}R_1+R_3 \to R_3 \end{matrix} \quad \begin{bmatrix} 3 & 2 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$n = 3 \quad \text{rank } A = 1 = r$

$n - r = 3 - 1 = 2$ nontrivial solu

solution 1 $\quad x = 0 \quad y = 1$

$3x + 2y + 4z = 0 \qquad 3 \times 0 + 2 \times 1 + 4z = 0$

$4z = -2$

$z = -0.5$

solution 2 $\quad x = 1 \quad y = 0$

$3 \times 1 + 2 \times 0 + 4 \times z = 0 \qquad 4z = -3 \quad z = -\frac{3}{4}$

4) Calculate the inverse of the following matrix by Gaus iteration Method.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 1 & 0 \\ 6 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{matrix} -6R_1+R_4 \to R_4 \\ \frac{R_2}{2} \to R2 \end{matrix}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -6 & 0 & 0 & 1 \end{array}\right] \quad -3R_4+R_3 \to R_3$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 18 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 & -6 & 0 & 0 & 1 \end{array}\right]$$

$$\underbrace{\hphantom{xxxxxxxx}}_{A^{-1}}$$

5)Examine the following equation systems
a,b,c are any number.

$$\begin{bmatrix} 2 & a & b \\ 0 & 4 & c \\ 0 & 8 & 2c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

State true or false.
a)This system may have multiple solution ....F....
b)This system has **always** unique solution ....F...
c)We cannot say anything unless we know the values of a,b,c ....F.....

$$\begin{bmatrix} 2 & a & b & 4 \\ 0 & 4 & c & 4 \\ 0 & 8 & 2c & 0 \end{bmatrix} -2R_2+R_3 \to R_3$$

$$\begin{bmatrix} 0 & a & b & 4 \\ 0 & 4 & c & 4 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

rank A = 2
rank $\bar{A}$ = 3

6) A,B,C,D,X are all matrices in the following equations. A,B,C,D are known matrices and X is unknown matrix. Obtain X from each matrix equation.
a) AX+BAX=CX+DA
b) XAB+XB+CA=A+XB
c) AX+X=B

a) $AX+BAX-CX = DA$

$[A+BA-C]X = DA$

$X = [A+BA-C]^{-1} DA$

b) $X[AB+B-B] = A-CA$

$X = [A-CA][AB]^{-1}$

c) $AX+X=B$

$[A+I]X=B$

$X=[A+I]^{-1}B$

7) Examine the following linear equation systems
A is any number.
x + y =3
4x+4 =A
State true or false
a)The system has multiple solution if A=12 ---T---
b)The system has always unique solution ---- F
c)Whatever the value of A is,
The system has multiple solution ----F----

d)The system has no solution if A≠12 ----T-----

$$\begin{bmatrix} 1 & 1 & 3 \\ 4 & 4 & A \end{bmatrix} -4R_1+R_2 \to R2$$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 0 & A-12 \end{bmatrix}$$

8) a) Calculate X from the following matrix equation.
A,B,C,X, are matrices
AX+BX=C
b) Calculate X if

$$A=\begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}, B=\begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}, C=\begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

a) $[A+B]X=C$

$X=[A+B]^{-1}C$

$$X=\left[\begin{bmatrix} 2 & 1 \\ 0 & 4 \end{bmatrix}+\begin{bmatrix} 3 & 5 \\ 0 & 2 \end{bmatrix}\right]^{-1}\begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

$$=\begin{bmatrix} 5 & 6 \\ 0 & 6 \end{bmatrix}^{-1}\begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\frac{1}{ad-bc}$$

$$\begin{bmatrix} 5 & 6 \\ 0 & 6 \end{bmatrix}^{-1} = \begin{bmatrix} 6 & -6 \\ 0 & 5 \end{bmatrix}\frac{1}{30}$$

$$X=\frac{1}{30}\begin{bmatrix} 6 & -6 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 16 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

9)Examine the following equation systems
a,b are any **nonzero** number. ($a\neq 0$, $b\neq 0$)

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 1 & 3 \\ 0 & 8 & 2 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} a \\ b \\ 2b \end{bmatrix}$$

State true or false.
b)This system has multiple solution ...True
a)This system has no solution ...f.........
c)We cannot say anything unless we know the exact values of a and b ...false

$$\begin{bmatrix} 4 & 0 & 0 & 0 & a \\ 0 & 4 & 1 & 3 & b \\ 0 & 8 & 2 & 6 & 2b \end{bmatrix} \quad -2R_2 + R_3 \to R_3$$

$$\begin{bmatrix} 4 & 0 & 0 & 0 & a \\ 0 & 4 & 1 & 3 & b \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rank $A = 2$

rank $\bar{A} = 2$

$n = 3$

10)Find the rank of the following matrices

$$A=\begin{bmatrix} 0 & 0 & 0 \\ 25 & 25 & 55 \\ 0 & 0 & 0 \end{bmatrix},\ B=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$C=\begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 1.5 & 1.5 & 1.5 & 1.5 \end{bmatrix}\quad D=\begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 4 & 4 & 12 \end{bmatrix}$$

Rank A=1 Rank B=2 Rank C= 1 Rank D= 2

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \xrightarrow{-R_1+R_3\to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 1.5 & 1.5 & 1.5 & 1.5 \end{bmatrix} \begin{matrix} -2R_1+R_2\to R_2 \\ -\frac{1.5}{2}R_1+R_3\to R_3 \end{matrix} \begin{bmatrix} 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 4 & 4 & 12 \end{bmatrix} \begin{matrix} -3R_1+R_2\to R_2 \\ -4R_1+R_3\to R_3 \end{matrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

11) $x=\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, y=\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, z=\begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

x and y are linearly Dep
x and z are linearly Ind
y and z are linearly Ind

12) The linearly independent vectors X,Y,Z and matrices P,Q are given as follows

$$X=\begin{bmatrix} a \\ b \\ c \end{bmatrix},\ Y=\begin{bmatrix} d \\ e \\ f \end{bmatrix},\ Z=\begin{bmatrix} g \\ h \\ k \end{bmatrix},\ P=\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & k \end{bmatrix},\ Q=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$$

State True or False
c) $P^T=Q$ ...T..... (T: Transpose)

a) det P=0 ...F....

b) det P = det Q ...T....

d) $P=Q^T$ ...T.....

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e) $P^{-1} = Q^{-1}$ ...F.......

f) $P^{-1} = [Q^{-1}]^T$ T

g) $P^{-1} = [Q^T]^{-1}$ T

13) The linearly dependent vectors X,Y,Z are given as follows

$$X=\begin{bmatrix} a \\ b \\ c \end{bmatrix},\ Y=\begin{bmatrix} d \\ e \\ f \end{bmatrix},\ Z=\begin{bmatrix} g \\ h \\ k \end{bmatrix},$$

The matrix Q and vectors M,N,B are given below

$$P=\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & k \end{bmatrix},\quad \begin{matrix} M=[a \ \ d \ \ g] \\ N=[b \ \ e \ \ h] \\ B=[c \ \ f \ \ k] \end{matrix}$$

State True or False
a) det P=0 ...T......
b) P^{-1} exists ...F....
c) The vectors M,N,B are linearly dependent...T
d) rank P=3 ...F

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