Analytical Methods in Engineering (EE300) Major1 **Computer ID**

1) Examine the following price list

Apple (kg)	Orange (kg)	Pear (kg)	Total Price (SR)
1	2	0	4
0	3	6	15
2	7	6	27

a) Write the necessary equations in matrix form.
b) Examine the existence of solution. (unique solution, multiple solution, no solution)

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 6 \\ 2 & 7 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 15 \\ 27 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 3 & 6 & 15 \\ 2 & 7 & 6 & 27 \end{array}\right] -2R_1+R_3 \to R_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 3 & 6 & 15 \\ 0 & 3 & 6 & 19 \end{array}\right] -R_2+R_3 \to R_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & 0 & 4 \\ 0 & 3 & 6 & 15 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

rank $A = 2$ No solution

rank $\bar{A} = 3$

$n = 3$

2) It is known that the vectors X,Y are linearly dependent. Calculate p, q.

$$X = \begin{bmatrix} 2 \\ p \\ 3 \end{bmatrix}, \quad Y = \begin{bmatrix} 0.7 \\ 7 \\ q \end{bmatrix},$$

$y = \alpha x$

$0.7 = 2\alpha \qquad \alpha = \frac{0.7}{2} = 0.35$

$7 = \alpha p \qquad p = \frac{7}{\alpha} = \frac{7}{0.35} = 20$

$q = 3\alpha$

3) Examine the following equation systems

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 6 & 9 & 12 & 15 \\ 1 & 1.5 & 2 & 2.5 \\ 4 & 6 & 8 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

a) Find two nontrivial solutions for this system
b) How many independent nontrivial solutions does this system have.

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 6 & 9 & 12 & 15 \\ 1 & 1.5 & 2 & 2.5 \\ 4 & 6 & 8 & 10 \end{bmatrix} \begin{array}{l} -3R_1+R_2 \to R_2 \\ -\frac{1}{2}R_1+R_3 \to R_3 \\ -2R_1+R_4 \to R_4 \end{array}$$

$$\begin{bmatrix} 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} \text{rank } A = 1 \\ n = 4 \end{array}$$

$4 - 1 = 3$ nontrivial solution

solution 1 $x=0$ $y=0$ $z=1$ $4z+5w=0$ $w=-\frac{4}{5}$

solution 2 $x=0$ $y=1$ $z=0$ $\Rightarrow w=-\frac{3}{5}$

4) Calculate the inverse of the following matrix by Gaus iteration Method.

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 7 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] R_3+R_4 \to R_4$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 7 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} -7R_4+R_1 \to R_1 \\ \frac{R_2}{2} \to R_2 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & 0 & -7 & -7 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array}\right]$$

$\underbrace{\qquad\qquad}_{A^{-1}}$

Dr R

5) A,B,C,D,X are all matrices in the following equations. A,B,C,D are known matrices and X is unknown matrix. Obtain X from each matrix equation.

a) XAB+XB+CA=A+XB

b) AX+BAX=CX+DA

c) XA+X=B

a) $XAB + XB + XB = A - CA$

$X[AB + B - \cancel{B}] = A - CA$

$X = [A - CA][AB]^{-1}$

b) $AX + BAX - CX = DA$

$[A + BA - C]X = DA$

$X = [A + BA - C]^{-1} DA$

c) $X[A + I] = B \qquad X = B[A + I]^{-1}$

6)Examine the following equation systems
a,b,c are any number.

$$\begin{bmatrix} 2 & a & b \\ 0 & 4 & c \\ 0 & 8 & 2c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

State true or false.

a)This system may have multiple solution ...false...

b)This system has **always** unique solution ...false

c)We cannot say anything unless we know the values of a,b,c ...false.

$$\left[\begin{array}{cccc} 2 & a & b & 4 \\ 0 & 4 & c & 4 \\ 0 & 8 & 2c & 0 \end{array}\right] \quad -2R_2 + R_3 \to R_3$$

$$\left[\begin{array}{cccc} 2 & a & b & 4 \\ 0 & 4 & c & 4 \\ 0 & 0 & 0 & -8 \end{array}\right]$$

7)Examine the following equation systems
a,b are any **nonzero** number. ($a \neq 0$, $b \neq 0$)

$$\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 1 & 3 \\ 0 & 8 & 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} a \\ b \\ 2b \end{bmatrix}$$

State true or false.

