Dr. Ramazan

1

**Analytical Methods in Engineering (EE300)** Major -1 5 March 2005 24-1-1426

**DO NOT USE CALCULATOR**

Any type of calculator is not allowed.

1) a) Calculate X from the following matrix equation. A,B,C, are matrices

AX+BX=C

b) Calculate X if

$$A=\begin{bmatrix}2 & 0\\0 & 4\end{bmatrix},\ B=\begin{bmatrix}3 & 3\\0 & 2\end{bmatrix},\ C=\begin{bmatrix}16\\12\end{bmatrix}$$

**2)** Examine the following price list

| Apple (kg) | Orange (kg) | Pear (kg) | Total Price (SR) |
|---|---|---|---|
| 1 | 2 | 0 | 4 |
| 0 | 3 | 6 | 15 |
| 2 | 7 | 6 | 23 |

a) Write the necessary equations in matrix form.

b) Examine the existence of solution. (unique solution, multiple solution, no solution)

c) Obtain a solution in the following range

Apple 1-5 SR

Orange 1-3 SR

Pear 1-3 SR

**3)** Examine the following linear homogenous system.

$$\begin{bmatrix}2 & 3 & -5\\4 & 10 & 7\\2 & 7 & 12\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

a) Find two nontrivial solutions for this system

b) Are your solutions linearly dependent or independent.

c) How many independent nontrivial solutions does this system have.

**4)** It is known that the vectors X,Y,Z are linearly dependent. Calculate q.

$$X=\begin{bmatrix}1\\2\\3\end{bmatrix},\quad Y=\begin{bmatrix}0\\5\\7\end{bmatrix},\quad Z=\begin{bmatrix}1\\12\\q\end{bmatrix}$$

**5)** The following matrix is given

$$A=\begin{bmatrix}a & b & c\\x & y & z\\p & q & r\end{bmatrix}$$

It is known that det(A)=$\Delta$A=20. Calculate the determinant of the following matrices.

$$B=\begin{bmatrix}a & b & c\\2a+x & 2b+y & 2c+z\\p & q & r\end{bmatrix},\quad C=\begin{bmatrix}a & b & c\\2a+2x & 2b+2y & 2c+2z\\p & q & r\end{bmatrix}$$

$$D=\begin{bmatrix}-a & b & -c\\x & -y & z\\-p & q & -r\end{bmatrix},\quad E=\begin{bmatrix}p & q & r\\a & b & c\\x & y & z\end{bmatrix},$$

**6)** Calculate the inverse of the following matrix using Gauss elimination technique.

$$A=\begin{bmatrix}1 & 2 & 0 & 0 & 0\\0 & 1 & 0 & 0 & 0\\0 & 0 & 0 & 1 & 0\\0 & 0 & 1 & 2 & 0\\0 & 0 & 0 & 0 & 0.5\end{bmatrix}$$

7) Calculate the rank of the following matrices

$$A=\begin{bmatrix}2 & 6 & 4 & 2\\0 & 0 & 1 & 2\\1 & 3 & 2 & 1\\3 & 3 & 3 & 3\end{bmatrix},\quad B=\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\\7 & 8\\9 & 0\end{bmatrix},\quad C=\begin{bmatrix}0 & 0 & 0\\1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\\4 & 4 & 4\end{bmatrix},$$

$$D=\begin{bmatrix}0 & 0\\0 & 0\\0 & 0\\0 & 0\end{bmatrix}$$

Dr Raman

(1)

$$Ax + Bx = C$$

$$[A+B]x = C \Rightarrow x = ?$$



$$\underbrace{[A+B]^{-1}[A+B]}_{I}\, x = [A+B]^{-1} C$$

$$I x = [A+B]^{-1} C$$

$$x = [A+B]^{-1} C$$

$$A+B = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 0 & 6 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 5 & 3 & 1 & 0 \\ 0 & 6 & 0 & 1 \end{array}\right] \quad -0.5 R_2 + R_1 \rightarrow R_1$$

$$\left[\begin{array}{cc|cc} 5 & 0 & 1 & -0.5 \\ 0 & 6 & 0 & 1 \end{array}\right] \quad \frac{R_1}{5} \rightarrow R_1 \quad \frac{R_2}{6} \rightarrow R_2$$

$$\left[\begin{array}{cc|cc} \frac{5}{5} & 0 & \frac{1}{5} & \frac{-0.5}{5} \\ 0 & \frac{6}{6} & 0 & \frac{1}{6} \end{array}\right]$$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0.2 & -0.1 \\ 0 & 1 & 0 & \frac{1}{6} \end{array}\right]$$

Dr Ramazan

(2)

$$x = (A+B)^{-1} c = \begin{pmatrix} 0.2 & -0.1 \\ 0 & \frac{1}{6} \end{pmatrix} \begin{bmatrix} 16 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$ (3)

$x + 2y = 4$

$3y + 6z = 15$

$2x + 7y + 6z = 23$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & 6 \\ 2 & 7 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 15 \\ 23 \end{pmatrix}$$

$$\begin{matrix} 1 & 2 & 0 & 4 \\ 0 & 3 & 6 & 15 \\ 2 & 7 & 6 & 23 \end{matrix} \xrightarrow{-2R_1+R_3 \to R_3} \begin{pmatrix} 1 & 2 & 0 & 4 \\ 0 & 3 & 6 & 15 \\ 0 & 3 & 6 & 15 \end{pmatrix} \to \begin{bmatrix} 1 & 2 & 0 & 4 \\ 0 & 3 & 6 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

rank $A = 2$ rank $\bar{A} = 2$ $n = 3$

multiple solution.