b)This system has multiple solution ...True...

a)This system has no solution ...false...

c)We cannot say anything unless we know the exact values of a and b ...false.

$$\left[\begin{array}{ccccc} 4 & 0 & 0 & 0 & a \\ 0 & 4 & 1 & 3 & b \\ 0 & 8 & 2 & 6 & 2b \end{array}\right] \quad -2R_2 + R_3 \to R_3$$

$$\left[\begin{array}{ccccc} 4 & 0 & 0 & 0 & a \\ 0 & 4 & 1 & 3 & b \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} \text{rank } A = 2 \\ \text{rank } \bar{A} = 2 \\ n = 4 \end{array}$$

8)Find the rank of the following matrices

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 1.5 & 1.5 & 1.5 & 1.5 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 4 & 4 & 12 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & 5 & 5 \\ 25 & 25 & 55 \\ 0 & 0 & 0 \end{bmatrix},$$

Rank A=1 Rank B=2 Rank C= 1 Rank D= 1

9) $x = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, y = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, z = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

x and y are linearly Dep

x and z are linearly Ind

y and z are linearly Ind

Dr Ramazan

10) The linearly independent vectors X,Y,Z and matrices P,Q are given as follows

$$X=\begin{bmatrix}a\\b\\c\end{bmatrix},\ Y=\begin{bmatrix}d\\e\\f\end{bmatrix},\ Z=\begin{bmatrix}g\\h\\k\end{bmatrix},\ P=\begin{bmatrix}a&d&g\\b&e&h\\c&f&k\end{bmatrix},\ Q=\begin{bmatrix}a&b&c\\d&e&f\\g&h&k\end{bmatrix}$$

State True or False

a) det P=0 ...False

b) det P = det Q ...True

c) $P^T=Q$ ...True (T: Transpose)

d) $P=Q^T$ ...True

---

e) $P^{-1} = Q^{-1}$ ...false

f) $P^{-1} = [Q^{-1}]^T$ True

g) $P^{-1} = [Q^T]^{-1}$ True

11) The linearly dependent vectors X,Y,Z are given as follows

$$X=\begin{bmatrix}a\\b\\c\end{bmatrix},\ Y=\begin{bmatrix}d\\e\\f\end{bmatrix},\ Z=\begin{bmatrix}g\\h\\k\end{bmatrix},$$

The matrix Q and vectors M,N,P are given below

$$P=\begin{bmatrix}a&d&g\\b&e&h\\c&f&k\end{bmatrix},\quad M=\begin{bmatrix}a&d&g\end{bmatrix},\ N=\begin{bmatrix}b&e&h\end{bmatrix},\ P=\begin{bmatrix}c&f&k\end{bmatrix}$$

State True or False

a) det P=0 ...True

b) P^{-1} exists ...false

c) The vectors M,N,P are linearly dependent...True

d) rank P=3 ...false

12) Examine the following linear equation systems A is any number.

$$x + y = 5$$
$$3x+3y = A$$

State true or false

a) The system has unique solution False

b) Whatever the value of A is, The system has multiple solution false

$$\begin{bmatrix}1&1&5\\3&3&A\end{bmatrix} \to \begin{bmatrix}1&1&5\\0&0&A-15\end{bmatrix}$$

c) The system has multiple solution if A=15 True

d) The system has no solution if $A\neq 15$ True

13) a) Calculate X from the following matrix equation. A,B,C,X, are matrices

$$AX+BX=C$$

b) Calculate X if

$$A=\begin{bmatrix}2&1\\0&4\end{bmatrix},\ B=\begin{bmatrix}3&5\\0&2\end{bmatrix},\ C=\begin{bmatrix}16\\6\end{bmatrix}$$

$$[A+B]x = C$$

$$x = [A+B]^{-1} C$$

$$= \begin{bmatrix}5&6\\0&6\end{bmatrix}^{-1}\begin{bmatrix}16\\6\end{bmatrix}$$

$$\begin{bmatrix}a&b\\c&d\end{bmatrix} = \frac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$$

$$\begin{bmatrix}5&6\\0&6\end{bmatrix} = \frac{1}{30}\begin{bmatrix}6&-6\\0&5\end{bmatrix}$$

$$x = \frac{1}{30}\begin{bmatrix}6&-6\\0&5\end{bmatrix}\begin{bmatrix}16\\6\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix}$$