$3y + 6z = 15$

Set $z = 1 \Rightarrow y = 3$ $x = -2$ (no)

Set $z = 2$. $y = 1$ $x = 2$ (correct)

3) $\begin{vmatrix} 2 & 3 & -5 \\ 4 & 10 & 7 \\ 2 & 7 & 12 \end{vmatrix} \rightarrow \begin{pmatrix} 2 & 3 & -5 \\ 0 & 4 & 17 \\ 0 & 4 & 17 \end{pmatrix} \rightarrow$

$$\rightarrow \begin{pmatrix} 2 & 3 & -5 \\ 0 & 4 & 17 \\ 0 & 0 & 0 \end{pmatrix}$$

rank $A = 2$

$n = 3$

$3 - 2 = 1$ Non trivial solution.

Set $z = 1 \Rightarrow y = -\frac{17}{7}$  $x = 8.875$

Set $z = 2 \Rightarrow y = -\frac{34}{7}$  $x = 17.75$

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$$V_1 = \begin{pmatrix} 8.875 \\ -\frac{17}{7} \\ 1 \end{pmatrix} \qquad V_2 = \begin{pmatrix} 17.75 \\ -\frac{34}{7} \\ 2 \end{pmatrix}$$

$V_2 = 2V_1$ dependent solution.

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$n - r = 3 - 2 = 1$ independent solution.

4)

5) Given in previous exam solutions



No explanation No **Grade**

6)
$$\left[\begin{array}{ccccc|ccccc} 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_3 \leftrightarrow R_4$

$$\left[\begin{array}{ccccc|ccccc} 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 & 0.5 \end{array}\right]$$

$-2R_2 + R_1 \rightarrow R_1 \quad -2R_4 + R_3 \rightarrow R_3 \quad 2R_5 \rightarrow R_5$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & \\ 0 & 1 & 0 & 0 & 0 & \\ 0 & 0 & 1 & 0 & 0 & \\ 0 & 0 & 0 & 1 & 0 & \\ 0 & 0 & 0 & 0 & 1 & \end{array} \underbrace{\begin{array}{ccccc} 1 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{array}}_{A^{-1}}\right]$$

7) $A = \begin{bmatrix} 2 & 6 & 4 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 3 & 3 & 3 & 3 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 2 & 1 \\ 2 & 6 & 4 & 2 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$ $\quad -2R_1 + R_2 \to R_2$

$\qquad -3R_1 + R_3 \to R_3$

$\begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & -6 & -3 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 2 & 1 \\ 0 & -6 & -3 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ echolen form

rank = 3

$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \\ 9 & 0 \end{bmatrix}$ $\quad -3R_1 + R_2 \to R_2 \quad \begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 5 & 6 \\ 7 & 8 \\ 9 & 0 \end{bmatrix}$ rank = 2

Here we do not need to make operations. Because it is clear that we can make 3, 4, 5 rows zero.

$\left[\begin{array}{cc} 1 & 2 \\ 0 & -2 \\ \hline 5 & 6 \\ 7 & 8 \\ 9 & 0 \end{array}\right] \begin{array}{l} \\ \\ \alpha R_1 + R_3 \to R_3 \\ \beta R_1 + R_4 \to R_4 \\ \gamma R_1 + R_5 \to R_5 \end{array} \qquad \begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 0 & A \\ 0 & B \\ 0 & C \end{bmatrix} \begin{array}{l} m R_2 + R_3 \to R_3 \\ n R_2 + R_4 \to R_4 \\ q R_2 + R_5 \to R_5 \end{array}$

$$\begin{bmatrix} 1 & 2 \\ 0 & -2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{rank } B = 2$$

we do not need to know the values of $\alpha, \beta, \gamma$ $m, n, q$, $A, B, C$.

Another way to find the rank is to use the formula

$$\text{rank } A = \text{rank } A^T$$

$$\text{rank of } \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \\ 9 & 0 \end{bmatrix} \equiv \text{rank of } \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \\ 2 & 4 & 6 & 8 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 5 & 7 & 9 \\ 2 & 4 & 6 & 8 & 0 \end{bmatrix} \xrightarrow{-2R_1 + R_2 \to R_2} \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \\ 0 & -2 & -4 & -6 & -18 \end{bmatrix}$$

The last matrix is in echelon form. rank $B = 2$

Dr Ramanan

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix} \quad \begin{matrix} -2R_2 + R_3 \to R_3 \\ -3R_2 + R_4 \to R_4 \end{matrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{rank } C = 1$$

Here we do not need to make row operations.

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

it is clear that

$R_3 = 2R_2$

$R_4 = 3R_2$

$R_1 = 0 \times R_2$

Thus it is only one row independent. rank $C = 1$

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$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{rank } D = 0$$